LD QCD sum rules for strong IB in decay constants of heavy mesons

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We propose a new method for calculating the dependences of the decay constants of heavy-light mesons on the light-quark mass m_q based on QCD sum rules at infinitely large Borel mass parameter (local-duality limit). For a specific choice of the correlation functions, all condensate contributions vanish and the m_q-dependence of the decay constants is shown to be mainly determined by the known analytic m_q-dependence of the diagrams of perturbative QCD.

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QCD sum rule for 2 – point function

A typical Borel QCD sum rule for the decay constant f_H of a heavy (pseudoscalar or vector) $\bar{Q}q$ meson H of mass M_H, (heavy quark Q with mass m_Q and light quark q with mass m_q):

$$f_H^2(M_H^2)^N \exp(-M_H^2\tau) = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}^{(N)}(\tau,m_Q,m_q,\alpha_s)} ds\,\exp(-s\tau)s^N\rho_{\rm pert}(s,m_Q,m_q,\alpha_s) + \Pi_{\rm power}^{(N)}(\tau,m_Q,m_q,\alpha_s,\langle\bar{q}q\rangle,...).$$

Nonperturbative effects appear at two places:

(i) power corrections

(ii) effective threshold $s_{\rm eff}^{(N)}(\tau,m_Q,m_q,\alpha_s)$.

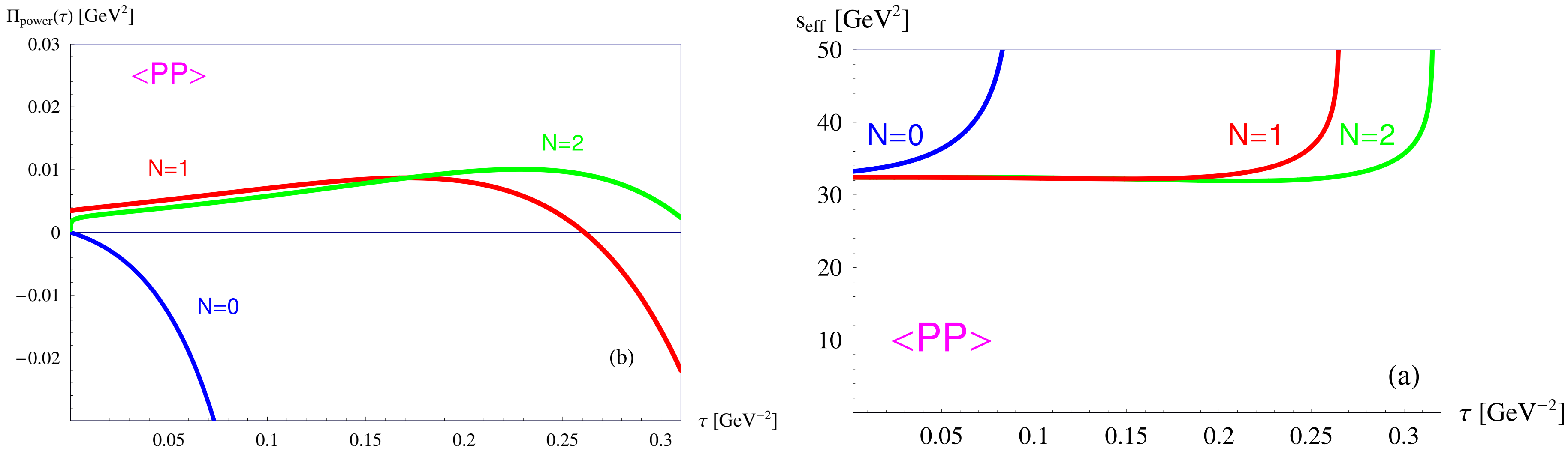
Depending on N, nonperturbative effects are distributed in a different way between power corrections and the effective threshold.

The usual procedure is aimed at obtaining hadron parameters *entirely* within QCD sum rules: work in a "windows" of (nonzero) τ, fix $s_{\rm eff}$ and obtain the decay constant.

If one is interested in particular dependence of the hadron observable on m_q and can make use of some "external" inputs, a different approach is promising: sum rule at $\tau = 0$ [local dulity (LD) limit]

LD limit $\tau \to 0$ in a sum rule : is it well defined ?

Power corrections vs τ and the "exact" τ-dependent threshold:



- $N = 0$ **and** $N = 1$ **power corrections are regular at** $\tau = 0$**;**
- $N = 2$ **power corrections contain** $\log(\tau)$

Clearly, for $N = 0$ and $N = 1$ (and also for $N = 2$) working at very small values of τ is eligible.

For $N = 0$ power corrections vanish at $\tau = 0$ and the sum rule takes the form

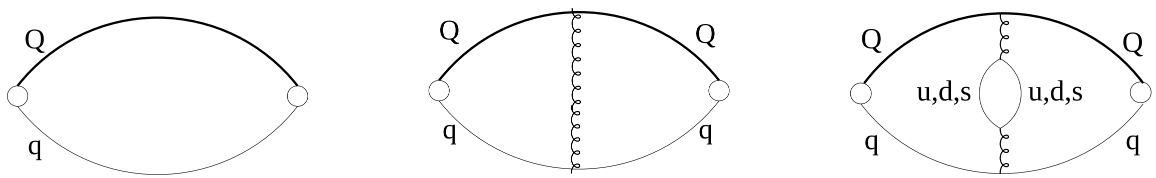
$$f_q^2 = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}^{(q)}} ds \rho_{\rm pert}(s, m_Q, m_q, \alpha_s|m_{\rm sea}) \equiv \Pi_{\rm dual}(s_{\rm eff}^q, m_q|m_{\rm sea})$$

Here $m_q = m_u$, m_d, or m_s, and $m_{\rm sea}$ denotes the set of values m_u, m_d, and m_s.
The IB is related to

$$\delta\Pi_{\rm dual} = \Pi_{\rm dual}(s_{\rm eff}^d, m_d|m_{\rm sea}) - \Pi_{\rm dual}(s_{\rm eff}^u, m_u|m_{\rm sea}).$$

- **In QCD, $\rho_{\rm pert}$ is calculated as expansion in powers of $a \equiv \alpha_s/\pi$:**

$$\rho_{\rm pert}(s, m_Q, m_q, \alpha_s) = \rho^{(0)}(s, m_Q, m_q) + a\rho^{(1)}(s, m_Q, m_q) + a^2\rho^{(2)}(s, m_Q, m_q|m_{\rm sea}) + \dots$$



Order a^2 is the first order where the "sea-quark" masses appear;
$\rho^{(2)}$ is known analytically for massless light quarks, $\rho^{(2)}(s, m_Q, m_q = 0|m_{\rm sea} = 0)$.

The correlation function calculated for a massless sea quark, $m_{\,sea} = 0$**, leads to:**

• **For decay constants of heavy mesons, the OPE error is** $O(a^2 m_s) \sim O($**a few MeV**$)$**,** $a \le 0.1$

• **However, for IB effects, the strange sea-quark contributions cancel in the difference and the OPE accuracy increases strongly:**

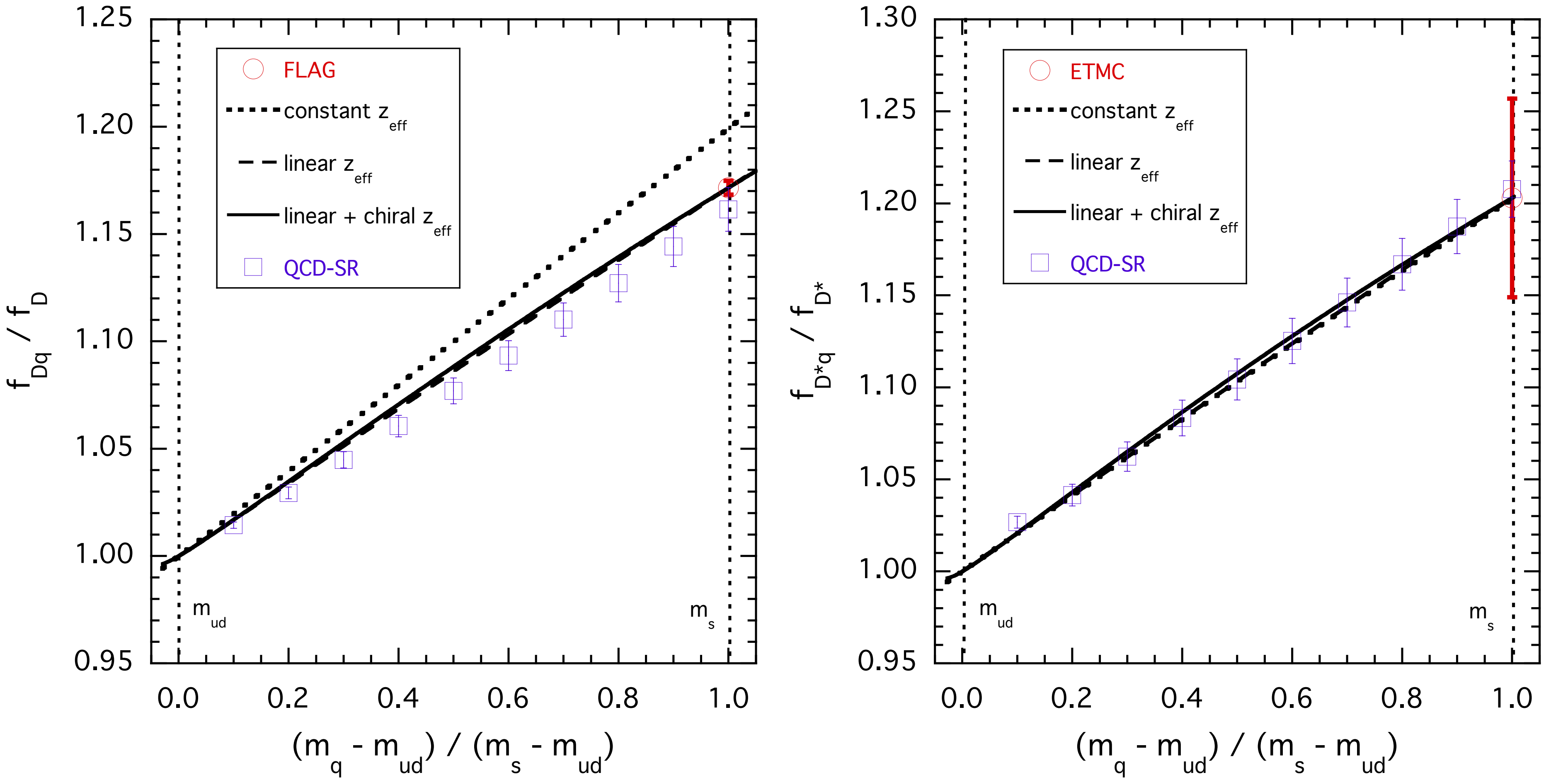
$$\begin{aligned}\delta\Pi_{\rm dual} &= \Pi_{\rm dual}(\tau, s^d_{\rm eff}, m_d|m_u, m_d, m_s) - \Pi_{\rm dual}(\tau, s^u_{\rm eff}, m_u|m_u, m_d, m_s)\\ &= \Pi_{\rm dual}(\tau, s^d_{\rm eff}, m_d|m_{\rm sea} = 0) - \Pi_{\rm dual}(\tau, s^u_{\rm eff}, m_u|m_{\rm sea} = 0) + O(a^2\delta m).\end{aligned}$$

What to do with the m_q*-dependence of* $s_{\rm eff}$*?*

Parametrize $s_{\rm eff} = s^{(0)}_{\rm eff} + m_q s^{(1)}_{\rm eff} + \cdots$ **and determine** $s^{(0)}_{\rm eff}$ **and** $s^{(1)}_{\rm eff}$ **using a few "external" lattice QCD results for strange mesons and isosymmetric mesons.**

We have *analytic* **representation for the decay constants and IB effects!**
(Few unknown parameters of the effective threshold are fixed by lattice QCD results).

How this works for D and D^*:



$m_{ud} = \frac{1}{2}(m_u + m_d)$.

- **$\sim$80% of IB comes from the m_q-depemdence of the perturbative spectral density**
- **The accuracy is limited mainly by the uncertainties of the available lattice results for f_H.**

Summary and conclusions

• We present the first application of QCD sum rules in LD limit to IB in the decay constants of heavy mesons and show that it is possible to obtain accurate predictions for $\delta f/f$. This was not obvious since the typical accuracy of the SR predictions for f_{B,B^*,D,D^*} is about 10-15 MeV.

• The known OPE (full m_q-dependence at LO and NLO, massless light quarks in NNLO) allows one to access IB with $O(a^2 m_u, a^2 m_d)$ accuracy, whereas the accuracy of the individual f is $O(a^2 m_s)$.

• Knowing the explicit dependence of the OPE on m_q and obtaining the decay constants as a function $f(m_q)$ opens the possibility to access the IB effects.

• Making use of lattice QCD results for strange and isosymmetric heavy mesons, we report the following IB effects:

$$f_{D^+} - f_{D^0} = (0.96 \pm 0.09)\ \text{MeV}\,, \qquad f_{B^0} - f_{B^+} = (1.01 \pm 0.10)\ \text{MeV}$$
$$f_{D^{*+}} - f_{D^{*0}} = (1.18 \pm 0.35)\ \text{MeV}\,, \qquad f_{B^{*0}} - f_{B^{*+}} = (0.89 \pm 0.30)\ \text{MeV}$$

(i) The main IB (~80%) is due to the m_q dependence of the spectral densities

(ii) The accuracy is limited mainly by the uncertainties of the available lattice results for f_H.