LD QCD sum rules for strong IB in decay constants of heavy mesons

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We propose a new method for calculating the dependences of the decay constants of heavy-light mesons on the light-quark mass \( m_q \) based on QCD sum rules at infinitely large Borel mass parameter (local-duality limit). For a specific choice of the correlation functions, all condensate contributions vanish and the \( m_q \)-dependence of the decay constants is shown to be mainly determined by the known analytic \( m_q \)-dependence of the diagrams of perturbative QCD.


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QCD sum rule for 2 – point function

A typical Borel QCD sum rule for the decay constant $f_H$ of a heavy (pseudoscalar or vector) $\bar{Q}q$ meson $H$ of mass $M_H$, (heavy quark $Q$ with mass $m_Q$ and light quark $q$ with mass $m_q$):

$$f_H^2(M_H^2)^N \exp(-M_H^2 \tau) = \int_{(m_Q+m_q)^2} \exp(-s \tau) s^N \rho_{\text{pert}}(s, m_Q, m_q, \alpha_s) + \Pi_{\text{power}}^{(N)}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, ...) .$$

Nonperturbative effects appear at two places:

(i) power corrections

(ii) effective threshold $s_{\text{eff}}^{(N)}(\tau, m_Q, m_q, \alpha_s)$. 

Depending on $N$, nonperturbative effects are distributed in a different way between power corrections and the effective threshold.

The usual procedure is aimed at obtaining hadron parameters entirely within QCD sum rules: work in a “windows” of (nonzero) $\tau$, fix $s_{\text{eff}}$ and obtain the decay constant.

If one is interested in particular dependence of the hadron observable on $m_q$ and can make use of some “external” inputs, a different approach is promising: sum rule at $\tau = 0$ [local duality (LD) limit]
LD limit $\tau \to 0$ in a sum rule: is it well defined?

Power corrections vs $\tau$ and the “exact” $\tau$-dependent threshold:

- $N = 0$ and $N = 1$ power corrections are regular at $\tau = 0$;
- $N = 2$ power corrections contain $\log(\tau)$

Clearly, for $N = 0$ and $N = 1$ (and also for $N = 2$) working at very small values of $\tau$ is eligible.
For $N = 0$ power corrections vanish at $\tau = 0$ and the sum rule takes the form

$$f_q^2 = \int_{s_{\text{eff}}^{(q)}}^{s_{\text{eff}}^{-1}} ds \rho_{\text{pert}}(s, m_Q, m_q, \alpha_s| m_{\text{sea}}) = \Pi_{\text{dual}}(s_{\text{eff}}^{q}, m_q| m_{\text{sea}})$$

Here $m_q = m_u, m_d, \text{ or } m_s$, and $m_{\text{sea}}$ denotes the set of values $m_u, m_d, \text{ and } m_s$.

The IB is related to

$$\delta \Pi_{\text{dual}} = \Pi_{\text{dual}}(s_{\text{eff}}^{d}, m_d| m_{\text{sea}}) - \Pi_{\text{dual}}(s_{\text{eff}}^{u}, m_u| m_{\text{sea}}).$$

• In QCD, $\rho_{\text{pert}}$ is calculated as expansion in powers of $a \equiv \alpha_s/\pi$:

$$\rho_{\text{pert}}(s, m_Q, m_q, \alpha_s) = \rho^{(0)}(s, m_Q, m_q) + a\rho^{(1)}(s, m_Q, m_q) + a^2\rho^{(2)}(s, m_Q, m_q| m_{\text{sea}}) + \ldots$$

Order $a^2$ is the first order where the “sea-quark” masses appear; $\rho^{(2)}$ is known analytically for massless light quarks, $\rho^{(2)}(s, m_Q, m_q = 0|m_{\text{sea}} = 0)$. 
The correlation function calculated for a massless sea quark, \( m_{\text{sea}} = 0 \), leads to:

- For decay constants of heavy mesons, the OPE error is \( O(a^2 m_s) \sim O(\text{a few MeV}) \), \( a \leq 0.1 \)

- However, for IB effects, the strange sea-quark contributions cancel in the difference and the OPE accuracy increases strongly:

\[
\delta \Pi_{\text{dual}} = \Pi_{\text{dual}}(\tau, s_{\text{eff}}^d, m_d|m_u, m_d, m_s) - \Pi_{\text{dual}}(\tau, s_{\text{eff}}^u, m_u|m_u, m_d, m_s) \\
= \Pi_{\text{dual}}(\tau, s_{\text{eff}}^d, m_d|m_{\text{sea}} = 0) - \Pi_{\text{dual}}(\tau, s_{\text{eff}}^u, m_u|m_{\text{sea}} = 0) + O(a^2 \delta m).
\]

What to do with the \( m_q \)-dependence of \( s_{\text{eff}} \)?

Parametrize \( s_{\text{eff}} = s_{\text{eff}}^{(0)} + m_q s_{\text{eff}}^{(1)} + \cdots \) and determine \( s_{\text{eff}}^{(0)} \) and \( s_{\text{eff}}^{(1)} \) using a few “external” lattice QCD results for strange mesons and isosymmetric mesons.

We have analytic representation for the decay constants and IB effects!
(Few unknown parameters of the effective threshold are fixed by lattice QCD results).
How this works for $D$ and $D^*$:

$m_{ud} = \frac{1}{2}(m_u + m_d)$.

- $\sim 80\%$ of IB comes from the $m_q$-dependence of the perturbative spectral density
- The accuracy is limited mainly by the uncertainties of the available lattice results for $f_H$. 
Summary and conclusions

- We present the first application of QCD sum rules in LD limit to IB in the decay constants of heavy mesons and show that it is possible to obtain accurate predictions for $\delta f/f$. This was not obvious since the typical accuracy of the SR predictions for $f_{B,B^*,D,D^*}$ is about 10-15 MeV.

- The known OPE (full $m_q$-dependence at LO and NLO, massless light quarks in NNLO) allows one to access IB with $O(a^2 m_u, a^2 m_d)$ accuracy, whereas the accuracy of the individual $f$ is $O(a^2 m_s)$.

- Knowing the explicit dependence of the OPE on $m_q$ and obtaining the decay constants as a function $f(m_q)$ opens the possibility to access the IB effects.

- Making use of lattice QCD results for strange and isosymmetric heavy mesons, we report the following IB effects:

  \[
  f_{D^+} - f_{D^0} = (0.96 \pm 0.09) \text{ MeV}, \quad f_{B^0} - f_{B^+} = (1.01 \pm 0.10) \text{ MeV} \\
  f_{D^{*+}} - f_{D^{*0}} = (1.18 \pm 0.35) \text{ MeV}, \quad f_{B^{*0}} - f_{B^{*+}} = (0.89 \pm 0.30) \text{ MeV}
  \]

  (i) The main IB ($\sim 80\%$) is due to the $m_q$ dependence of the spectral densities

  (ii) The accuracy is limited mainly by the uncertainties of the available lattice results for $f_H$. 