Rare FCNC radiative leptonic $B \to \gamma l^+ l^-$ decays in the SM

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1. Motivation: tensions with SM predictions in $b \to s, d$ decays induced by flavour-changing neutral current (FCNC)
2. $H_{\rm eff}$ for $b \to s, d$ and the $\langle \gamma l^+ l^- | H_{\rm eff} | B \rangle$ amplitude
3. Top-quark contribution and $B \to \gamma$ form factors
4. Charm-quark contributions
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Based on A.Kozachuk, D.Melikhov, N.Nikitin, Phys. Rev. D97, 053007 (2018), *Rare FCNC radiative leptonic $B(s,d) \to \gamma l^+ l^-$ decays in the standard model*.

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Motivation

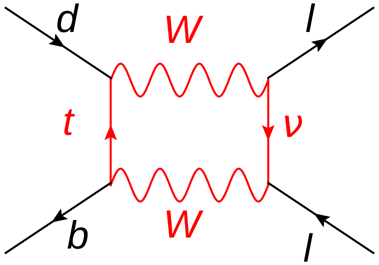
Rare radiative leptonic decays $B_{d,s} \to \gamma\ell^+\ell^-$:

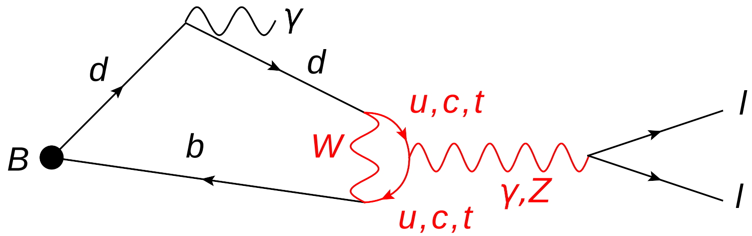
- **Induced by weak flavour changing neutral currents (FCNCs)**
- **In SM forbidden at tree level and occur only via loop diagrams**
- **Have small branching ratios of order** $10^{-8} - 10^{-10}$
- **New particles can contribute to the loops**

Similar B-decays are actively studied experimentally ($B \to K^{(*)}\ell^+\ell^-$ and others), and 2-3 σ deviations from the Standard Model have been observed.

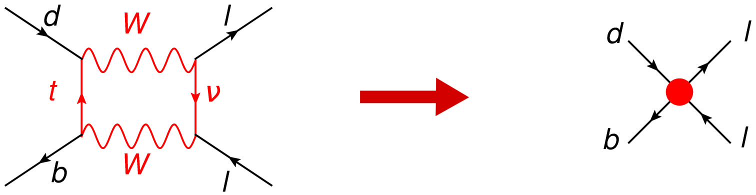
Effective Hamiltonian for FCNC $B-$decays

FCNC transitions in SM proceed via boxes and penguins





For B-decays W, Z, t are integrated out



$$\mathcal{H}_{\rm eff}^{b\to q} = \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_i C_i(\mu) O_i(\mu),$$

$C_i(\mu)$ - **Wilson coefficients**, $O_i(\mu)$ - **basis operators.**

Contributions of top and W to $\mathcal{H}_{\rm eff}$:

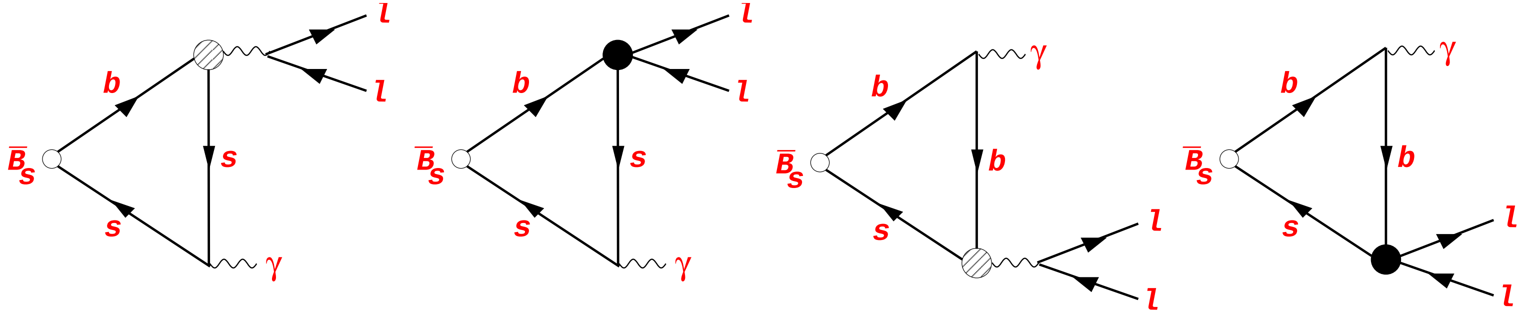
$$\begin{aligned}\mathcal{H}_{\rm eff}^{b\to d\ell^+\ell^-} &= \frac{G_F}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi} V_{tb}V_{tq}^*[-2im_b\frac{C_{7\gamma}(\mu)}{q^2}\cdot\bar{d}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\cdot\bar{\ell}\gamma^\mu\ell + \\ &+ C_{9V}(\mu)\cdot\bar{d}\gamma_\mu(1-\gamma_5)b\cdot\bar{\ell}\gamma^\mu\ell + C_{10A}(\mu)\cdot\bar{d}\gamma_\mu(1-\gamma_5)b\cdot\bar{\ell}\gamma^\mu\gamma_5\ell]\end{aligned}$$

But b, c, u, d, s-quarks are dynamical!

Top – quark contributions

Several distinct contributions induced by top [integrated out and described via $H_{\rm eff}(b \to s)$]:

• Diagrams with *virtual* photon emitted by FCNC vertex [similar to $B \to V l^+ l^-$ transition]:



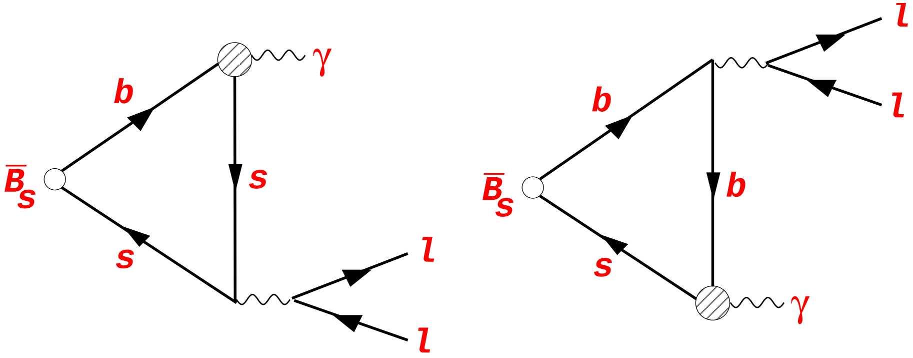
Dashed blob: penguin operator $O_{7\gamma}$; full blob: four-fermion operators O_{9V} and O_{10A}; point: e-m photon emission.

The corresponding contributions to the $B \to \gamma l^+ l^-$ amplitude contains several $B \to \gamma$ form factors: $F_A(q^2)$, $F_V(q^2)$, $F_{TA}(q^2, 0)$ and $F_{TV}(q^2, 0)$. These have no poles and cuts at $0 < q^2 < M_B^2$.

These formfactors were calculated via relativistic dispersion approach based on constituent quark picture. The parameters of the model (quark masses and parameters of the hadron wave functions) were fixed by reproducing weak decay constants. Comparison with other approaches (lattice QCD, sum rules) showed the accuracy of 10%.

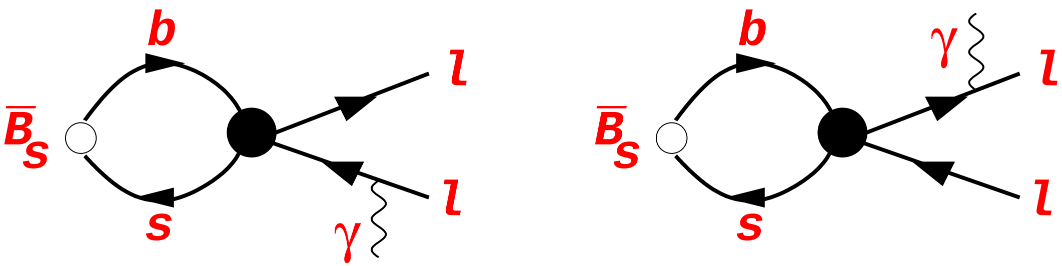
New diagrams compared to $B \to V l^+ l^-$:

- **Diagrams with *real* photon emitted from FCNC vertex [contributions induced by $H_{\rm eff}(b \to s\gamma)$]:**



Described by form factors $F_{TA,TV}(0, q^2)$, contain hadron poles and cuts are in the physical q^2-range. For calculations we combined our relativistic quark model with approaches based on VMD.

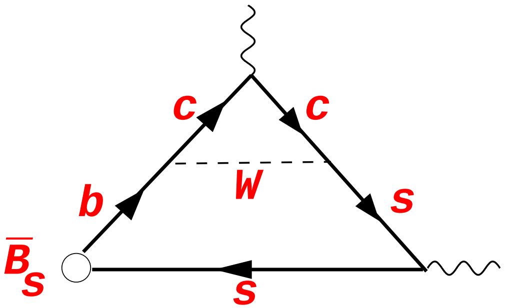
- **Bremsstrahlung (only operator O_{10A} contributes, the amplitude $\sim f_{B_s}$)**



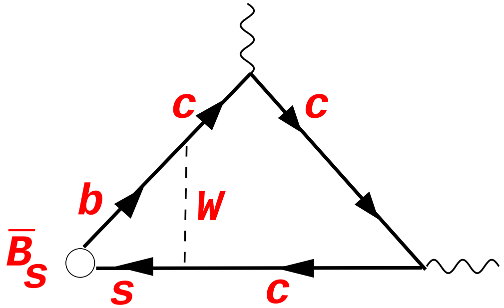
Charm – loop contributions

Charm is dynamical, and q^2 covers the range of all charmonia. Difficult!

Two topologies of charm contributions:



(a)



(b)

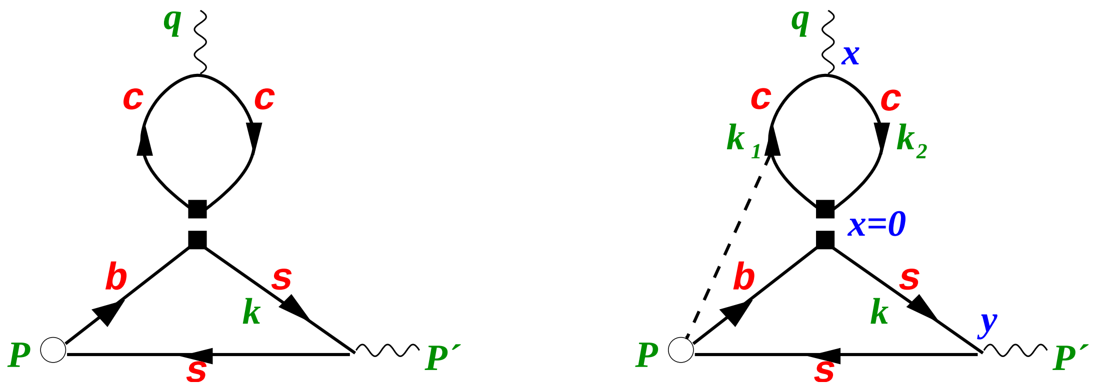
When the W boson line is shrinked to a point ($M_W \gg M_B$),

diagram (a) produces a penguin diagram (dominant),

diagram (b) produces weak annihilation diagram (numerically small compared to penguin)

Factorizable gluon exchanges (left plot)

are equal to charm contribution to vacuum polarization (known from $l^+l^- \to c\bar{c}$). They are universal (i.e. the same in $B \to K l^+l^-$, $B \to K^* l^+l^-$, $B \to \gamma l^+l^-$).



Nonfactorizable gluon exchanges (right plot)

are small compared to factorizable gluon exchanges at $q^2 \ll 4m_c^2$ and here can be calculated in QCD (QCD sum rules).

BUT!

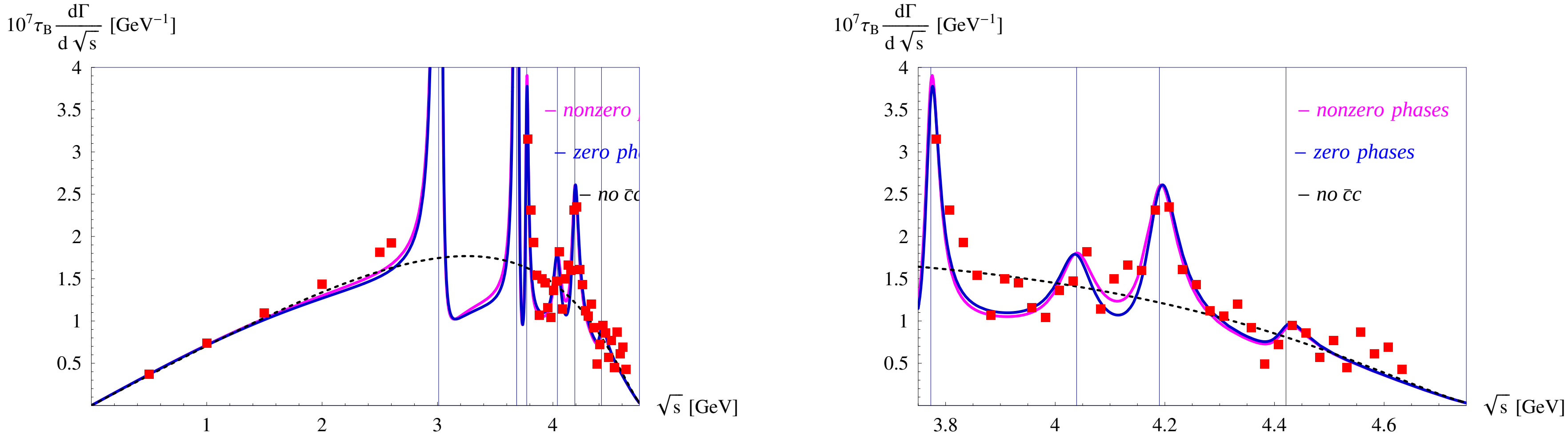
(i) They increase with q^2 and are (at least) comparable with factorizable contributions in the resonance region (know from experiment)

(ii) They may lead to relative phases between charmonium resonances

(iii) They are not universal (i.e. may differ in $B \to K l^+l^-$, $B \to K^* l^+l^-$, $B \to \gamma l^+l^-$)

Our ignorance of nonfactorizable corrections in the charmonia region leads to uncertainties in theoretical predictions.

LHCb data for $B \to K\mu^+\mu^-$:



At $q^2 = M_{R_i}^2$, the amplitude $A(B \to K l^+ l^-)$ has poles.

From data we know the modulus of its residue - from $\Gamma(B \to RK), R = \psi, \psi'$.

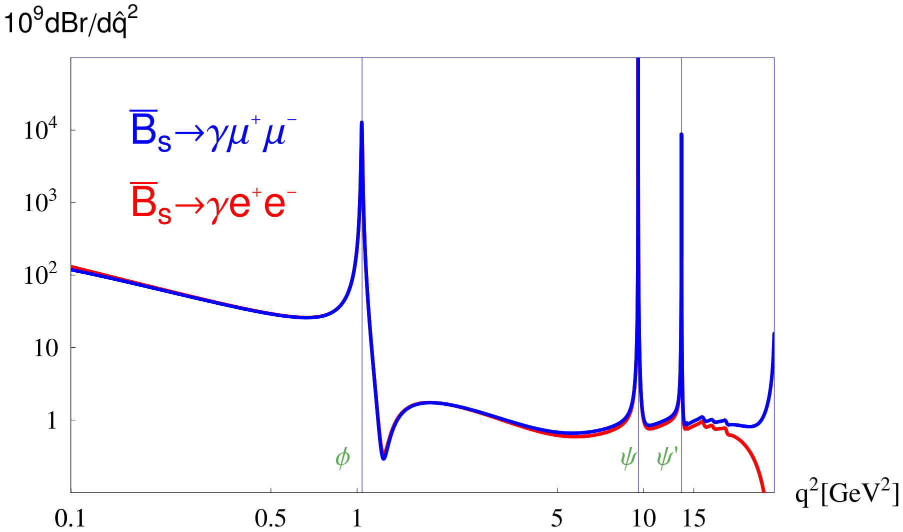
(i) The latter says that nonfactorizable corrections are large

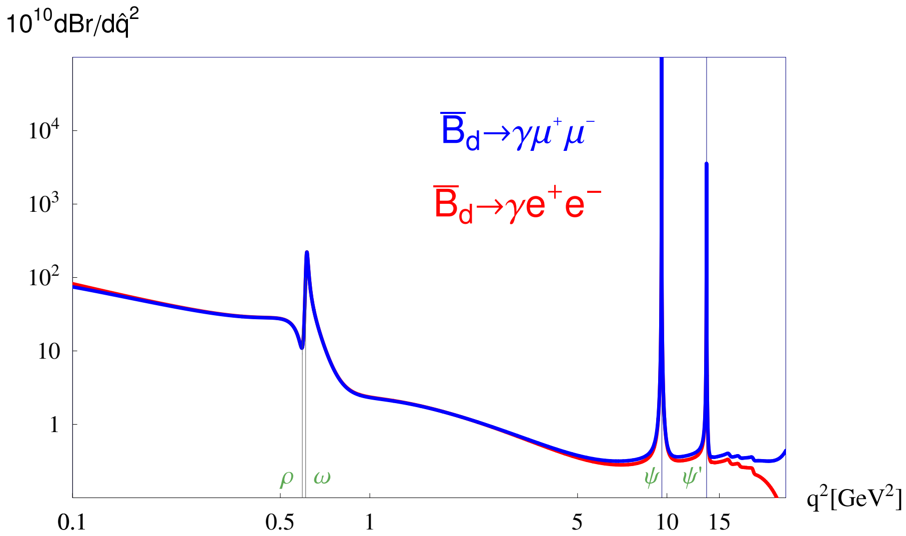
(ii) We can expect relative phases between different resonance contributions to $A(B \to K l^+ l^-)$, but we cannot extract well these relative phases.

Numerical results

Obtained many differential distributions in $B_s \to \gamma l^+ l^-$ vs q^2.

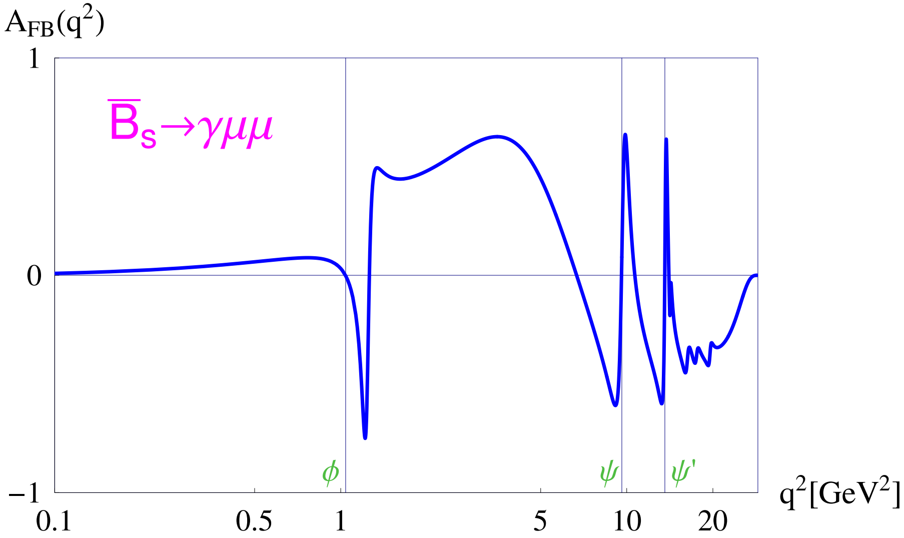
• **Differential branching ratios:**

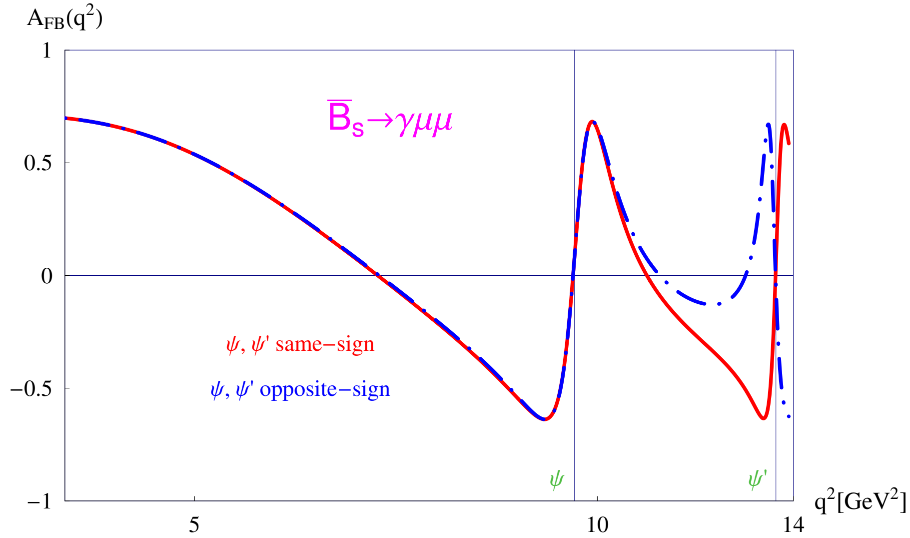




Compared to semileptonic $B \to V l^+ l^-$ decays:
(i) additional contributions of light vector resonances (ii) Bremsstrahlung at large q^2.

• Forward-backward asymmetry A_{FB}



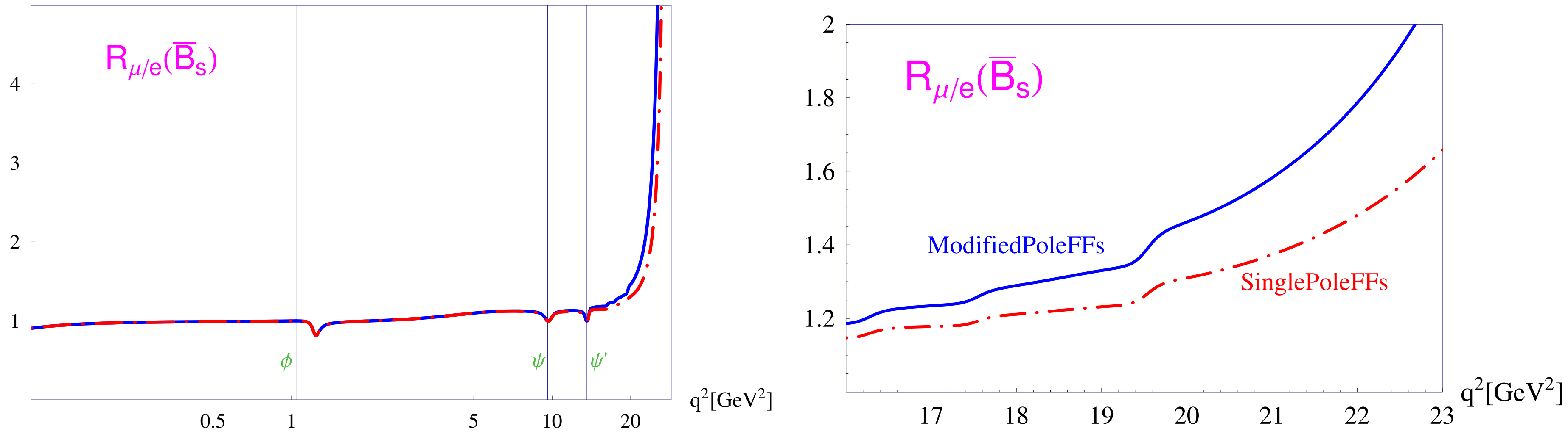


Has a tricky q^2 dependence. In the region between ψ and ψ' its shape is very sensitive to the ψ and ψ' relative signs.

• **Ratio** $R_{\mu/e} = [d\Gamma(B_s \to \gamma\mu^+\mu^-)/dq^2]/[d\Gamma(B_s \to \gamma e^+e^-)/dq^2]$

In $B \to (P, V)l^+l^-$, a similar ratio provides a clear test of lepton universality.

For radiative decays the picture is more complicated and contains a *strong interference between Bremsstrahlung and the form factor contribution***:**



As the result, at large q^2 one observes strong sensitivity to the q^2 dependence of the $B \to \gamma$ form factors F_A and F_V.

Summary and conclusions

A detailed theoretical analysis of QCD effects in $B_{d,s} \to \gamma\ell^+\ell^-$ decays was presented:

• All $B \to \gamma$ form factors were calculated using relativistic dispersion approach based on constituent quark picture. The form factors from dispersion approach satisfy rigorous constraints (e.-m. gauge invariance, LEET, HQET). We expect a 10% accuracy of our form factor results (based on comparison with available results from lattice QCD and QCD sum rules).

• Theoretical analysis of charm-loop contributions (factorizable and nonfactorizable) was done.

• Predictions for numerous differential distributions in $B \to \gamma\ell^+\ell^-$ were obtained:
(i) it was shown that the experimental measurement of A_{FB} in the region between the ψ and ψ' may clarify the relative phases of the narrow cc-resonances;
(ii) the ratio $R_{\mu/e}(q^2)$ in the region of large q^2 was shown to give direct access to the q^2 dependence of $B \to \gamma$ formfactors.

• We gave predictions for branching ratios over the region $q^2 \in [1,6]$ GeV^2, where the formfactors are reliably known and the contribution of charm is at the level of few percent:

$$\mathcal{B}(\bar{B}_s \to \gamma l^+ l^-)|_{q^2\in[1,6]\,\mathrm{GeV}^2} = (6.01 \pm 0.08 \pm 0.70)10^{-9},$$
$$\mathcal{B}(\bar{B}_d \to \gamma l^+ l^-)|_{q^2\in[1,6]\,\mathrm{GeV}^2} = (1.02 \pm 0.15 \pm 0.05)10^{-11}$$

(first error due to $B \to \gamma$ form factor uncertainty; second error due to uncertainty in light vector resonance contributions).

Much more results are in our Phys. Rev. D97, 053007 (2018).