Heavy Tetraquarks from Lattice QCD

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Discovery of doubly-charmed baryon

\[ \Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+ \]

LHCb: Phys. Rev. Lett. 119, 112001

Life time: 

\[ \text{3621.40} \pm 0.72 \, \text{(stat)} \pm 0.27 \, \text{(syst)} \pm 0.14 \, \text{(} \Lambda_c^+ \text{) MeV/c}^2 \]

\[ \text{0.256}^{+0.024}_{-0.022} \, \text{(stat)} \pm 0.014 \, \text{(syst) ps.} \]

arXiv:1806.02744
\( \Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+ \)

LHCb : arXiv:1807.01919

3620.6 \pm 1.5 \text{ (stat)} \pm 0.4 \text{ (syst)} \pm 0.3 \text{ (}\Xi_c^+\text{) MeV}/c^2.

\( \Xi_{cc}^{++} \) 
\[
\begin{array}{c}
\text{u} \\
\text{c} \\
\text{c} \\
\end{array}
\]
\[
\begin{array}{c}
\text{u} \\
\text{c} \\
\text{s} \\
\end{array}
\]
\[
\begin{array}{c}
\text{W}^+ \\
\text{u} \\
\text{d} \\
\end{array}
\]
\[
\begin{array}{c}
\Xi^+ \\
\end{array}
\]

\( \Xi_c^+ \)

\( \pi^+ \)
Does the existence of doubly heavy baryons suggest the existence of heavy tetraquarks?

Quark models, HQET prediction $\Rightarrow$ YES

Phys. Rev. Lett. 119, 202001 (2017),

What about LQCD?
Z(4430) in $B \rightarrow \psi(2S)K^+\pi^-$

- Decay $B^0 \rightarrow \psi(2S)K^+\pi^-$
- Signal yield: 25k events
- Combinatorial background: $\sim 4\%$
- 4D amplitude analysis: $(m^2(K\pi), m^2(\psi(2S)\pi), \theta_{\psi'}, \phi_{\psi'})$

Difficult to study on lattice except using a large basis of operators (@HADSPEC), multiple large volumes, resonance study
The strong force within *ud* diquark in baryon is more attractive with $I = J = 0, C = \bar{3}$ (good diquark) than with $I = J = 1, C = \bar{3}$ (bad diquark).

Spin dependent interaction vs color diquark color interaction

Good *ud* diquark is more attractive than good *us* diquark (attraction increase with decrease of quark mass)
At sufficiently large heavy quark mass diquark interaction wins over spin-spin interaction. It is possible to form a deeply bound tetraquark!
How to build a stable tetraquark (which can relatively be easier to study on lattice)?

Hints from Manohar & Wise (1992) (see T. Cohen@Thursday)

• Two heavy quarks with two light quarks
• $C_l = 3$, good light diquark

$F = 3, J_l = 0 \Rightarrow J_h = 1, C_h = 3, J^p = 1^+$

Possible states: $\bar{b}b\bar{u}d, \bar{b}b\bar{u}s, \bar{b}b\bar{u}c, \bar{b}b\bar{s}c,$
$\bar{b}\bar{c}ud, \bar{b}\bar{c}us$ etc.
Decay

• Decay into two mesons:

\[ E(Q\bar{q}) + E(Q\bar{q}') - E(Q^2q\bar{q}') \approx \frac{2}{9} m_\pi^2 \left[ 1 + O(m^{-1}) \right] > 0 \text{ (Positive)} \]

For sufficiently large \( m \) this should be bound!  
No decay to two mesons

Carlson, Heller, Tjon PRD 37, 744 (1988)

• Decay into 2 baryons:

\[
E(qq'q'') + E(\bar{q}''\bar{Q}\bar{Q}) - E(q\bar{q}'\bar{Q}\bar{Q}) = E(qq'q'') + E(\bar{q}''Q) - E(qq'Q) \\
\geq E(\text{proton}) + E(\bar{q}''Q) - E(qq'Q)
\]

For known charm and bottom masses right hand side is positive
Therefore, no strong decay to two baryons

Eichten, Quigg, PRL 199, 202002 (2017)
LQCD for heavy quark physics

\[ ma \ll 1 \]

- **Charm**: \( ma = 1.275 \text{ GeV} \),
  - \( ma = 0.5 \Rightarrow a \sim 0.075 \text{ fm} \)
  - \( ma = 0.3 \Rightarrow a \sim 0.046 \text{ fm} \)
- **Bottom**: \( ma = 4.66 \text{ GeV} \)
  - \( ma = 0.5 \Rightarrow a = 0.021 \text{ fm} \)
  - \( ma = 0.3 \Rightarrow a = 0.013 \text{ fm} \)

Being heavy lattice correlation functions for heavy quarks decay rapidly.

Relativistic charm quark calculations are now possible.

However, relativistic bottom-quark is still prohibitively costly.
Doubly charmed-strange baryons ($\Omega_{cc}(ccs)$)

Is it the next doubly charmed baryon to be discovered?

Decay to: $\Sigma^0 K^+ \pi^+ \pi^+$ and $\Omega_{c\pi^+}$

NM and Padmanath: arXiv:1807.00174
$bc$ hadrons

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\end{figure}

<table>
<thead>
<tr>
<th>Mesons ($q_1q_2$)</th>
<th>Baryons ($<a href="J%5EP">q_1q_2q_3</a>$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$J^P \equiv 1/2^+$</td>
</tr>
<tr>
<td>$B_c(bc)(0^-)$</td>
<td>$\Xi_{cb}[cbu]$</td>
</tr>
<tr>
<td>$B_c^*(bc)(1^-)$</td>
<td>$\Omega_{cb}[cbs]$</td>
</tr>
<tr>
<td>$B_c(bc)(0^+)$</td>
<td>$\Omega_{ccb}[ccb]$</td>
</tr>
<tr>
<td>$B_c(bc)(1^+)$</td>
<td>$\Omega_{cbb}[bbc]$</td>
</tr>
</tbody>
</table>

NM, Padmanath and Mondal : arXiv:1806.04151
Set up

- **Gauge Configurations**: HISQ 2+1+1

- **Valence quark propagators**:
  
  -- Light to charm: Overlap

  -- Bottom: NRQCD with improved coefficients

  -- Wall source propagators
Overlap fermions on 2+1+1 Flavors
HISQ Configurations

Lattices used for this study:
HISQ gauge configurations from MILC

- $24^3 \times 64$, $a = 0.1207$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.54$, $m_\pi = 305$ MeV
- $32^3 \times 96$, $a = 0.0888$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.5$, $m_\pi = 312$ MeV
- $48^3 \times 144$, $a = 0.0582$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.51$, $m_\pi = 319$ MeV

PHYSICAL REVIEW D 87, 054505 (2013) (MILC)

- Charm mass is tuned by $\bar{1S}_{cc} = \frac{1}{4}(\eta_c + 3J/\psi)$

$m_c a = 0.528$ ($a = 0.1192$ fm)
  = 0.428 ($a = 0.0888$ fm),
  = 0.290 ($a = 0.0582$ fm)

Considering kinetic masses
of mesons
(a la Fermilab formulation)
How to build a stable tetraquark (which can relatively be easier to study on lattice)?

Hints from Manohar & Wise (1992) (see T. Cohen@Thursday)

• Two heavy quarks with two light quarks
• $C_l = \bar{3}$, good light diquark

$F = \bar{3}, J_l = 0 \Rightarrow J_h = 1, C_h = 3, J^P = 1^+$

Possible states: $\bar{b}b\bar{u}d, \bar{b}b\bar{u}s, \bar{b}b\bar{u}c, \bar{b}b\bar{s}c, \bar{b}\bar{c}\bar{u}d, \bar{b}\bar{c}\bar{u}s$ etc.
1. **Compact Tetraquark type:**

\[
(l_1 l_2 \bar{h} \bar{h}) = (l_1)_\alpha^a(x) (C\gamma_5)_{\alpha\beta} (l_2)_\beta^b(x) \bar{h}^a_\kappa(x) (C\gamma_i)_{\kappa\rho} \bar{h}^b_\rho(x)
\]

2. **Two mesons type:**

\[
M(x) = M_1(x)M_2^*(x) - M_2(x)M_1^*(x)
\]

\[
M_{1,2}(x) = (l_{1,2})_\alpha^a(x) (\gamma_5)_{\alpha\beta} \bar{h}_\beta^a(x)
\]

\[
M_{1,2}^*(x) = (l_{1,2})_\alpha^a(x) (\gamma_i)_{\alpha\beta} \bar{h}_\beta^a(x)
\]

\[
M^d(x) = B^+(x)B^{0*}(x) - B^0(x)B^{+*}(x)
\]

\[
M^s(x) = B^+(x)B_s^{0*}(x) - B_s^0(x)B^{+*}(x)
\]

\[
M^c(x) = B^+(x)B^*_c(x) - B_c^+(x)B^{+*}(x)
\]

Related by Fierz transformation!
\( J = 0 \)

\[(l, l) \rightarrow (6_c, 0, F_S), \quad (\bar{h} h) \rightarrow (\bar{6}_c, 0, F_S).\]

\[l \in (u, s, c) \quad h \in (c, b).\]

1. **Compact Tetraquark type:**

\[(ll\bar{h}h) = \bar{l}^a_\alpha(x)(C\gamma_5)_{\alpha\beta}l^b_\beta(x) \bar{h}^b_\kappa(x)(C\gamma_5)_{\kappa\rho}\bar{h}^a_\rho(x).\]

2. **Two mesons type:**

\[\mathcal{L}_T(x) = \bar{h}^a_\alpha(x)(\gamma_5)_{\alpha\beta}l^a_\beta(x) \bar{h}^b_\kappa(x)(\gamma_5)_{\kappa\rho}l^b_\rho(x).\]

\[
\begin{array}{lllll}
(l_1l_2\bar{h}h) & (M_1M_2M'_1M'_2) & I & m_\pi [\text{MeV}] \\
\hline
uu\bar{b}\bar{b} & (B\bar{B}^0 B^* \bar{B}^{0*}) & 1 & 215 - 685 \\
ss\bar{b}\bar{b} & (B_s B_s^0 B_s^* \bar{B}_s^{0*}) & 0 & 150 - 685 \\
c\bar{c}\bar{c}\bar{b} & (B_c B_c^0 B_c^* \bar{B}_c^{0*}) & 0 & 150 - 685 \\
uu\bar{c}\bar{c} & (D\bar{D}^0 D^* \bar{D}^{0*}) & 1 & 215 - 685 \\
ss\bar{c}\bar{c} & (D_s \bar{D}_s^0 D_s^* \bar{D}_s^{0*}) & 0 & 215 - 685 \\
\end{array}
\]
Two eigenvalues:

\[ \lambda_1(t) \sim e^{-E_1t} \text{ and } \lambda_2(t) \sim e^{-E_2t} \]

\[ \Delta E_i^q = E_i^q - \left( m_{B^+} + m_{B_{q^+}} \right) \]
\[ \Delta E \text{(MeV)} \]

\[ I(J^P) = 0(1^+) \]

\[ m_\pi^2 \text{(GeV)}^2 \]
As light diquark mass increases \( \Delta E \) decreases.
$s_c b \bar{b} \quad J^P = 0(1^+)$
Lattice results for $u\bar{d}\bar{b}\bar{b}$ and $\ell_s\bar{b}\bar{b}$ tetraquarks

Results are from a correlation matrix of two $I(J^P) = 0(1^+)$ operators:

$$(u_a^T C \gamma_5 d_b)(\bar{b}_a C \gamma_i \bar{b}_b) \quad \text{and} \quad (\bar{b}_a \gamma_5 u_a)(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)(\bar{b}_b \gamma_i u_b)$$

Francis, Hudspith, Lewis, Maltman, PRL118(2017)142001

R. Lewis@Charm 2018
No significant excess for $b\bar{b}b\bar{b}$

See also: C. Hughes@Thursday
There is tremendous interest in the study of bound states with heavy quark(s).

New discovery of a doubly-charmed baryon further boosted the interest in the study of heavy tetraquark states.

More than one lattice QCD studies suggest there may exist deeply bound doubly bottom tetraquarks with quantum number \(1^+\).

Volume study (resonance study) is ongoing to confirm the presence of such states.

Will look forward to discoveries of these new hadrons.
$u \bar{s} b \bar{b} \ m_\pi = 497 \text{ MeV}$

$E_0 = -109.2 \pm 14.52 \text{ MeV}$
\[ V_C = \sum_{i>j} \alpha_s \frac{F_i \cdot F_j}{r_{ij}} \]

\[ F_i \cdot F_j = -\frac{2}{3} \]

Bohr radius \( a = \frac{3}{ma_s} \)

\[ E(Q^2) \approx -\frac{1}{9} m\alpha_s^2 \]

\[ E(Q\bar{q}) + E(Q\bar{q}') - E(Q^2\bar{q}\bar{q}') \approx \frac{2}{9} m\alpha_s^2 \left[ 1 + O(m^{-1}) \right] \]