NiPS Laboratory
Noise in Physical Systems

Superinsulators: one color QCD with Cooper pairs

M. Cristina Diamantini

Nips laboratory, INFN and Department of Physics and Geology University of Perugia

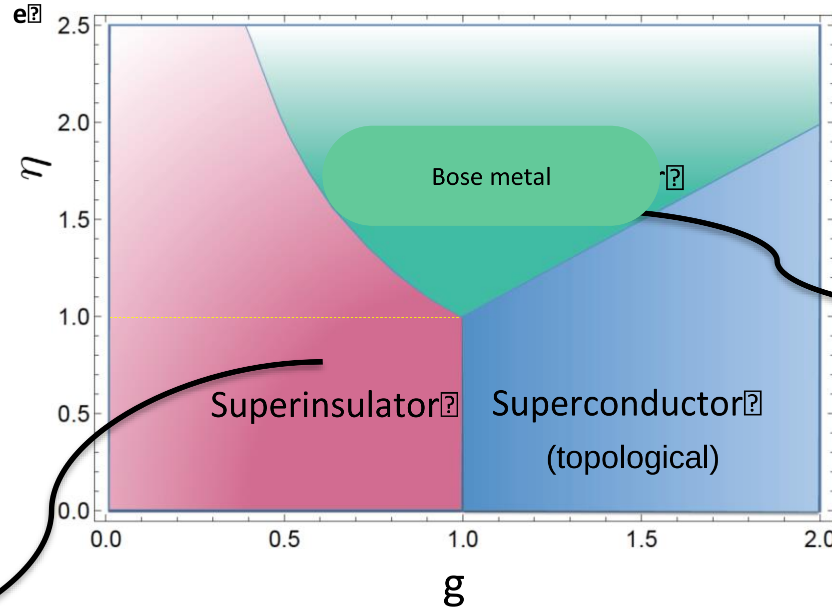
Coll:

- **Luca Gammaitoni, University of Perugia**
- **Carlo A. Trugenberger, SwissScientific**
- **Valerii Vinokur, Argonne National Laboratory**

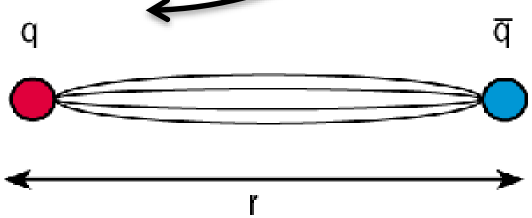
arXiv:1806.00823 **arXiv:1807.01984**

XIIIth Quark Confinement and the Hadron Spectrum, Maynooth August 2018

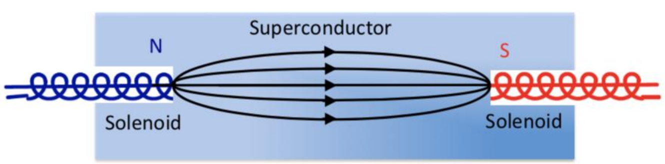
T=0



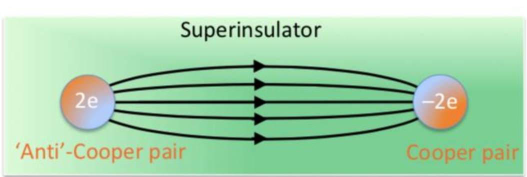
topological insulator (bosonic)



quarks bound by (chromo)-electric strings in a condensate of magnetic monopoles
(Mandelstam, ‘t Hooft, Polyakov)



mirror analogue to vortex formation in type II superconductors



Polyakov's magnetic monopole condensation⟹ electric string
⟹ **linear confinement** of Cooper pairs
one color QCD

Superconductor

$R = 0$

$G = \infty$

S duality

Mandelstam 'tHooft
Polyakov

Superinsulator

$R = \infty$

$G = 0$

- theoretically predicted in 1996

  P. Sodano, C.A. Trugenberger, MCD, Nucl. Phys. B474 (1996) 641
- experimentally observed in TiN films in 2008

  Vinokur et al, Nature 452 (2008) 613
- confirmed in NbTin films in 2017

  Vinokur et al, Scientific Reports 2018

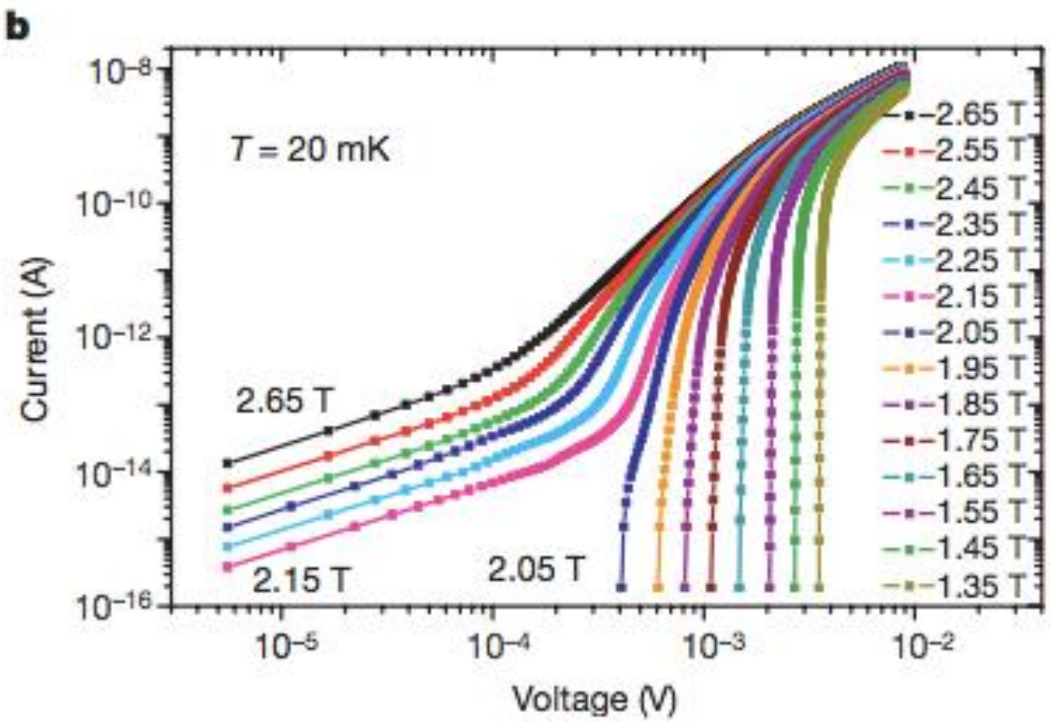
Superinsulation: realization and proof of confinement by monopole condensation and asymptotic freedom in solid state materials

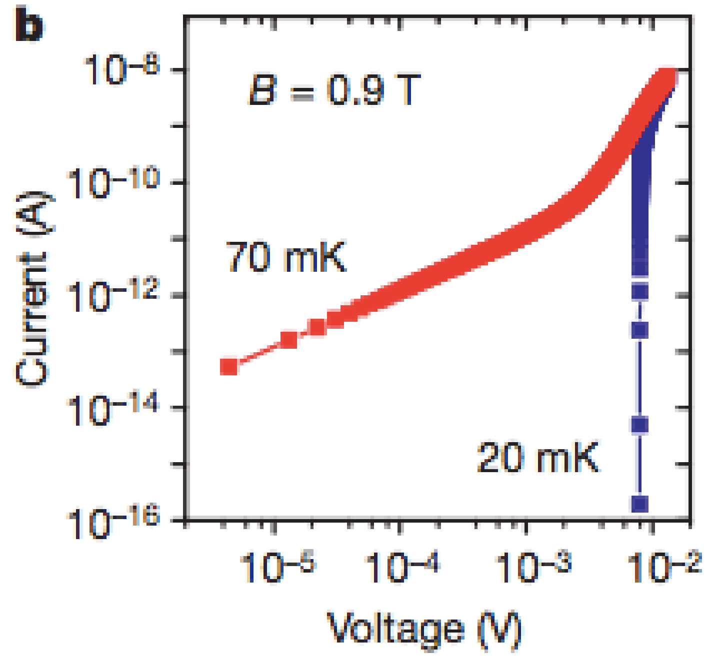
Cooper pairs ⟷ Quarks

Results for homogeneously disordered TiN film (2D)

transition driven by:

- tuning disorder (thickness of the film)
- external magnetic field





(Vinokur et al. Nature)

- superinsulating state **dual** to the superconducting state

Superconductor	**Superinsulator**
Cooper pair condensate and pinned vortices	Vortex condensate and localized Cooper pairs

- **Cooper pairs** and **vortices** are the relevant degrees of freedom
- SIT is driven by the competition between charge (Cooper pairs) and vortex **degrees of freedom:**

  topological interactions, Aharonov-Bohm and Aharonov-Casher

  gauge invariance

  disorder plays a role only as tuning mechanism

PT INVARIANT TOPOLOGICAL STATES OF MATTER

- **Sodano, Trugenberger, MCD (1996)**

  two fluids model, **2d**

$$\boxed{j_\mu \propto \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha}$$

conserved charge current, b_μ is a pseudovector

$$\boxed{\phi_\mu \propto \epsilon_{\mu\nu\alpha}\partial_\nu a_\alpha}$$

conserved vortex current a_μ is a vector

$$\boxed{\mathrm{S} = i\int d^3x \frac{k}{2\pi} a_\mu \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha} \longrightarrow$$

mixed CS ≡ BF , P(T) invariant
U(1) x U(1)

- add kinetic term for the emergent gauge fields

$$\boxed{\mathrm{S_{TM}} = \int d^3x \, \frac{1}{4e_f^2} f_{\mu\nu} f_{\mu\nu} + i\frac{k}{2\pi} a_\mu \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha + \frac{1}{4e_g^2} g_{\mu\nu} g_{\mu\nu}}$$

these are the terms that are allowed by gauge invariance with minimum number of derivatives

3d

$$\boxed{\begin{array}{l} j_\mu \propto \epsilon_{\mu\nu\alpha\beta}\partial_\nu b_{\alpha\beta} \\ \phi_{\mu\nu} \propto \epsilon_{\mu\nu\alpha\beta}\partial_\alpha a_\beta \end{array}}$$

$\longrightarrow$ charge current, $b_{\mu\nu}$ pseudotensor

$\longrightarrow$ vortex current, a_μ vector

$$\boxed{S_{BF} = i\frac{k}{2\pi}\int d^4x\; b_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}\partial_\alpha a_\beta}$$

$\longrightarrow$ **BF , P(T) invariant**
U(1) x U(1)

+ kinetic for a_μ and $b_{\mu\nu}$

$$\boxed{S_{TM} = \int d^4x \frac{1}{12e_g^2} h_{\mu\nu\rho}h_{\mu\nu\rho} - \frac{i\kappa}{8\pi} b_{\mu\nu}\epsilon_{\mu\nu\lambda\rho}f_{\lambda\rho} + \frac{1}{4e_f^2} f_{\mu\nu}f_{\mu\nu}}$$

BF theory provides a generalization of fractional statistics to arbitrary dimensions (Semenoff, Szabo), (3+1): particles around vortex strings

S_{TM} was first proposed in 2 and 3d as a field theory description of topological phases of condensed matter systems in 1996 (Sodano, Trugenberger, MCD)

$$S_{TM} = \int d^4x \frac{1}{12e_g^2} h_{\mu\nu\rho} h_{\mu\nu\rho} - \frac{i\kappa}{8\pi} b_{\mu\nu} \epsilon_{\mu\nu\lambda\rho} f_{\lambda\rho} + \frac{1}{4e_f^2} f_{\mu\nu} f_{\mu\nu}$$

S_{TM} : generalization of CS mass to BF theory, **topological mass generation**

- a_μ and b_μ ($b_{\mu\nu}$) acquire a topological mass $m = (k\, e_f\, e_g)/2\pi$
- k is a dimensionless parameter, it determines the ground state degeneracy on manifold with non trivial topology and the statitistics
- $[e_f^2] = m^{-d+3}$ $[e_g^2] = m^{d-1}$ naively irrelevant (first one marginal in 3d) but necessary to correctly define the limit $m \to \infty$ (pure CS limit)

  (Dunne, Jackiw, Trugenberger, 1990)
- they enter in the phase structure of the theory

2d

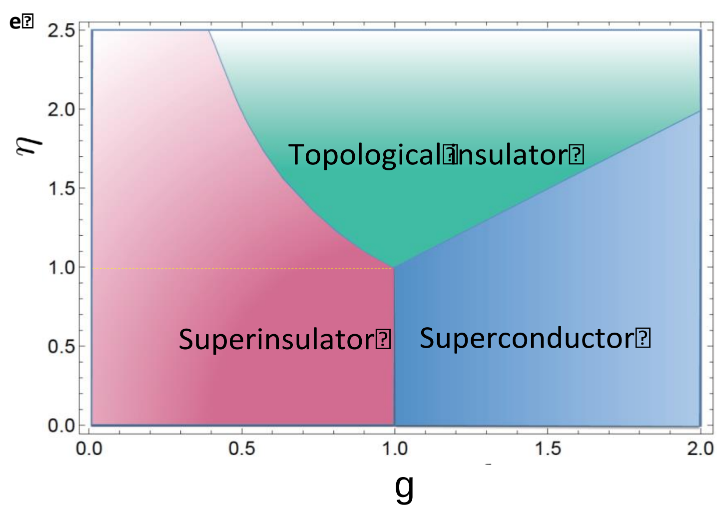
compact gauge U(1) X U(1)⇒lattice regularization, lattice spacing l

$$\boxed{Z = \sum_{\{M_\mu, Q_\mu\}} \int \mathcal{D}a_\mu \mathcal{D}b_\mu \exp -S}$$

$$S = \sum_x \frac{\ell^3}{4e_f^2} f_{\mu\nu} f_{\mu\nu} + i\frac{\ell^3}{\pi} a_\mu \epsilon_{\mu\nu\alpha} d_\nu b_\alpha + \frac{\ell^3}{4e_g^2} g_{\mu\nu} g_{\mu\nu} + \\ + i\ell\sqrt{2} a_\mu Q_\mu + i\ell\sqrt{2} b_\mu M_\mu$$

charge world-lines Q_μ represent the singularities in the dual field strength f_μ

vortex world-lines M_μ represent singularities in the dual field strength g_μ



$$\eta = \frac{\pi m \ell G(m\ell)}{\mu}$$

with G(ml) diagonal part of the lattice kernel:

$l^2 (m^2 - \nabla^2) G(x-y) = \delta(x-y)$

$g = e_f/e_g$

SUPERINSULATING PHASE

induced effective action $S^{eff}(A_\mu)$ for the electromagnetic gauge potential A_μ

2d : M_μ condense, Q_μ diluted

$$\boxed{\exp(-S^{eff}) = \sum_{M\mu} \exp \sum_x [- \gamma (M_\mu - e\, l^2 F_\mu \backslash \pi)^2]}$$

$\gamma = g\mu\eta$

Villain approximation of compact QED in 2d

$$\boxed{S_{QED} = \gamma/2\pi^2 \sum_x [\, 1 - \cos(\, 2e\, l^2\, F_\mu\,)]}$$

(Polyakov)

M_μ can be open ending in magnetic monopole (instanton) $\quad d_\mu M_\mu = m$

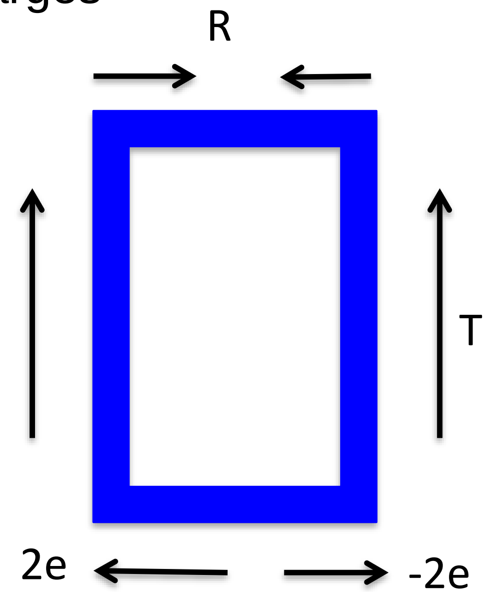
m  tunneling event

dense phase ⇒ monopole gas

$$S_{\text{Top}} = \sum_x \frac{2\pi^2}{l^3 \gamma} m \frac{1}{-\nabla^2} m$$

Wilson loop: its expectation value measures the potential between static external test charges q (2e) and anti-charges

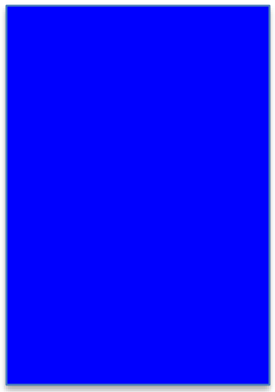
$$W(C) \equiv \exp\ i\, I\, q \sum_x q_\mu A_\mu$$



rectangular loop, for $T \to \infty$

$$\langle W(C)\rangle \propto_{T\to\infty} \exp -V(R)\, T$$

$$\langle W(C)\rangle \propto \exp -\sigma A \Rightarrow V(R) = \sigma R$$



A = area enclosed by the loop C

σ emergent scale, string tension

dual Meissner effect, charge confinement in a monopole condensate, ***true also in 3d***

⇒ superinsulation can exist also in 3d

string tension (Polyakov; Kogan and Kovner; Quevedo, Trugenberger, and MCD)

2d:

$$\boxed{\sigma = \frac{\pi^{3/2}}{l^2} \exp\left(-\frac{\gamma}{16\pi e^2}\right)}$$

3d:

$$\boxed{\sigma = \frac{1}{64\pi l^2} K_0(\gamma/2)}$$

confining string picture like QCD :

linear confinement of Cooper pairs into neutral "U(1) mesons"

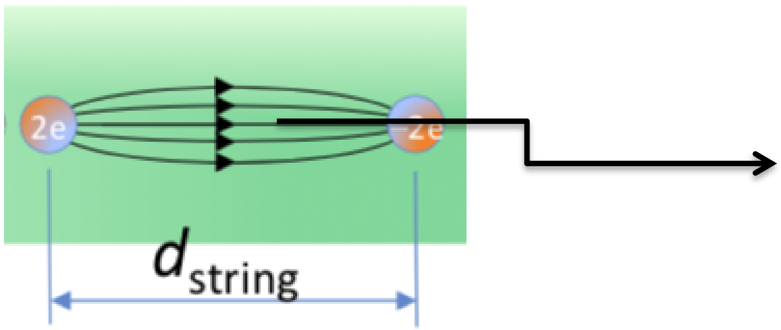
typical size $\boxed{d_{string} \simeq ((v_c)/\sigma)^{1/2}}$

2d:

$$d_{\mathrm{string}} \simeq \ell \; \exp\left(K \frac{g\eta c}{v_{\mathrm{c}}}\right)$$

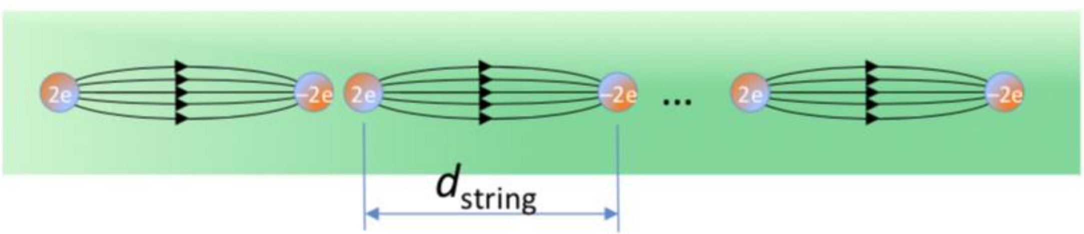
K numerical constant

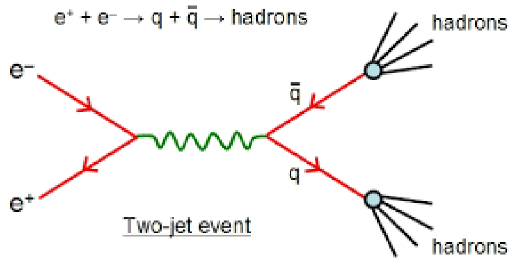
near SIT : g≈ 1/η $v_{\mathrm{c}} = 1/\sqrt{\mu_{\mathrm{P}} \epsilon_{\mathrm{P}}} \ll c$ **d_{string} >> l**



$W_{string} = 1/m_\gamma$
(Caselle et al)

long strings unstable, string fragmentation via creation of charge-anticharge pairs like formation of hadron jets at LHC $\Longrightarrow$ creation of neutral mesons ($V \approx 2m_{CP}$, V applied voltage)

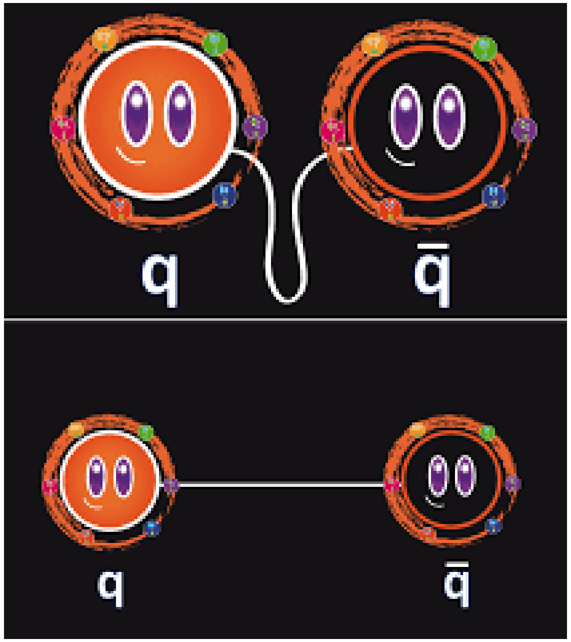




deconfinement phase transition (T=0): increase V to $V_t \approx \sigma L$, L size of the system $\Longrightarrow$ breaking of neutral mesons:

- creation of strips of 'normal' insulator where current can flow
- large current fluctuations
- large current fluctuations recently observed in InO_x films (Tamir et al)

HINT OF ASYMPTOTIC FREEDOM



reverse of the confinement on scales smaller than the typical string size

SIT: string scale can be inferred from experimental data

$d_{string} = \hbar\, v_c / KT_{CBKT}$ (v_c speed of light in the material)

KT_{CBKT} energy required to break up the string

d_{string} scale associated with this energy

TiN films: $T_{CBKT} = 60$ mK^O

$v_c = c/4.10$^5 (Baturina and Vinokour)

$d_{string} \approx 60\ \mu m$

study of formation of superinsulators in TiN films of different sizes (Kalok et al): samples of size ≤ 20μm **metallic behaviour**

Θ-TERM

superinsulators **(3d)** :

add an additional coupling

$$\frac{\theta e}{16\pi}\phi_{\mu\nu}F_{\mu\nu} = \frac{\theta e}{16\pi}\epsilon_{\alpha\beta\mu\nu}\partial_\alpha a_\beta F_{\mu\nu}$$

electromagnetic response, extra term in the action:

$$\int d^4x \frac{i\theta}{2\pi^2}\mathbf{E}\bullet\mathbf{B} = \int d^4x \frac{i\theta}{16\pi^2}F_{\mu\nu}\tilde{F}_{\mu\nu}$$

θ =0, π due to T symmetry,

θ = π **partition function factor $(-1)^v$**

v = self intersection number of confining electric flux tubes worldsheets

spin term (Polyakov) ⇒ Cooper pairs "mesons" become fermions

θ-term present in 3d topological insulators, distinguishes between strong and weak

θ = π strong top. ins

DECONFINEMENT CRITICALITY

critical behavior of confining string at finite temperature:

$\sigma(T)$ = string tension (same thing as mass for a particle) vanishes at a critical temperature $T_C \Rightarrow$ ***deconfinement:***
infinitely long strings on the microscopic scale and Cooper pairs at the end are liberated

correlation length **$\xi \propto 1/\sqrt{\sigma}$**

$$\boxed{\sigma \propto \exp\left(\frac{\text{const}}{\sqrt{|T/T_C - 1|}}\right) \quad , \quad \mathrm{R} \propto \exp\left(\frac{\text{const}}{\sqrt{|T/T_C - 1|}}\right) \quad , 2\mathrm{d}}$$

(Yaffe and Svetiski) $\Longrightarrow$ **Berezinski-Kosterlitz-Thouless critical scaling**

$$\boxed{\sigma \propto \exp\left(\frac{\text{const}}{|T/T_C - 1|}\right) \quad , \quad \mathrm{R} \propto \exp\left(\frac{\text{const}}{|T/T_C - 1|}\right) \quad , 3\mathrm{d}}$$

InO_x films: “more” 3d than TiN and NbTiN films ($d \gg \xi \equiv$ superconducting coherence lenght) (Shahar et al)***Vogel-Fulcher-Tamman criticality ≡ behaviour of confining strings in 3d*** (Gammaitoni, Trugenberger, Vinokour, MCD)

THANK YOU!!!!

$$\boxed{S^{\mathrm{2D}} = \int d^3x \frac{ik}{2\pi} a_\mu \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha + \frac{1}{2e_f^2 \mu_P} f_0^2 + \frac{\epsilon_P}{2e_f^2} f_i^2 + \frac{1}{2e_g^2 \mu_P} g_0^2 + \frac{\epsilon_P}{2e_g^2} g_i^2 + i\sqrt{k} a_\mu Q_\mu + i\sqrt{k} b_\mu M_\mu}$$

e^2_g charge energy

e^2_f vortex energy

ε_P dielectric permittivity

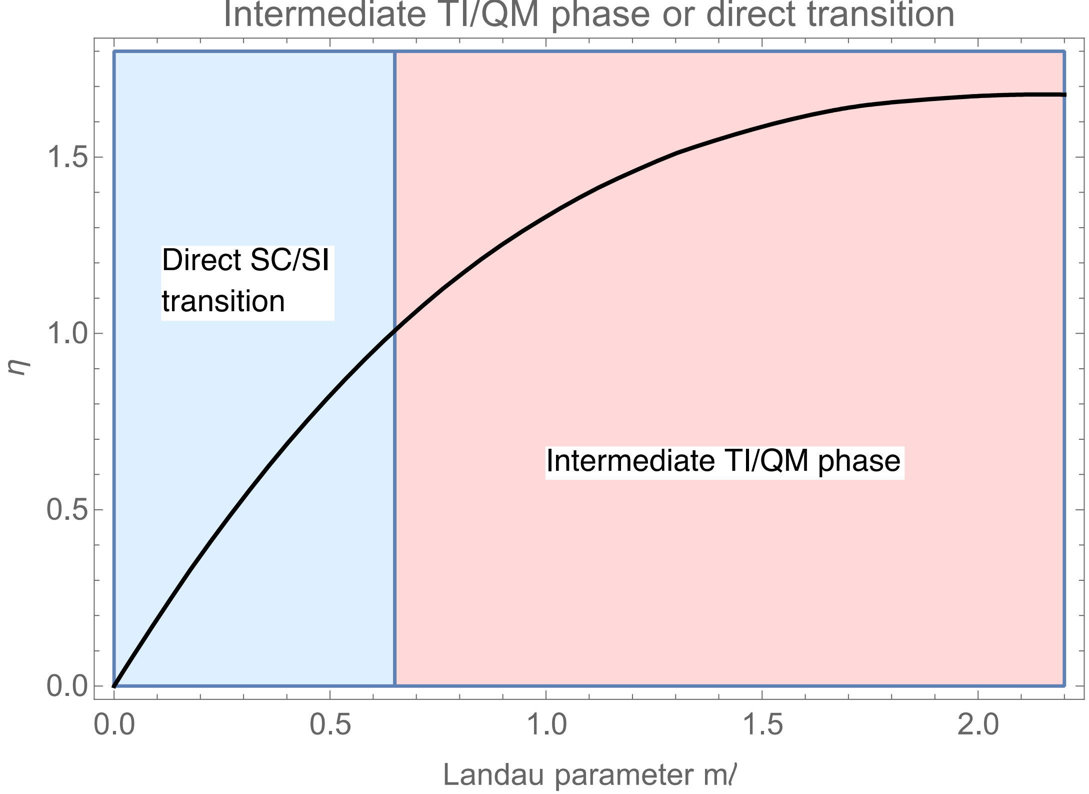
μ_P magnetic permittivity

$g \equiv e_f/e_g$ tuning parameter

$g \Leftrightarrow 1/g$ duality

light velocity in the medium $\boxed{v_c = \frac{1}{\sqrt{\varepsilon_P \mu_P}}}$

$$m^{\mathrm{CS}} = \mu_P \frac{e_f e_g}{\pi}$$



$e^2_g = e^2/d$

$e^2_f = \pi^2/(e^2\,\lambda_\perp)$

$K = \lambda_\perp/\xi$ **Landau parameter**

$\lambda_\perp = \lambda_L^2/d$ Pearl length 2d

λ_L = London penetration depth

$\alpha = e^2/(\hbar\, c)$

d = film thickness

$l \approx \xi$ coherence length

$$\eta = (1/\alpha)\text{ל}(K, v_c)$$

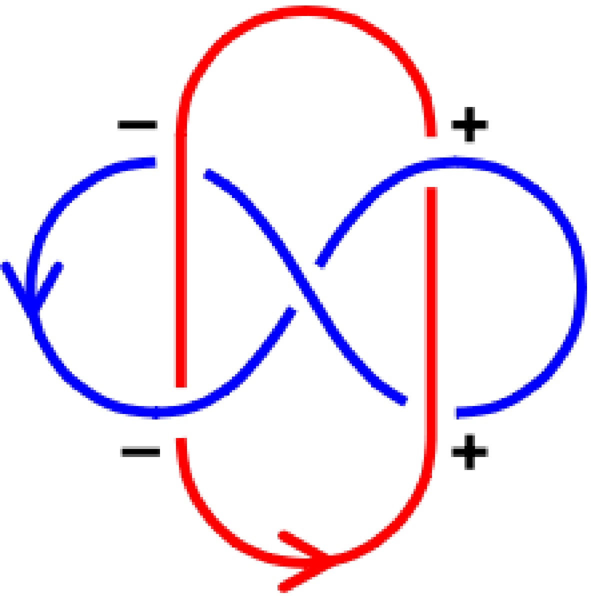
quantum behavior

characteristic of the material

$ml \propto 1/(\alpha K)$ close to the SIT

2d

Q_μ M_μ world lines of elementary charges (Cooper pairs) and vortices



$$\boxed{S_{\text{linking}} = \int d^3x \; i \; 2\pi \; Q_\mu \epsilon_{\mu\alpha\nu} \frac{\partial_\alpha}{-\nabla^2} M_\nu}$$

integer linking numbers do not contribute to the partition function; contribution for infinitely extended world-lines of charges and vortices

local formulation:

$$\boxed{S^{\text{CS}} = \int d^3x \left[i \frac{\kappa}{2\pi} a_\mu \epsilon_{\mu\alpha\nu} \partial_\alpha b_\nu + i\sqrt{\kappa} a_\mu Q_\mu + i\sqrt{\kappa} b_\mu M_\mu \right]}$$

two emergent gauge fields a_μ (vector) and b_μ (pesudovector); emergent mixed Chern –Simons term

need regularization

2d

$\langle W(C) \rangle \equiv \langle \exp\ i\, l\, q \sum_x q_\mu A_\mu \rangle$

expansion in power of q $\Longrightarrow$

$$\Longrightarrow \quad \langle F_\mu(x) F_\nu(y) \rangle_0 \propto \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\nabla^2} \right) \delta_{x,y}$$ **without monopoles**

$$\langle F_\mu(x) F_\nu(y) \rangle \propto \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\nabla^2 - m_\gamma^2} \right) \delta_{x,y}$$ **with monopoles**

(Polyakov)

instantons disorder the system $\Longrightarrow$ short range correlations

photon acquires a dynamical mass m_γ

$$\boxed{m_\gamma = \frac{1}{\ell}\, 4\pi^{3/2} \beta \exp(-\pi\beta)}$$

<W(C)> = $\boxed{\int \mathcal{D}B_{\mu\nu} \exp\left(-S\left(B_{\mu\nu}\right) + i\int_{\text{surface}} B_{\mu\nu}\, d\sigma_{\mu\nu}\right)}$

$B_{\mu\nu} \neq b_{\mu\nu}$

$\sigma_{\mu\nu}$ parametrize the surface enclosed by the loop C

integration over $B_{\mu\nu}$ leads to an induced action for $\sigma_{\mu\nu}$

confining string action: (Quevedo and Trugenberger, Polyakov)

<W(C)> = exp – $S_{\text{conf. string}}(\sigma_{\mu\nu})$

first term in the derivative expansions of $S_{\text{conf. string}}(\sigma_{\mu\nu}) \propto \sigma A$

σ = string tension