CHIRAL AND U(1)$_A$ RESTORATION: WARD IDENTITIES AND EFFECTIVE THEORIES

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OUTLINE:

- $U(3)$ **Ward Identities:** $O(4)$ **vs** $O(4) \times U(1)_A$**, chiral partners**
- **WI and scaling of meson screening masses**
- **Hadron realization:** $U(3)$ **ChPT, role of thermal** $\sigma/f_0(500)$ **pole**

AGN, R.Torres Andrés, J.Ruiz de Elvira, PRD88, 076007 (2013)

AGN, J.Ruiz de Elvira, JHEP 1603 (2016) 186; PRD97, 074016 (2018); PRD98, 014020 (2018)

S.Ferreres-Solé, AGN, A.Vioque-Rodríguez, in prep. 2018

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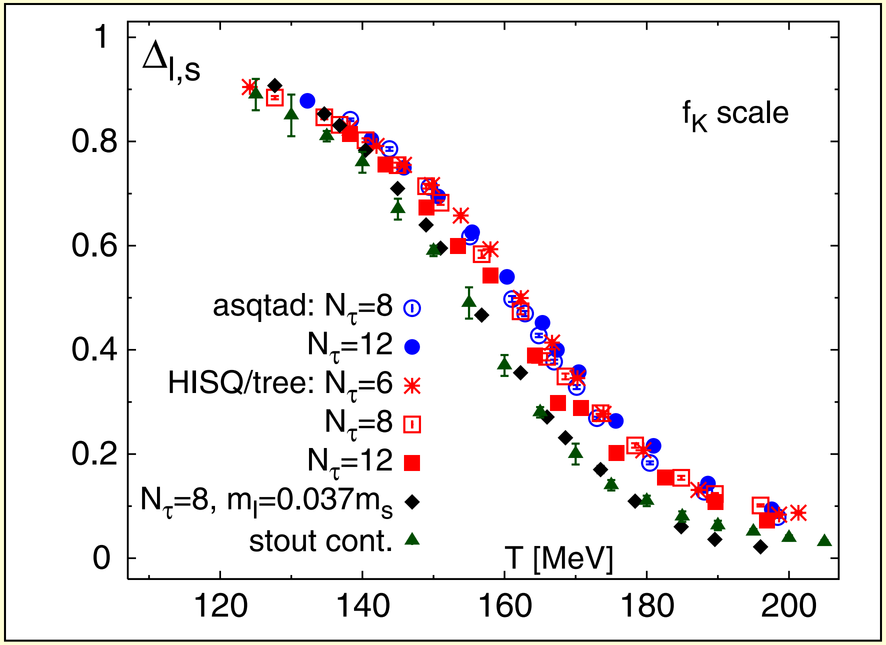
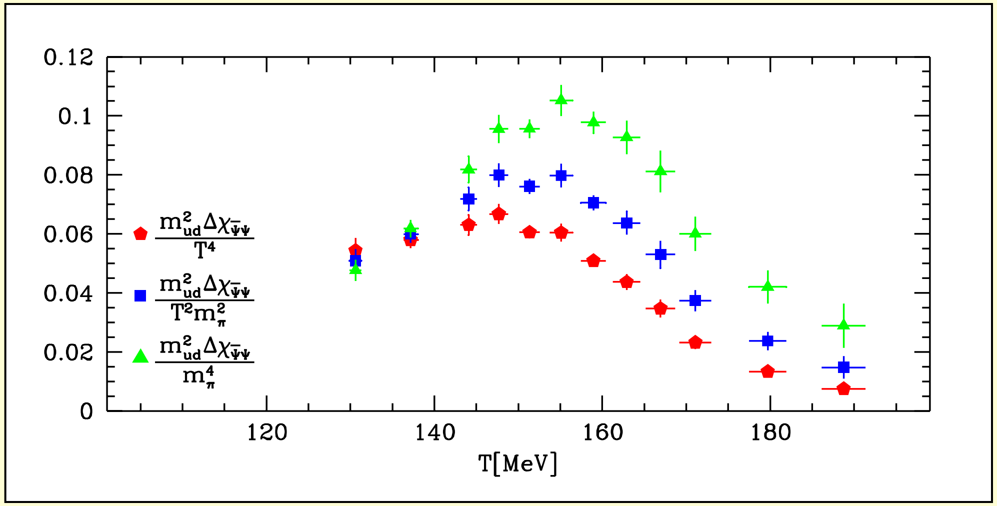
Chiral Symmetry Restoration in QCD

A.Bazavov et al (Hot QCD), 2012, 2014



$$\Delta_{l,s} = \frac{\langle \bar{q}q \rangle_T - (2m_q/m_s)\, \langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - (2m_q/m_s)\, \langle \bar{s}s \rangle_0}$$

$$\chi_S = -\frac{\partial}{\partial m_q} \langle \bar{q}q \rangle_T = \int_x \left[\langle \bar{q}q(x) \bar{q}q(0) \rangle_T - \langle \bar{q}q \rangle_T^2 \right]$$



Y.Aoki, S. Borsanyi et al (Budapest-Wuppertal) 2009, 2010

CROSSOVER Transition @ $T_c \approx$ 155 MeV for N_f=2+1 and physical masses

Exact restoration ➔ Phase transition for N_f=2 in chiral limit

R.D.Pisarski, F.Wilczek 1984

Chiral Patterns and Partners

- $U(1)_A$ **asymptotic restoration close to** T_c **would lead to** $O(4) \times U(1)_A$ **pattern instead of** $O(4) \sim SU(2)_V \times SU(2)_A$

  Gross, Pisarski, Yaffe 1981

  Shuryak 1994. Cohen 1996. Lee-Hatsuda 1996

- **Affects the transition order, critical end point, etc**

  Pisarski, Wilczek, 1984. Pelissetto, Vicari 2013. Mitter, Schaefer 2014. Esser, Grahl, Rischke 2015

- **Observed** $M_{\eta'}$ **reduction points to** $U(1)_A$ **restoration.**
  Increase of η' **production would affect dileptons&diphotons**

  Kapusta, Kharzeev, McLerran 1996. Csorgo, Vertesi, Sziklai 2010

- **Chiral pattern not fully settled in lattice** ⇨

Chiral Patterns and Partners

$O(4)$ and $U(1)_A$ partners for scalar/pseudoscalar nonets:

$$\begin{array}{ccc} \pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi_l & \overset{SU_A(2)}{\longleftrightarrow} & \sigma_l = \bar{\psi}_l\psi_l \\ \updownarrow U_A(1) & & \updownarrow U_A(1) \\ \delta^a = \bar{\psi}_l\tau^a\psi_l & \overset{SU_A(2)}{\longleftrightarrow} & \eta_l = i\bar{\psi}_l\gamma_5\psi_l \end{array}$$

$$\pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi, \quad \delta^a = \bar{\psi}_l\tau^a\psi_l \sim a_0(980)$$

$$\sigma_l = \bar{\psi}_l\psi_l, \ \sigma_s = \bar{s}s \to f_0(500), f_0(980) \ \textbf{(mixed)}$$

$$\eta_l = i\bar{\psi}_l\gamma_5\psi_l, \ \eta_s = i\bar{s}\gamma_5 s \to \eta, \eta' \ \textbf{(mixed)}$$

Chiral Patterns and Partners: lattice

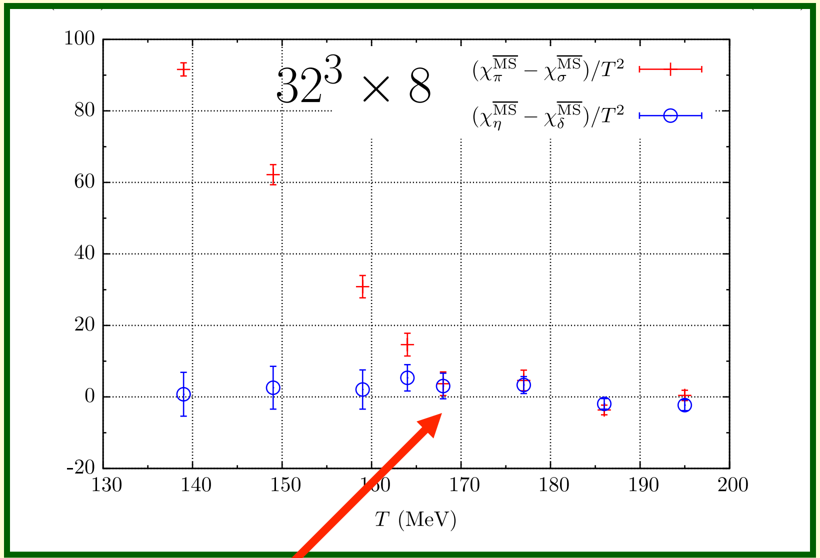
Buchoff et al (LLNL/RBC coll) PRD89 (2014)

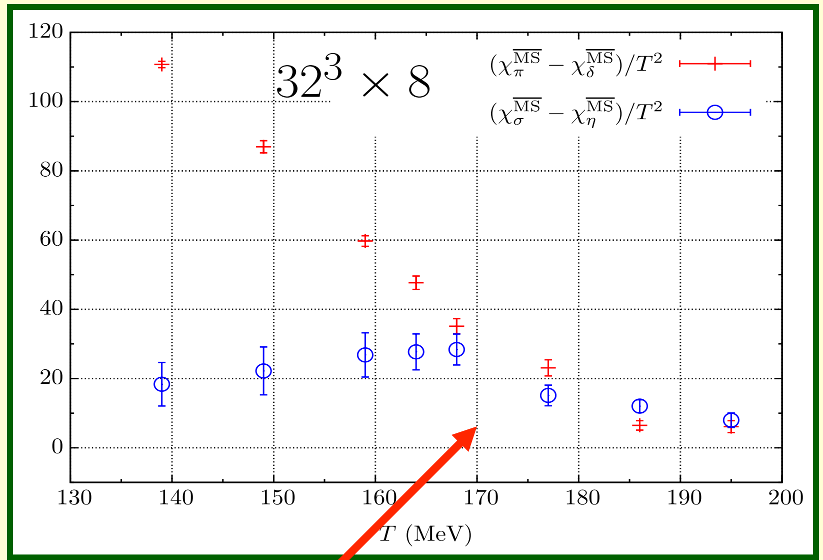
$N_f = 2+1$

$$\pi^a = i\bar{\psi}_l \gamma_5 \tau^a \psi_l \overset{SU_A(2)}{\longleftrightarrow} \sigma_l = \bar{\psi}_l \psi_l$$

$$\updownarrow U_A(1) \qquad\qquad \updownarrow U_A(1)$$

$$\delta^a = \bar{\psi}_l \tau^a \psi_l \overset{SU_A(2)}{\longleftrightarrow} \eta_l = i\bar{\psi}_l \gamma_5 \psi_l$$





$O(4)$ **OK** (with large uncertainties in $\chi_\eta - \chi_\delta$)

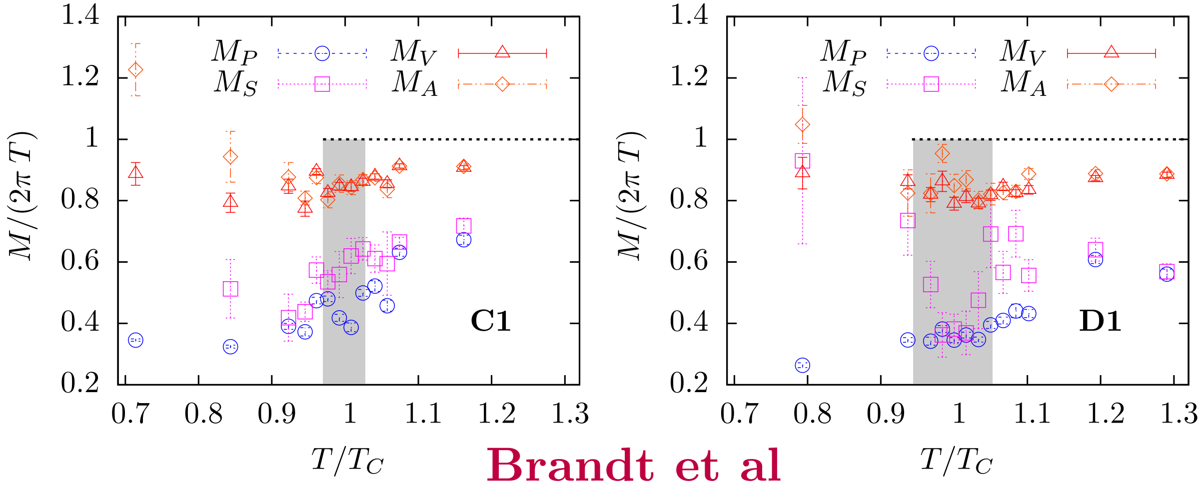
$U(1)_A$ **NOT restored at** T_c

Chiral Patterns and Partners: lattice

Brandt et al, JHEP 1612 (2016) $(N_f = 2, \hat{m} \neq 0)$

Aoki et al, PRD86 (2012),
Cossu et al, PRD87 (2013) $(N_f = 2, \hat{m} \to 0)$.

$$\pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi_l \overset{SU_A(2)}{\longleftrightarrow} \sigma_l = \bar{\psi}_l\psi_l$$
$$\updownarrow_{U_A(1)} \qquad \updownarrow_{U_A(1)}$$
$$\delta^a = \bar{\psi}_l\tau^a\psi_l \overset{SU_A(2)}{\longleftrightarrow} \eta_l = i\bar{\psi}_l\gamma_5\psi_l$$



Brandt et al

$U_A(1)$ **restored at** $T \gtrsim T_c \Longrightarrow O(4) \times U_A(1)$ **pattern**

Ward Identities: quark condensates vs P suscept.

- π **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) = -\hat{m}\chi_P^\pi(T)$ **(1)**
- K **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) + 2\langle \bar{s}s \rangle (T) = -(\hat{m}+m_s)\chi_P^K(T)$
- η, A **SECTOR** $\rightarrow$ η/η' **mixing &** $U_A(1)$ **anomaly enter:**

$$\chi_P^{\eta_l}(T) = -\frac{\langle \bar{q}q \rangle_l (T)}{\hat{m}} - \frac{4}{\hat{m}^2}\chi_{top}(T) \qquad (2)$$

$$\chi_P^{ss}(T) = -\frac{\langle \bar{s}s \rangle (T)}{m_s} - \frac{1}{m_s^2}\chi_{top}(T) \qquad (1,2)$$

$$\chi_P^{ls} = -\frac{2}{\hat{m}m_s}\chi_{top}(T)$$

$\frac{1}{m_s}$ **suppressed**

(1) Checked in lattice Buchoff et al PRD89 (2014)
(2) OK with V.Azcoiti, PRD94 (2016)

$$\chi_P^{ab} \equiv \int_T dx \langle P^a(x)P^b(0)\rangle, \quad \langle \bar{q}q \rangle_l = \langle \bar{u}u + \bar{d}d \rangle, \quad \hat{m} = m_u = m_d$$

$$A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$\chi_{top} \equiv -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x)A(0)\rangle$$

Chiral Patterns and Partners from WI

Crossed ls correlator nonzero due to 08 mixing. From WI:

$$\chi_P^{ls}(T) = -2\frac{\hat{m}}{m_s}\chi_{5,disc}(T) = -\frac{2}{\hat{m}m_s}\chi_{top}(T)$$

where $\chi_{5,disc} = \frac{1}{4}\left(\chi_P^{\pi} - \chi_P^{\eta_l}\right)$ **measures** $O(4) \times U(1)_A$ **restoration**

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$\Rightarrow$ $SU(2)_A$ **transforms (**η_s **invariant)** $P_{ls} \xrightarrow{(*)} \langle \delta\eta_s \rangle = 0$ **by parity**

$$\chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^{\delta} \Rightarrow \chi_P^{ls} \overset{O(4)}{\sim} 0 \Rightarrow \chi_{5,disc} \overset{O(4)}{\sim} 0, \ \chi_{top} \overset{O(4)}{\sim} 0$$

$(*)\ \eta_l \to i\bar{\psi}_l\gamma_5 e^{i\frac{\pi}{2}\gamma_5\tau^b}\psi_l = -\delta^b$

Chiral Patterns and Partners from WI

Crossed ls correlator nonzero due to 08 mixing. From WI:

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$$\chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^{\delta} \Rightarrow \chi_P^{ls} \overset{O(4)}{\sim} 0 \Rightarrow \chi_{5,disc} \overset{O(4)}{\sim} 0, \ \chi_{top} \overset{O(4)}{\sim} 0$$

$\Longrightarrow$ $O(4)\times U(1)_A$ **pattern at *exact* chiral restoration (for above partners)**

(hence consistent with Cossu, Aoki, Brandt et al $N_f = 2$ **)**

efficient $\eta_l - \delta$ **degeneration** $@T_c$

Chiral Patterns and Partners from WI

$I = 1/2$ **SECTOR** $(K-\kappa)$ **DEGENERATION**

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2m_s}{m_s^2-\hat{m}^2}\left[\langle\bar{q}q\rangle_l(T) - 2\frac{\hat{m}}{m_s}\langle\bar{s}s\rangle(T)\right]$$

$\Rightarrow$ **Phys.case: dictated by subtracted condensate, measurable in lattice through** $\Delta_{l,s}$

Chiral Patterns and Partners from WI

$I = 1/2$ SECTOR $(K-\kappa)$ DEGENERATION

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2m_s}{m_s^2 - \hat{m}^2}\left[\langle \bar{q}q \rangle_l (T) - 2\frac{\hat{m}}{m_s}\langle \bar{s}s \rangle (T)\right]$$

⇒ **Phys.case: dictated by subtracted condensate, measurable in lattice through $\Delta_{l,s}$**

⇒ $\chi_S^\kappa \overset{O(4)}{\sim} \chi_P^K$ **degeneration for exact chiral rest.** $(\hat{m}, \langle \bar{q}q \rangle_l \to 0^+)$

⇒ **Also degenerate by $U(1)_A$ rotation, hence both effects encoded in $\Delta_{l,s}$ in physical case**

WI and Screening Masses

Assuming soft T behavior for residues and M_{sc}/M_{pole} of correlators $K_{P,S}$:

$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow$ **measured in lattice**

WI and Screening Masses

Assuming soft T behavior for residues and M_{sc}/M_{pole} of correlators $K_{P,S}$:

$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \to$ **measured in lattice**

$$\frac{M_\pi^{sc}(T)}{M_\pi^{sc}(0)} \sim \left[\frac{\chi_P^\pi(0)}{\chi_P^\pi(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)}{\langle\bar{q}q\rangle_l(T)}\right]^{1/2}$$

$$\frac{M_K^{sc}(T)}{M_K^{sc}(0)} \sim \left[\frac{\chi_P^K(0)}{\chi_P^K(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0) + 2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T) + 2\langle\bar{s}s\rangle(T)}\right]^{1/2}$$

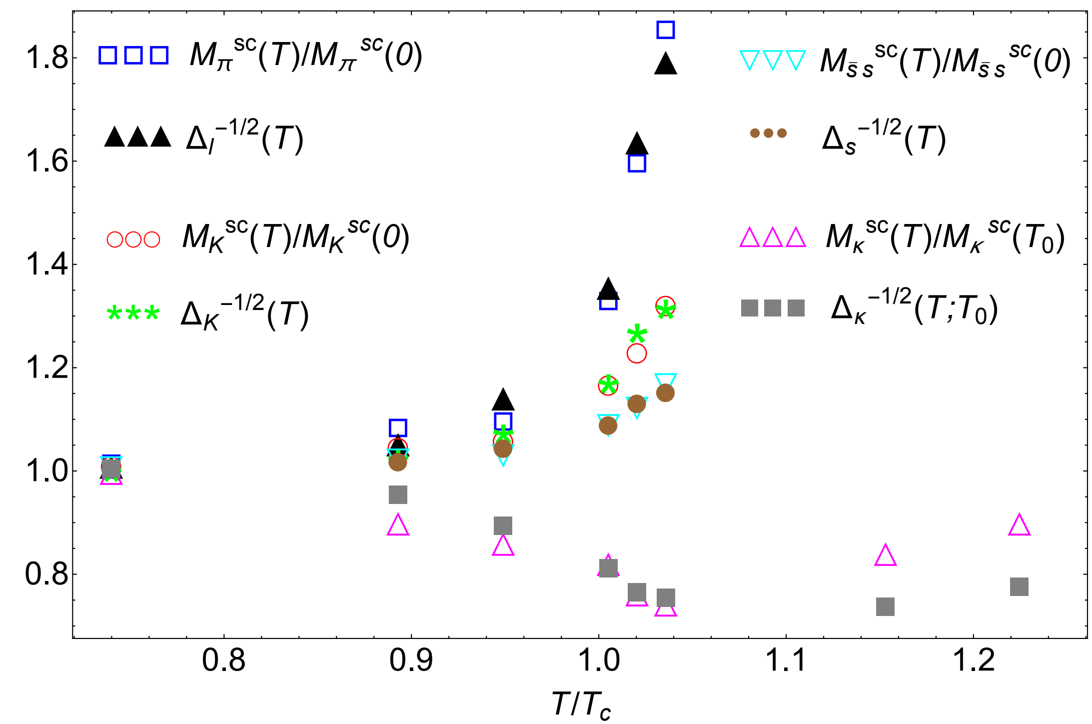
$$\frac{M_{\bar{s}s}^{sc}(T)}{M_{\bar{s}s}^{sc}(0)} \sim \left[\frac{\chi_P^{\bar{s}s}(0)}{\chi_P^{\bar{s}s}(T)}\right]^{1/2} \sim \left[\frac{\langle\bar{s}s\rangle(0)}{\langle\bar{s}s\rangle(T)}\right]^{1/2}$$

$$\frac{M_\kappa^{sc}(T)}{M_\kappa^{sc}(0)} \sim \left[\frac{\chi_S^\kappa(0)}{\chi_S^\kappa(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0) - 2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T) - 2\langle\bar{s}s\rangle(T)}\right]^{1/2}$$

Anomalous contrib. $\frac{\hat{m}}{m_s}$ **suppressed**

WI and Screening Masses

Same lattice setup for masses (Cheng et al EPJC'11) and condensates (PRD'08)



- $<5\%$ **deviations** below T_c from predicted WI scaling
- Δ_i **subtracted condensates** with two fit parameters to eliminate $T=0$ lattice divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$
- **Rapid** T_c **increase** due to $M_\pi^{sc} \sim \langle \bar{q}q \rangle_l^{-1/2}$. **Softer** $M_K^{sc} \sim (\langle \bar{q}q \rangle_l + 2\langle \bar{s}s \rangle)^{-1/2}$ (soft T-dep $\langle \bar{s}s \rangle$). **Even softer** $M_{\bar{s}s}^{sc} \sim \langle \bar{s}s \rangle^{-1/2}$ (no light contrib.)
- κ **minimum** from condensate diff. (last two points not fitted)

Low-energy hadron realization: effective theories

- Effective Theories needed below the transition to study consistently WI and partner degeneration within a hadron picture.

- U(3) ChPT model-independent framework for π, K, η, η' [(1)].

- Light meson scattering ($\pi\pi$,...) dominant interactions in the thermal bath.

  Unitarized scattering generates (thermal) resonances [(2)].

- HRG approach to include (free) heavier states as eff. interact.[(3)]

(1) Gasser, Leutwyler, Gerber, Kaiser, Herrera-Siklody et al, ...

(2) Cabrera, Dobado, Fernández-Fraile, AGN, Llanes-Estrada, Peláez, Ruiz de Elvira, Torres-Andrés

(3) Karsch, Tawfik, Redlich, Tawfik-Toublan, Huovinen, Petreczcky, Jankowski, Blaschke, Spalinski, ...

Low-energy hadron realization: effective theories

$\Rightarrow$ **WI verified in $U(3)$ ChPT* to NNLO in $\delta \sim 1/N_c \sim m_q \sim T^2$**

* **to account consistently for $U_A(1)$ anomaly and η'**

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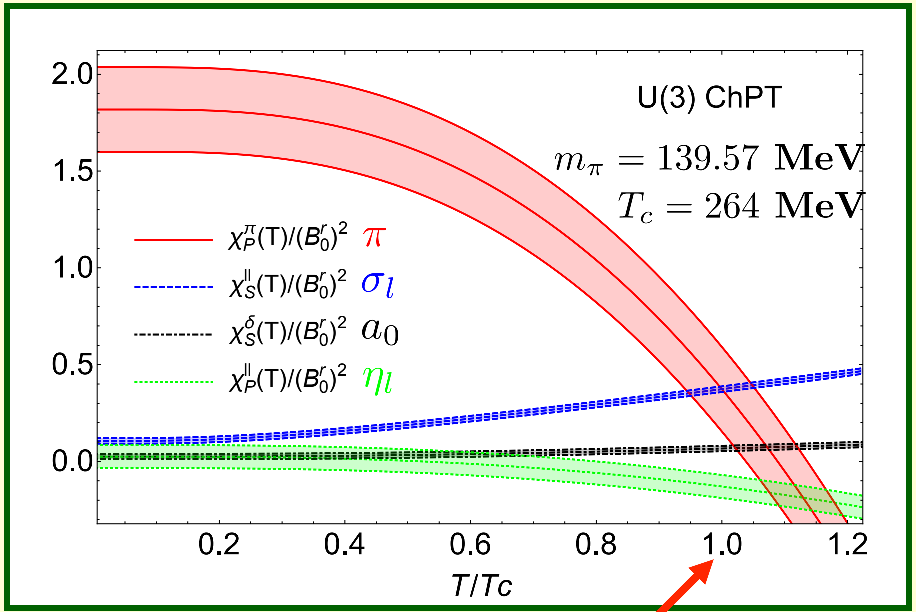

Differences within ChPT uncertainty in massive case.

Low-energy hadron realization: effective theories

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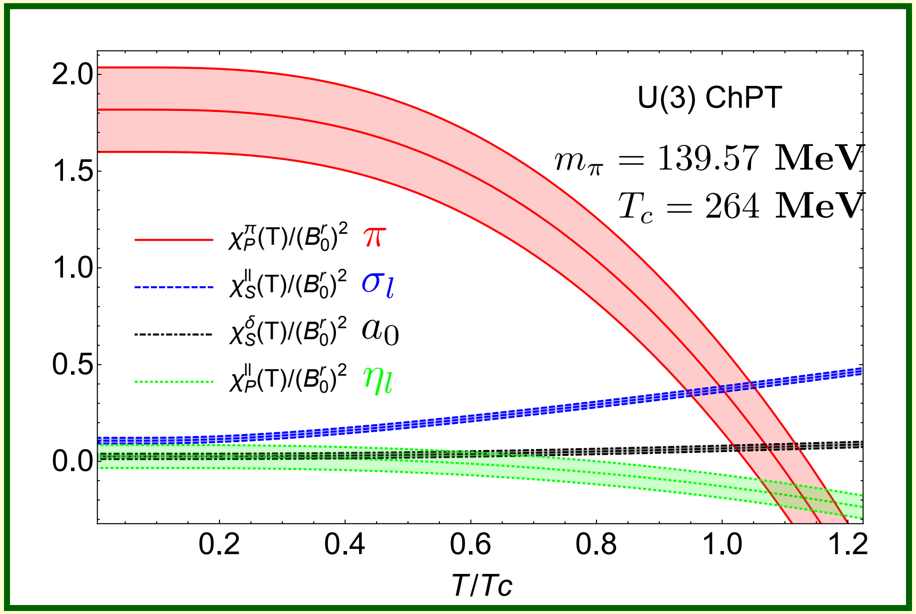
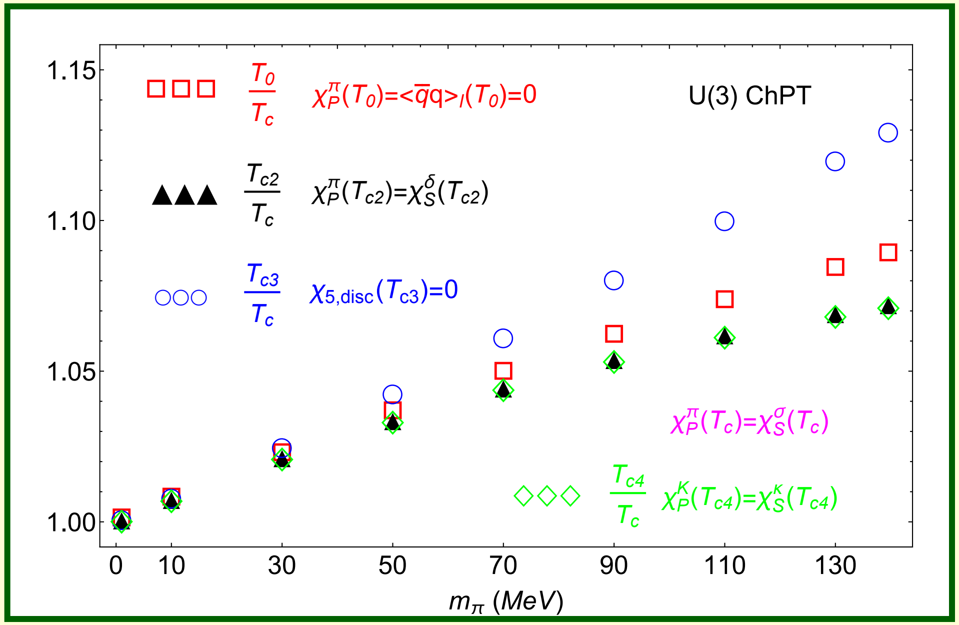
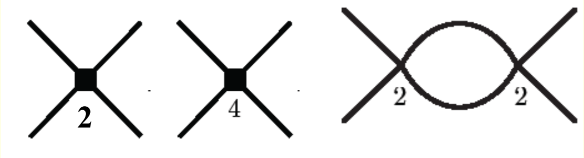




$\rightarrow O(4) \times U_A(1)$ in chiral limit

with $$\frac{\chi_{5,disc}(T)}{\chi_{5,disc}(0)} \sim \frac{\langle \bar{q}q\rangle_l(T)}{\langle \bar{q}q\rangle_l(0)}$$ **(holds reasonably also in $N_f = 2+1$ lattice)**

Unitarizing scattering: resonances



ChPT Partial waves $t^{IJ} = t_2^{IJ} + t_4^{IJ} + \dots$

Unitarity $\rightarrow$ **Im** $t(s) = \sigma(s)|t(s)|^2 \;\; (s \geq 4M^2) \Rightarrow$ **Im** $t^{-1} = -\sigma$

$$\sigma(s) = \sqrt{1 - \frac{4M^2}{s}} = \frac{p_{CM}}{\sqrt{s}}$$ **two-particle phase space**

$$t^U(s;T) = \frac{\left[t_2(s)\right]^2}{t_2(s) - t_4(s;T)}$$

(IAM)

Exactly proven for large NGB and chiral limits:
S.Cortés, AGN, J.Morales '16

FINITE TEMPERATURE:

$$t_4(s) \rightarrow t_4(s;T)$$

$$\sigma \rightarrow \sigma\left[1 + 2n_B(\sqrt{s}/2)\right] \equiv \sigma_T$$

Thermal phase Space.
Bose net enhancement $(1+n)^2 - n^2$

A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez, E.Tomás-Herruzo, '02 '05 '07

The σ/f0(500) and chiral symmetry restoration

⇒ **Assume χ_S thermal dependence dominated by $f_0(500)$ $I=J=0$ pole (II Riemann sheet) saturation:**

$$\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}$$

$$M_S^2(T) = \mathbf{Re}(s_{pole}(T)) \sim \mathbf{Re}\Sigma_{f_0}$$

See also A.Vioque-Rodríguez poster

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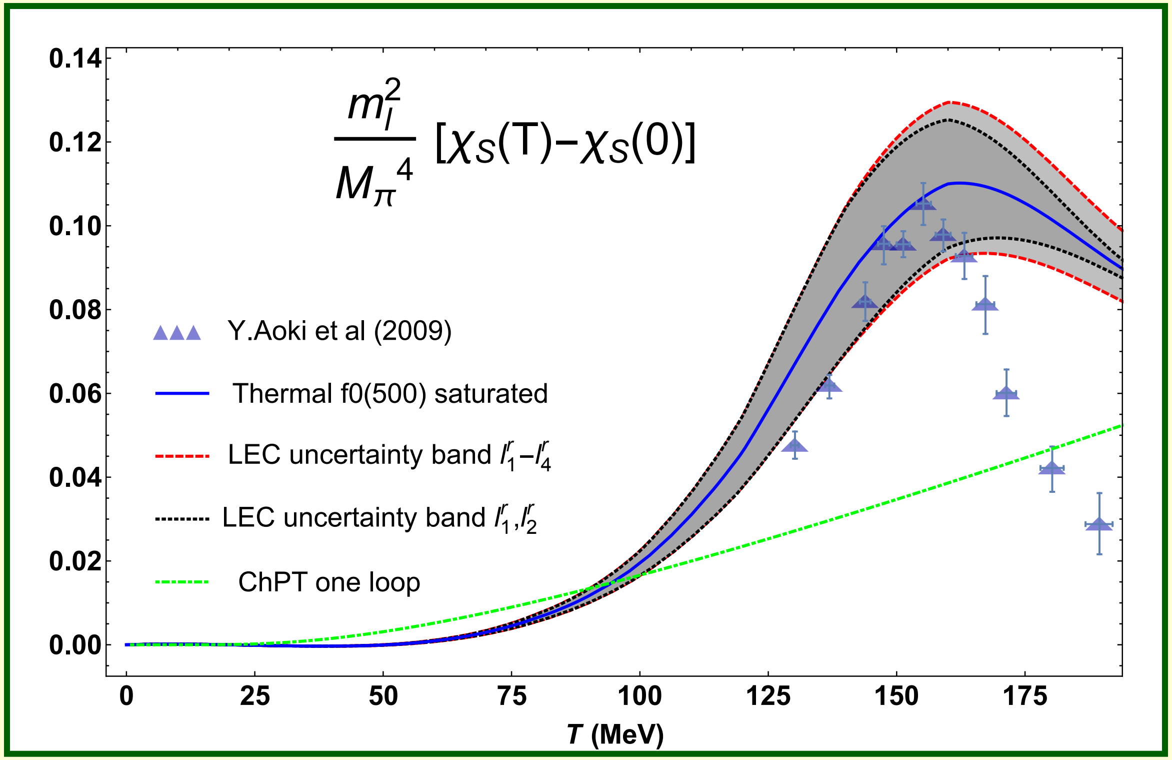
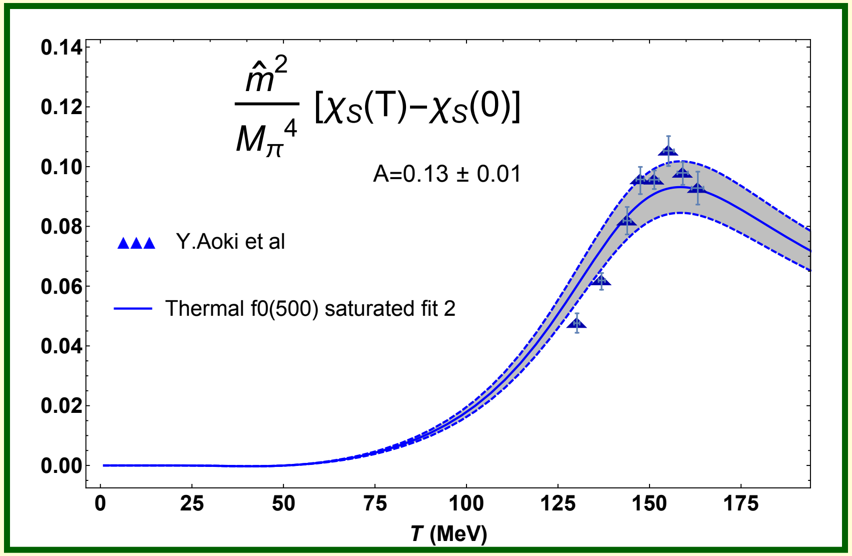


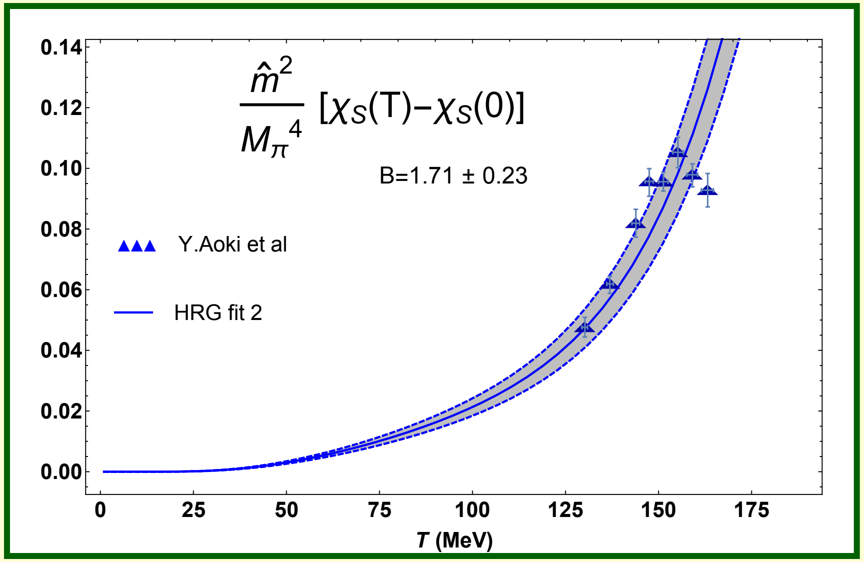
- **Consistent with lattice transition peak.**
- **LECs and uncertaintities from unitarized $T = 0$ fit in Hanhart, Peláez, Ríos PRL100 (2008)**

$$s_p = 446.5 - i220.4 \text{ MeV}$$

- **Consistent T_c reduction and χ_S growth near chiral limit**

Unitarized susceptibility fits vs HRG

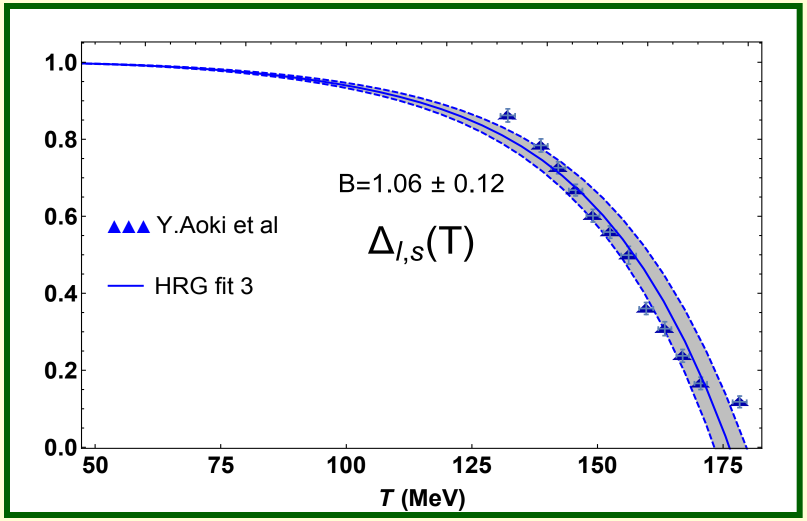




$$\chi_S^U(T) = A\frac{m_\pi^4}{4m_q^2}\frac{M_S^2(0)}{M_S^2(T)} \qquad (A_{ChPT} \simeq 0.14)$$

HRG Jankowski et al [1]
normalization par. B fitted.

Fit	A	B	χ^2/dof	T_{max}(MeV)
Thermal f_0 fit 1	0.13± 0.02	—	6.25	155
Thermal f_0 fit 2	0.13± 0.01	—	4.93	165
HRG fit 1	—	1.90± 0.02	1.33	155
HRG fit 2	—	1.71± 0.23	10.30	165
HRG fit 3	—	1.06± 0.12	3.77	155



- **Thermal f_0 approach better around T_c**
- **HRG fits of $\Delta_{l,s}$ and χ_S incompatible**

[1] **PRD87, 105018 (2013)**

CONCLUSIONS

- ★ **WI allow for study of chiral pattern and related partner degeneration. Benchmark for lattice&model analysis.**
- ★ **From WI&ChPT** $\rightarrow O(4)\times U(1)_A$ **for exact restoration of scalar/pseudoscalar nonet. OK** $N_f=2$ **lattice.**
- ★ **In physical** $N_f=2+1$ **case,** $\delta-\eta$ **distortion in lattice.** $\chi_{5,disc}\sim\langle\bar{q}q\rangle_l$ **scaling in ChPT.**
- ★ **WI** $\Rightarrow$ **scaling of meson screening masses consistent with lattice.**
- ★ **Thermal** $f_0(500)\Rightarrow$ **saturated** χ_S^U **in agreement with lattice data.**

BACKUP SLIDES

Ward Identities

Formally from QCD through A/V transformations:

$$\left\langle \frac{\delta \mathcal{O}_P(y)}{\delta \alpha_A^a(x)} \right\rangle = -\left\langle \mathcal{O}_P(y)\bar{\psi}(x)\left\{\frac{\lambda^a}{2},\mathcal{M}\right\}\gamma_5\psi(x)\right\rangle + i\,\frac{\delta_{a0}}{\sqrt{6}}\left\langle \mathcal{O}_P(y)A(x)\right\rangle$$

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y)\bar{\psi}(x)\left[\frac{\lambda^a}{2},\mathcal{M}\right]\psi(x)\right\rangle$$

$$\lambda^0 = \sqrt{2/3}\,\mathbb{1},\quad A(x) = \frac{3\alpha_s}{4\pi}Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Ward Identities

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$$\left\langle \frac{\delta \mathcal{O}_P(y)}{\delta \alpha_A^a(x)} \right\rangle = -\left\langle \mathcal{O}_P(y) \bar{\psi}(x) \left\{ \frac{\lambda^a}{2}, \mathcal{M} \right\} \gamma_5 \psi(x) \right\rangle + i \frac{\delta_{a0}}{\sqrt{6}} \langle \mathcal{O}_P(y) A(x) \rangle$$

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y) \bar{\psi}(x) \left[\frac{\lambda^a}{2}, \mathcal{M} \right] \psi(x) \right\rangle$$

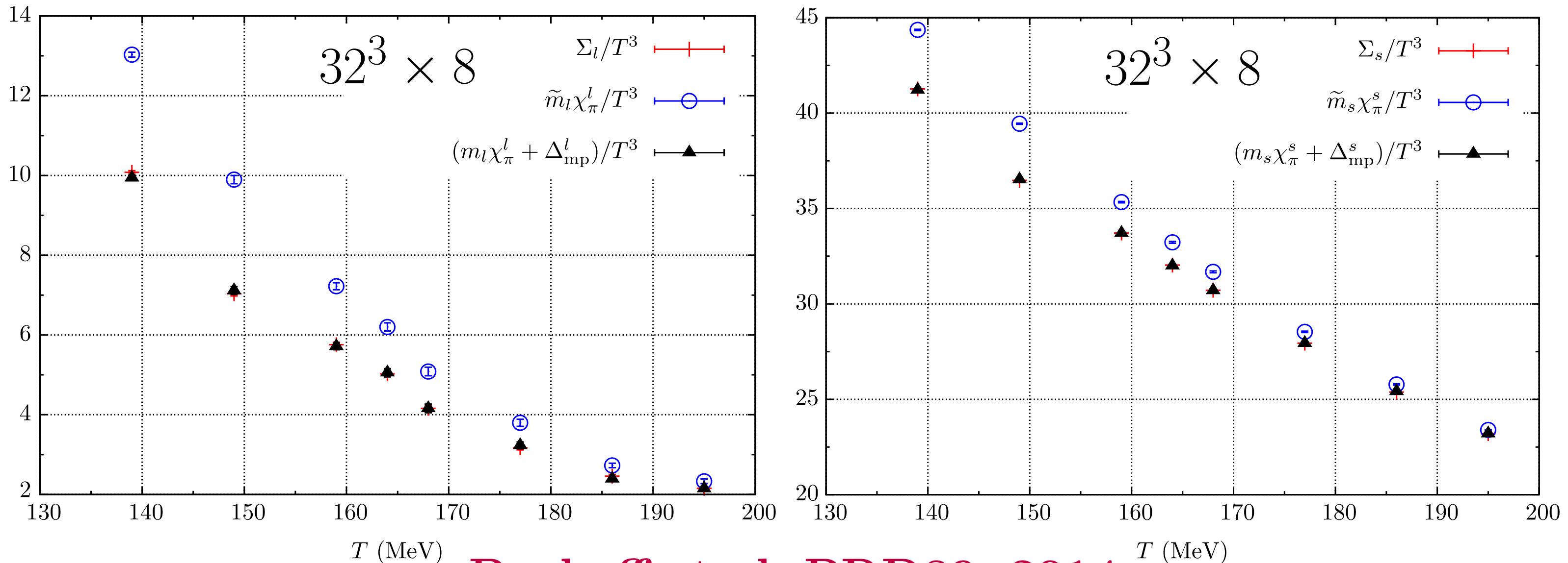
$\lambda^0 = \sqrt{2/3}\, \mathbb{1}$, $A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu} \tilde{G}^{\mu\nu}$

$\mathcal{O}_P^b = i\bar{\psi}\gamma_5\lambda^b\psi \equiv P^b \rightarrow$ **1p vs 2p fns** $\rightarrow \langle \bar{q}q \rangle$ **vs** χ_P

$\mathcal{O}_P^{bc} = P^b S^c \rightarrow$ **2p vs 3p** $\rightarrow$ **ch.partners vs meson vertices (e.g.** $\chi^\sigma - \chi^\pi \sim \sigma\pi\pi, \ldots$**)**

$\mathcal{O}_S^b = \bar{\psi}\lambda^b\psi \equiv S^b \rightarrow \langle \bar{q}q \rangle$ **vs** χ_S **for** κ **sector** $b = 4, \ldots, 7$

Check of WI in lattice



Buchoff et al, PRD89, 2014

- ⋆ **Both π and $\bar{s}s$ channel need compensating lattice current to reduce finite-size effects**
- ⋆ **Small deviations in $\bar{s}s$ channel compatible with anomaly suppression**
- ⋆ **No results for K channel (so far) which would test $\langle\bar{q}q\rangle_l + 2\langle\bar{s}s\rangle$ combination**

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m}\int_T dx\,\langle \mathcal{T}\sigma_l(y)\pi(x)\pi(0)\rangle$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m}\int_T dx\,\langle \mathcal{T}\delta(y)\pi(x)\eta_l(0)\rangle$$

$$P_{ls}(y) = \frac{1}{3}\hat{m}\int_T dx\,\langle \mathcal{T}\eta_s(y)\pi(x)\delta(0)\rangle$$

$$S_{ls}(y) = -\frac{1}{3}\hat{m}\int_T dx\,\langle \mathcal{T}\sigma_s(y)\pi(x)\pi(0)\rangle$$

$$d^{abc}\left[P_{KK}(y) - S_{\kappa\kappa}(y)\right] = \hat{m}\int_T dx\,\left\langle \mathcal{T}K^b(y)\kappa^c(x)\pi^a(0)\right\rangle$$

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$\sigma\pi\pi$ **vertex**

$\rightarrow \pi\pi$ **scattering** $I = J = 0$

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \sigma_l(y) \pi(x) \pi(0) \rangle$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \delta(y) \pi(x) \eta_l(0) \rangle$$

$$P_{ls}(y) = \frac{1}{3} \hat{m} \int_T dx \, \langle \mathcal{T} \eta_s(y) \pi(x) \delta(0) \rangle$$

$$S_{ls}(y) = -\frac{1}{3} \hat{m} \int_T dx \, \langle \mathcal{T} \sigma_s(y) \pi(x) \pi(0) \rangle$$

$$d^{abc} \left[P_{KK}(y) - S_{\kappa\kappa}(y) \right] = \hat{m} \int_T dx \, \left\langle \mathcal{T} K^b(y) \kappa^c(x) \pi^a(0) \right\rangle$$

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$U(1)_A$ **partners**

$$
\begin{aligned}
P_{\pi\pi}(y) - S_{\delta\delta}(y) &= \int_T dx \, \langle \mathcal{T} \pi(y) \delta(0) \tilde{\eta}(x) \rangle \\
P_{ll}(y) - S_{ll}(y) &= \int_T dx \, \langle \mathcal{T} \eta_l(y) \sigma_l(0) \tilde{\eta}(x) \rangle \\
P_{ls}(y) - S_{ls}(y) &= \int_T dx \, \langle \mathcal{T} \eta_l(y) \sigma_s(0) \tilde{\eta}(x) \rangle \\
P_{ss}(y) - S_{ss}(y) &= \int_T dx \, \langle \mathcal{T} \eta_s(y) \sigma_s(0) \tilde{\eta}(x) \rangle \\
P_{KK}(y) - S_{\kappa\kappa}(y) &= \int_T dx \, \langle \mathcal{T} K(y) \kappa(0) \tilde{\eta}(x) \rangle
\end{aligned}
$$

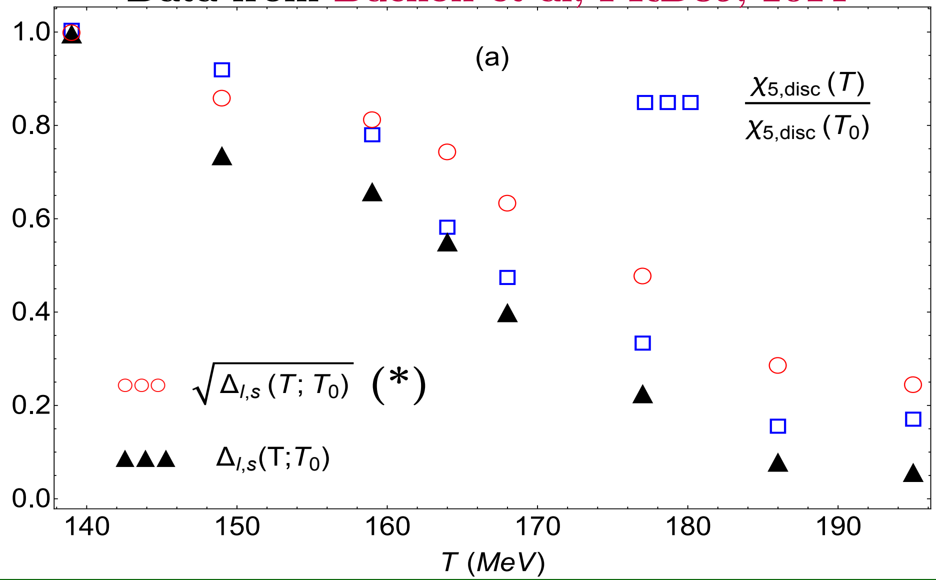
$\tilde{\eta}(x) = \hat{m}\eta_l(x) + m_s \eta_s(x) + \frac{1}{2} A(x)$ **three sources of** $U(1)_A$ **breaking**

Chiral Patterns and Partners from WI

Physical case ($N_f = 2+1$, $\hat{m} \neq 0$):

- **m_s distortion.**
- **Worse $\chi_P^{\eta_l} - \chi_S^{\delta}$ degeneration in lattice.**
- **$\chi_{5,disc}$ would scale dictated by quark condensate:** **(1)**

Data from Buchoff et al, PRD89, 2014



$\Delta_{l,s}(T;T_o)$ relative to $T_0 = 139$ MeV

$32^3 \times 8$ lattice size

$\hat{m}/m_s = 0.088$

(1) $\chi_{top} \sim \hat{m}\langle\bar{q}q\rangle_l$ in ch. limit
V.Azcoiti, PRD94 (2016)

(*) **from χ_P^{ls} WI and normalization $\pi \sim \sqrt{-\langle\bar{q}q\rangle_l G_\pi^{-1}(p^2=0)}$**
compatible with $\chi_P^{\pi} = -\langle\bar{q}q\rangle_l/\hat{m}$

WI and Screening Masses

Subtracted Condensates have the right critical behavior in lattice, avoiding $T=0$ finite-size divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$:

$$\Delta_l(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + \langle \bar{q}q \rangle_l^{ref}}{\langle \bar{q}q \rangle_l^{ref}}$$

$$\Delta_K(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}$$

$$\Delta_s(T) = \frac{2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{s}s \rangle^{ref}}{\langle \bar{s}s \rangle^{ref}}$$

$$\Delta_\kappa(T;T_0) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l (T_0) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T_0) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}$$

$$\begin{aligned} r_1^3 \langle \bar{q}q \rangle_l^{ref} &= 0.750 \\ r_1^3 \langle \bar{s}s \rangle^{ref} &= 1.061 \\ r_1 &\simeq 0.31 \text{ fm} \end{aligned}$$

Connected/Disconnected susceptibilities

$$\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis} \quad (m_u = m_d)$$

On the other hand,

$$\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^{\pi}(T) - \chi_S^{\sigma}(T)\right] + \frac{1}{4}\left[\chi_S^{\delta}(T) - \chi_P^{\eta_l}(T)\right]$$

$\Rightarrow$ **Is the vanishing of $\chi_{5,disc}$ in conflict with χ_S^{dis} peaking at the chiral transition?**

Connected/Disconnected susceptibilities

$$\chi_S^{ll} \equiv \chi_S^\sigma = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^\delta + 4\chi_S^{dis} \quad (m_u = m_d)$$

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From ChPT in the chiral limit $M_\pi \to 0^+$ (IR), $\Rightarrow T_{c3} = T_c + \mathcal{O}\left(M_\pi^2\right)$

$$\Rightarrow \chi_{5,disc}(T_c) \sim \mathcal{O}\left(\frac{T_c}{M_\pi}\right) \sim \chi_S^{dis}(T_c)$$

"peak" with same coeff.

$$\chi_{5,disc}(T_c) = \tilde{\chi}_S^{dis}(T_c) + \frac{1}{4}\left[\chi_P^\pi(T_c) - \chi_S^\sigma(T_c)\right] + \underbrace{\frac{1}{4}\left[\chi_S^\delta(T_c) - \chi_P^{\eta_l}(T_c)\right]}_{IR\ regular}$$

Connected/Disconnected susceptibilities

$$\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis} \quad (m_u = m_d)$$

On the other hand,

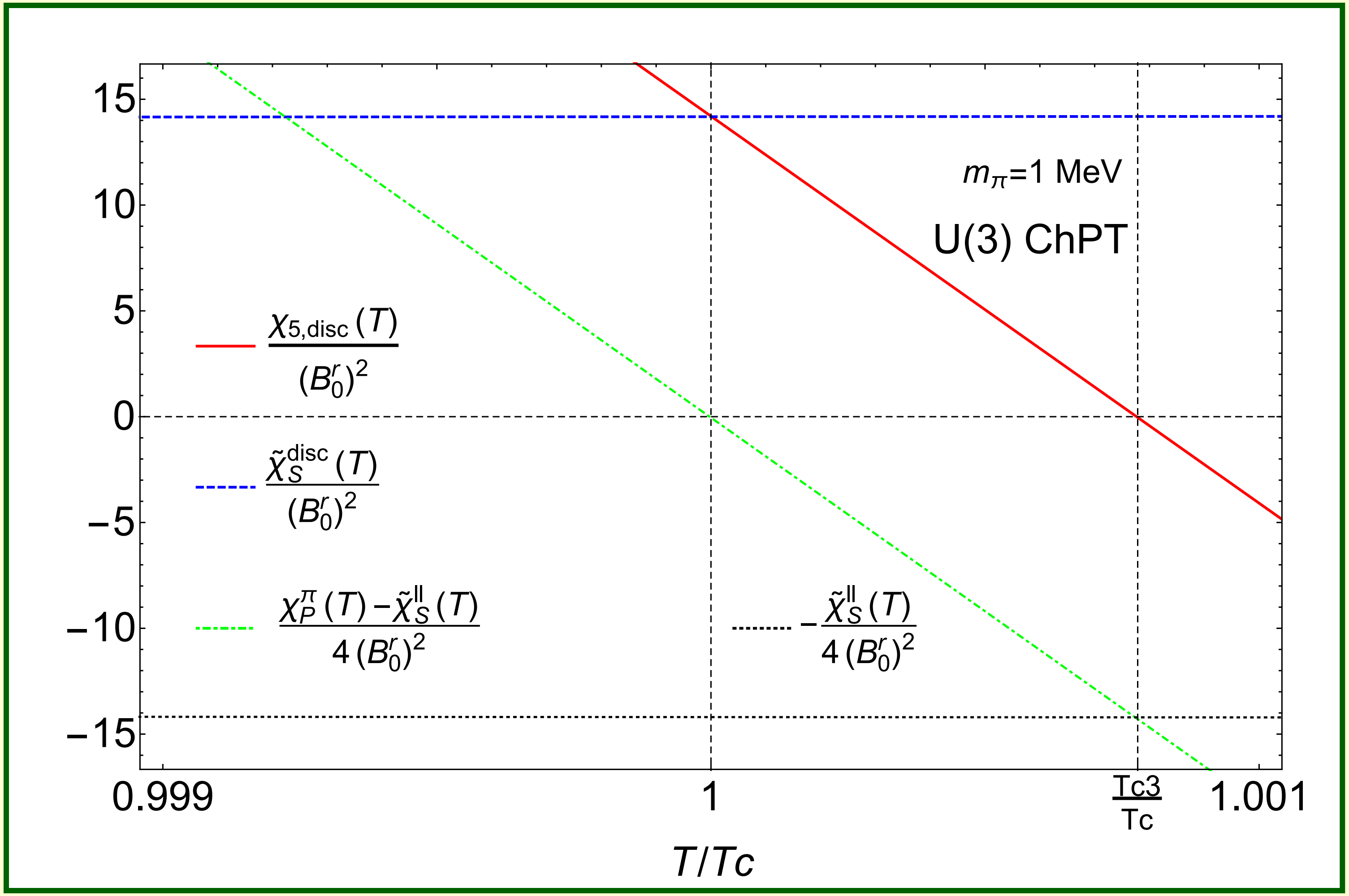
$$\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^{\pi}(T) - \chi_S^{\sigma}(T)\right] + \frac{1}{4}\left[\chi_S^{\delta}(T) - \chi_P^{\eta_l}(T)\right]$$

From ChPT in the chiral limit $M_\pi \to 0^+$ (IR), $\Rightarrow T_{c3} = T_c + \mathcal{O}\left(M_\pi^2\right)$

$$\Rightarrow \chi_S^{disc}(T_{c3}) \sim \mathcal{O}\left(\frac{T_c}{M_\pi}\right) \sim \frac{1}{4}\chi_S^{\sigma}(T_{c3})$$ **"peak" with same coff.**

$$\chi_{5,disc}(T_{c3}) = 0 = \tilde{\chi}_S^{dis}(T_{c3}) + \frac{1}{4}\left[\chi_P^{\pi}(T_{c3}) - \chi_S^{\sigma}(T_{c3})\right] + \frac{1}{4}\left[\underbrace{\chi_S^{\delta}(T_{c3})}_{IR\ regular} - \chi_P^{\eta_l}(T_{c3})\right]$$

Connected/Disconnected susceptibilities



Connected/Disconnected susceptibilities

$$\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis} \quad (m_u = m_d)$$

- **In general, only the total $\chi_S^{\sigma} \sim \frac{\partial}{\partial \hat{m}} \langle \bar{q}q \rangle_l$ expected to peak**

A. V. Smilga and J. J. M. Verbaarschot, PRD54 1996

$\Rightarrow$ χ_S^{δ} **could peak at $U(1)_A$ restoration**

Actually χ_S^{δ} grows for $T < T_c$ and should vanish asymptotically if $\chi_S^{\delta} \sim \chi_P^{\pi} = -\langle \bar{q}q \rangle_l / \hat{m} \to 0$

From Bazavov et al, PRD85, 2012

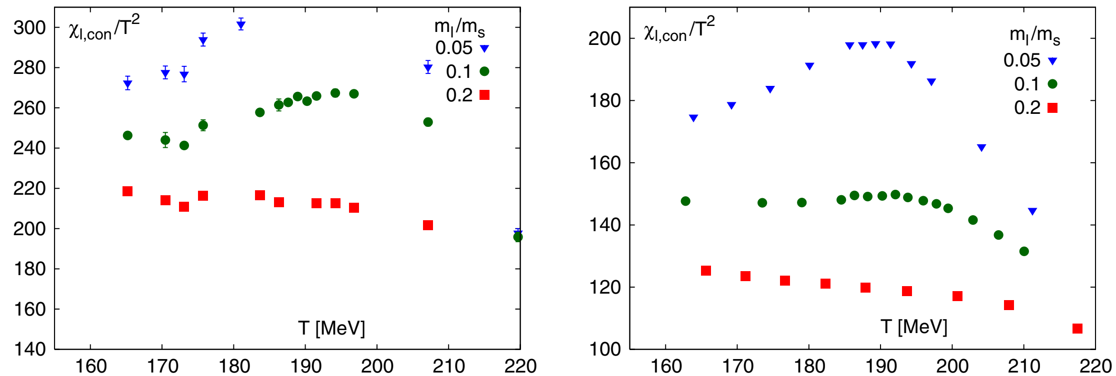
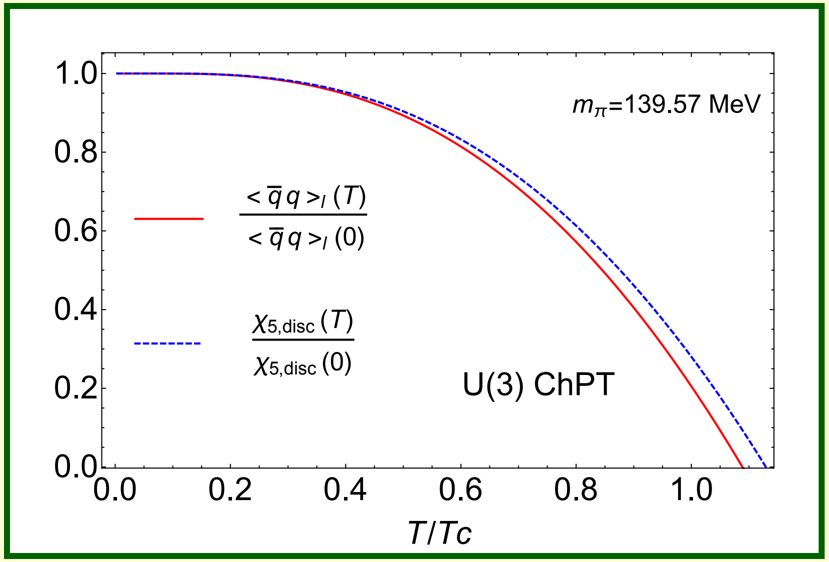
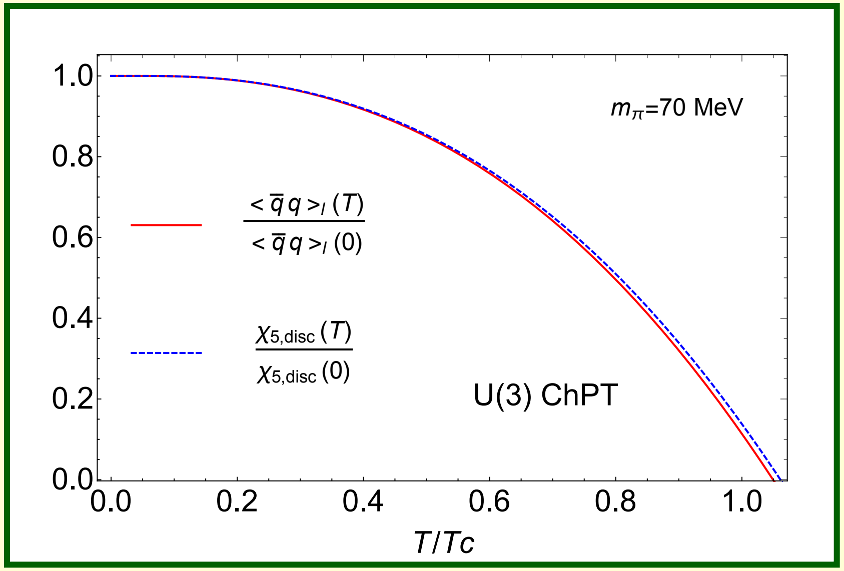
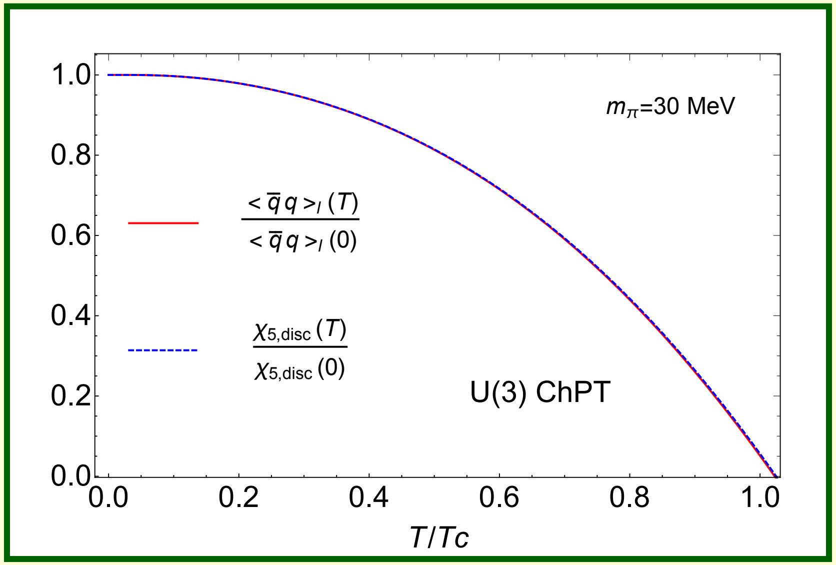


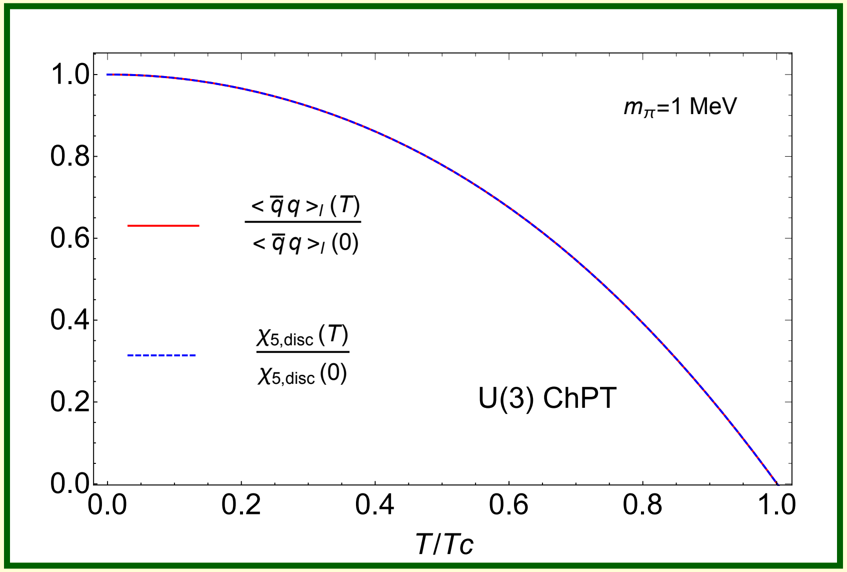
FIG. 12 (color online). The connected part of the chiral susceptibility for the p4 (left) and asqtad (right) actions for different quark masses on $N_\tau = 8$ lattices.

$\chi_{5,disc}$ vs $\langle \bar{q}q \rangle_l$ scaling in $U(3)$ ChPT









Large-N_{GB} NLSM at finite temperature (chiral limit)

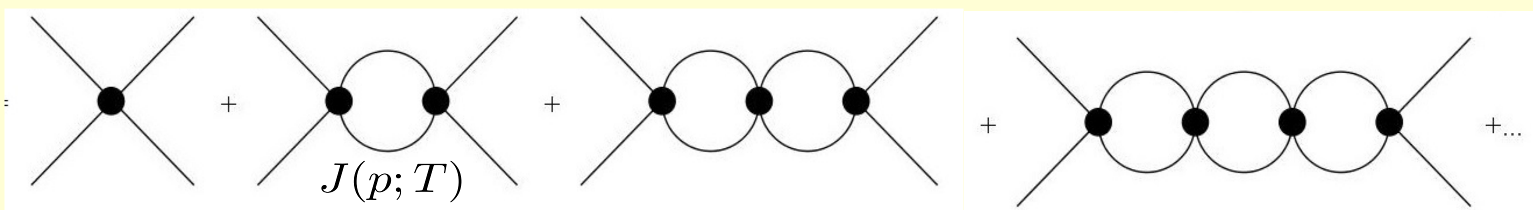
S.Cortés, AGN, J.Morales, PRD93 (2016) 036001

- $S^N = \dfrac{O(N+1)}{O(N)}$ **formulation:**

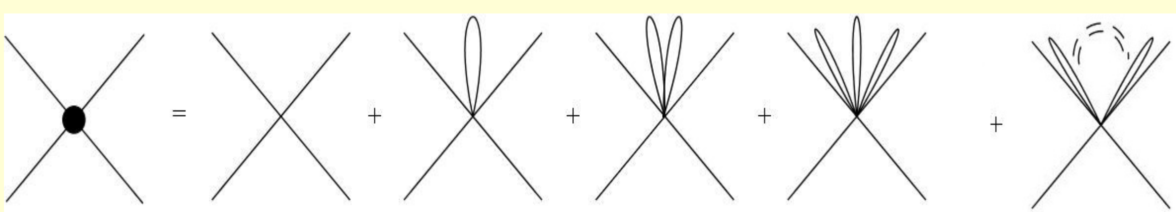
$$\mathcal{L}_{NLSM} = \frac{1}{2} g_{ab}(\pi) \partial_\mu \pi^a \partial^\mu \pi^b; \qquad g_{ab}(\pi) = \delta_{ab} + \frac{1}{NF^2} \frac{\pi_a \pi_b}{1 - \pi^2/NF^2}$$

- **Leading order scattering at finite T:**

$A(p;T) =$



$$= \boxed{\frac{s}{NF^2} \frac{f(T)}{1 - \frac{s}{2F^2} f(T) J(p;T)}} \qquad f(T) = \frac{1}{1 - \frac{T^2}{12F^2}}$$

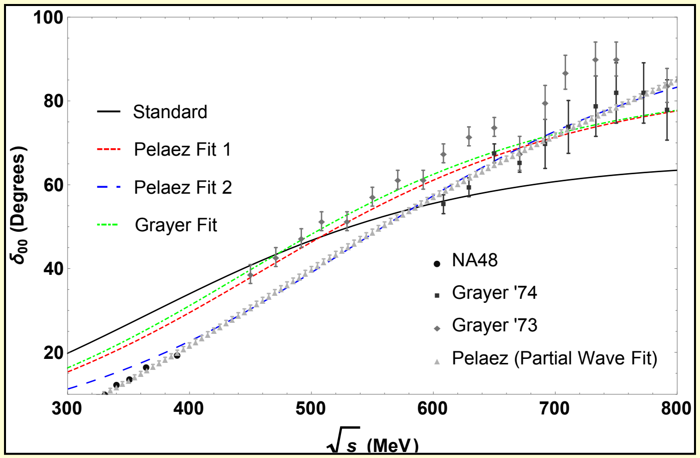


$$= \frac{s}{NF^2} f(T)$$

Large-N_{GB} NLSM at finite temperature (chiral limit)

S.Cortés, AGN, J.Morales, PRD93 (2016) 036001

- **Thermal Unitarity exact:** $\text{Im}\, t_{IJ}(s;T) = \sigma_T |t_{IJ}(s;T)|^2$
- **Renormalizable** with $T=0$ scheme $\rightarrow$ two free parameters F, μ
- $I=J=0$ **phase shift and** $f_0(500)$ **thermal pole** consistent with data and 2nd order chiral symmetry restoration (chiral limit)



Parameter set	T_c (MeV)
Grayer	92.33
Peláez 1	96.00
Peláez 2	129.07
IAM	118.23