Thermal effective potential for the Polyakov loop to higher loop order

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<Work in progress>

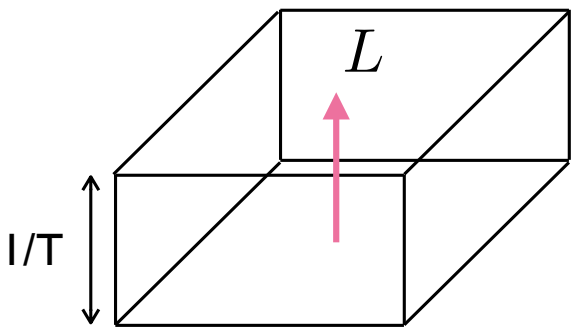
Outline

1. Introduction: the effective potential for the Polyakov loop, L

$$-V_{\rm eff}(L) = T^4 \left[c_0(L) + c_2(L) g^2 + c_3(L) g^3 + c_4(L) g^4 + \ldots \right]$$

2. Higher loop order

- Two loop
- Ring
- Three loop

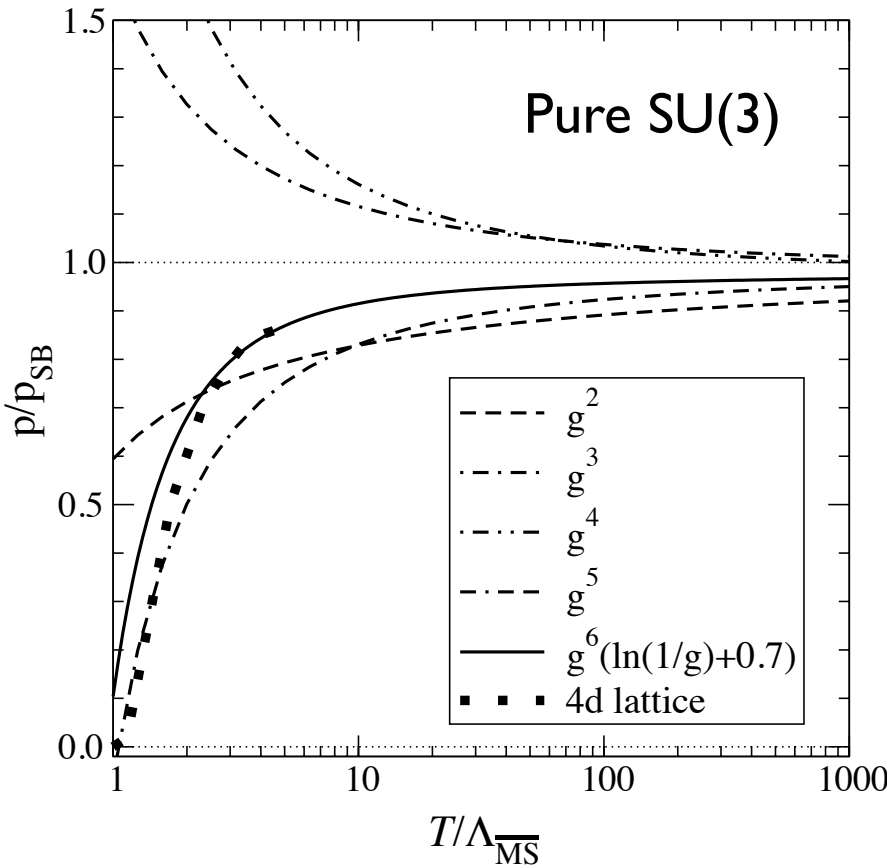


3. Summary

Perturbation theory

- Perturbative expansion at high T

$$\text{pressure} = \frac{T}{V} \ln Z = T^4 \left[c_0 + c_2 g^2 + c_3 g^3 + c_4 g^4 + c_5 g^5 + c_6 g^6 \right]$$



c_0: Stefan-Boltzman pressure

c_2: <E.Shuryak, 1978>

c_3: <J. Kapusta, 1979>

c_4: <T. Toimela, 1983>;
<P. Arnold and C. Zhai, 1994>

c_5: <C. Zhai and B. Kastening, 1995>
<E. Braaten and A. Nieto, 1996>

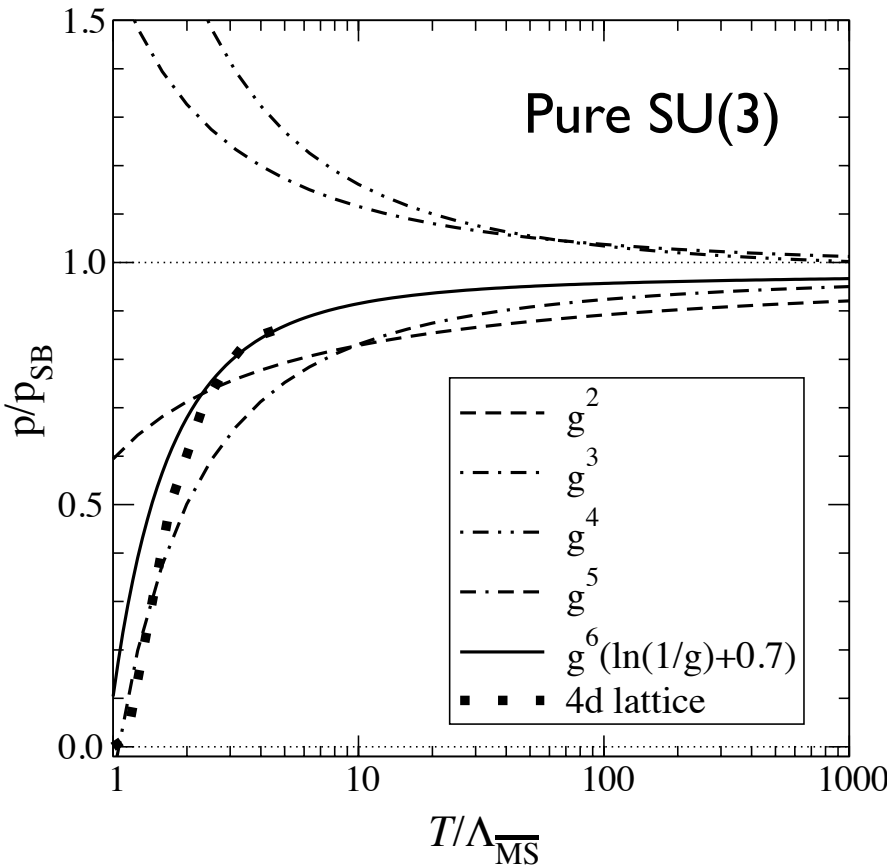
c_6: <K. Kajantie, M. Laine, K. Rummukainen, and Y. Schroder, 2001>

Perturbation theory

- Perturbative expansion at high T

Our Goal

$$\text{pressure} = T^4 \left[c_0(L) + c_2(L) g^2 + c_3(L) g^3 + c_4(L) g^4 \ldots \right]$$



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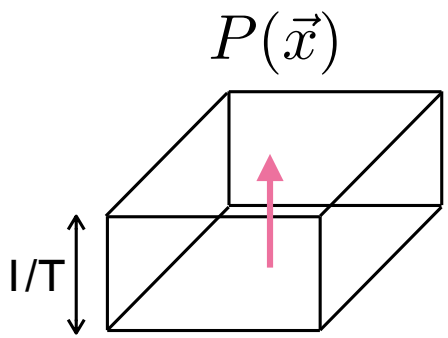
Effective potential

- Polyakov loop, L

$$P(\vec{x}) = \mathcal{P} e^{i \int_0^{1/T} dx_4 A_4(x)}$$



$$L \equiv \frac{1}{N} \mathrm{tr} P \begin{cases} = & 0 \quad \text{Confined} \\ \neq & 0 \quad \text{Deconfined} \end{cases}$$

m-th winding: $$L_m \equiv \frac{1}{N} \mathrm{tr} P^m$$

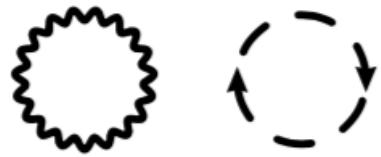
- Background field gauge

$$Z = \int [dA_\mu] \exp\left[-S_{\mathrm{YM}}(A_i, A_4 + \bar{A}_4)\right] = \exp\left[-\beta \mathcal{V} V_{\mathrm{eff}}(\bar{A}_4)\right]$$

Holonomy: $$P = e^{ig\bar{A}_4/T} = \mathrm{diag}\left(e^{i\theta_1}, e^{i\theta_2}, \ldots e^{i\theta_N}\right)$$

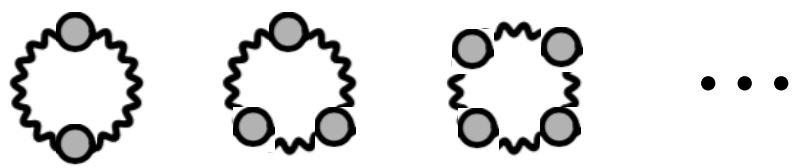
Loop expansion

1 loop:



2 loop:

Ring:



3 loop:

Same as in pQCD, but with the holonomy: $p_4 = 2\pi n T \longrightarrow p_4^a = (2\pi n + \theta^a)\, T$

where $\theta^a \equiv \theta^{ij} = \theta^i - \theta^j$

Two loop (g^2)

- Two-loop free energy in R-xi gauge <K. Enqvist and K. Kajantie, 1990>

$$\frac{V_{\rm pert}}{T^4} = -\frac{1}{\pi^2}\sum_{m\neq 0}\frac{L_m L_{-m}}{m^4} + \frac{Ng^2}{(2\pi)^4}\left[\sum_{m_1,m_2\neq 0}\frac{L_{m_1+m_2}L_{-m_1}L_{-m_2}}{m_1^2 m_2^2} + (1-\xi)\,F(L)\right]$$

Gauge dependent? A puzzle at that time.

- Renormalization of the Polyakov loop <V.M. Belyaev, 1990>

$$L_m^{(\rm ren)} = L_m + g^2 L_m^{(2)} + \ldots$$

Need to express the free energy in terms of $L_m^{(\rm ren)}$, not of L_m.

- Gauge-independent effective potential at two loop

Use

$$L_m = L_m^{(\mathrm{ren})} - g^2 L_m^{(2)}(L_m) + \mathcal{O}(g^4) = L_m^{(\mathrm{ren})} - g^2 L_m^{(2)}(L_m^{(\mathrm{ren})}) + \mathcal{O}(g^4)$$

into V_{pert}:

$$\begin{aligned}
\frac{V_{\mathrm{pert}}}{T^4} &= -\frac{1}{\pi^2}\sum_{m\neq 0}\frac{L_m^{(\mathrm{ren})}L_{-m}^{(\mathrm{ren})}}{m^4} \\
&\quad + \frac{Ng^2}{(2\pi)^4}\sum_{m_1,m_2\neq 0}\left(\frac{1}{m_1^2 m_2^2} - \frac{4}{m_1 m_2^3}\right) L_{m_1+m_2}^{(\mathrm{ren})} L_{-m_1}^{(\mathrm{ren})} L_{-m_2}^{(\mathrm{ren})} \\
&= \left[-\frac{1}{\pi^2} + \frac{5Ng^2}{(2\pi)^4}\right]\sum_{m\neq 0}\frac{L_m^{(\mathrm{ren})}L_{-m}^{(\mathrm{ren})}}{m^4}
\end{aligned}$$

Double-trace

<C. Korthals-Altes, 1994>

Accident or Symmetry?!

Higher loop

- Propagator with nontrivial holonomy

$$\frac{1}{(2\pi nT + \theta^a T)^2 + p^2} \qquad \text{where} \qquad \theta^a = \begin{pmatrix} 0 & \theta^{1,2} & \dots & \theta^{1,N} \\ \theta^{2,1} & 0 & \dots & \theta^{2,N} \\ \dots & \dots & \dots & \dots \\ \theta^{N,1} & \theta^{N,2} & \dots & 0 \end{pmatrix}$$

- There are N^2-N charged (off-diagonal) gluons and N-1 neutral (diagonal) gluons.

- No need to resum if $\theta_a = \theta_i - \theta_j \sim 1$

- Pressure at large N is in the series of g^2

$$\text{pressure} = T^4 \left[c_0(L) + c_2(L) g^2 + c_4(L) g^4 + \dots \right]$$

Does double-trace structure persist at higher loops? Relevant for GWW.

<H. N., R. Pisarski, and V. Skokov, 2017>

Resummation (g^3)

- Neutral gluons

Free energy: $$g^3 V_{\rm eff}^{(3)} = -\frac{1}{12\pi} T \sum_{a=1}^{N-1} m_a^3$$

Renormalized Loop: $$g^3 L_n^{(3)} = g^2 \frac{n^2}{8\pi N} \sum_a \mathrm{tr}\left[e^{ing\beta \bar{A}_4} T^a T^a\right] \left|\frac{m_a}{T}\right|$$

where m_a^2 is the eigenvalue of $\Pi_{44}^{a\bar{a}} = 4g^2 T^2 f_{abc} f_{\bar{b}\bar{c}\bar{a}} B_2(\frac{\theta_b}{2\pi})$

<Work in progress>

- Reduction

Using the momentum-color conservation and the symmetry of diagrams,

we can reduce them down to three sum-integrals at large N:

$$
\begin{aligned}
I_1 &= \sum_{ijkl} \int_{pqr} \frac{1}{(p_{ij}-q_{ik})^2\, q_{ik}^2\, (p_{ij}-r_{il})^2\, r_{il}^2} = \text{“}I_{\rm ball}\text{”} \\
I_2 &= \sum_{ijkl} \int_{pqr} \frac{q_{ik}\cdot r_{il}}{p_{ij}^2\, (p_{ij}-q_{ik})^2\, q_{ik}^2\, (p_{ij}-r_{il})^2\, r_{il}^2} \\
I_3 &= \text{“}I_{\rm QCD}\text{”}
\end{aligned}
$$

where $\displaystyle \int_p \equiv T \sum_{n_p\in\mathbb{Z}} \int \frac{d^3p}{(2\pi)^3}$

- Duality between the Matsubara modes and winding numbers.

Given $$f(\theta) = \sum_{n\in\mathbb{Z}} F(2\pi n + \theta)$$

we have $$f(\theta) = \sum_{m\in\mathbb{Z}} \tilde{f}_m e^{im\theta}$$

where $$\tilde{f}_m = \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} e^{-im\theta} F(\theta)$$

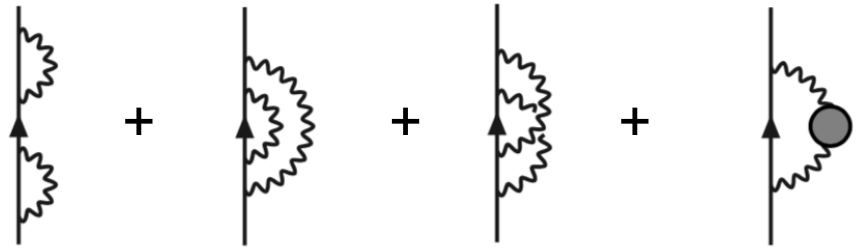
- Advantages over other methods.

  - Polyakov loop comes out naturally: $V_{\rm eff}(A_4) \to V_{\rm eff}(L)\,.$
  - Momentum integration done first, and summation replaced by integral.
  - Zero winding (m=0) contains the divergence.

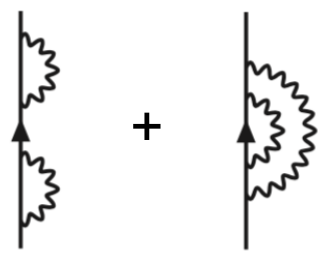
Three loop (N^2g^4)

- Renormalization of the Polyakov loop

$$g^4 L_m^{(4)} =$$



- Partial results



$$= -\frac{g^4}{N_c} \sum_{i\neq j\neq l} \sum_{m_p,m_q\in\mathbb{Z}} \frac{n e^{in\theta_i}}{(2\pi)^4} \left[\left(\frac{\log(|m_p m_q|)}{m_q} - \frac{i}{2}\frac{n}{m_p m_q} + \frac{\frac{1}{2\epsilon}+\tilde{c}}{m_q} \right) e^{im_p\theta_{ij}} e^{im_q\theta_{il}} \right.$$
$$\left. - \left(\frac{\log(|m_p/m_q|)}{m_p - m_q} + \frac{\log(|m_p m_q|)}{m_q} + \frac{\frac{1}{2\epsilon}+\tilde{c}}{m_q} \right) e^{im_p\theta_{ij}} e^{im_q\theta_{jl}} \right]$$

Stay tuned!

Summary

- The Polyakov loop is nontrivial near Tc and should not be ignored.

- Perturbative calculation becomes simpler at large N.

- Poisson resummation formula is the natural way to compute the sum integral with the nontrivial holonomy.