Vacuum-like jet fragmentation in a dense QCD medium

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Introduction

- Jets are very important probes of the quark-gluon plasma (QGP) produced in heavy-ions collisions at LHC or RHIC.

- Understanding observables such that the jet suppression or the jet fragmentation function will help to better characterize the QGP.

- From a theoretical point of view, a complete picture of the evolution of a jet in a dense medium is still lacking.
Motivations and goal of the talk

- Jet evolution in a dense medium: medium induced emissions versus vacuum-like emissions. How can we include both mechanisms?

- Our solution is to work with the simplest possible approximation in parton shower: the **leading double-logarithm** approximation (DLA).
Vacuum emissions vs ...

Bremsstrahlung spectrum $\implies$ logarithmic enhancement for soft and collinear emissions.

Formation time due to the virtuality of the parent parton:

$$t_{vac} \sim \frac{\omega}{k_{\perp}^2} \sim \frac{1}{(\omega \theta^2)}$$

$p_z, p_\perp = 0$

$$k_z = x p_z, k_\perp$$

$$d\mathcal{P} \sim \frac{\alpha_s C_R}{\pi} \frac{dx}{x} \frac{d\theta^2}{\theta^2}$$
... medium induced radiation

BDMPS-Z spectrum (Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov 1996–97) → NOT DOUBLE LOG!

Medium-induced formation time and broadening characteristic time scale: $t_{med} \sim \sqrt{\omega/\hat{q}}$ from $\langle k_{\perp}^2 \rangle = \hat{q} t$ and $t_f = \omega/k_{\perp}^2$.

$$dP \simeq \bar{\alpha}_s \frac{d\omega}{\omega} \frac{L}{t_{med}(\omega)} \simeq \bar{\alpha}_s L \sqrt{\frac{\hat{q}}{\omega^3}} d\omega$$
Vacuum-like emission inside the medium

If $t_{vac} \lesssim t_{med}$: emission triggered by the virtuality and not yet affected by the momentum broadening.

$\implies$ double-logarithmic enhancement of the probability.

Equivalent condition

$$\omega \geq (\hat{q}/\theta^4)^{1/3} \equiv \omega_0(\theta)$$
Vacuum-like emission outside the medium

- \( t_{\text{vac}} \geq L \implies \) vacuum-like emission outside the medium triggered by the virtuality of the parent parton.

- In terms of energy: \( \omega \leq \frac{1}{(L\theta^2)}. \)
Lund diagram: double logarithmic phase space with a QGP for one emission

The energy scale \( \omega_c \)

The condition \( t_{med} = L \) defines the energy scale \( \omega_c = 1/2\hat{q}L^2 \). Gluons with energy greater than \( \omega_c \) are always vacuum like.
Iteration of vacuum-like emissions

Large $N_c$ limit

Emission of a soft gluon by an antenna $\Leftrightarrow$ splitting of the parent antenna into two daughter antennae.

Decoherence time

- Color coherence is responsible for **angular ordering** in **vacuum** parton cascades.

- In the medium, an antenna loses its color coherence after a time $t_{coh} = (\hat{q}^2 \theta_{q\bar{q}})^{-1/3}$.

  (Mahtar-Tani, Salgado, Tywoniuk, 2010-11 ; Casalderrey-Solana, Iancu, 2011)

- In principle, angular ordering could be violated by cascades inside the medium.
Coherence in vacuum vs (de)coherence in the medium

The angular scale $\theta_c$

The condition $t_{\text{coh}} = L$ gives the definition of the critical angle $\theta_c = 2/\sqrt{\hat{q}L^3}$. Antennae with angles greater than $\theta_c$ always lose their coherence propagating over a distance $L$. 
Parton cascade in the medium

But decoherence is **impossible** for vacuum-like emissions (VLE)!

\[ t_{\text{vac}}(\omega_i, \theta_i) \geq t_{\text{coh}}(\theta_{i-1}) \text{ and } \theta_i \geq \theta_{i-1} \Rightarrow t_{\text{vac}}(\omega_i, \theta_i) \geq t_{\text{med}}(\omega_i) \]

\[ \Rightarrow \text{not a VLE} \]

In the leading double-logarithmic approximation, successive in-medium vacuum-like emissions form angular-ordered cascades.

**Consequence:** at DLA successive VLEs are strongly ordered in

- energy \( \omega_i \ll \omega_{i-1} \) because of energy conservation
- angle \( \theta_i \ll \theta_{i-1} \) by color coherence at DLA.
Parton shower inside the medium
Last antenna inside the medium

- The precedent proof does not apply if the parent antenna is the last inside the medium.
- In that case, the formation time of the next antenna is larger than $L$.

Last emission inside the medium

- If $\theta \leq \theta_c$ : the coherence time is also larger than $L \Rightarrow$ angular ordering is preserved.
- If $\theta \geq \theta_c$ : the antenna has lost its coherence during the formation time of the next antenna $\Rightarrow$ no constraint on the angle of the next antenna.

Parton shower inside and outside the medium

Three important (leading-twist) effects:

- Reduction of the available phase space due to the VLE constraint.
- Angular ordering in the in-medium region.
- One violation of angular ordering by the first emission outside the medium.
Analytical study of jets at DLA

Double differential gluon distribution

\[ T(\omega, \theta^2 \mid E, \theta^2_{q\bar{q}}) \equiv \omega \theta^2 \frac{d^2 N}{d\omega d\theta^2} \]

⇒ probability of emission of a gluon with energy \( \omega \) and angle \( \theta^2 \) from an antenna with energy \( E \) and opening angle \( \theta^2_{q\bar{q}} \).

In the vacuum at DLA, this quantity satisfies the simple master equation

\[ T_{vac}(\omega, \theta^2 \mid E, \theta^2_{q\bar{q}}) = \bar{\alpha}_s + \int_{\theta^2}^{\theta^2_{q\bar{q}}} \frac{d\theta_1^2}{\theta^2_1} \int_{\omega/E}^{1} \frac{dz_1}{z_1} \bar{\alpha}_s T_{vac}(\omega, \theta^2 \mid z_1 E, \theta^2_1) \]

With a medium, this equation holds only inside the medium

⇒ mathematically, one must take into account “jumps” over the vetoed region.
Numerical results: ratio $T(\omega, \theta^2)/T_{\text{vac}}(\omega, \theta^2)$

$T/T_{\text{vac}}$

$\omega$ [GeV]

$0.1 \quad 1 \quad 10 \quad 100$

0.4

0.2

0.1

0.05

0.02

0.01

$\theta$

$0.02 \quad 0.05 \quad 0.2 \quad 0.4 \quad 0.01 \quad 0.1 \quad 0.1 \quad 1 \quad 10 \quad 100$

$E=200$ GeV, $\theta_{qq}=0.4, \tilde{\alpha}_s=0.3, \hat{q}=2$ GeV$^2$/fm, $L=3$ fm
Fragmentation function with fixed-coupling

Definition
Integral over angle between the $k_\perp$ cut-off and $\theta_{q\bar{q}}$

$\Rightarrow D(\omega) \equiv \omega \frac{dN}{d\omega} = \int_{\Lambda^2/\omega^2}^{\theta_{q\bar{q}}^2} \frac{d\theta^2}{\theta^2} T(\omega, \theta^2)$

(CMS collaboration, Phys. Rev. C 90, 2014)
Numerical results for the fragmentation function
Results beyond DLA

Preliminary results

- Running coupling + DLA: \( \bar{\alpha}_s P_{gg}(z) \rightarrow \bar{\alpha}_s (k^2_{\perp})^{1/z} \).
- Running coupling + NDLA:
  \[ \bar{\alpha}_s P_{gg}(z) \rightarrow \bar{\alpha}_s (k^2_{\perp})^{1/z} \left( 1 - \frac{11}{12} z \right). \]

Fragmentation function ratios

- Running coupling + NDLA, \( C_{\text{reg}} = 0.001, \Lambda = 100 \) [MeV]
- Running coupling, \( C_{\text{reg}} = 0.001, \Lambda = 100 \) [MeV]
- Fixed coupling, \( \bar{\alpha}_s = 0.3, \Lambda = 100 \) [MeV]

Medium with \( q = 1 \) [GeV^2/fm] and \( L = 3 \) [fm]
Jet with \( E = 200 \) [GeV] and \( R = 0.4 \)
What about the energy loss?

Energy loss is **negligible** for any parton of the cascade inside the medium (except for the last one)

- \( \omega_{\text{loss}} \sim \hat{q} t^2 \) energy of the hardest medium induced emission that can develop during \( t \).
- By the inequality \( t_{\text{vac}}(\omega_i, \theta_i^2) \ll t_f(\omega_i, \theta_i^2) \), one finds that \( \omega_{\text{loss}} \ll \omega_i \).

However...

- Energy loss is not negligible for the last antenna inside the medium since it will cross the medium along a distance of order \( L \).
- Partons produced inside the medium via VLEs act as sources for medium-induced democratic branching processes.
Estimation of the energy loss by a jet at NDLA

Preliminary results

Model

All the partons with energy $\omega$ produced by the shower inside the medium act as new sources for medium induced cascades so lose an energy typically equal to $\min(\omega, \omega_{br} = \alpha_s^2 \omega_c)$. 

![Graph showing the energy loss by a jet at NDLA](image)

- **Jet - $R = 0.4$, $\hat{q} = 1$ [GeV$^2$/fm], $L = 3$ [fm]**
- **Jet - $R = 0.4$, $\hat{q} = 2$ [GeV$^2$/fm], $L = 4$ [fm]**
- **Parton**
Conclusion

Summary

- Vacuum-like emissions inside the medium can be factorized from the medium-induced radiations within the double-log approximation.
- DLA is fine for intrajet multiplicity at small energy but it is not accurate enough for experimental observables relying on energy because energy is not exactly conserved through the shower.

In perspective

Monte-Carlo simulation: build an event generator which will include the full splitting functions (hence, energy conservation) for the vacuum-like cascades and the medium-induced cascades.
Thank you for listening!