Forward particle production in proton-nucleus collisions at next-to-leading order

Bertrand Ducloué
(IPhT Saclay)

XIIIth Quark Confinement and the Hadron Spectrum

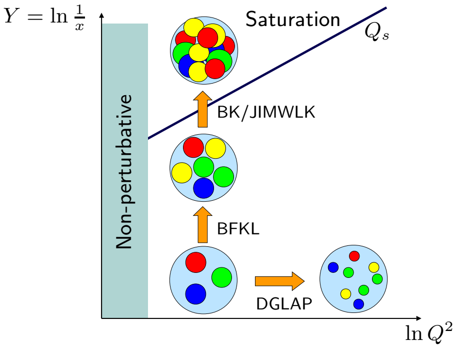
Maynooth University, August 2, 2018

QCD: theory difficult to study in the general case

Presence of a hard scale ($p_\perp$, M): possible to use perturbative expansion

One can then study the evolution of parton densities in hadrons:

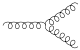
- as a function of Q^2: DGLAP
- as a function of x: BFKL (dilute) / BK, JIMWLK (dense)



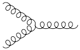
Our goal here is to study the dense limit of QCD (saturation)

Motivations

The linear small x evolution (BFKL) is governed by the emission of multiple soft gluons ($1 \to 2$ scatterings):

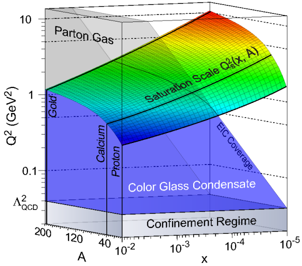


At large densities, recombination effects become important: Non-linear evolution (BK, JIMWLK)



The importance of non-linear effects is quantified by the size of the saturation scale Q_s, which corresponds to the typical transverse momentum of the gluons in the target $\Rightarrow$ need Q_s as large as possible

Roughly we have $Q_s^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$



Therefore we need:

- Small x values
- Large nucleus

} to enhance saturation effects

- A hard scale

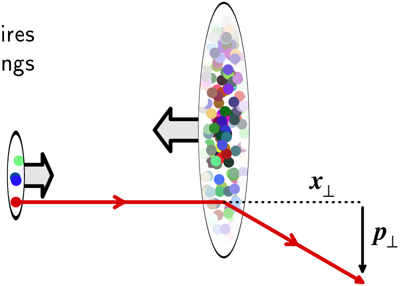
} to be in the perturbative regime

The process we will study here: forward hadron production in high energy proton-nucleus collisions, e.g. at LHC

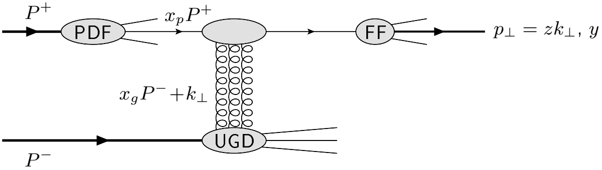
A quark initially collinear with the proton acquires a transverse momentum $p_\perp$ via multiple scatterings off the saturated gluons

in the dilute proton: $x_p = \frac{p_\perp}{\sqrt{s}} e^{y} \sim 1$

in the dense nucleus: $x_g = \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$



Single inclusive forward hadron production at LO in the $q \to q$ channel:



Dilute projectile: $x_p = \frac{k_\perp}{\sqrt{s}} e^{y}$, described by a collinear PDF

Dense target: $x_g = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution $\mathcal{S}$

LO quark multiplicity: $\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} \propto \mathrm{PDF} \otimes \mathcal{S} \otimes \mathrm{FF}$ $\left(\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} = \int \mathrm{d}^2\mathbf{b} \frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} \right)$

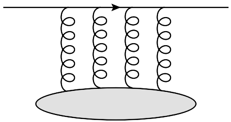
$\mathcal{S}$ is the Fourier transform of the dipole correlator $S(\mathbf{r})$:

$$\mathcal{S}(k_\perp) = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r}), \quad S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$$

Where $V(\mathbf{x})$ is a fundamental representation Wilson line in the target color field

At high densities the target can be described as a classical color field

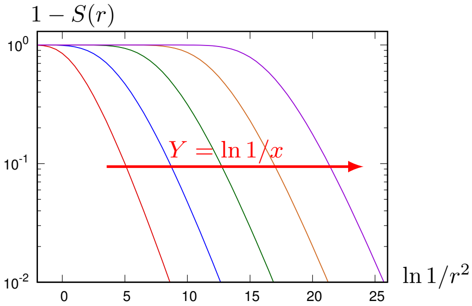
The eikonal interaction of the dilute probe (quark) with the dense target (nucleus) is described by a Wilson line V



The evolution of the dipole correlator $S(\mathbf{r}=\mathbf{x}-\mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$ is governed by the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial S(\mathbf{r},x)}{\partial \ln x} = 2\alpha_s N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r},x) - S(\mathbf{x},x) S(\mathbf{r}-\mathbf{x},x) \right]$$

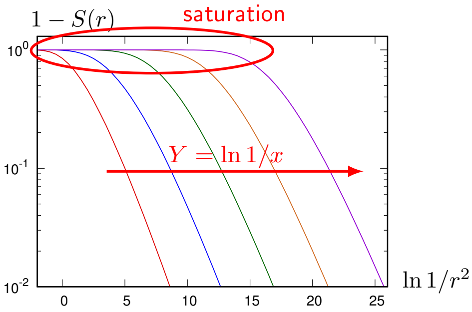
Solving this equation numerically, we can evolve perturbatively S to larger values of $Y = \ln 1/x$:



We need an initial condition $S(\mathbf{r}, x_0)$ to start the evolution

The initial condition cannot be computed perturbatively. It can be extracted by a fit to data (e.g. DIS)

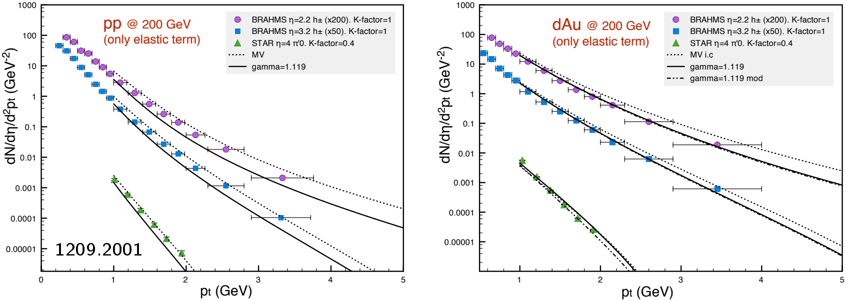
Solving this equation numerically, we can evolve perturbatively S to larger values of $Y = \ln 1/x$:



We need an initial condition $S(\mathbf{r}, x_0)$ to start the evolution

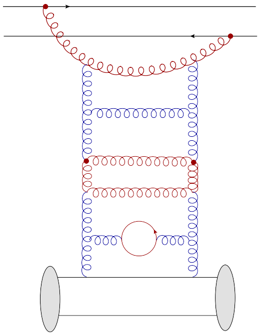
The initial condition cannot be computed perturbatively. It can be extracted by a fit to data (e.g. DIS)

Several groups were able to obtain a reasonable description of RHIC data, but sometimes large K factors are needed to describe the normalization (Albacete, Dumitru, Fujii, Nara, 2012; Lappi, Mäntysaari, 2013):



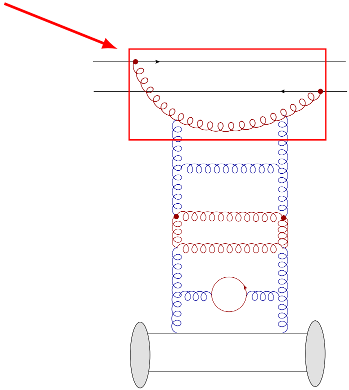
This is only leading order. Can this formalism be systematically extended to higher orders to improve the reliability of the predictions?

This process receives NLO corrections from two sources:



This process receives NLO corrections from two sources:

- Corrections to the process-dependent hard part (impact factor)
  Chirilli, Xiao, Yuan, 2012



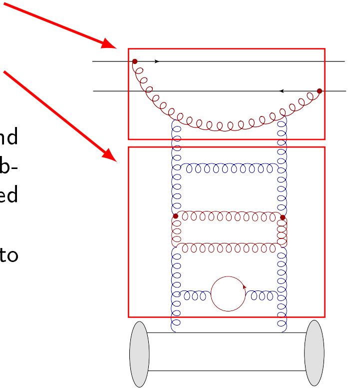
NLO corrections

This process receives NLO corrections from two sources:

- Corrections to the process-dependent hard part (impact factor)
  Chirilli, Xiao, Yuan, 2012
- Corrections to the BK evolution
  Balitsky, Chirilli, 2007

At first both sources of corrections were found to lead to unphysical results. But these problems were progressively understood and solved

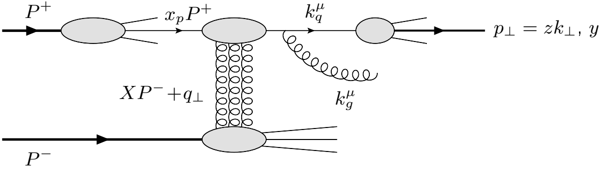
This talk will focus on the NLO corrections to the impact factor



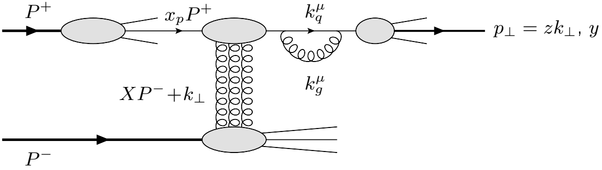
NLO corrections

NLO corrections to the impact factor for this process:
(Chirilli, Xiao, Yuan, 2012)

Example of real $q \to q$ contribution:



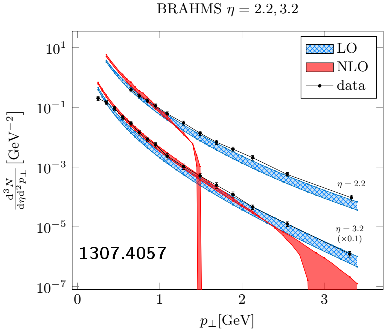
Example of virtual $q \to q$ contribution:



$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

NLO corrections

First numerical implementation of these expressions:
(Staśto, Xiao, Zaslavsky, 2013)



Negative cross section above some $p_\perp \sim Q_s$!

The issue appears in the range of semi-hard transverse momenta where this formalism is expected to apply

NLO corrections

Many works devoted to understanding and solving this issue:

- Kang, Vitev, Xing, 2014
- Staśto, Xiao, Yuan, Zaslavsky, 2014
- Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 2014
- Watanabe, Xiao, Yuan, Zaslavsky, 2015
- B.D., Lappi, Zhu, 2016

Can push the positivity to higher $p_\perp$ but doesn't solve the problem completely

It turned out that the negativity is related to the subtraction of the LO evolution from the NLO corrections

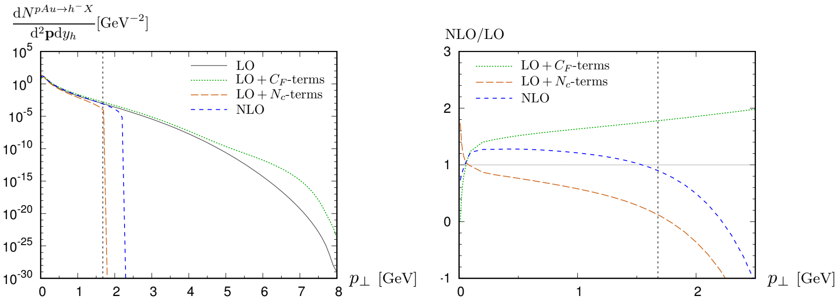
Avoided by a new factorization scheme in which there is no subtraction
Iancu, Mueller, Triantafyllopoulos, 2016

In both real and virtual corrections, there are terms proportional to the C_F color factor and terms proportional to N_c:

$$\frac{\mathrm{d}N^{\mathsf{NLO}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = \frac{\mathrm{d}N^{\mathsf{LO}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} + \frac{\mathrm{d}N^{C_\mathsf{F}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} + \frac{\mathrm{d}N^{N_\mathsf{c}}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y}$$

- The terms proportional to C_F are divergent when the additional gluon at NLO is collinear to the initial or final state quark
  These divergences are absorbed in the DGLAP evolution of the PDFs and fragmentation functions
- The terms proportional to N_c are related to the high energy evolution (recall BK: $\partial_Y S = 2\alpha_s N_\mathsf{c} \int \mathrm{d}^2\mathbf{x}\,[...]$)

At high $p_\perp$ the C_F NLO corrections are positive while the N_c ones are negative: (B.D., Lappi, Zhu, 2016)

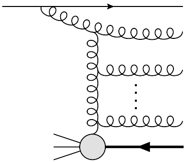


The negativity issue comes from the N_c terms, and more precisely the subtraction of the LO evolution from the NLO terms

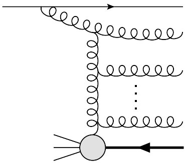
LO vs. NLO

Balitsky-Kovchegov (BK) evolution: resummation of $(\alpha_s \ln 1/x)^n$, corresponding to any number of soft gluons already at LO

LO: all gluons are soft:



NLO impact factor: the first gluon can be hard:

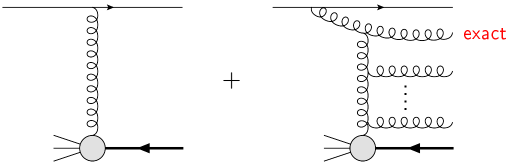


The case where the first gluon is soft is already included in the leading order
$\Rightarrow$ Need to avoid double counting between LO and NLO

Two possible solutions to avoid double counting:

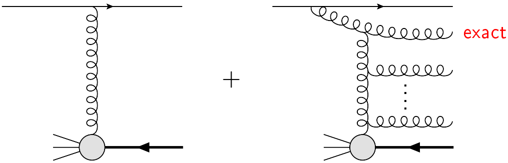
1) Subtract the case where the gluon in the NLO impact factor is soft
Chirilli, Xiao, Yuan ('CXY'), 2012

2) Rearrange the terms to avoid doing a subtraction
Iancu, Mueller, Triantafyllopoulos, 2016



These two choices should be equivalent

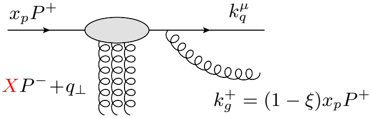
LO vs. NLO



The cross section can be schematically written as

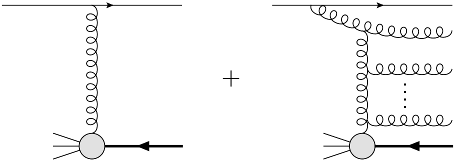
$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = \mathcal{S}(k_\perp, x_0) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \mathcal{K}(\xi)\mathcal{S}(k_\perp, X(\xi))$$

Where the rapidity scale $X(\xi)$ is the P^- fraction needed from the target:



$$X = \frac{k_\perp}{\sqrt{s}} e^{-y} \left(1 + \frac{\xi}{1-\xi} \frac{(q_\perp - k_\perp)^2}{k_\perp^2}\right)$$

$$\approx \frac{x_g}{1-\xi} \equiv X(\xi) \text{ when } k_\perp \gtrsim Q_s$$



The cross section can be schematically written as

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = \mathcal{S}(k_\perp, x_0) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \mathcal{K}(\xi)\mathcal{S}(k_\perp, X(\xi))$$

Soft gluon limit ($\xi \to 1$): recover LO BK

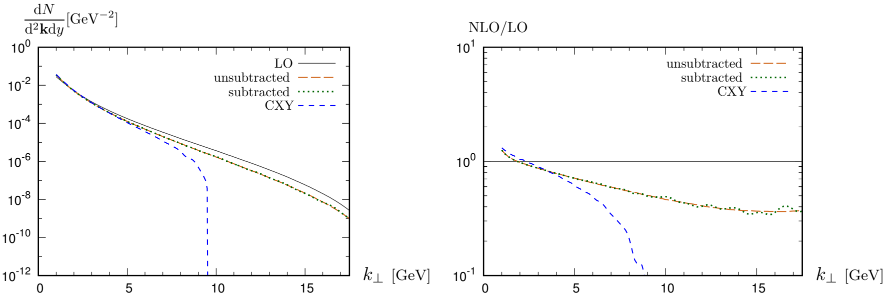
$$\mathcal{S}(k_\perp, x_g) = \mathcal{S}(k_\perp, x_0) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \mathcal{K}(1)\mathcal{S}(k_\perp, X(\xi))$$

We can thus rewrite

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = \mathcal{S}(k_\perp, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \left[\mathcal{K}(\xi) - \mathcal{K}(1)\right] \mathcal{S}(k_\perp, X(\xi))$$

Up to NLO accuracy, one can take $X(\xi) \to x_g$ and $1 - \frac{x_g}{x_0} \to 1$ in this subtracted version. Local in rapidity: k_T-factorization as presented by CXY

Results for the total NLO multiplicity at fixed coupling ($\alpha_s = 0.2$):
(B.D., Lappi, Zhu, 2017)



The 'subtracted' and 'unsubtracted' expressions give the same positive results

The 'CXY' approximation leads to negative results for $k_\perp \gtrsim 10$ GeV.

The negativity issue observed in the first implementation of the NLO impact factor can be attributed to approximations made in the LO subtraction

In the 'subtracted' formulation, we add and subtract a large contribution. If we use the CXY approximation what we add and subtract is no longer the same which can make the final result negative

Without this approximation the cross section has a physical behavior at all $k_\perp$ $\Rightarrow$ Problem solved? No!

So far we discussed only the fixed coupling case. But the running of the coupling is an important effect that has to be taken into account in realistic calculations

The equivalence between the 'subtracted' and 'unsubtracted' formulations holds only if one uses the same coupling α_s when computing the cross section and when solving the BK equation

At semi-hard transverse momenta $k_\perp \gtrsim Q_s$ the natural choice for the scale is $\alpha_s(k_\perp)$. But the BK equation is usually solved in coordinate space

Fixed coupling BK equation:

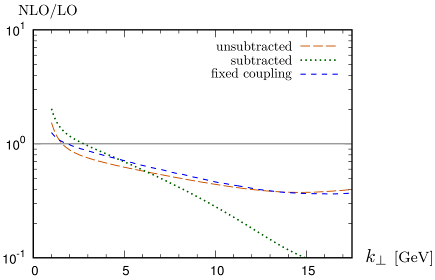
$$\frac{\partial S(\mathbf{r}, X)}{\partial \ln X} = 2\alpha_s N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r}, X) - S(\mathbf{x}, X) S(\mathbf{r}-\mathbf{x}, X)\right]$$

rcBK with the simple parent dipole running coupling prescription:

$$\frac{\partial S(\mathbf{r}, X)}{\partial \ln X} = 2\alpha_s(r_\perp) N_c \int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r}, X) - S(\mathbf{x}, X) S(\mathbf{r}-\mathbf{x}, X)\right]$$

The choice $\alpha_s(r_\perp)$ seems to be reasonable since $\mathbf{r}$ is Fourier-conjugate to $\mathbf{k}$

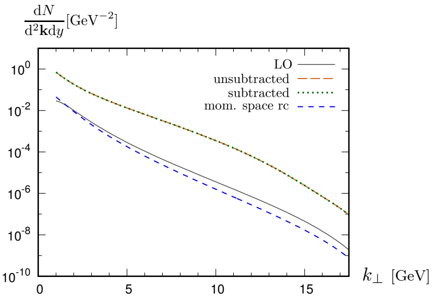
Using $\alpha_s(r_\perp)$ when solving BK and $\alpha_s(k_\perp)$ for the explicit α_s factors in the cross section:



The 'subtracted' and 'unsubtracted' expressions are no longer equivalent since we don't use exactly the same α_s in the cross section and when solving BK

- 'Subtracted' version: can become negative again at large $k_\perp$
- 'Unsubtracted' version: does not reduce to the correct LO result when $\xi \to 1$

Possible way to use consistently a coordinate-space running coupling: rewrite the cross section expression in coordinate space by Fourier transform. Then we can use the same coupling $\alpha_s(r_\perp)$ everywhere



The 'subtracted' expression gives the same results as the 'unsubtracted' one

But completely different results compared to fixed coupling or $\alpha_s(k_\perp)$, absurdly large NLO corrections

Similar situation with other prescriptions → the problem is more general

Running coupling

To illustrate the problem, let's look at the following simple quantities:

$$\mathcal{N}_k \equiv \bar{\alpha}_s(k_\perp)\,\mathcal{S}(k_\perp) = \bar{\alpha}_s(k_\perp)\int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$$

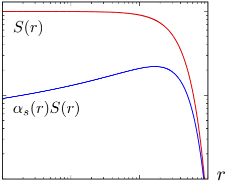
$$\mathcal{N}_r \equiv \int \mathrm{d}^2\mathbf{r}\,\bar{\alpha}_s(r_\perp) e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})$$

These two quantities do not differ by only a small factor. Indeed, using the McLerran-Venugopalan model $S(r_\perp) = \exp\left(-\frac{r_\perp^2 Q_s^2}{4}\ln\frac{1}{r_\perp^2\Lambda^2}\right)$, we find at large $k_\perp$

$$\mathcal{N}_k \sim \frac{4\pi\bar{\alpha}_s(k_\perp)Q_s^2}{k_\perp^4} \qquad \text{while} \qquad \mathcal{N}_r \sim -\frac{4\pi}{\bar{b}[\ln(k_\perp^2/\Lambda^2)]^2}\,\frac{1}{k_\perp^2}$$

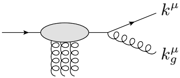
which are opposite in sign and have different tails: the choice of the running coupling prescription and the Fourier transform do not 'commute'

Note that the problem comes from the perturbative region $\mathbf{r} \to 0$, not the IR region



To avoid the problem we must choose a scale which does not depend on $r_\perp$

The target can only give a $k_\perp \sim Q_s$ → a large $k_\perp$ quark can only be produced with a recoil gluon with $\mathbf{k}_g \sim -\mathbf{k}$



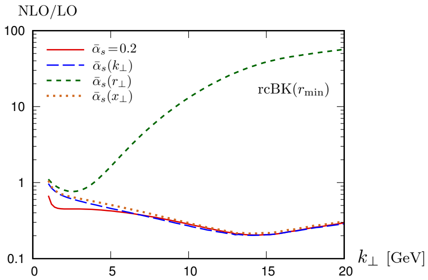
In coordinate space this corresponds to $x_\perp \sim r_\perp$ → we propose to use $\alpha_s(x_\perp)$:

- $x_\perp \sim r_\perp$: same as $\alpha_s(r_\perp)$, with $r_\perp$ Fourier-conjugate to $k_\perp$
- $x_\perp \gg r_\perp$: unphysical contributions eliminated by the Fourier transform

Such severe issues don't appear when solving BK with the parent dipole prescription: the spurious contributions cancel between real and virtual terms

But in the NLO cross section the real and virtual terms are multiplied by different quark distributions $q(x_p/\xi)$ and $q(x_p)$ ⇒ No cancellation

With this daughter dipole prescription the cross section indeed has a physical behavior, similar to the results with fixed or momentum space running coupling: (B.D., Iancu, Lappi, Mueller, Soyez, Triantafyllopoulos, Zhu, 2017)

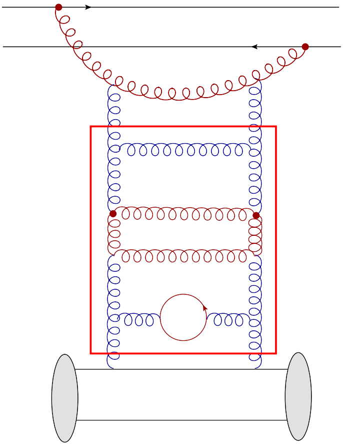


Other possibility: do the whole calculation in momentum space?

So far we discussed only the NLO corrections to the impact factor

A complete NLO calculation must also include the NLO corrections to the BK evolution

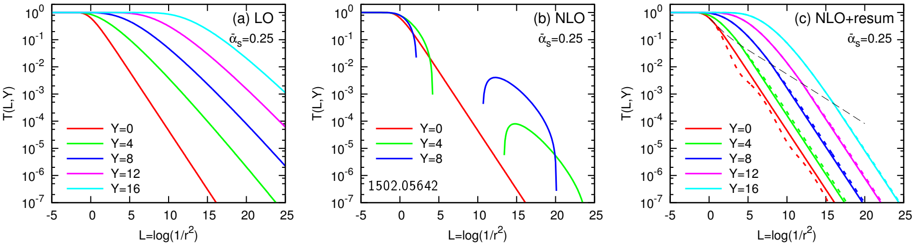
Balitsky, Chirilli, 2007



The first numerical implementation of the NLO corrections to BK showed that they lead to instabilities in the evolution because of large collinear contributions (Lappi, Mäntysaari, 2015)

A similar observation was made for BFKL a long time ago and was solved by resumming these contributions to all orders (Salam et al., 98-2003)

This resummation was done in coordinate space for NLO BK which restores the stability of the perturbative expansion:
(Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, 2015)



Conclusions

The saturation formalism is being pushed to NLO accuracy. The issues met in the first implementation of the NLO forward hadron production impact factor are now understood:

- Fixed coupling: negativity due to approximations made in the original implementation of the NLO impact factor
- Running coupling: new complications (mismatch between coordinate and momentum space couplings, spurious contributions due to F.T.)

  Can be avoided by using the daughter dipole prescription $\alpha_s(x_\perp)$

So far proof of principle. Now time to do phenomenology:

- Add the $q \to g$, $g \to q$ and $g \to g$ channels + fragmentation functions
- Use (resummed) NLO BK for high energy evolution
- The initial condition for the BK evolution of the target must be obtained by a fit (e.g. to HERA DIS data) also performed at NLO accuracy