Universität Bielefeld

Spectral and transport properties from Lattice QCD

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I) Light quark vector meson spectral function

- thermal photon, dilepton rates, electrical conductivity

[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

[H-T.Ding, F.Meyer, OK, PRD94 (2016) 034504]

II) Heavy quark momentum diffusion coefficient

[A.Francis, OK, et al., PRD92(2015)116003]

III) Thermal quarkonium physics in the pseudoscalar channel

[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

Quark Confinement and the Hadron Spectrum
Maynooth, 31.07.-06.08.2018

CRC-TR 211
Strong-interaction matter under extreme conditions

Spectral and transport properties in the QGP

Thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T}-1)} \rho_{\mathbf{V}}(\omega, \mathbf{T})$$

Thermal photon rate

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x \mathrm{d}^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T}-1} \rho_V(\omega = |\vec{k}|, T)$$

Transport coefficients are encoded in the same spectral function

→ Kubo formulae

Diffusion coefficients:

$$DT = \frac{T}{2\chi_q} \lim_{\omega \to 0} \frac{\rho_{\mathrm{ii}}(\omega)}{\omega}$$

On the lattice only correlation functions can be calculated

→ spectral reconstruction required

This talk: continuum extrapolated lattice correlation functions compared to perturbation theory

for Bayesian reconstruction techniques and lattice NRQCD see talks by A. Rothkopf

for a comparison of Bayesian and stochastic reconstructions of spectral functions see

[H.-T. Ding, OK, S. Mukherjee, H. Ohno, H.-T. Shu, PRD97(2018)094503]

Vector-meson spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

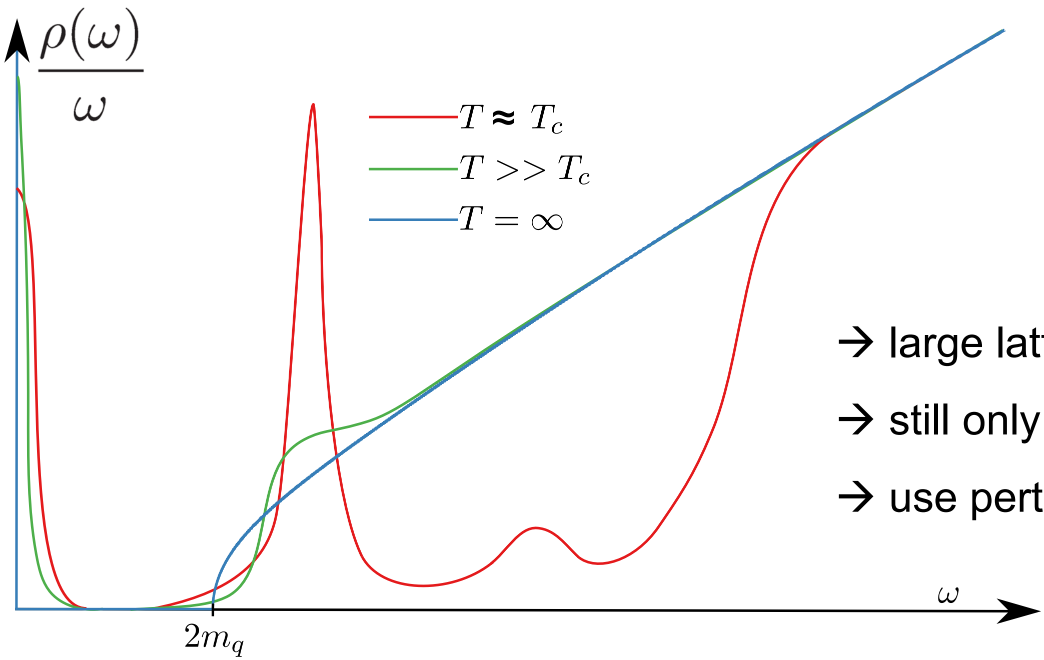
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- **in addition cut-off effects on the lattice**

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

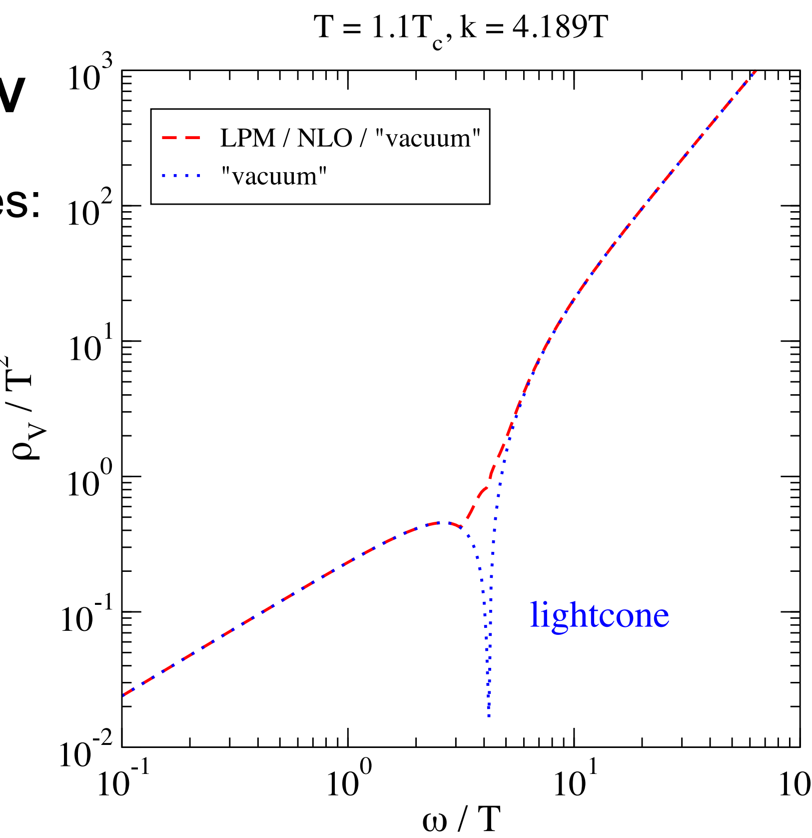
→ use perturbation theory to constrain the UV behavior

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}$, $\eta = \frac{T}{MD}$

Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\rho_V(\omega = |\vec{k}|, T)$$

pQCD spectral function used to constrain the UV



interpolation between different (perturbative) regimes:

3T < ω < 10T: [J.Ghiglieri, G.D.Moore, JHEP 1412 (2014) 029]

ω > 10T: [I. Ghisoiu, M.Laine, JHEP 10 (2014) 84]

ω >> 10T: [M.Laine, JHEP 1311 (2013) 120]

to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators

Vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$:

$$\frac{\rho_{\mathrm{V}}(\omega, \mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2 \right) \chi_{\mathrm{q}} D$$

with the quark number susceptibility: $\chi_{\mathrm{q}} \equiv \int_0^{\beta} \mathrm{d}\tau \int_{\mathbf{x}} \langle V^0(\tau, \mathbf{x}) V^0(0) \rangle$

and the diffusion coefficient: $D \equiv \frac{1}{3\chi_{\mathrm{q}}} \lim_{\omega \to 0^+} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega, \mathbf{0})}{\omega}$

which relate to the electric conductivity: $\sigma = e^2 \sum_{f=1}^{N_{\mathrm{f}}} Q_f^2 \chi_{\mathrm{q}} D$

In this limit the (soft) photon rate becomes: $\frac{\mathrm{d}\Gamma_\gamma(\mathbf{k})}{\mathrm{d}^3\mathbf{k}} \overset{k \lesssim \alpha_{\mathrm{s}}^2 T}{\approx} \frac{2T\sigma}{(2\pi)^3 k}$

In the AdS/CFT framework the vector spectral function has the same infrared structure

and here numerical result can make predictions beyond the hydro regime

[S.Caron-Huot, JHEP12 (2006) 015]

Although not QCD, it may give qualitative insight into the structures at small ω and k

Modeling the spectral function

$(5+2\, n_{max})^{th}$ order polynomial Ansatz at small ω:

$$\rho_{\rm fit} \;\equiv\; \frac{\beta\,\omega^3}{2\omega_0^3}\left(5-\frac{3\omega^2}{\omega_0^2}\right)-\frac{\gamma\,\omega^3}{2\omega_0^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)+\sum_{n=0}^{n_{\rm max}}\frac{\delta_n\omega^{1+2n}}{\omega_0^{1+2n}}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2$$

with the constraints to match smoothly with pQCD at ω_o

$$\rho_{\rm V}(\omega_0,\mathbf{k}) \;\equiv\; \beta\;, \quad \rho_{\rm V}'(\omega_0,\mathbf{k}) \;\equiv\; \gamma\;,$$

and n_{max}+1 free parameters

starting with a linear behavior at $\omega \ll$ T

smoothly matched to the perturbative spectral function at $\omega_0 \simeq \sqrt{k^2+(\pi T)^2}$

In the following we will use n_{max} = 0 and n_{max} = 1 for the fits to the lattice data

and to estimate the systematic uncertainties

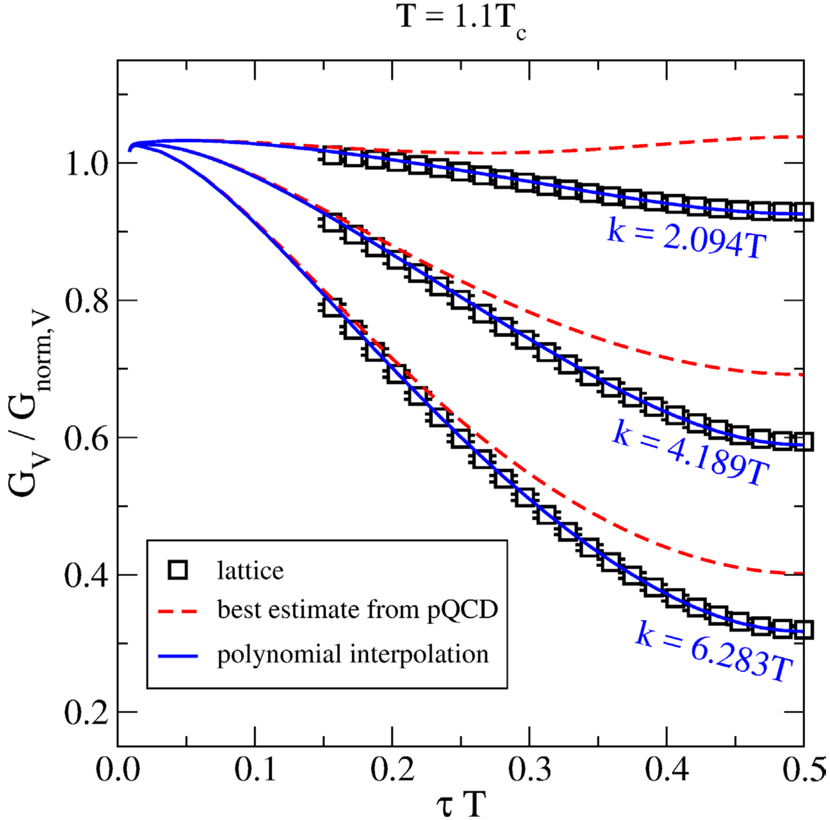
[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

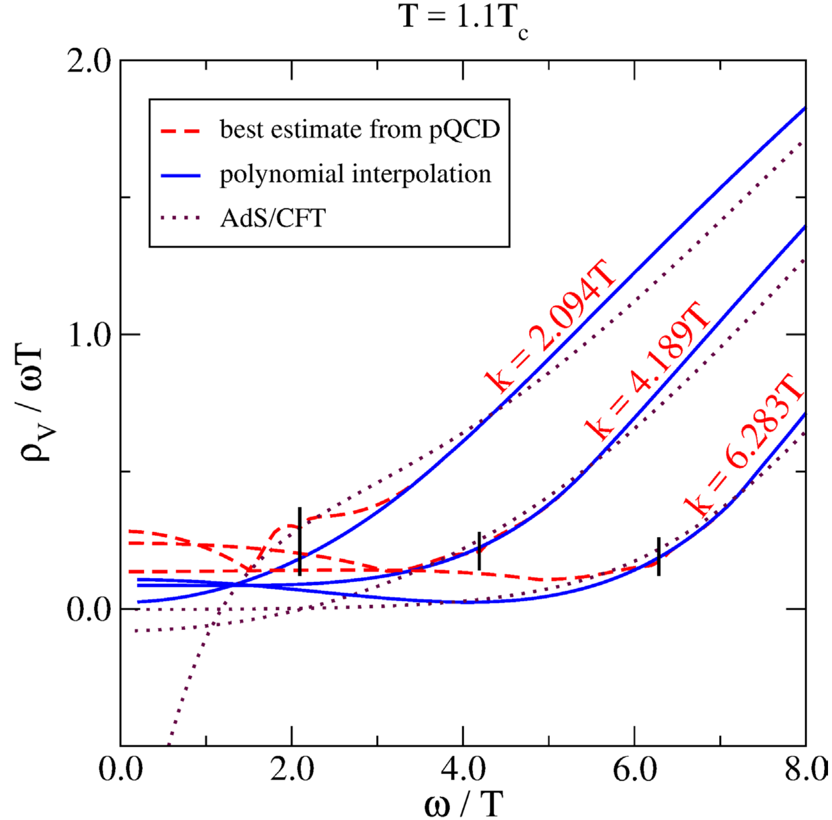
Fixed aspect ratio used to perform continuum extrapolation at finite p

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$
7.192	$96^3 \times 32$	314	0.2796	1.12	0.816	1.09
7.544	$144^3 \times 48$	358	0.2843	1.14	0.817	1.10
7.793	$192^3 \times 64$	242	0.2862	1.15	0.813	1.09
7.192	$96^3 \times 28$	232	0.3195	1.28	0.933	1.25
7.544	$144^3 \times 42$	417	0.3249	1.31	0.934	1.25
7.793	$192^3 \times 56$	273	0.3271	1.31	0.929	1.25

use perturbation theory at large ω and fit a polynomial at small ω to extract the spectral function





[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

Fixed aspect ratio used to perform continuum extrapolation at finite p

$$\frac{\vec{p}}{T} = 2\pi \vec{k} \frac{N_\tau}{N_\sigma}$$

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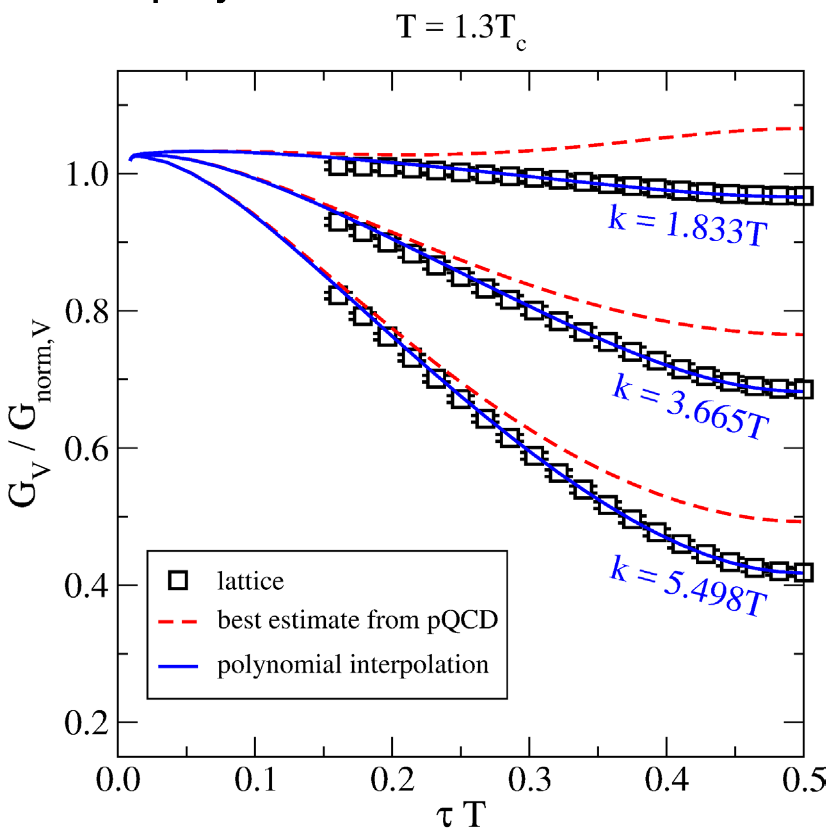
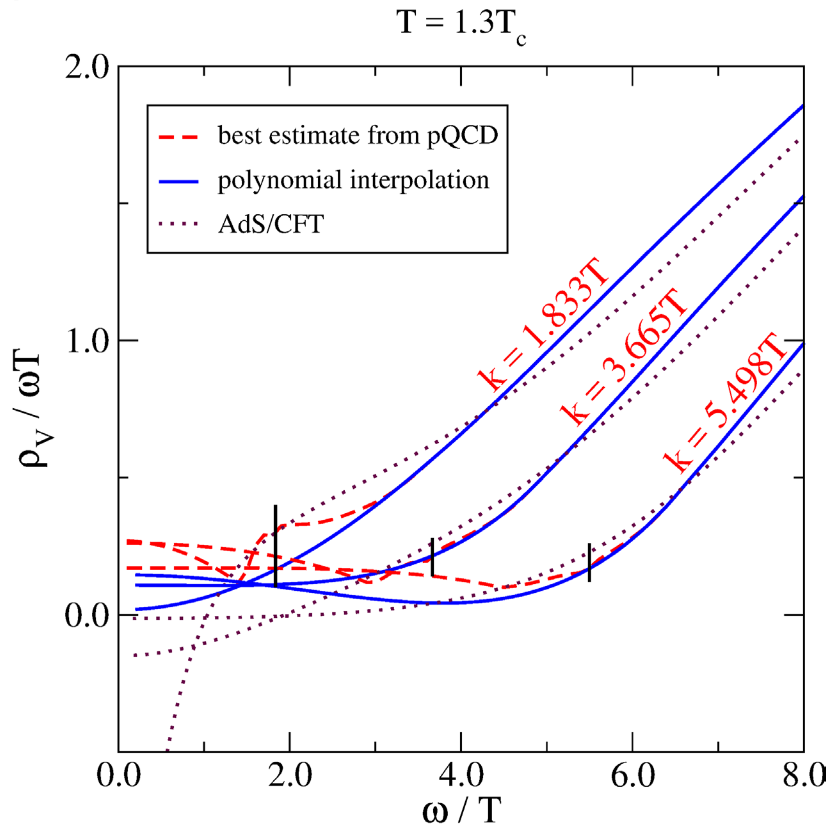
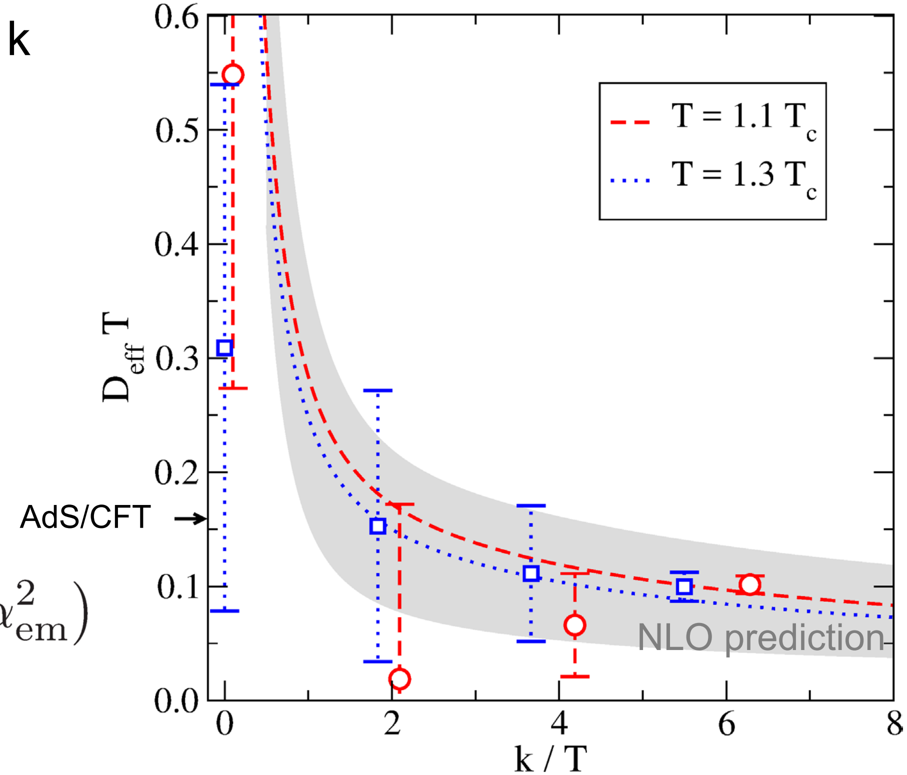




[J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94(2016)016005]

The spectral function at the photon point ω = k

$$D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q} k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases} .$$



can be used to calculate the photon rate

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} = \frac{2\alpha_{\rm em}\chi_{\rm q}}{3\pi^2} n_{\rm B}(k) D_{\rm eff}(k) + \mathcal{O}\left(\alpha_{\rm em}^2\right)$$

becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)

[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

Electrical conductivity obtained in the limit k→0 between the results from

AdS/CFT: $DT = \dfrac{1}{2\pi}$

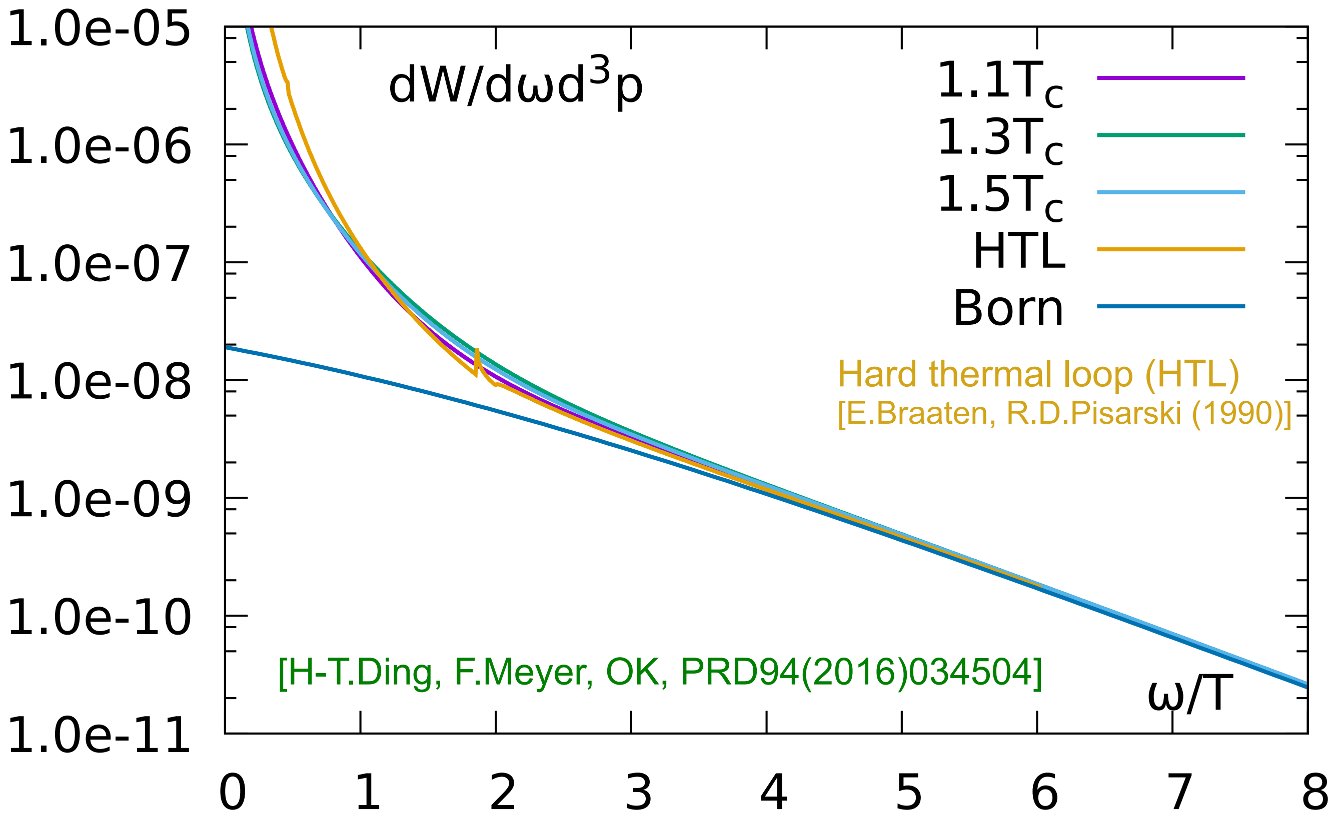
[G.Policastro, D.T.Son,A.O.Starinets, JHEP09(2002)043]

LO perturbation theory [Arnold, Moore Yaffe, JHEP 05 (2003)]

using lattice value for χ_q/T^2: $DT = 2.9 - 3.1$

Dileptonrate directly related to vector spectral function:

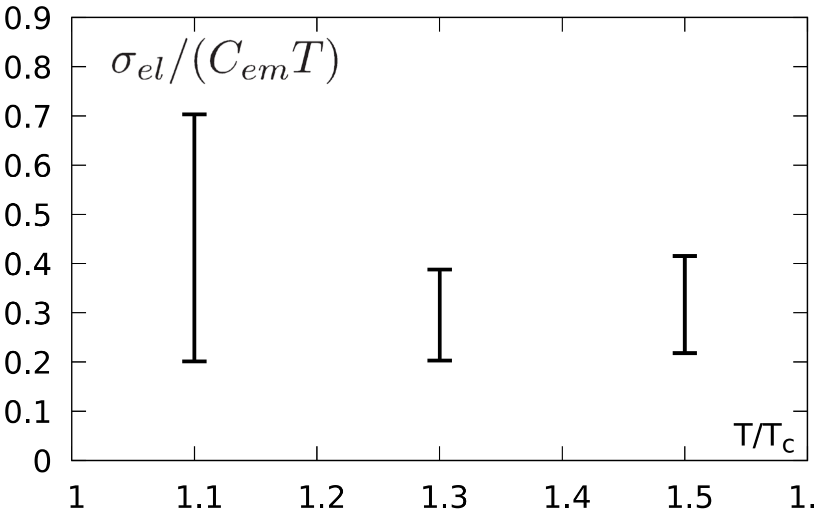
$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\mathbf{T})$$



continuum estimate for the of the electrical conductivity

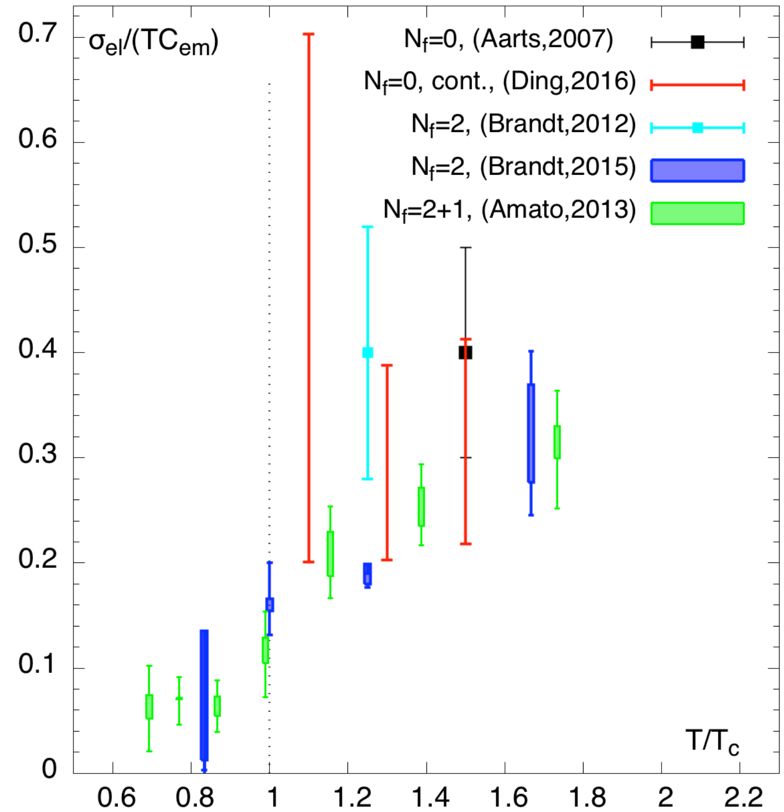
lower and upper limits from analysis of different classes of spectral functions:

$$\frac{\sigma_{el}}{C_{em}T} = \frac{1}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$



[H-T.Ding, F.Meyer, OK, PRD94(2016)034504]

comparison of different lattice results (Plot courtesy of A.Francis)



[G.Aarts et al., PRL 99 (2007) 022002,
H-T.Ding, F.Meyer, OK, PRD94(2016)034504,
B.B.Brandt et al., JHEP 1303 (2013) 100,
Brandt et al., PRD93 (2016) 054510,
A.Amato et al., PRL 111 (2013) 172001]

Progress in determining transport coefficients, although systematic uncertainties still need to be reduced in the future.

Heavy Quark Momentum Diffusion Constant κ

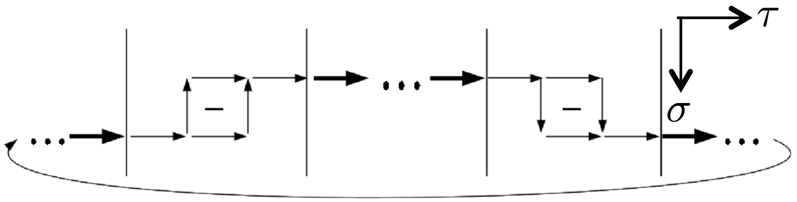
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]

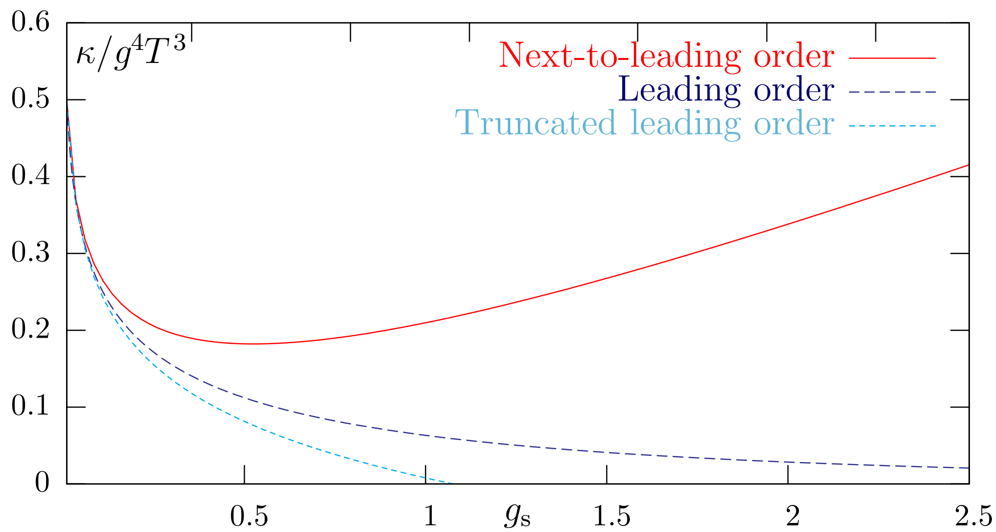
$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, g\, E_i(\tau,\mathbf{0})\, U(\tau;0)\, g\, E_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

NLO perturbative calculation:

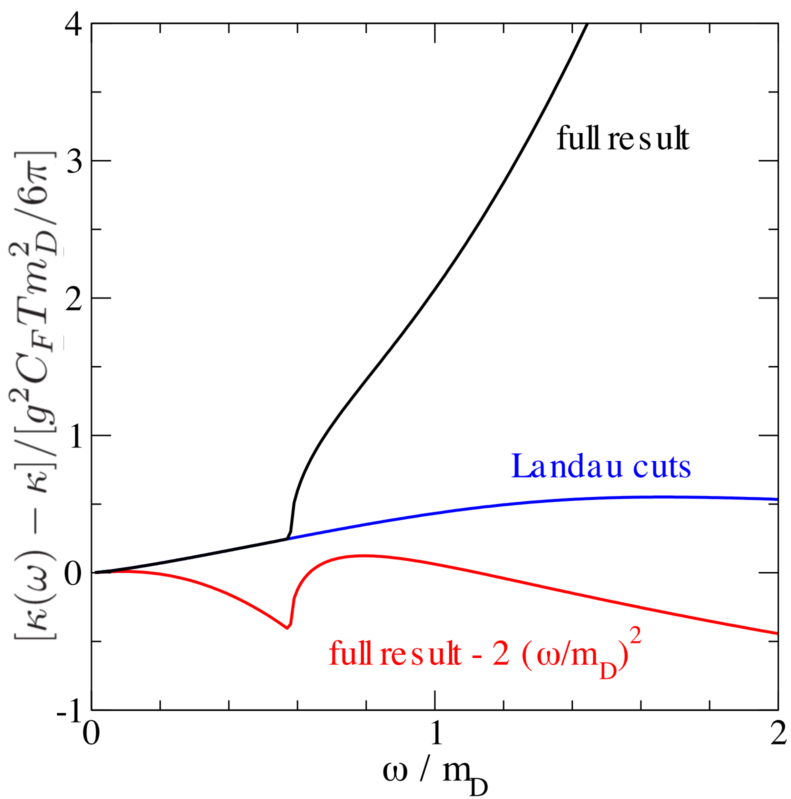
[Caron-Huot, G. Moore, JHEP 0802 (2008) 081]



→ large correction towards strong interactions

→ non-perturbative lattice methods required

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



in contrast to a narrow transport peak, from this a smooth limit

$$\kappa/T^3 = \lim_{\omega \to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

is expected

qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

[A.Francis, OK, M.Laine, T.Neuhaus, H.Ohno, PRD92(2015)116003]

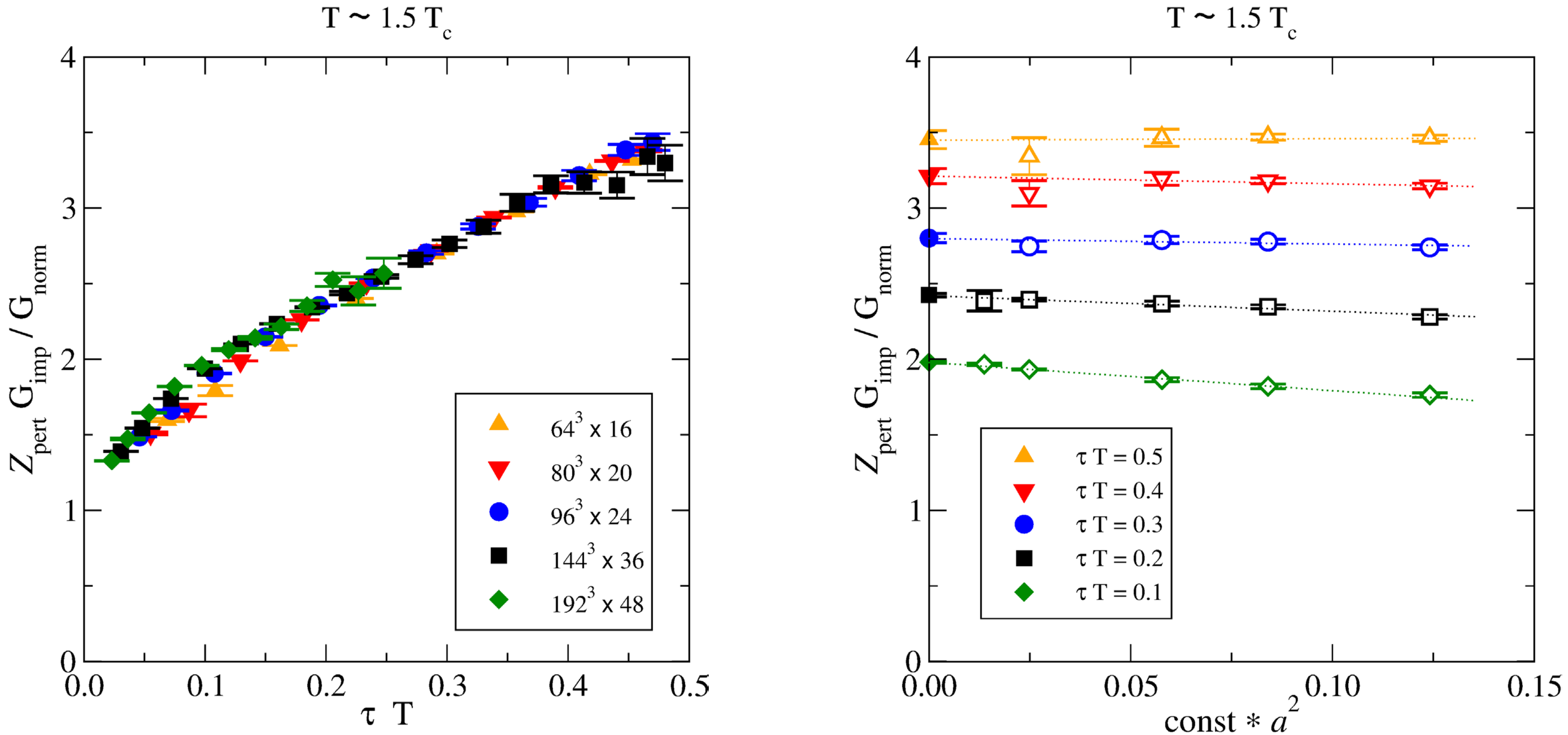
Quenched Lattice QCD on large and fine isotropic lattices at T$\simeq$ 1.5 T_c

- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration N_s/N_t = 4, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, a$\rightarrow$ 0 $\leftrightarrow$ $N_t \rightarrow \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$
- scale setting using r_0 and t_0 scale [A.Francis,OK,M.Laine, T.Neuhaus, H.Ohno, PRD91(2015)096002]

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}^{(\text{imp})}$	$T/T_c\|_{t_0}^{(\text{imp})}$	$T\sqrt{t_0}^{(\text{clov})}$	$T/T_c\|_{t_0}^{(\text{clov})}$	Tr_0	$T/T_c\|_{r_0}$
6.872	$64^3 \times 16$	172	0.3770	1.52	0.3805	1.53	1.116	1.50
7.035	$80^3 \times 20$	180	0.3693	1.48	0.3739	1.50	1.086	1.46
7.192	$96^3 \times 24$	160	0.3728	1.50	0.3790	1.52	1.089	1.46
7.544	$144^3 \times 36$	693	0.3791	1.52	0.3896	1.57	1.089	1.46
7.793	$192^3 \times 48$	223	0.3816	1.53	0.3955	1.59	1.084	1.45

similar studies by [Banerjee,Datta,Gavai,Majumdar, PRD85(2012)014510]
and [H.B.Meyer, New J.Phys.13(2011)035008]

we performed a continuum extrapolation, $a \to 0 \leftrightarrow N_t \to \infty$, at fixed $T=1/a N_t$



well behaved continuum extrapolation for $0.05 \leq \tau T \leq 0.5$

finest lattice already close to the continuum

coarser lattices at larger τT show almost no cut-off effects

how to extract the spectral function from the correlator?

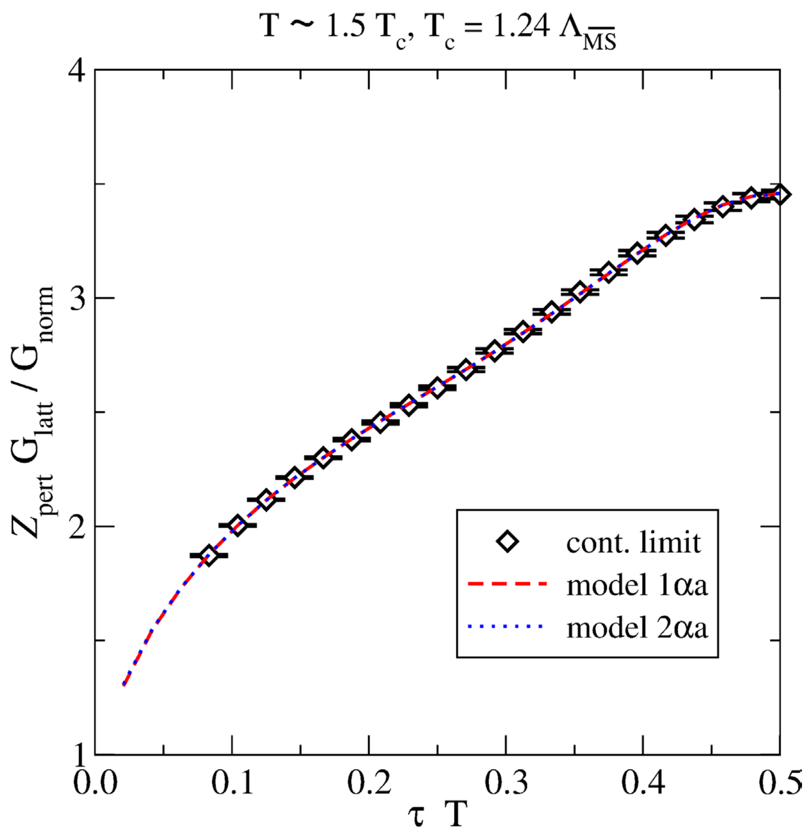
Heavy Quark Momentum Diffusion Constant – systematic uncertainties

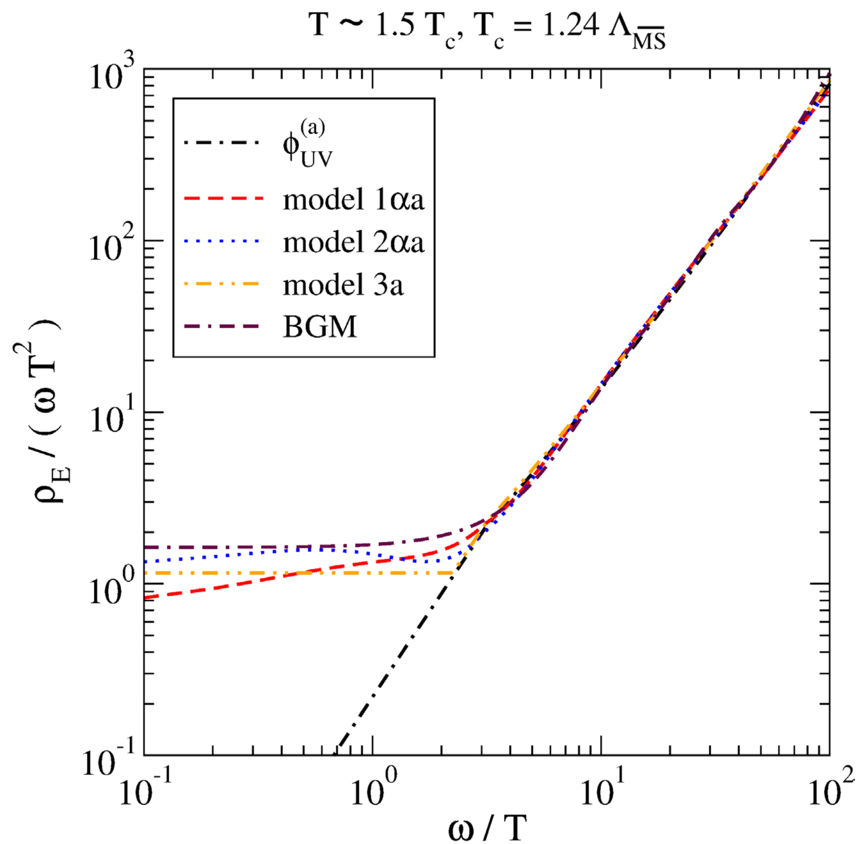
Spectral function models with correct asymptotic behavior

$$\rho_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

modeling corrections to ρ_{IR} by a power series in ω

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$



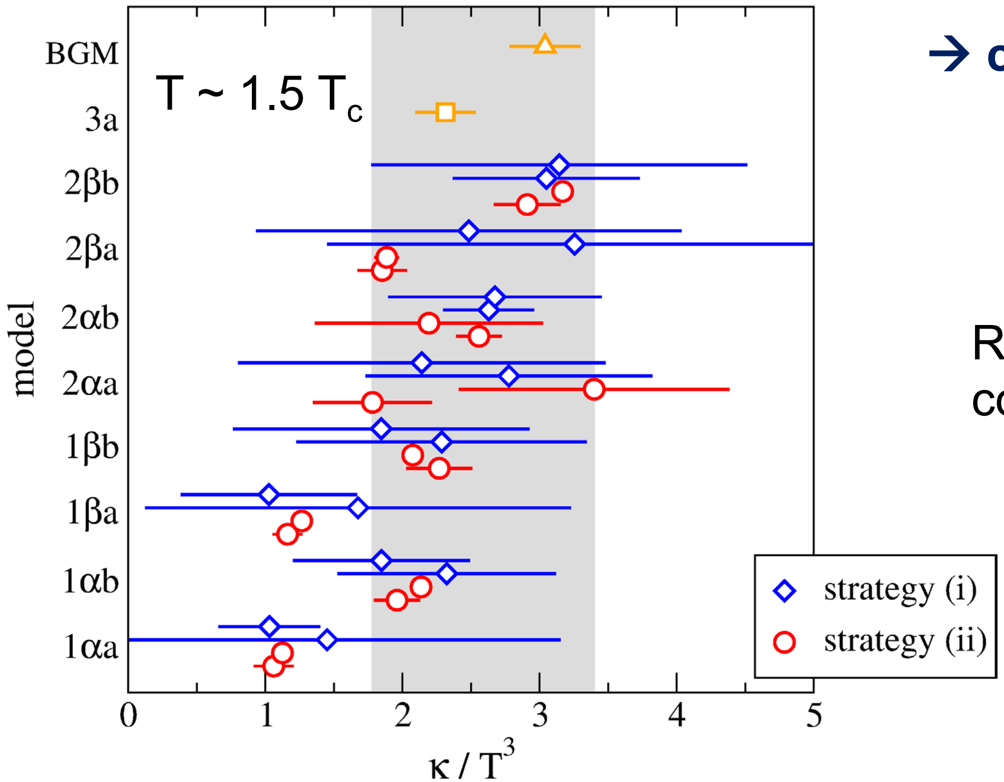


$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

[A.Francis, OK et al., PRD92(2015)116003]



Detailed analysis of systematic uncertainties

→ continuum estimate of κ :

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega} = 1.8 \ldots 3.4$$

Related to diffusion coefficient D and drag coefficient η_D (in the non-relativistic limit)

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7 \ldots 7.0$$

$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

time scale associated with the kinetic equilibration of heavy quarks:

$$\tau_{\mathrm{kin}} = \frac{1}{\eta_D} = (1.8\ldots 3.4)\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ \mathrm{GeV}}\right)\ \mathrm{fm/c}$$

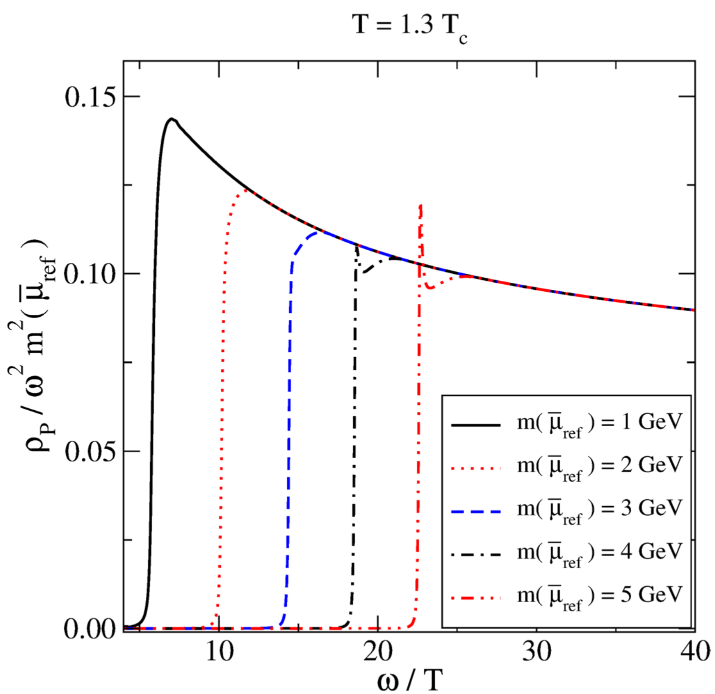
→ close to T_c, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.

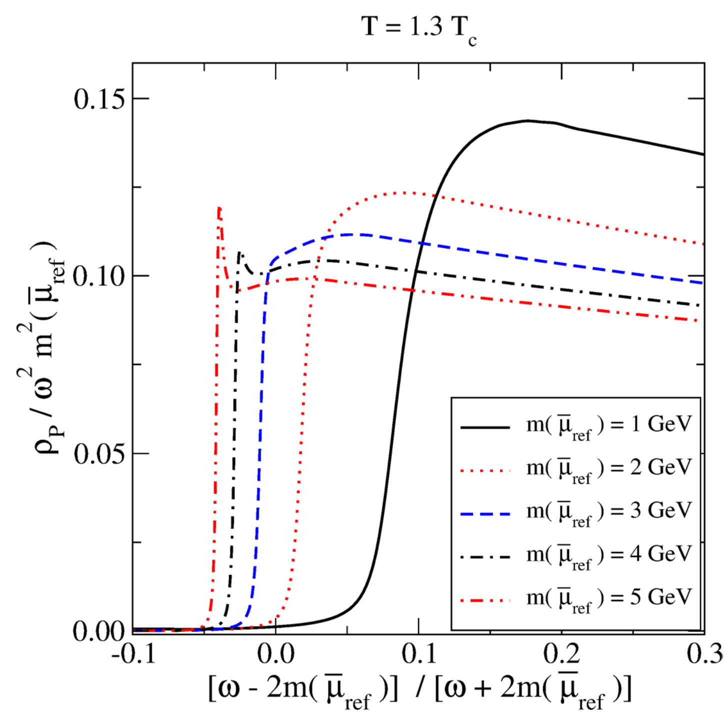
[Y. Burnier, H.-T. Ding, OK et al. JHEP11 (2017) 206]

Using **continuum extrapolated correlation functions** from Lattice QCD

$$G_{\rm P}(\tau) \;\equiv\; M_{\rm B}^2 \int_{\vec{x}} \Big\langle (\bar\psi i\gamma_5\psi)(\tau,\vec{x})\; (\bar\psi i\gamma_5\psi)(0,\vec{0}) \Big\rangle_{\rm c}\,, \quad 0 < \tau < \frac{1}{T}\,,$$

and best knowledge on the spectral function from **perturbation theory and pNRQCD** interpolated between different regimes





we will focus on the pseudo-scalar channel (no transport contribution in this channel) quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

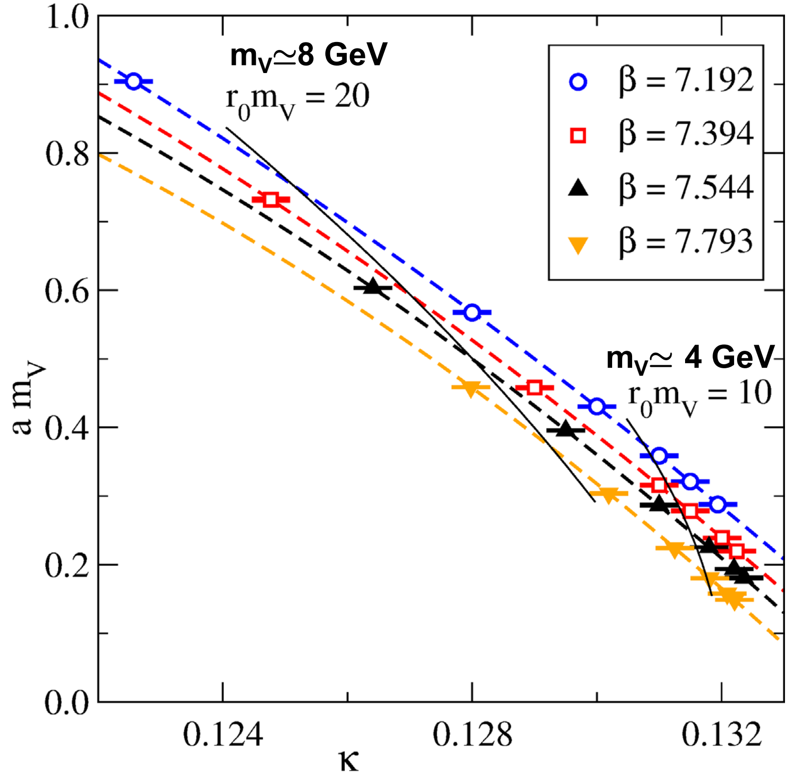
6 quark masses between charm and bottom → interpolate to physical c and b mass

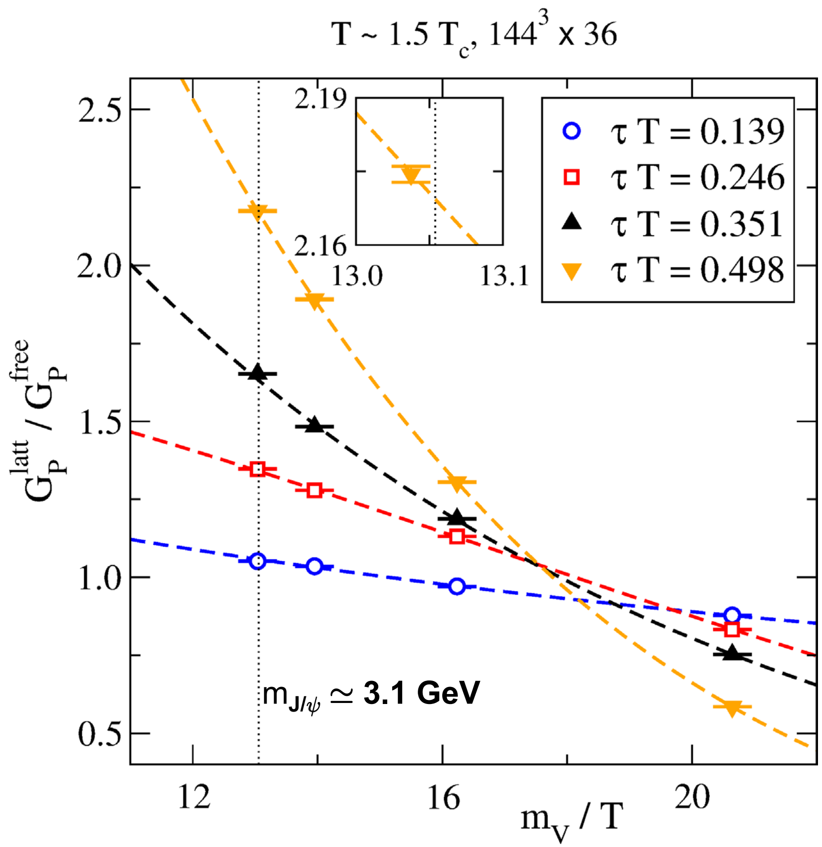
β	N_s	N_τ	confs	r_0/a	T/T_c	c_{SW}	κ_c	κ	$\frac{m^2(1/a)}{m^2(\bar{\mu}_{ref})}$
7.192	96	48	237	26.6	0.74	1.367261	0.13442	0.12257, 0.12800, 0.13000, 0.13100, 0.13150, 0.13194	0.6442
		32	476		1.12				
		28	336		1.27				
		24	336		1.49				
		16	237		2.23				
7.394	120	60	171	33.8	0.76	1.345109	0.13408	0.124772, 0.12900, 0.13100, 0.13150, 0.132008, 0.132245	0.6172
		40	141		1.13				
		30	247		1.51				
		20	226		2.27				
7.544	144	72	221	40.4	0.75	1.330868	0.13384	0.12641, 0.12950, 0.13100, 0.13180, 0.13220, 0.13236	0.5988
		48	462		1.13				
		42	660		1.29				
		36	288		1.51				
		24	237		2.26				
7.793	192	96	224	54.1	0.76	1.310381	0.13347	0.12798, 0.13019, 0.13125, 0.13181, 0.13209, 0.13221	0.5715
		64	249		1.13				
		56	190		1.30				
		48	210		1.51				
		32	235		2.27				

Interpolation to physical c + b bare quark masses required to perform continuum extrap.

→ lines of constant physics defined by vector meson mass at 0.75 T_c

→ interpolate correlators to the physical lines of constant physics





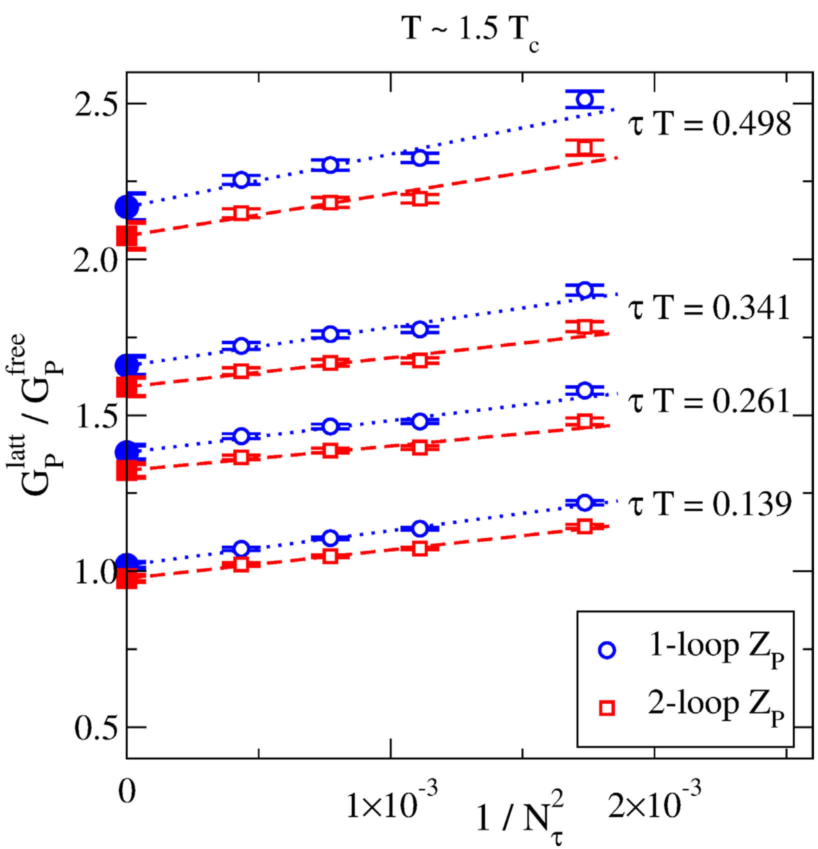
→ continuum extrapolation on lines of constant physics for c and b quark masses

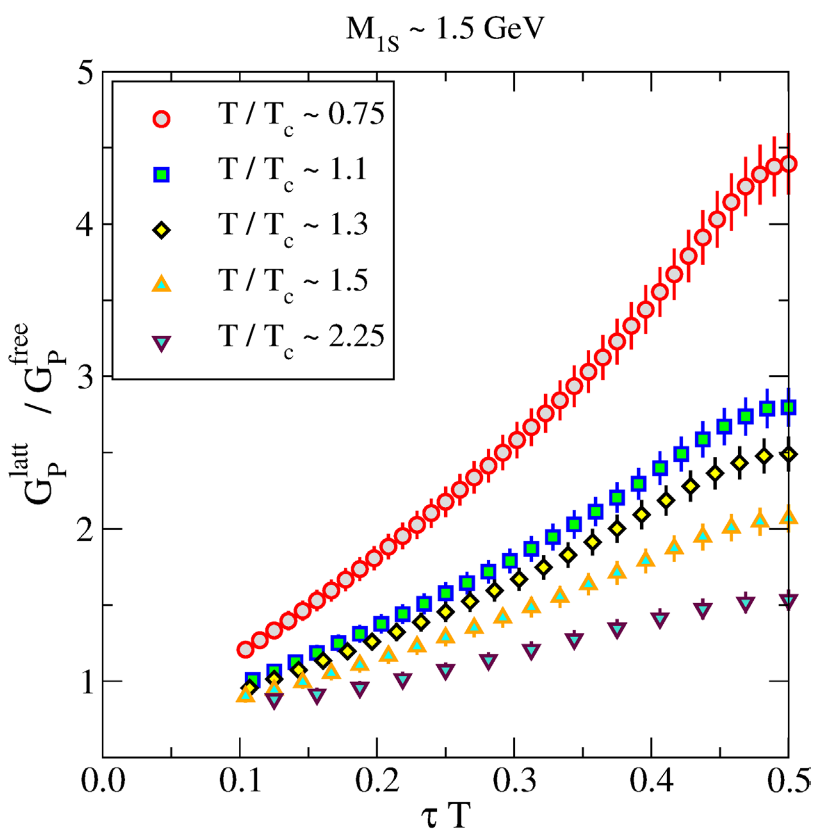
Continuum extrapolation

Continuum limit of the correlation functions in a^2

continuum extrapolation well behaved

some uncertainty in renormalization → well defined continuum correlators



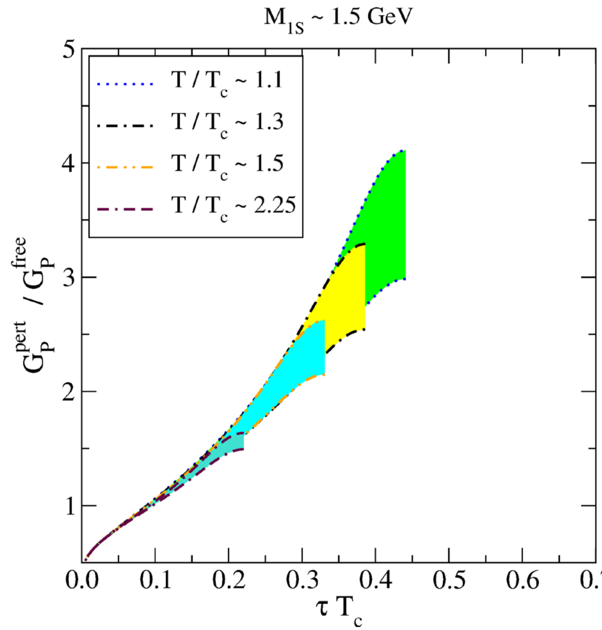


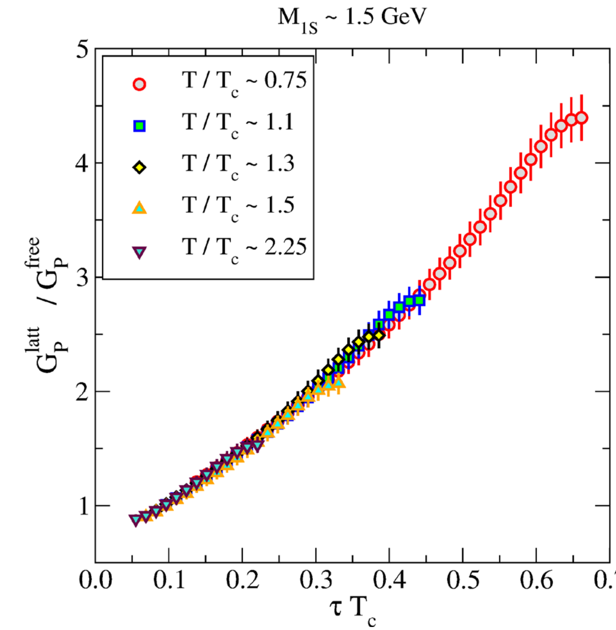
→ comparison to perturbation theory and determination of continuum spectral functions

Continuum extrapolated lattice vs. perturbative correlators

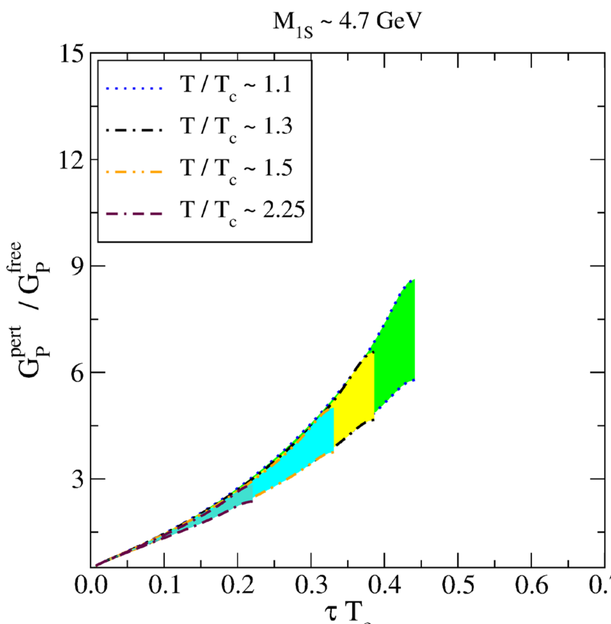
perturbative correlators: lattice correlators (continuum extrap.):

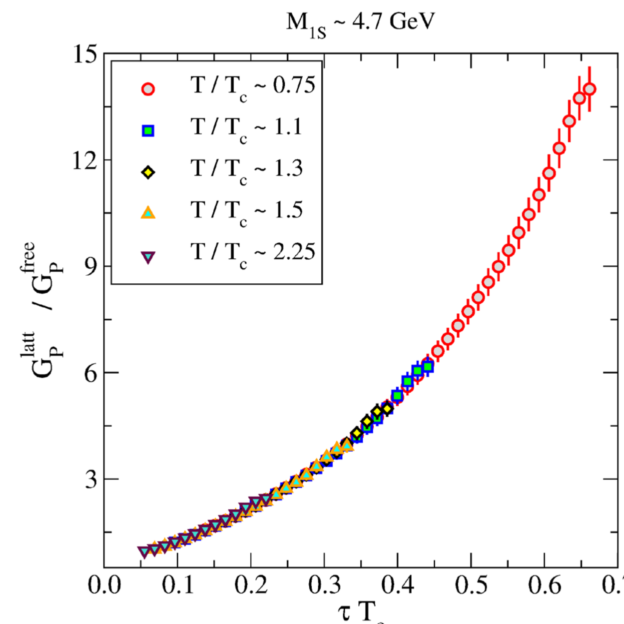
charm:

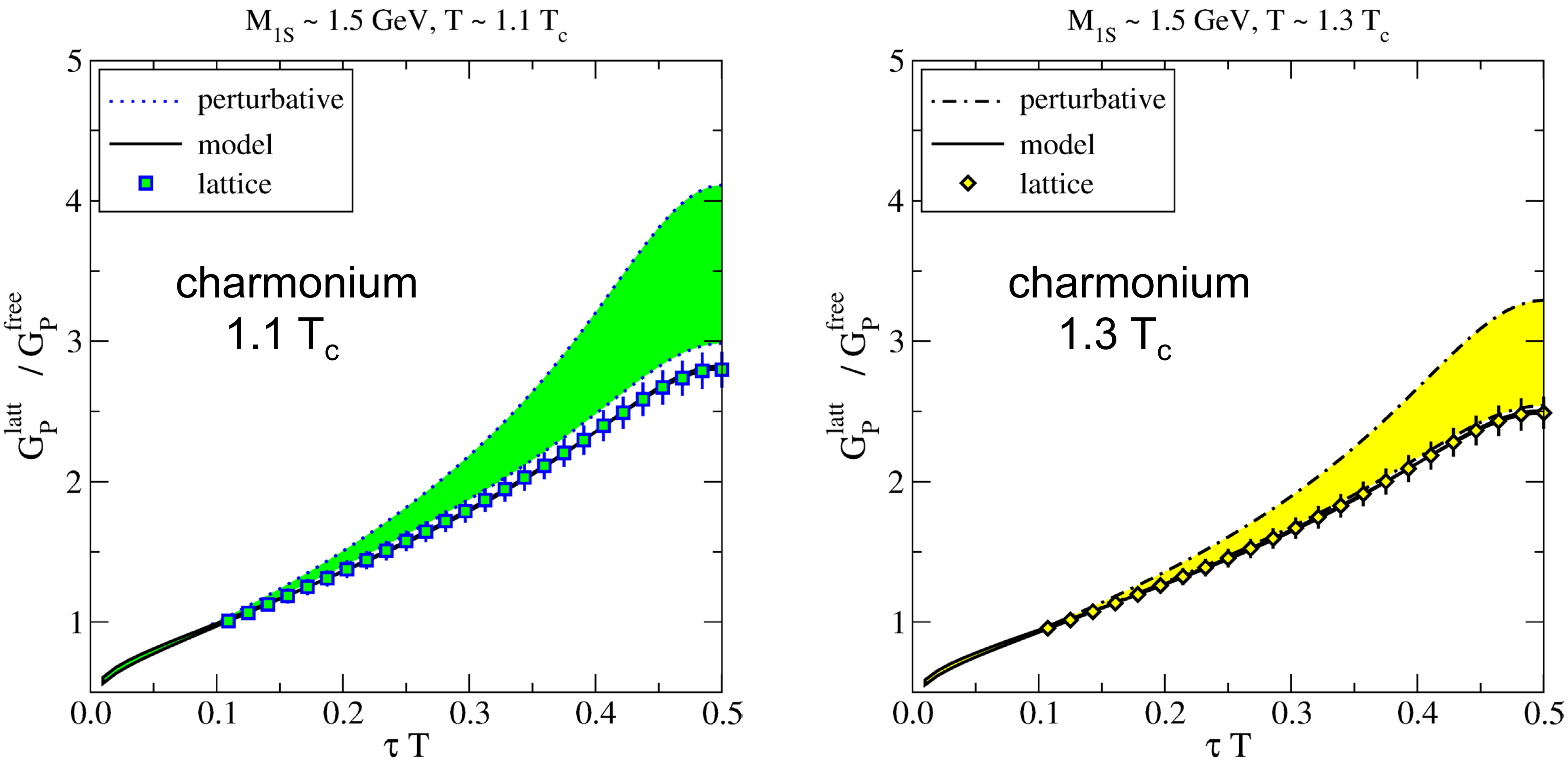




bottom:







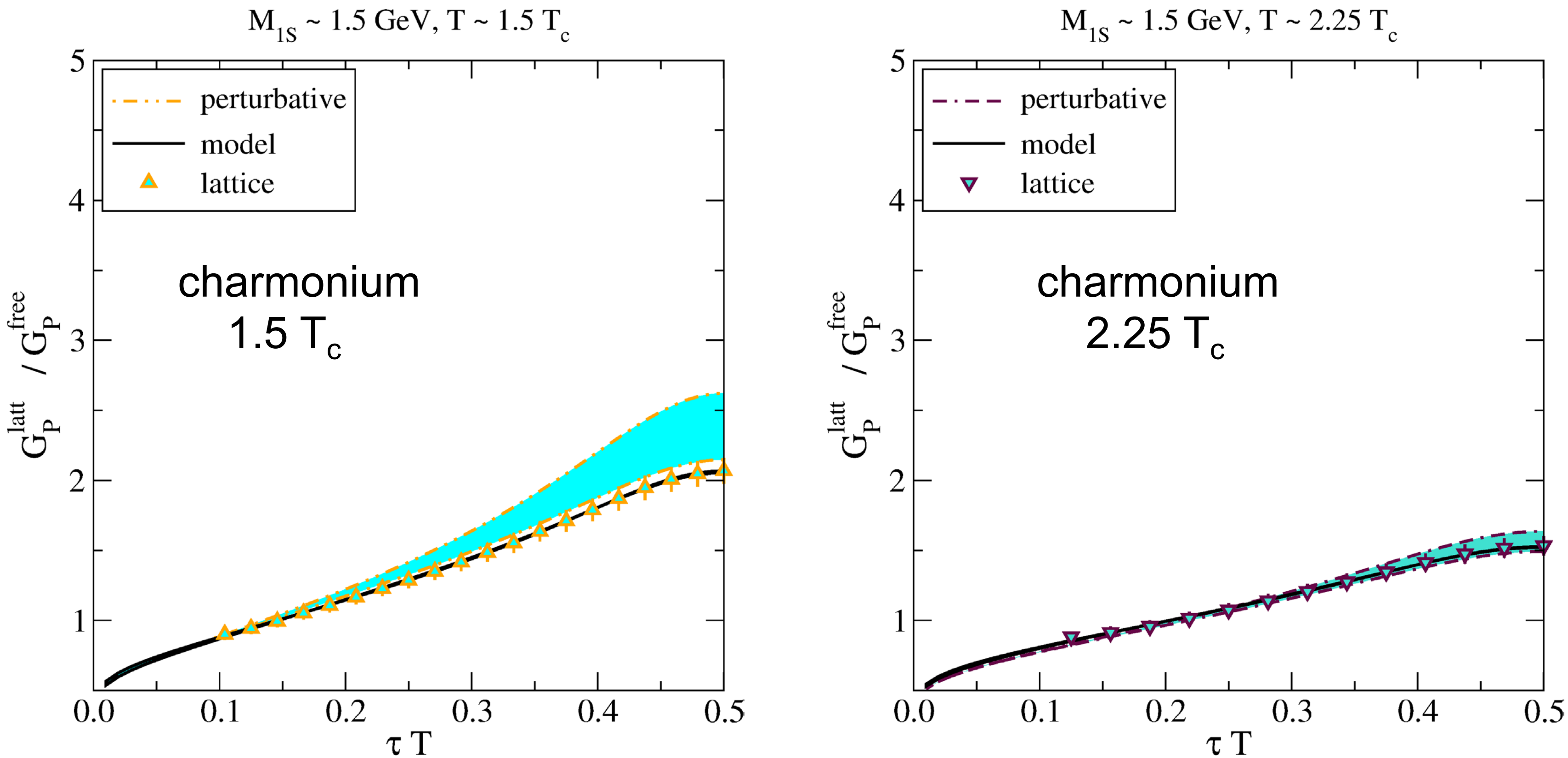
differences between lattice and perturbation theory may have a simple explanation

A: uncertainties related to the perturbative renormalization factors

B: non-perturbative mass shifts

$$\rho_{\mathrm{P}}^{\mathrm{model}}(\omega) \equiv A\,\rho_{\mathrm{P}}^{\mathrm{pert}}(\omega - B)\ .$$

→ continuum lattice data well described by this model with χ^2/d.o.f < 1



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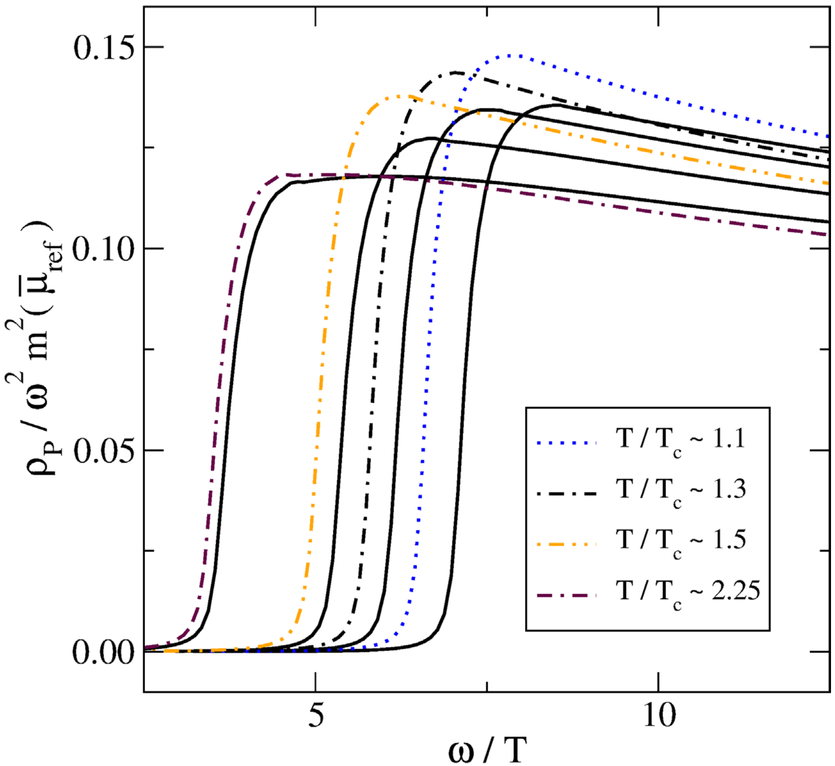
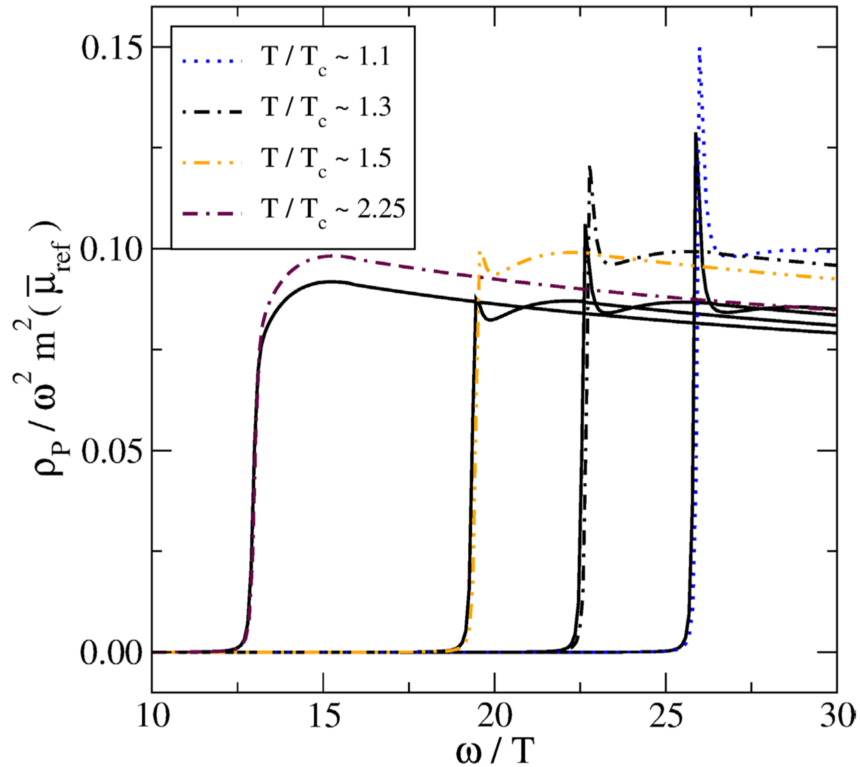
Pseudo-scalar spectral functions

charmonium: $m(\bar{\mu}_{ref}) = 1$ GeV



bottomonium: $m(\bar{\mu}_{ref}) = 5$ GeV



charmonium:

no resonance peaks are needed for representing the lattice data even for 1.1 T_c

modest threshold enhancement sufficient in the analyzed temperature region

bottomonium:

thermally broadened resonance peak present

up to temperatures around 1.5 T_c

next steps:

analysis of the vector channel

heavy quark diffusion coefficient

Conclusion – Disclaimer - Outlook

Continuum extrapolated correlators from quenched lattice QCD are well described by perturbative model spectral functions down to $T \approx T_c$ for observable with an external scale (mass, momentum) $\gtrsim \pi T$

All results in this talk were obtained in the quenched approximation

What may change when going to full QCD?

$$\Lambda_{\overline{MS}}|_{N_f=0} \approx 255\text{MeV} \qquad \Lambda_{\overline{MS}}|_{N_f=3} \approx 340\text{MeV}$$

$$T_c|_{N_f=0} \approx 1.24\Lambda_{\overline{MS}}|_{N_f=0} \qquad T_c|_{N_f=3} \approx 0.45\Lambda_{\overline{MS}}|_{N_f=3}$$

$$\alpha_s^{EQCD}|_{T\simeq T_c} \simeq 0.2 \qquad \alpha_s^{EQCD}|_{T\simeq T_c} > 0.3$$

1st order deconfinement transition **chiral crossover transition**

Physics may become more non-perturbative, more interesting, more complicated…

Quenched theory is a nice playground but full QCD studies crucial!