QCD Thermodynamics at Finite Density in Deconfinement Phase ($T > T_c$).

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Talk by M.A. Andreichikov
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Plan of the talk

- Introduction to Field correlator method
- QCD thermodynamics with FCM
- Effects of finite density in complex μ plane
- Resulting curves for $P(T, \mu)$

Introduction to FCM

We start from Wilson loop

$$\frac{1}{N_c}\langle trP_A e^{-ig\int_C A_\mu dx_\mu}\rangle = \frac{1}{N_c}\langle trP_F e^{-ig\int_C G_{\mu\nu}(x,x_0)d\sigma_{\mu\nu}(x)}\rangle$$

from non-Abelian Stokes theorem with connected F's

$$G_{\mu\nu}(x,x_0) = U(x,x_0)F_{\mu\nu}(x)U^+(x,x_0)$$

where parallel transporters are (fundamental for quarks and adjoint for gluons)

$$U(x,x_0) = P_A exp(ig\int_{x_0}^{x} A_\mu dx'_\mu)$$

One can perform cumulant series (van Kampen)

$$\frac{1}{N_c}\langle trP_F e^{-ig\int_C G_{\mu\nu}(x,x_0)d\sigma_{\mu\nu}(x)} = \frac{1}{N_c}e^{\sum_{n=1}^{\infty}\frac{(i)^n}{n!}\int d\sigma_1 \ldots d\sigma_n \langle\langle G_1 \ldots G_n\rangle\rangle}$$

Bilocal cumulant is defined as

$$\langle\langle G_1 G_2\rangle\rangle = \langle G_1 G_2\rangle - \langle G_1\rangle\langle G_2\rangle$$

Introduction to FCM

If one supposes short correlation length $\lambda \sim 1\ GeV^{-1}$ bilocal cumulant (BC) becomes dominant.

Color-electric (CE) part of the BC - static confinement potential for $T < T_c$ region

$$\langle W(C)\rangle \sim e^{-V(r)T}$$

$$\langle W(C) = \frac{1}{N_c} e^{\frac{g^2}{2}\int\int\langle\langle G_1 G_2\rangle\rangle d\sigma_1 d\sigma_2} \sim \frac{1}{N_c} e^{\frac{g^2}{2}\langle G^2(0)\rangle\lambda^2 S} \to e^{-\sigma S_{min}}$$

where $G^2(0)$ is gluon condensate and $\sigma = 0.18\ GeV^2$ - string tension

Introduction to FCM

Closer look to the BC

$$\langle\langle G_1 G_2\rangle\rangle_{\mu\nu\rho\sigma} = \langle trU(x_0, x_1)F_{\mu\nu}(x_1)U(x_1, x_0)U(x_0, x_2)F_{\rho\sigma}(x_2)U(x_2, x_0)\rangle \to$$

$$\langle trU(x_2, x_1)F_{\mu\nu}(x_1)U(x_1, x_2)F_{\rho\sigma}(x_2)\rangle$$

When the correlation length λ is short enough, one can replace contours with straight lines + gauge invariance of BC

In coordinate gauge $x_\mu A_\mu = 0$ and with $O(4)$-symmetry (Euclidean) one has

$$\langle\langle G_1 G_2\rangle\rangle_{\mu\nu\rho\sigma} = (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(x_1 - x_2)+$$

$$+\frac{1}{2}\left[\frac{\partial}{\partial x_\mu}(x_\rho\delta_{\nu\sigma} - x_\sigma\delta_{\nu\rho}) + perm.\right] D_1(x_1 - x_2)$$

D generates scalar-like interaction, D_1 - vector-like interaction.

Introduction to FCM

Separation of color-electric (CE) and color-magnetic (CM) d.o.f.

$$\frac{g^2}{N_c}\langle trE_i(x_1)UE_j(x_2)U\rangle = \delta_{ij}\left(D^E(u) + D_1^E(u) + u_4^2\frac{\partial D_1^E(u^2)}{\partial u^2}\right) + u_i u_j \frac{\partial D_1^E(u^2)}{\partial u^2}$$

$$\frac{g^2}{N_c}\langle trH_i(x_1)UH_j(x_2)U\rangle = \delta_{ij}\left(D^H(u) + D_1^H(u) + \mathbf{u}^2\frac{\partial D_1^H(u^2)}{\partial \mathbf{u}^2}\right) + u_i u_j \frac{\partial D_1^H(u^2)}{\partial u^2}$$

In hadronic region $T < T_c$ one has $\langle EE\rangle = \langle HH\rangle$ because of symmetry.
An estimate gives cross-correlator $\langle EH\rangle$ to be small in comparison with $\langle EE\rangle$ and $\langle HH\rangle$

Introduction to FCM

Static potentials in a Wilson loop are provided by D and D_1 correlators

$$V_D(r) = 2c_a \int_0^r (r-\lambda) d\lambda \int_0^\infty d\nu D(\lambda,\nu) = V_D^{lin}(r) + V_D^{sat}(r)$$

$$V_1(r) = c_a \int_0^r \lambda d\lambda \int_0^\infty d\nu D_1(\lambda,\nu) = V_1^{sat}(r) + V_{oge}(r)$$

Important: For $T < T_c$ $V_D^{sat}(r)$ and $V_1^{sat}(r)$ compensate each other.
For $T > T_c$ $\langle EE \rangle$ vanish - $V_1^{sat}(r)$ gives Polyakov line $L = e^{-\frac{V_1(\infty)}{2T}}$.

- 4 correlators define thermodynamics is $T < T_c$ phase
- In $T > T_c$ phase one has only D_1^E, D^H, D_1^H

D^H gives gluon Debye mass m_D through the magnetic (spatial) confinmenet mechanism (see below).

Introduction to FCM

Background perturbation theory (hep-ph/9311216)
(color is not changed by the background)
where a_μ is a valence gluon with a propagator

$$A_\mu = B_\mu + a_\mu, \; (G^{-1})^{ab}_{\mu\nu} = -D^2(B)^{ab}\delta_{\mu\nu} - 2gF^c_{\mu\nu}f^{abc}$$

Information about D and D_1 from gluelumps
1-gluon gluelump Green function connected with D^1

$$G^{(1g)}_{\mu\nu}(x,y) = \langle tr \; a_\mu(x)U^{adj}(x,y)a_\nu\rangle_B$$

$$D^1_{\mu\nu\lambda\sigma} = \frac{g^2}{2N_c^2}\left(\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\nu}G^{(1g)}_{\nu\sigma}(x,y) + perm.\right) + \ldots$$

2-gluon gluelump Green function connected with D

$$G^{(2g)}_{\mu\nu\lambda\sigma} = \langle tr \; f^{abc}f^{def}a^a_\mu(x)a^b_\nu(x)T^cU^{adj}(x,y)T^f a^d_\lambda(x)a^e_\sigma(x)\rangle_B$$

$$D(x-y) = \frac{g^4(N_c^2-1)}{2}G^{(2g)}(x-y)$$

Introduction to FCM

Taking the lowest contribution ($T \to \infty$) one has

$$G^{(g)} \sim e^{-M_0^{(g)} T} |\psi_0(0)|^2$$

As a result one has an analytic expressions for D and D_1

$$D(z) \sim e^{-M_0^{(2g)}|z|}$$

$$D^1(z) \sim \frac{M_0^{(1g)} \sigma^{adj}}{|z|} e^{-M_0^{(1g)}|z|}$$

where M_0 are gluelump ground states.

Thermodynamics in FCM approach

let's introduce periodic boundary conditions

$$B_\mu(z_4, \mathbf{z}) = B_\mu(z_4 + n\beta, \mathbf{z});\ \ a_\mu(z_4, \mathbf{z}) = a_\mu(z_4 + n\beta, \mathbf{z});$$

Using the Feynman-Fock-Schwinger proper time formalism

$$\langle Z(B) \rangle = N \langle e^{-\frac{F_0(B)}{T}} \rangle_B$$

$$\frac{F_0(B)}{T} = \frac{1}{2} \ln\ det\ G^{-1} = tr \left[-\frac{1}{2} \int_0^\infty \xi(s) \frac{ds}{s} e^{-sG^{-1}} \right]$$

"Loops" are defined on the cylinder (Matsubara frequences) up to gauge transformation.

Lowest one-loop expression for the free energy takes the form

$$\langle \frac{F_0(B)}{T} \rangle_B = - \int \frac{ds}{s} \xi(s) d^4x (Dz)^w_{xu} e^{-K} \left[\frac{1}{2} \langle tr\ U_F^{adj}(x,x) \rangle_B \right]$$

Thermodynamics in FCM approach

As a result one has an expression for the gluonic pressure

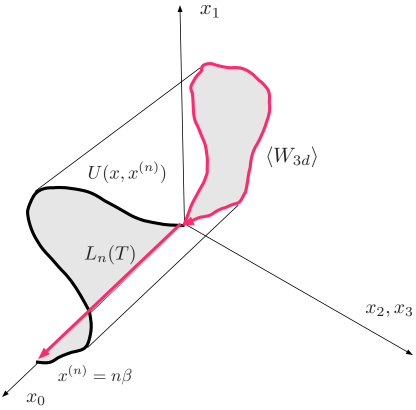
$$P_{gl} = -\frac{\langle F_0^{gl}(B)\rangle_B}{V}$$

$$P_{gl} = (N_c^2 - 1)\int_0^\infty \frac{ds}{s} \sum_{n=0,\pm 1,\dots} G^{(n)}(s)$$

$$G^{(n)}(s) = \int (Dz) e^{-K} \langle tr_a \; W(C_n)\rangle,$$

$$\langle tr_a \; W(C_n)\rangle = \frac{tr_a \; U^{adj}(x, x^{(n)})}{N_c^2 - 1} =$$

$$= (L^{adj}(T))^{(n)} \langle W_3\rangle$$



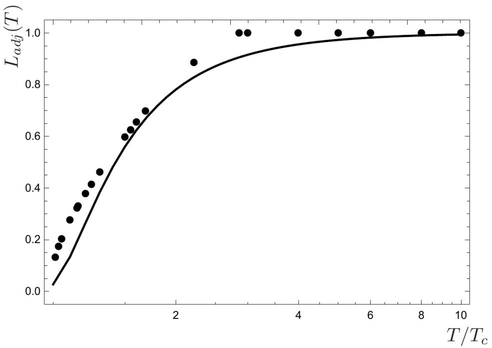
Calculation of loops $G^{(n)}$

- Closing contour with Polyakov line
- CE and CM d.o.f. separation - spatial 3d loop + Polyakov line

Polyakov line vs Lattice

Polyakov line is defined by color-electric the vector-like interaction V_1 with D_1 obtained from gluelumps

$$(L^{adj})^{(n)} = e^{-\frac{9}{4}\frac{V_1(\infty,T)}{2}n}, \;\; V_1^{np} = \frac{A_1}{(M^{1g})^2}\left(1 - \frac{T}{M^{1g}}(1 - e^{-\frac{M^{1g}}{T}})\right)$$



S. Gupta et. al., hep-lat/0608014

Spatial loop

To calculate the 3d Wilson loop, let's define $z_3 = t$, then in FSF representation one has 2d dynamics

$$G_3(s) = \int (D^3 z)_{3d} e^{-K} \langle trW_3(C) \rangle =$$

$$= \int (D^2 z)_{xu} d^2 u (D^2 z)_{ux} e^{-K_1 - K_2 - V^{CM}_{conf} t} \frac{dt}{2\pi s} = \frac{1}{\sqrt{\pi s}} \sum_{\nu=0,1,\dots} |\psi_\nu(0)|^2 e^{-M_\nu^2 s}$$

where M_ν are the eigenvalues of the light-light object (adjoint meson). For linear CMC potential one has

$$G_3(s) = \frac{1}{(4\pi s)^{3/2}} \sqrt{\frac{M_0^2 s}{sh(M_0^2 s)}} \sim \frac{1}{(4\pi s)^{3/2}} e^{-\frac{M_0^2 s}{4}}$$

And the CMC provides color-magnetic Debye mass in a natural way

$$M_0 = 4\sqrt{\sigma_s} = 2m_D$$

Spatial loop and colormagnetic confinement

Summarize: one has an area law for the 3d spatial loop that is provided by the scalar potential V^{lin} from the color-magnetic cumulant $\langle HH \rangle$ with function D^H

$$\langle W_3 \rangle = e^{-\sigma_s S_{3d}}$$

Spatial string tension grows! with T with asymptotics $\sigma_s(T) \sim g^4(T)T^2$ (arXiv: 1605.07060) for $T \gg Tc$ - QGP equation of state never reach Stefan-Boltzmann limit!

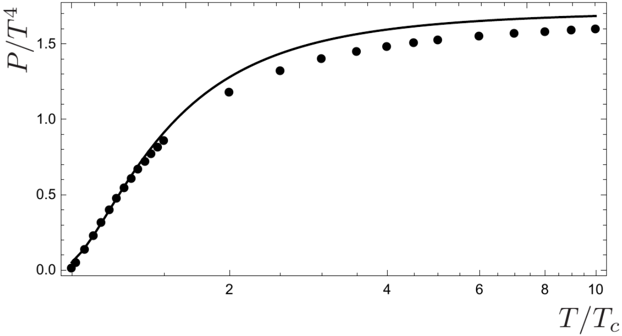
Differences between the CE and the CM confinement

- Both are from area law and scalar interactions
- CE is in $0i$ coordinates and gives potential $e^{-\sigma|r|T}$ and is absent in QGP phase.
- CM is in ij coordinates and gives screening Debye mass for valence gluon (quark) in QGP phase.

Gluon pressure for $T > T_c$

Final expression for the gluonic pressure in compariosn with the lattice data

$$P_{gl} = \frac{2(N_c^2-1)}{(4\pi)^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{ds}{s^2}e^{-\frac{n^2}{4T^2 s}}\sqrt{\frac{m_D^2}{sh(m_D^2 s)}}(L^{adj})^n$$



Sz. Borsanyi et. al., arXiv:1204.6184 [hep-lat]

Quark pressure for $T > T_c$

The same approach hold for quarks f = 2+1 up to fundamental representation

$$P_q^{(f)} = \frac{4N_c}{\sqrt{4\pi}} \int_0^\infty \frac{ds}{s^{3/2}} e^{-m_f^2 s} G_3^{(q)}(s) \sum_{n=1,2,\dots} (-1)^{(n+1)} e^{-\frac{n^2}{4T^2 s}} ch\left(\frac{\mu^f n}{T}\right) (L^{fund})^n$$

where μ^f are flavour chemical potentials.
Total quark pressure is

$$P_q = \sum_f P_q^{(f)}$$

As in the case of gluons, one can estimate spatial loop analytically for linear confinement interaction as

$$G_3^{(q)}(s) = \frac{1}{(4\pi s)^{3/2}} \sqrt{\frac{(m_D^2 s)}{sh(m_D^2 s)}} \simeq \frac{1}{(4\pi s)^{3/2}} e^{-\frac{m_D^2}{4} s}$$

Quark pressure for $T > T_c$

Summing up over the Matsubara frequences and introducing the u variable

$$\frac{P_q^{(f)}}{T^4} = \frac{N_c}{\pi^2}(\xi^+ + \xi^-)$$

with

$$\xi^{\pm} = \frac{4}{3}\left(\frac{\bar{M}}{2T}\right)^2 \int_0^\infty \frac{u^4 du}{\sqrt{1+u^2}} \frac{1}{1 + exp\left(\frac{\bar{M}}{T}\sqrt{1+u^2} + \frac{V_1}{2T} \pm \frac{\mu}{T}\right)}$$

And the magnetic Debye mass plays role of the quark effective mass

$$\bar{M} = \sqrt{m_f^2 + \frac{m_D^2}{4}}$$

Roberge-Weiss singularities in complex μ-plane

The denominator bacame singular at $u = 0$

$$1 + exp\left(\frac{\bar{M}}{T}\sqrt{1+u^2} + \frac{V_1}{2T} \pm \frac{\mu}{T}\right)$$

Polyakov loop has complex phase $\phi_k = \frac{2\pi}{3}k,\ \ k = 0, \pm 1, ...$ One can reach a singularity for

$$\frac{Im(\mu)}{T} = \frac{\pi}{3}(2n+1),\ \ n = 0, \pm 1, ...$$

These are exactly Roberge-Weiss singularities observed on the lattice A.Roberge, N.Weiss, Nucl. Phys. **B 275**, 734 (1986).

Roberge-Weiss singularities in complex μ-plane

To determine the position of the Roberge-Weiss points for arbitrary CMC interaction one can return to the initial expression for the spatial loop through the Hamiltonian eigenvalues

$$D_3^{(q)}(s) = \frac{1}{\sqrt{\pi s}} \sum_{\nu=0,1,\dots} |\psi_\nu(0)|^2 e^{-m_\nu^2 s}$$

After the summation one has

$$\frac{P_q^{(f)}}{T^4} = \frac{8N_c}{\pi T^4} \sum_\nu |\psi_\nu(0)|^2 M_\nu^2 \int_0^\infty sh^2 t dt \frac{1}{2} \left(\frac{c_+}{1+c_+} + \frac{c_-}{1-c_-} \right)$$

with 3d spatial loop bound state mass provides an effective quark mass

$$M_\nu = \sqrt{m_f^2 + m_\nu^2}$$

and functions

$$c_\pm = e^{-\frac{M_\nu}{T} ch(t) - \frac{V_1}{2T} \pm \frac{\mu}{T}}$$

define the singularities in the denomiantor at $t = 0$ as in the previous case

Roberge-Weiss singularities in complex μ-plane

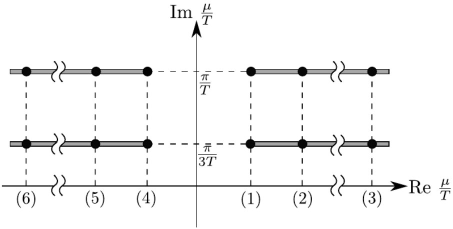


Figure 1: Roberge-Weiss singular points and cuts in the complex plane of μ. Points 1,2,3,4,5, and 6 are, respectively, $\left(\frac{V_1}{2T}+\frac{M_0}{T}\right)$, $\left(\frac{V_1}{2T}+\frac{M_0\sqrt{3}}{T}\right)$, $\left(\frac{V_1}{2T}+\frac{M_\nu}{T}\right)$, $-\left(\frac{V_1}{2T}+\frac{M_0}{T}\right)$, $-\left(\frac{V_1}{2T}+\frac{M_0\sqrt{3}}{T}\right)$, and $-\left(\frac{V_1}{2T}+\frac{M_\nu}{T}\right)$. In the lower half plane the points are mirror-reflected of the axis $\mathrm{Re}\,(\mu/T)$.

y defines the coordinate in complex μ-plane in the vicinity of the singularity

$$\frac{\mu}{T} = i\pi + \frac{M_\nu + \frac{V_1}{2}}{T} + \frac{M_\nu}{T} y$$

In the vicinity of the singularities an intgral is proportional to

$$f(y) \sim \int_0^\infty \frac{t^2 F(t) dt}{t^2 - 2y};\ f(y+i\delta) - f(y-i\delta) = i\sqrt{2y}F(\sqrt{2y})$$

And one has square-root-type singularities with branching points.

Results: total pressure vs T and μ

$$P(\mu, T) = P_{gl}(T) + \sum_f P_q^{(f)}(\mu^f, T)$$

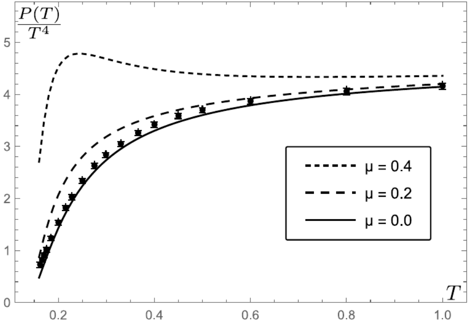


Figure 3: The pressure $\frac{P(T)}{T^4}$ with $M_0 = 3.5\sqrt{\sigma_s}$ for $\mu = 0.0,\ 0.2,\ 0.4$ (from bottom to top), – filled dots are for the lattice data from [16].

Sz. Borsanyi et. al., arXiv:1007.2580

Results: Dependence of the pressure on the spatial string tension

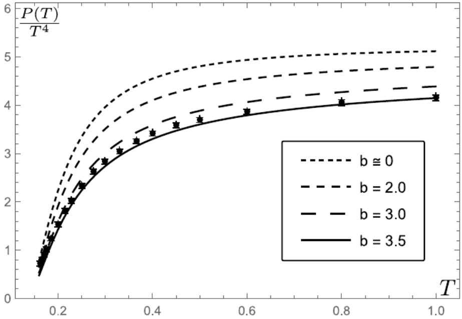


Figure 2: The pressure $\frac{P(T)}{T^4}$ with $M_0 = b\sqrt{\sigma_s}$ ($\mu = 0$), where $b = 0,\ 2.0,\ 3.0,\ 3.5$ (from top to bottom), – filled dots are for the lattice data from [16].

Spatial string tension depends on T with asymptotics $\sigma_s(T) \sim g^4(T)T^2$ (arXiv: 1605.07060) for $T \gg Tc$ makes SB limit unreachable in $T \geq T_c$ region.