Exotic states and their properties in large – N_c QCD

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We will discuss properties of those Green functions that may contain tetraquark poles in large-N_c QCD: diagram selection criteria and consistency conditions.

Based on W.L., D.M, H.S.,

Narrow exotic tetraquark mesons in large-N_c QCD, **PRD96, 014022, 2017;**

Tetraquark and two-meson states at large-N_c, **EPJC77, 866, 2017.**

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QCD at large Nc

$SU(N_c)$ gauge theory with $N_c \to \infty$ and $\alpha_s \sim 1/N_c$. At leading order, QCD Green functions have only non-interacting mesons as intermediate states; tetraquark bound states may emerge only in N_c-subleading diagrams. This fact was believed to provide the theoretical explanation of the non-existence of exotic tetraquarks.

However, even if the exotic tetraquark bound states appear only in subleading diagrams, the crucial question is their width: if narrow, they might be well observed in nature.

We discuss four-point Green functions of bilinear quark currents, depend on 6 variables p_1^2, p_2^2, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, and the two Mandelstam variables $s = p^2$ and $t = (p_1 - p_1')^2$.

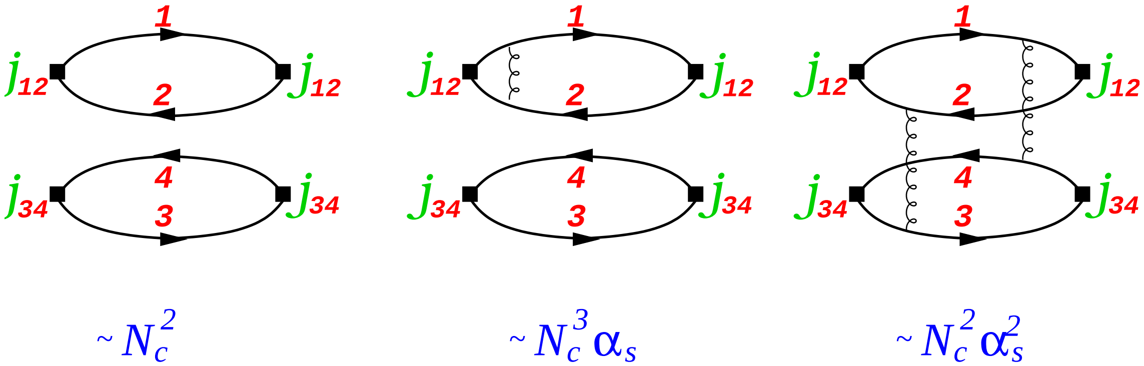
Criteria for selecting diagrams which potentially contribute to the tetraquark pole at $s = M_T^2$:

1. **The diagram should have a nontrivial (i.e., non-polynomial) dependence on the variable s.**
2. **The diagram should have a four-particle cut (i.e. threshold at $s = (m_1 + m_2 + m_3 + m_4)^2$), where m_i are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.**

Flavour – exotic tetraquarks (u $\bar{c}$ d $\bar{s}$)

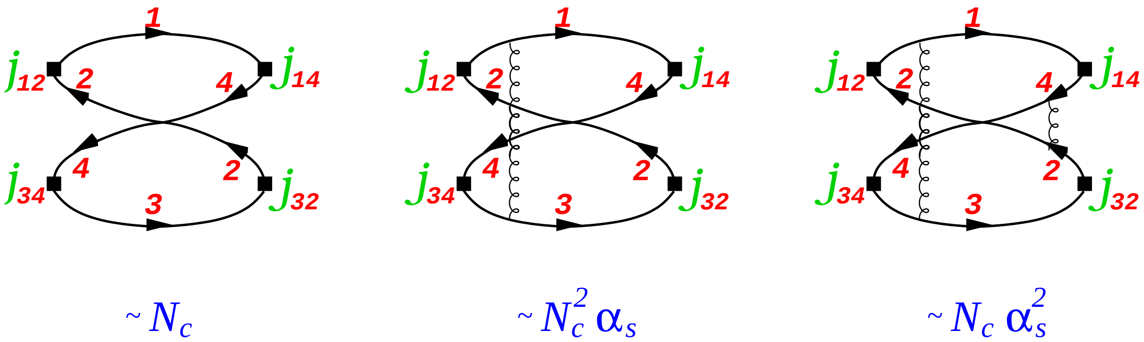
Bilinear quark currents $j_{ij} = \bar{q}_i q_j$ **producing** M_{ij} **from the vacuum,** $\langle 0|j_{ij}|M_{ij}\rangle = f_{M_{ij}}$, $f_M \sim \sqrt{N_c}$.

“Direct” 4-point functions $\Gamma_{\rm I}^{\rm (dir)} = \langle j_{12}^\dagger j_{34}^\dagger j_{12} j_{34}\rangle$ **and** $\Gamma_{\rm II}^{\rm (dir)} = \langle j_{14}^\dagger j_{32}^\dagger j_{14} j_{32}\rangle$**:**

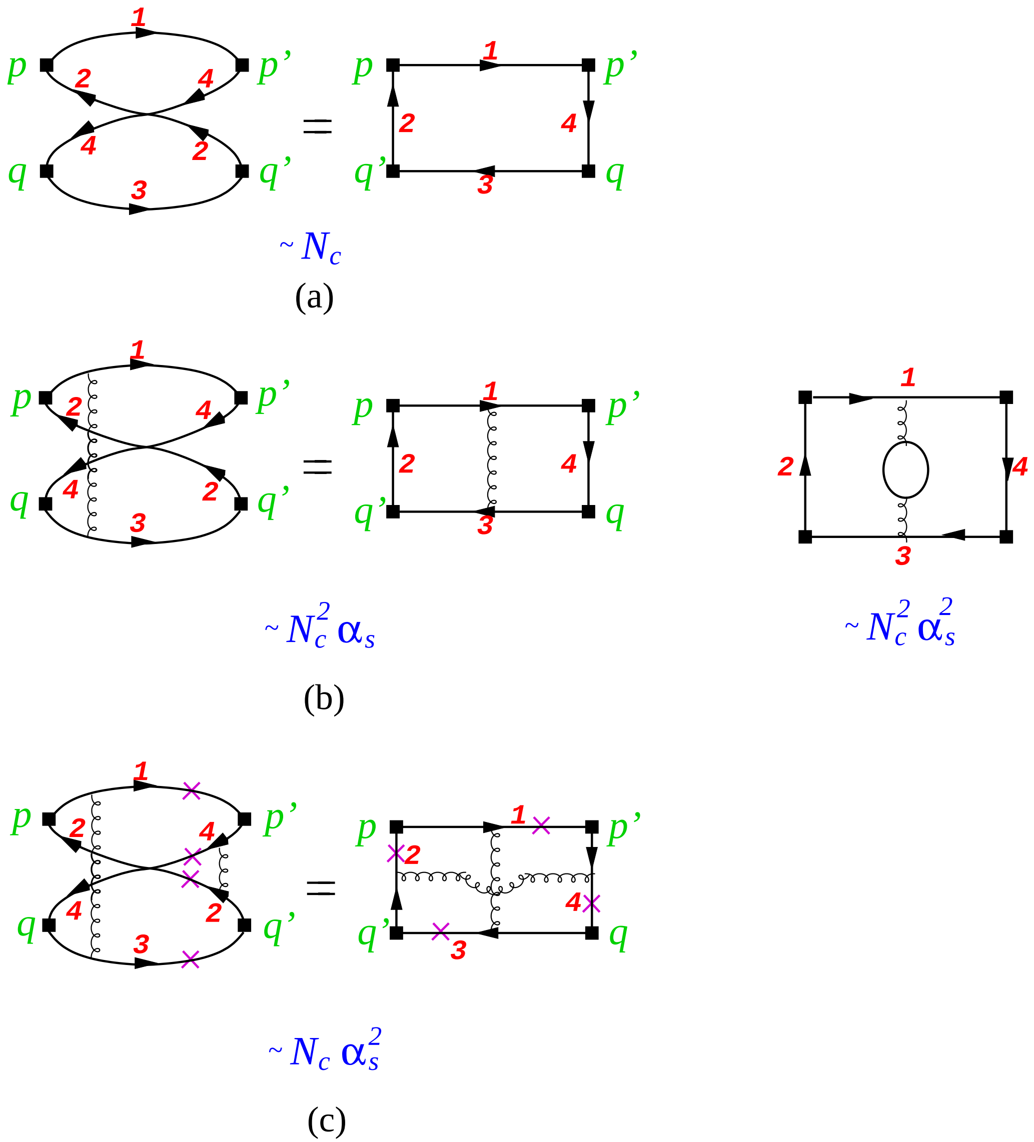


(a) (b) (c)

“Recombination” functions $\Gamma^{\rm (rec)} = \langle j_{12}^\dagger j_{34}^\dagger j_{14} j_{32}\rangle$ **and** $\Gamma^{{\rm (rec)}\dagger}$**:**



(a) (b) (c)



p
p'
q
q'
1
2
3
4
=
~ N_c
(a)
~ $N_c^2 \alpha_s$
~ $N_c^2 \alpha_s^2$
(b)
~ $N_c \alpha_s^2$
(c)

Direct and recombination Green functions have different large – Nc behaviour

$$\Gamma^{(\rm dir)}_{\rm I,T} = \langle j^\dagger_{12} j^\dagger_{34} j_{12} j_{34}\rangle = O(N_c^0), \quad \Gamma^{(\rm dir)}_{\rm II,T} = \langle j^\dagger_{14} j^\dagger_{32} j_{14} j_{32}\rangle = O(N_c^0), \quad \Gamma^{(\rm rec)}_{T} = \langle j^\dagger_{12} j^\dagger_{34} j_{14} j_{32}\rangle = O(N_c^{-1}).$$

If one has a bound state, its contribution cannot exceed these bounds at large N_c ($f_M \sim \sqrt{N_c}$):

$$\Gamma^{(\rm dir)}_{\rm I,T} = O(N_c^0) = f_M^4\left(\frac{|A(M_{12}M_{34}\to T_A)|^2}{p^2-M^2_{T_A}} + \frac{|A(M_{12}M_{34}\to T_B)|^2}{p^2-M^2_{T_B}}\right) + \cdots,$$

$$\Gamma^{(\rm dir)}_{\rm II,T} = O(N_c^0) = f_M^4\left(\frac{|A(M_{14}M_{32}\to T_A)|^2}{p^2-M^2_{T_A}} + \frac{|A(M_{14}M_{32}\to T_B)|^2}{p^2-M^2_{T_B}}\right) + \cdots,$$

$$\Gamma^{(\rm rec)}_{T} = O(N_c^{-1}) = f_M^4\left(\frac{A(M_{12}M_{34}\to T_A)A(T_A\to M_{14}M_{32})}{p^2-M^2_{T_A}} + \frac{A(M_{12}M_{34}\to T_B)A(T_B\to M_{14}M_{32})}{p^2-M^2_{T_B}}\right) + \cdots.$$

Molecular states

(bound states in effective meson-meson theory)

$$L_{\rm eff} = \frac{\lambda_1}{N_c}\phi_{1\bar 2}\phi_{3\bar 4}(\phi_{1\bar 2}\phi_{3\bar 4})^\dagger + \frac{\lambda_2}{N_c^2}\phi_{1\bar 2}\phi_{3\bar 4}(\phi_{1\bar 4}\phi_{3\bar 2})^\dagger + h.c.$$

Summation of meson blobs in a two-channel problem leads to two poles of the masses $M^2 = O(N_c)$.

Molecular four-quark states are broad at large N_c.

Narrow states with finite mass

Are bound states with a finite mass at large N_c allowed? Yes, but need 2 states T_a and T_b with different preferred decay channels.

T_A **couples stronger to** $M_{12}M_{34}$ **channel,** T_B **couples stronger to** $M_{14}M_{32}$ **channel.**

$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}),$$
$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}).$$

The widths $\Gamma(T_{A,B}) = O(N_c^{-2})$**.**

What could be the structure of these states? **Most likely, diquark-antidiquark.**

But we have one flavour combination $ud\ \bar{s}\bar{c}$**. How can one form out of this one flavour combination two states with different preferred channels? (other quantum numbers are unlikely to help)**

Hard to find a sensible scenario.

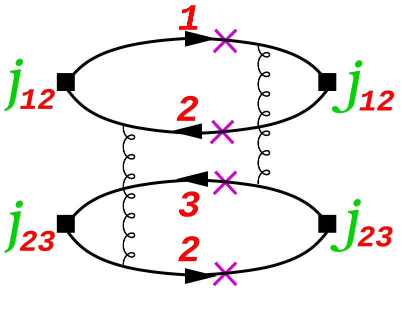
One may speculate that flavour-exotic narrow states do not exist.

(In fact such candidates have not been seen yet).

Crypto – exotic tetraquarks ($c\,\bar{c}\,u\,\bar{s}$)

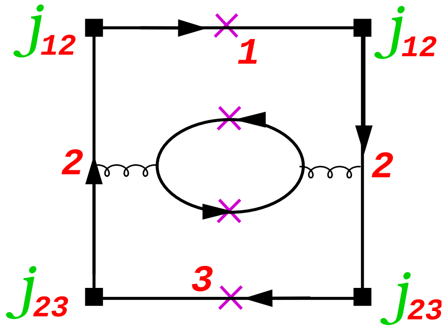
Diagrams of new topologies emerge.

For direct functions $\Gamma^{(\mathrm{dir})}_{(\mathrm{I,II}),T}$, new diagrams do not change leading large-N_c behavior:



$\sim N_c^2\,\alpha_s^2$

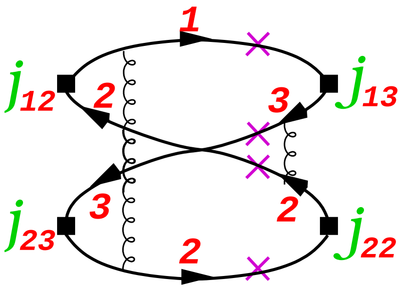
(a)



$\sim N_c^2\,\alpha_s^2$

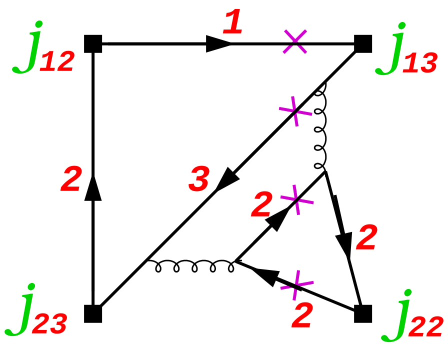
(b)

For recombination functions, the new diagram modifies leading large-N_c behavior



$\sim N_c\,\alpha_s^2$

(a)



$\sim N_c^2\,\alpha_s^2$

(b)

The new diagram modifies the leading large-N_c behavior:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{23} j_{12} j_{23} \rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{13} j^\dagger_{22} j_{13} j_{22} \rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{23} j_{13} j_{22} \rangle = O(N_c^0).$$

"dir" and "rec" functions have the same behavior, and one exotic state T is enough:

$$A(T \to M_{12} M_{23}) \; = \; O(N_c^{-1}), \qquad A(T \to M_{13} M_{22}) = O(N_c^{-1}).$$

Its width is $\Gamma(T) = O(N_c^{-2})$.

T can mix with the ordinary meson M_{13}. The restriction on the mixing parameter $g_{TM_{13}}$:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = O(N_c^0) = f_M^4 \left(\frac{A(M_{12} M_{23} \to T)}{p^2 - M_T^2} g_{TM_{13}} \frac{A(M_{13} \to M_{12} M_{23})}{p^2 - M_{M_{13}}^2} \right) + \cdots .$$

$A(M_{13} \to M_{12} M_{23}) \sim 1/\sqrt{N_c}$**, so** $g_{TM_{13}} \leq O(1/\sqrt{N_c})$**.**

The analysis of Green functions in large-N_c QCD allows one to restrict some properties of the possible exotic states.

Conclusions

• *QCD at large N_c*:

Large-N_c QCD restricts properties of the exotic poles if such poles exist.

E.g., two exotic $\bar{q}_1 q_2 \bar{q}_3 q_4$ narrow states $\Gamma \sim O(1/N_c^2)$, each decaying preferrably into one meson-meson channel are needed on the basis of N_c-leading behaviour of Green functions. This could be a non-molecular state with $D\bar{D}$ structure. Since for a flavour-exotic state one has only one $D\bar{D}$ configuration, it is hard to imagine that two states with different preferred decay channels may be formed. The existence of flavour-exotic narrow tetraquarks is unlikely.

One cryptoexotic state $\bar{q}_1 q_2 \bar{q}_2 q_3$ $\Gamma \sim O(1/N_c^2)$ decaying into various meson-meson channels with similar probabilities.

• *Dynamics of exotic-state decays and implications for QCD sum rules*:

Dynamics of fall-apart decays of exotic resonances has fundamental difference from dynamics of ordinary-meson decays: the appropriate contributions to Green functions describing decays of exotic states emerge only at subleading α_s orders; the leading order disconnected diagrams are not related to strong decays of exotic hadrons. This makes the calculation of α_s-corrections mandatory.