Pseudoscalar pole contribution to the hadronic light-by-light piece of $a_\mu$

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The main purpose of this work is to reduce the theoretical uncertainty in the computation of the $a_\mu$, in which the main source of uncertainty comes from the hadronic contributions. This is why we decided to analyze the hadronic light-by-light contribution using $\chi$PT extended to include resonances.
Magnetic moment

- The Dirac equation predicts a magnetic moment for a particle with EM charge $Q$ and mass $m$

\[
\mu_\ell = g_\ell \frac{Q}{2m} \mathbf{s}
\]

- such that $g_\ell = 2$. This is obtained for a classic EM field.

- The deviation from $g_\ell = 2$ defines the anomalous magnetic moment, which will happen due to loop corrections.

\[
a_\ell := \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) \approx 0.00116.
\]
Contributions to $a_\mu$

- The computation of $a_\mu$ can be split in different contributions, whose values can be found in PDG$^1$

$$a_\mu = a^{QED}_\mu + a^{EW}_\mu + a^{Had}_\mu$$

- $a^{QED}_\mu$ are all corrections$^2$ that might come from QED

$$a^{QED}_\mu = 116584718.95(0.08) \times 10^{-11} + \mathcal{O}\left(\frac{\alpha}{\pi}\right)^6$$

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$^1$C. Patrignani et al. (Particle Data Group), Chin.Phys.C40(2016)

$^2$T. Aoyama et al. PRL 109(2012)
$a_{\mu}^{EW}$ and contributions to $a_{\mu}$

- $a_{\mu}^{EW}$ are Electroweak contribution that are not $a_{\mu}^{QED}$ ($W^\pm, Z, H$) at two loops\(^3\). Three loops contribution is negligible ($\lesssim 0.4 \times 10^{-11}$).

$$a_{\mu}^{EW} = 153.6(1.0) \times 10^{-11}$$

Hadronic contributions

- \( a^{\mu}_{\text{Had}} \) can be split into two parts, the PDG values are\(^4\)

\[ a^{\mu}_{\text{HVP}} = 6845(33)(7) \times 10^{-11} \]

- Hadronic Vacuum Polarization (HVP) contribution.

\[ a^{\mu}_{\text{HLbL}} = 105(26) \times 10^{-11} \]

- Hadronic light-by-light (HLbL) contribution.

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Hadronic contributions to $a_\mu$

- All the contributions and their uncertainties are shown in the next table.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$\times 10^{11}$</th>
<th>Uncertainty $\times 10^{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>116 584 718.95</td>
<td>0.08</td>
</tr>
<tr>
<td>EW</td>
<td>153.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Had</td>
<td>6950 (34)</td>
<td>(34) Vac. Pol. (26) Light-by-Light</td>
</tr>
<tr>
<td>Total</td>
<td>116 591 823</td>
<td>(34)(26)</td>
</tr>
<tr>
<td>Exp</td>
<td>116 592 091</td>
<td>(54)(33)</td>
</tr>
</tbody>
</table>

- Clearly, the largest uncertainty comes from the hadronic contribution.

- With these values there is a discrepancy

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 268(63)(43) \times 10^{-11} \sim 3.5\sigma$$
Hadronic contributions to $a_\mu$

- The main uncertainty comes from hadronic contributions\(^5\), which give $4.3 \times 10^{-10}$.

- The current experimental\(^6\) error is $6.3 \times 10^{-10}$.

- Fermilab & J-Parc are planning to lower\(^7\) the error to $1.6 \times 10^{-10}$. It is necessary to reduce theoretical uncertainty.

- A reanalysis of $R_{\text{had}}$ from Lattice QCD may reduce\(^8\) the HVP error ($3.3 \times 10^{-10}$) below that of the HLbL piece.

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\(^6\)G. W. Bennet et al., [Muon g-2 Collab.],PRD73(2006)


\(^8\)Talks given at the Muon g-2 Theory Initiative Workshops held during the last year at FNAL, Tsukuba, Connecticut Univ. and Mainz Univ.
Hadronic Light by Light

- We decided to analyze the HLbL piece since, nowadays, it can’t be obtained from experimental data.

- It can be separated into three parts.

- The sum of (b) and (c) is\(^9\) one order of magnitude smaller than (a).

Pseudoscalar pole

- Our contribution to $a_\mu$ comes from diagram (a)

- To compute the pion transition form factor $F_{\pi\gamma^*\gamma^*}$ we rely on Resonance Chiral Theory\(^\text{10}\) (R\(\chi\)T) with $U(3)$ breaking.

P.D. Ruíz-Femenía et al., JHEP 0307 (2003)
K. Kampf and J. Novotný PRD84 (2011)
We include corrections up to $O(m_P^2)$. Some of this give\(^\text{11}\)
\[
M^2_\rho = M^2_\omega = M^2_V - 4e^V_m m^2_\pi, \quad M^2_\phi = M^2_V - 4e^V_m (2m^2_K - m^2_\pi)
\]

Where $e^V_m$ is the $U(3)$ breaking parameter.

We can constrain parameters by imposing high energy conditions on $F_{P\gamma^*\gamma^*}$.

After constraining parameters we find\(^\text{12}\)
\[
F_{\pi\gamma^*\gamma^*}(q^2_1, q^2_2) = \frac{32\pi^2 m^2_\pi F^2_V d^*_1 d^*_2}{12\pi^2 F_\pi D_\rho(q^2_1) D_\rho(q^2_2)} - N_C M^2_V M^2_\rho,
\]
where $D_R(q^2) = M^2_R - q^2$.


\(^{12}\)AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)
The $\eta^{(i)}$-TFF

- For the $\eta^{(i)}$ we find\(^\text{13}\)

$$
\mathcal{F}_{\eta^{*}\gamma^{*}}(q_1^2, q_2^2) = \frac{1}{12\pi^2 F D_\rho(q_1^2) D_\rho(q_2^2) D_\phi(q_1^2) D_\phi(q_2^2)} \times
\left\{ - \frac{N_C M_V^2}{3} \left[ 5 C_q M_\rho^2 D_\phi(q_1^2) D_\phi(q_2^2) - \sqrt{2} C_s M_\phi^2 D_\rho(q_1^2) D_\rho(q_2^2) \right] \\
+ \frac{32\pi^2 F_V^2 d_{123} m_\eta^2}{3} \left[ (5 C_q D_\phi(q_1^2) D_\phi(q_2^2) - \sqrt{2} C_s D_\rho(q_1^2) D_\rho(q_2^2) \right] \\
- \frac{256\pi^2 F_V^2 d_{2}^*}{3} \left[ (5 C_q \Delta_{\eta\pi}^2 D_\phi(q_1^2) D_\phi(q_2^2) + \sqrt{2} C_s \Delta_{2K\pi\eta}^2 D_\rho(q_1^2) D_\rho(q_2^2) \right] \right\}.
$$

- The $\eta'$-TFF can be obtained from $\mathcal{F}_{\eta\gamma^{*}\gamma^{*}}$ by making $C_q \rightarrow C'_q$, $C_s \rightarrow -C'_s$ and $m_\eta \rightarrow m_{\eta'}$.

\(^{13}\)AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)
Fit to experimental TFF

- We fit $e_m^V, M_V, d_{123}^*, d_2^*$ and $\eta - \eta'$ mixing parameters to experimental determinations of $\mathcal{F}_{\pi\gamma\gamma^*}$ and $\mathcal{F}_{\eta^{(i)}\gamma\gamma^*}$.

- We avoid observables involving $q^2 > 0$ since radiative corrections might have a large effect.\(^\text{14}\)

- BaBar $\pi^0$-TFF is at odds with the asymptotic QCD limit, with Belle data and $\eta^{(i)}$-TFF related through chiral symmetry.

- Neglecting BaBar $\pi^0$-TFF data reduces $\chi^2$/dof from $150./101 \rightarrow 69./84$.

- Therefore, our best fit will exclude BaBar $\pi^0$-TFF.

After fitting we get\(^\text{15}\).

*BaBar data is shown in red.*

\(^\text{15}\)AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)
\[ \mathcal{F}_{\eta \gamma \gamma^*} \text{ and } \mathcal{F}_{\eta' \gamma \gamma^*} \]

- Our fit for the $\eta$-TFF gives\textsuperscript{16}

\textsuperscript{16}AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)
\( F_{\eta\gamma\gamma^*} \) and \( F_{\eta'\gamma\gamma^*} \)

- While for the \( \eta' \)-TFF gives\(^{17}\)

\(^{17}\) AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)
We get a total pseudoscalar exchange contribution of

\[ a_{\mu, HLbL}^P = (8.47 \pm 0.16) \cdot 10^{-10} \]

For TFFs in the chiral limit we get \( a_{\mu, HLbL}^P = 8.27 \cdot 10^{-10} \).

This shows that NNLO corrections, which will be suppressed by further powers of \( m_P^2 \), must be negligible.

NLO effects from \( 1/N_C \) can be estimated from \( \pi\pi \) and \( K\bar{K} \) loops contribution to \( D_\rho: (\Delta a_{\mu, HLbL}^P)_{1/N_C} = \pm 0.09 \times 10^{-10} \).

Our TFF \( \sim 1/Q^4 \) when \( Q^2 \to \infty \) for doubly off-shell photon. A rough estimate of uncertainty is \( (\Delta a_{\mu, LbL}^P)_{\text{asym}} = ^{+0.5}_{-0.0} \cdot 10^{-10} \).
Now we can compare our results with earlier results.

<table>
<thead>
<tr>
<th>$a^{P,HLbL}_\mu \cdot 10^{10}$</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.3 ± 1.2</td>
<td>M. Knecht and A. Nyffeler, PRD 65(2002)</td>
</tr>
<tr>
<td>8.5 ± 1.3</td>
<td>J. Bijnens, E. Palante and J. Prades, Phys.Lett.75(1995)</td>
</tr>
<tr>
<td>8.60 ± 0.25</td>
<td>P. Roig, AG and G. López Castro, PRD 89 (2014)</td>
</tr>
<tr>
<td>9.4 ± 0.5</td>
<td>P. Masjuan and P. Sánchez Puertas, PRD 95 (2017)</td>
</tr>
<tr>
<td>8.28 ± 0.34</td>
<td>H. Czyż, P. Kisza and S. Tracz, PRD 97 (2018)</td>
</tr>
</tbody>
</table>

Our contribution gives

$$a^{P,HLbL}_\mu = (8.47 \pm 0.16_{\text{sta}} \pm 0.09_{1/N_C}^{0.5 \text{ asym}}) \cdot 10^{-10}$$
Conclusions

• Our determination of the $a_{\mu}^{P,HLbL}$ has an improved theoretical accuracy with lower uncertainty compared with previous determinations.

• We found that BaBar $\pi^0$-TFF data is incompatible with measurements of $\eta^{(l)}$ form factors.

• Excluding fitting data in the $q^2 > 0$ region we avoid large uncertainties due to EM radiative corrections.

• We find that further chiral corrections to $F_{P\gamma*\gamma*}$ must be negligible.
Back up
Short Distance constraints

- One finds by taking the limits
  \[
  \lim_{Q^2 \to \infty} F_{\pi \gamma^* \gamma^*}(Q^2, 0) \quad \text{and} \quad \lim_{Q^2 \to \infty} F_{\pi \gamma^* \gamma^*}(Q^2, Q^2),
  \]

- $\mathcal{O}(m_\pi^0)$:
  \[
  C_{22}^W = 0, \quad c_{125} = 0, \quad c_{1256} = -\frac{N_C M_V}{32 \sqrt{2} \pi^2 F_V}, \quad d_3 = \frac{c_{1256}}{\sqrt{2}} \frac{M_V}{F_V}
  \]

- $\mathcal{O}(m_\pi^2)$:
  \[
  \lambda_V = -\frac{32 \pi^2 F_V}{N_C} C_7^W, \quad c_{1235}^* = \frac{N_C M_V}{4 \sqrt{2} \pi^2 F_V} \left( \frac{e_m^V}{2} + \frac{M_V^2 \lambda_V}{F_V} \right)
  \]

- From $F_{\eta \gamma^* \gamma^*}$:
  \[
  C_8^W = 0, \quad c_3 = \frac{c_{1235}}{8}
  \]

- From VVP Green’s function: $C_7^W = \lambda_V = 0$. 
We get for $\pi^0$

$$a_{\mu}^{\pi, LbL} = 5.81 \pm 0.09 \times 10^{-10}$$

While we get for $\eta$

$$a_{\mu}^{\eta, LbL} = 1.51 \pm 0.06 \times 10^{-10}$$

And for $\eta'$

$$a_{\mu}^{\eta', LbL} = 1.15 \pm 0.07 \times 10^{-10}$$

Getting a total pseudoscalar exchange contribution of

$$a_{\mu}^{P, HlLbL} = 8.47 \pm 0.16 \times 10^{-10}$$
### Fitted parameters

<table>
<thead>
<tr>
<th></th>
<th>With $\pi^0$-BaBar</th>
<th>Without $\pi^0$-BaBar</th>
<th>Fixing $M_V$ and $e_m^V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$-0.2 \pm 1.0$</td>
<td>$0.0 \pm 1.0$</td>
<td>$0.0 \pm 1.0$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$0.5 \pm 1.0$</td>
<td>$0.0 \pm 0.5$</td>
<td>$0.0 \pm 1.0$</td>
</tr>
<tr>
<td>$\bar{d}_2$</td>
<td>$(-2.9 \pm 1.7) \cdot 10^{-2}$</td>
<td>$(-2.7 \pm 1.7) \cdot 10^{-2}$</td>
<td>$(-3 \pm 2) \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$\bar{d}_{123}$</td>
<td>$(-2.5 \pm 1.5) \cdot 10^{-1}$</td>
<td>$(-2.3 \pm 1.5) \cdot 10^{-1}$</td>
<td>$(-3 \pm 2) \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$M_V$</td>
<td>$(799 \pm 5)$ MeV</td>
<td>$(791 \pm 6)$ MeV</td>
<td>$764.3$ MeV</td>
</tr>
<tr>
<td>$e_m^V$</td>
<td>$-0.35 \pm 0.10$</td>
<td>$-0.36 \pm 0.10$</td>
<td>$-0.228$</td>
</tr>
<tr>
<td>$\theta_8$</td>
<td>$(-19.5 \pm 0.9)\degree$</td>
<td>$(-19.5 \pm 0.9)\degree$</td>
<td>$(-21.7 \pm 0.9)\degree$</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>$(-9.5 \pm 1.6)\degree$</td>
<td>$(-9.5 \pm 1.6)\degree$</td>
<td>$(-10.4 \pm 1.6)\degree$</td>
</tr>
<tr>
<td>$f_8$</td>
<td>$(118 \pm 4)$ MeV</td>
<td>$(118 \pm 3)$ MeV</td>
<td>$(118 \pm 3)$ MeV</td>
</tr>
<tr>
<td>$f_0$</td>
<td>$(108 \pm 3)$ MeV</td>
<td>$(107.5 \pm 1.0)$ MeV</td>
<td>$(107 \pm 3)$ MeV</td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td>$150./101$</td>
<td>$69./84$</td>
<td>$101./86$</td>
</tr>
</tbody>
</table>

$P_{1/2}$ are related to $\bar{d}_{123}$ and $\bar{d}_2$ through a rotation that reduces correlation between the two latter.
Asymptotic behavior

- We obtained the correct behavior for an on-shell photon,

\[
\lim_{Q^2 \to \infty} F_{\pi \gamma^* \gamma^*} (Q^2, 0) \approx \frac{2F}{Q^2}.
\]

- The correct behavior for \( F_{\pi \gamma^* \gamma^*} (Q^2, Q^2) \) can be obtained considering another vector multiplet. In the chiral limit we get

\[
F_{\pi^0 \gamma^* \gamma^*} (q_1^2, q_2^2) = \frac{-1}{12 \pi^2 F (M_\rho^2 - q_1^2) (M_\rho^2 - q_2^2) (M_{\rho'}^2 - q_1^2) (M_{\rho'}^2 - q_2^2)}
\]

\[
\times \left[ -q_1^2 q_2^2 \left( N_C M_{\rho'}^4 - 48 \pi^2 F^2 M_{\rho'}^2 + 4 \pi^2 F^2 (q_1^2 + q_2^2) \right) + N_C M_\rho^4 M_{\rho'}^4 - 8 \pi^2 F^2 M_\rho^2 \left( 3 (q_1^2 + q_2^2) M_{\rho'}^2 - q_1^2 q_2^2 \right) \\
+ 64 \pi^2 F_\rho^2 d_3^{(\rho, \rho)} M_\rho^2 q_1^2 q_2^2 \left( 1 - \frac{M_{\rho'}^2}{M_\rho^2} \right)^2 \\
- \frac{16 \pi^2 \sqrt{2} F_\rho c_{125}^{(\rho)}}{M_\rho} q_1^2 q_2^2 (q_1^2 - q_2^2)^2 \left( 1 - \frac{M_{\rho'}^2}{M_\rho^2} \right) \right].
\]
Asymptotic behavior

- Only two parameters remain unconstrained after matching with high energy QCD behavior, which we choose $c_{125}^\rho$ and $d_3^\rho$.

- Since contributions from the second multiplet are considered subleading, and one constraint is

$$F_\rho c_{125}^\rho M_\rho + F_\rho' c_{125}^\rho M_\rho' = 0$$

we assume that $c_{125}^\rho = c_{125}^\rho' = 0$.

- For $d_3$ we use the SD constraint from previous analysis.

$$F_\rho^2 d_3^\rho = \frac{N_C M_\rho^2}{64\pi^2}$$

- Comparison is done in the chiral limit, using $M_\rho = 770$ MeV.
A NLO effect from $1/N_C$ terms will be the intermediate $\pi\pi$ and $K\bar{K}$ contribution\(^{18}\) to $D_\rho$.

This gives

$$M_\rho^2 - q^2 \rightarrow M_\rho^2 - q^2 + \frac{q^2 M_\rho^2}{96\pi^2 F_\pi^2} \left( A_\pi(q^2) + \frac{1}{2} A_K(q^2) \right),$$

where

$$A_P(q^2) = \ln \frac{m_P^2}{M_\rho^2} + 8 \frac{m_P^2}{q^2} - \frac{5}{3} + \sigma_P^3(q^2) \ln \left( \frac{\sigma_P(q^2) + 1}{\sigma_P(q^2) - 1} \right),$$

and

$$\sigma_P(q^2) = \sqrt{1 - \frac{4m_P^2}{q^2}}.$$

Since for $a_\mu^{H,LbL}$ the photon momenta are $q^2 < 0$, $D_\rho$ is real.

Beyond Standard Model (BSM) probe

- Precise measurements of $a_\ell$ make feasible the search of BSM effects.

- Contributions to BSM interactions, like chiral $d=5$ operator
  $\mathcal{O}_{d=5} = \frac{g}{\Lambda} \psi \sigma^{\mu\nu} F_{\mu\nu} \psi$ mixes helicities of $\ell$.

- Helicity flips are allowed only for massive particles, so $\mathcal{O}_{d=5}$
  must be suppressed by a factor $\sim \frac{g m_\ell}{\Lambda^2}$.

- If current discrepancy is from BSM contribution to $a_\mu$,

  $\Lambda \approx \sqrt{g} \ 100 \ \text{TeV}$
Why not $\ell = \tau$?

- Since transition probability is squared modulus of the amplitude, BSM effects will be easier to detect with $\ell = \mu$

$$\left(\frac{m_\mu}{m_e}\right)^2 \sim 4 \times 10^4$$

- Therefore, BSM effects should be larger on $a_\tau$. Nevertheless, $\tau_\tau$ is so small that experimental results\(^{19}\) are still compatible with $a_\tau = 0$.

$$\tau_\mu = 2.197 \times 10^{-6} \text{s}, \quad \tau_\tau = 2.906 \times 10^{-13} \text{s} \quad \Rightarrow \quad \frac{\tau_\tau}{\tau_\mu} \sim 10^{-7}$$

$a_e$ vs $a_\mu$ precision

- Even though measurements of $a_e$ are 2250 times more precise$^{20}$, $a_\mu$ is
  \[ \frac{1}{2250} \left( \frac{m_\mu}{m_e} \right)^2 \sim 19 \]
  times more sensitive to BSM contributions.

- Therefore, it would be more plausible to find such a deviation in the $a_\mu$.

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$^{20}$R.S. Van Dyck et al., PRL59(1987);
Electromagnetic current

- The way to compute $a_\mu$ is through the interaction Lagrangian

$$\mathcal{L}_{int}^{QED}(x) = -e \bar{\psi}(x) \gamma^\mu A_\mu(x) \psi(x),$$

- where $A = A^{QED} + A^{ext}$. $A^{QED}$ will give the radiative corrections as that given by Schwinger and $A^{ext}$ is a classic EM field.

- Through Gordon identity, the lepton current in momentum space can be written as

$$\tilde{j}^\alpha = (-ie) \bar{u}(p+q) \left[ \gamma^\alpha F_E(q^2) + i \frac{\sigma^{\alpha\beta} q^\beta}{2m_\mu} F_M(q^2) \right] u(p),$$

- where $F_E(q^2)$ is called the Dirac (or electric charge) form factor and $F_M(q^2)$ is the Pauli (or magnetic) form factor.
Magnetic moment

• Then, $\vec{\mu}$ is the part interacting with the $\vec{B}$ from $A^{\text{ext}}$, $\vec{\mu} \cdot \vec{B}$.

• This gives

$$\vec{\mu} = g \left( \frac{e}{2m} \right) \vec{s},$$

• where

$$g = 2[F_1(0) + F_2(0)].$$

• By neglecting contributions from $A^{QED}_\mu$ one gets $F_1(0) = 1$ and $F_2(0) = 0$, recovering Dirac's result $g = 2$.

• Therefore, the $\vec{\mu} \cdot \vec{B}$ interaction is needed to measure $a_\mu$. 
How to measure $a_\mu$?

- If $\vec{B}$ is constant, the problem reduces to determining the helicity.

- However, one big issue arises. Muons are unstable!

- Thanks to maximal parity violation of weak interactions one can determine the helicity of the muon.

- To see this one needs to know how to generate muons.
The $\pi$ decay

- Charged pions decay 99.99% of the time to muons

$$\mathcal{B}(\pi^+ \rightarrow \mu^+ \nu_\mu) \approx 99.99\%.$$ 

- Therefore, one can produce muons by first producing $\pi^\pm$, generated by hitting a fixed target with a proton beam.

\[ \begin{array}{c}
\pi^- \rightarrow \bar{u} \ W^- \rightarrow \mu^- \bar{\nu}_\mu \\
da \ W^- \rightarrow \mu^- \bar{\nu}_\mu.
\end{array} \]

\[ \pi^-\text{decay} \]

- The lepton current coupling to the weak gauge boson, $W^\alpha$, is

$$j^W_\alpha(x) = \bar{\psi}_\nu_L(x)\gamma_\alpha\psi_{\mu L}(x),$$

- where $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ is a left eigenstate of helicity.

* Figure treacherously stolen from F. Jegerlehner & A. Nyffeler, Phys.Rep.477(2009).
Helicity of muons.

- This means that muons obtained from $\pi$ decays have a determined helicity.

- From $\pi^+$ decays results right anti-muons, where from $\pi^-$ decays results left muons.

The muon also decays through a weak gauge boson exchange. This means that the helicity of the electron (positron) can also be determined. Therefore, in wherever direction the electron is ejected, it must be parallel (\(e^+\)) or antiparallel (\(e^-\)) to its momentum. An additional electric quadrupole field normal to the muon orbit is used to focus the beam.

* Same as before, Jegerlehner and Nyffeler, Phys.Rep.33(2009)
Experimental summary

- To summarize, this is the experimental setup.

- All remaining is to determine the Larmor precession.

Who TF Larmor?

- The Larmor precession is defined as the precession of a magnetic moment about a magnetic field.

\[ \vec{\omega} = -\frac{e}{m_\mu} \left[ a_\mu \vec{B} - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) (\vec{\nabla} \cdot \vec{B}) \vec{v} + \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{E} \times \vec{v} \right]. \]

“Who is That Famous Larmor?”
It's magic?

- One can *magically disappear* the electric quadrupole field contribution.
Magic? Always believe it’s not so

- It is done by choosing the *magic* Lorentz factor to be \( \gamma^\infty = 29.3 \), corresponding to a *magic* energy \( E^\mu_\infty \approx 3.098 \) GeV.

- \( \vec{E} \) generates an oscillation in the beam direction and in \( \vec{B} \) direction.

- The reason to disregard the contribution from \( \vec{E} \) is to minimize \( \vec{\omega} \). This will reduce the error for \( a_\mu \).
• The relevant degrees of freedom are\(^{21}\) the octet of the lightest pseudoscalar \((\pi, K, \eta \text{ and } \eta')\).

• The expansion parameter in this theory is \(1/N_C\), and in large \(N_C\) the \(U(1)_A\) broken symmetry is restored, that is the reason for taking \(\eta'\) at the same level as the other resonances.

\( F_{\pi\gamma\gamma} \) parameters

- \( R_{\chi T} \) parameters can be found using short distance behavior of QCD, which predicts an asymptotic behavior of \( s^{-1} \) for this process.
- Thus, short distance relationships\textsuperscript{22} ensure a convergent behavior

\[
d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2} - \frac{4\sqrt{2}P_2}{F_V}; \quad c_{125} = 0; \quad d_{123} = \frac{1}{24};
\]

\[
F_V = \sqrt{3}F; \quad c_{125} = 0; \quad c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V}
\]

Restored $U(1)_A$

- Within t’Hooft’s large $N_C$, the anomaly term is suppressed by a factor $1/N_C$ with respecto to the rest of the QCD lagrangian
  \[ \frac{g^2}{8\pi^2} \frac{\theta}{N_C} \mathrm{Tr} F^{\mu\nu} \tilde{F}_{\mu\nu}, \]

- Therefore in the limit $N_C \to \infty$ the $U(1)_A$ symmetry is restored.
Wess-Zumino-Witten

- A fundamental part of the analysis is the WZW term, which is order $p^4$ in the chiral counting and describe intrinsic odd interactions $^{23}$.

\[ Z[U, l, r] = - \frac{i N_c}{240 \pi^2} \int_{M^5} d^5 x \varepsilon^{ijklm} \langle \Sigma^L_i \Sigma^L_j \Sigma^L_k \Sigma^L_l \Sigma^L_m \rangle \]

\[ - \frac{i N_c}{48 \pi^2} \int d^4 x \varepsilon_{\mu \nu \rho \sigma} (W(U, l, r)^{\mu \nu \rho \sigma} - W(1, l, r)^{\mu \nu \rho \sigma}) \]

\[
W(U, l, r)^{\mu \nu \rho \sigma} = \langle U_l U_\mu l_\nu l_\rho U_r^\dagger r_\sigma + \frac{1}{4} U_l U_\mu U_r^\dagger r_\nu U_\rho U_r^\dagger r_\sigma + i U_\mu \partial_\nu l_\rho \partial_\rho l_\sigma + i \Sigma^L_\mu l_\nu \partial_\rho l_\sigma + \Sigma^L_\mu U_r^\dagger \partial_\nu r_\rho U_l \sigma \\
\quad + \Sigma^L_\mu \Sigma^L_\nu U_r^\dagger r_\rho U_l \sigma + \Sigma^L_\mu \Sigma^L_\nu \partial_\rho l_\sigma + \Sigma^L_\mu \partial_\nu l_\rho l_\sigma - i \Sigma^L_\mu l_\nu l_\rho l_\sigma \\
\quad + \frac{1}{2} \Sigma^L_\mu l_\nu \Sigma^L_\rho l_\sigma - i \Sigma^L_\mu \Sigma^L_\nu \Sigma^L_\rho l_\sigma - (L \leftrightarrow R) \rangle, \tag{1} \]

\[
\Sigma^L_\mu = U_r^\dagger \partial_\mu U, \Sigma^R_\mu = U_\mu \partial_\mu U^\dagger, \]

Contribución de resonancias a las LEC de \( \chi PT \) a \( \mathcal{O}(p^4) \)

- El lagrangiano de interacción de las resonancias vectoriales es

\[
\mathcal{L}(V) = \langle V_{\mu\nu} J_{\mu\nu} \rangle; \quad J_{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_{\mu\nu}^\pm + i \frac{G_V}{2\sqrt{2}} [u^\mu, u^\nu]
\]

- Con \( f_{\mu\nu}^\pm = uF_{L}^{\mu\nu} u^\dagger \pm u^\dagger F_{R}^{\mu\nu} u \), donde

\[
F_{R,L}^{\mu\nu} = \partial^\mu (r, \ell)^{\nu} - \partial^{\nu} (r, \ell)^{\mu} - i [(r, \ell)^{\mu}, (r, \ell)^{\nu}]
\]

- siendo \( r \) y \( \ell \) las corrientes vectoriales y axiales externas, respectivamente.

- \( u^{\mu} = i \left[ u^\dagger (\partial^\mu - ir^{\mu}) u - u (\partial^\mu - i\ell^{\mu}) u^\dagger \right] = iu^\dagger D^\mu U u^\dagger \)

- \( F_V \) y \( G_V \) son parámetros reales.
• Así, se encuentra que $V$ debe cumplir una ecuación de constricción

$$\nabla^\alpha \nabla^\rho V^{\alpha\beta} - \nabla^\beta \nabla^\rho V^{\rho\alpha} + M_V^2 V^{\alpha\beta} = -2J^{\alpha\beta}$$

• Donde $\nabla_\mu R = \partial_\mu R + [\Gamma_\alpha, R]$ y

$$\Gamma_\alpha = \frac{1}{2} [u^\dagger (\partial_\alpha - ir_\alpha) u + u (\partial_\alpha - i\ell_\alpha) u^\dagger].$$

Al sustituir $V$ y a orden $p^4$ se tiene que

$$L_1^V = \frac{G_V^2}{8M_V^2} \quad L_2^V = 2L_1^V \quad L_3^V = -6L_1^V$$

$$L_9^V = \frac{F_V G_V}{2M_V^2} \quad L_{10}^V = -\frac{F_V^2}{4M_V^2}$$

• y de igual forma para las demás resonancias.