Pseudoscalar pole contribution to the hadronic light-by-light piece of a_μ

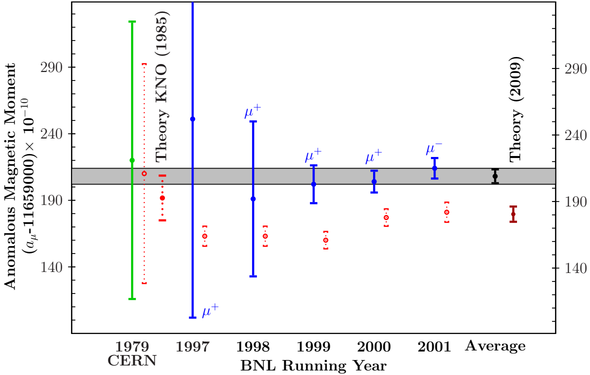
Adolfo Guevara

Departamento de Física Teórica
Facultad de CC Físicas, Madrid, Spain

Quark Confinement and the Hadron Spectrum, Maynooth
August 2, 2018

Purpose

- The main purpose of this work is to reduce the theoretical uncertainty in the computation of the a_μ, in which the main source of uncertainty comes from the hadronic contributions. This is why we decided to analyze the hadronic light-by-light contribution using χPT extended to include resonances.



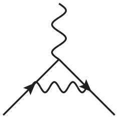
Magnetic moment

- The Dirac equation predicts a magnetic moment for a particle with EM charge Q and mass m

$$\boldsymbol{\mu}_\ell = g_\ell \frac{Q}{2m}\mathbf{s}$$

- such that $g_\ell = 2$. This is obtained for a classic EM field.

- The deviation from $g_\ell = 2$ defines the anomalous magnetic moment, which will happen due to loop corrections.



$$a_\ell := \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) \approx 0.00116.$$

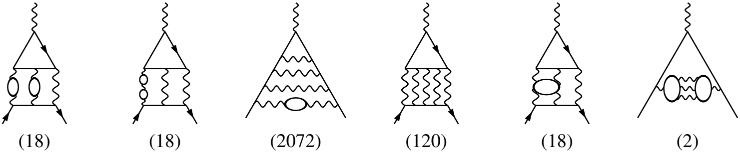
Contributions to a_μ

- The computation of a_μ can be splitted in different contributions, whose values can be found in PDG[1]

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had}$$

- a_μ^{QED} are all corrections[2] that might come from QED

$$a_\mu^{QED} = 116584718.95(0.08) \times 10^{-11} + \mathcal{O}\left(\frac{\alpha}{\pi}\right)^6$$

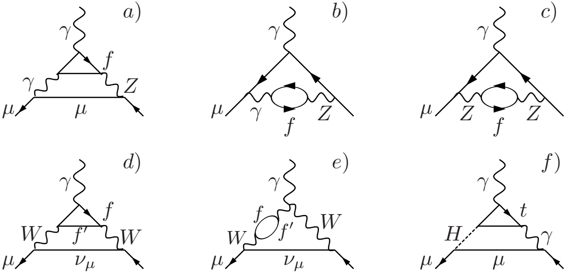


[1] C. Patrignani *et al.* (Particle Data Group), Chin.Phys.C40(2016)
[2] T. Aoyama *et al.* PRL 109(2012)

a_μ^{EW}

- a_μ^{EW} are Electroweak contribution that are not a_μ^{QED} $(W^\pm, Z, H)$ at two loops[3]. Three loops contribution is negligible ($\lesssim 0.4 \times 10^{-11}$).

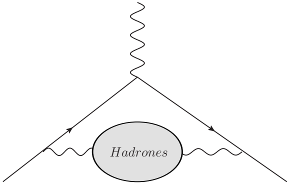
$$a_\mu^{EW} = 153.6(1.0) \times 10^{-11}$$



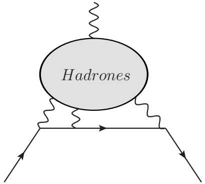
[3]C. Gnendiger *et al.*, Phys.Rev.D88 (2013)

Hadronic contributions

- a_μ^{Had} can be splitted in two parts, the PDG values are[4]



Hadronic Vacuum Polarization (HVP) contribution.
$a_\mu^{HVP} = 6845(33)(7) \times 10^{-11}$



Hadronic light-by-light (HLbL) contribution. $a_\mu^{HLbL} = 105(26) \times 10^{-11}$

[4]C. Patrignani *et al.* (Particle Data Group), Chin.Phys.C40(2016)
For HVP, M. Davier *et al.* Eur.Phys.J. C71 (2011); B. Krauze, Phys.Lett. B390 (1997); A. Kurz *et al.* Phys.Lett. B734 (2014)
For HLbL J. Prades *et al.* Advanced series on directions in HEP Vol20.

Hadronic contributions to a_μ

- All the contributions and their uncertainties are shown in the next table.

Contribution	$\times 10^{11}$	Uncertainty$\times 10^{11}$
QED	116 584 718.95	0.08
EW	153.6	1.0
Had	6 950	$(34)_{\text{Vac. Pol.}}$ $(26)_{\text{Light-by-Light}}$
Total	116 591 823	(34)(26)
Exp	116 592 091	(54)(33)

- Clearly, the largest uncertainty comes from the hadronic contribution.
- With these values there is a discrepancy

$$a_\mu^{exp} - a_\mu^{SM} = 268(63)(43) \times 10^{-11} \quad \sim 3.5\sigma$$

Hadronic contributions to a_μ

- The main uncertainty comes from hadronic contributions[5], which give 4.3×10^{-10}.
- The current experimental[6] error is 6.3×10^{-10}.
- Fermilab & J-Parc are planning to lower[7] the error to 1.6×10^{-10}. It is necessary to reduce theoretical uncertainty.
- A reanalysis of $R_{\rm had}$ from Lattice QCD may reduce[8] the HVP error (3.3×10^{-10}) below that of the HLbL piece.

---

[5] M. Davier *et al.*, Eur.Phys.J.C71(2011)

[6] G. W. Bennet *et al.*, [Muon g-2 Collab.],PRD73(2006)

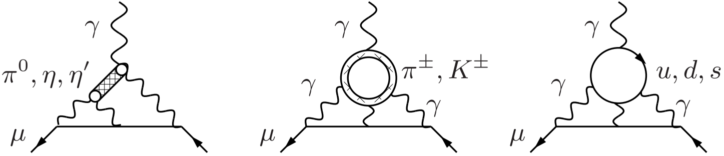
[7] H. Inuma *et al*, Nucl. Instrum. Meth. A **832** (2016); W. Gohn, FERMILAB-CONF-17-602-PPD, Muon g-2 collaboration, (2017)

[8] Talks given at the Muon g-2 Theory Initiative Workshops held during the last year at FNAL, Tsukuba, Connecticut Univ. and Mainz Univ.

Hadronic Light by Light

- We decided to analyze the HLbL piece since, nowadays, it can't be obtained from experimental data.

- It can be separated into three parts.



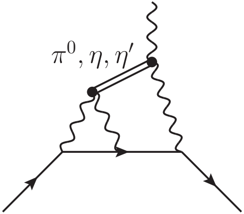
(a) [L.D.] (b) [L.D.] (c) [S.D.]

- The sum of (*b*) and (*c*) is[9] one order of magnitude smaller than (*a*).

[9] F. Jegerlehner & A. Nyffeler, Phys.Rep.477(2009)

Pseudoscalar pole

- Our contribution to a_μ comes from diagram (a)



- To compute the pion transition form factor $F_{\pi\gamma^\star\gamma^\star}$ we rely on Resonance Chiral Theory[10] (RχT) with $U(3)$ breaking.

[10] G. Ecker, J. Gasser A. Pich & E. De Rafael Nucl.Phys. B321(1989)
P.D. Ruíz-Femenía *et al.*, JHEP 0307 (2003)
K. Kampf and J. Novotný PRD84 (2011)

$U(3)$ breaking and $\mathcal{F}_{\pi\gamma^\star\gamma^\star}$

- We include corrections up to $\mathcal{O}(m_P^2)$. Some of this give[11]

$$M_\rho^2 = M_\omega^2 = M_V^2 - 4e_m^V m_\pi^2 \,, \qquad M_\phi^2 = M_V^2 - 4e_m^V(2m_K^2 - m_\pi^2)$$

- Where e_m^V is the $U(3)$ breaking parameter.

- We can constrain parameters by imposing high energy conditions on $\mathcal{F}_{P\gamma^\star\gamma^\star}$.

- After constraining parameters we find[12]

$$\mathcal{F}_{\pi\gamma^\star\gamma^\star}(q_1^2, q_2^2) = \frac{32\pi^2 m_\pi^2 F_V^2 d_{123}^\star - N_C M_V^2 M_\rho^2}{12\pi^2 F_\pi D_\rho(q_1^2) D_\rho(q_2^2)},$$

where $D_R(q^2) = M_R^2 - q^2$.

[11] V. Cirigliano, G. Ecker, H. Neufeld and T. Pich, JHEP 0306 (2006)
[12] AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)

The $\eta^{(\prime)}$-TFF

- For the $\eta^{(\prime)}$ we find[13]

$$\mathcal{F}_{\eta\gamma^\star\gamma^\star}(q_1^2,q_2^2) = \frac{1}{12\pi^2 F D_\rho(q_1^2)D_\rho(q_2^2)D_\phi(q_1^2)D_\phi(q_2^2)}\times$$

$$\Bigg\{-\frac{N_C M_V^2}{3}\left[5C_q M_\rho^2 D_\phi(q_1^2)D_\phi(q_2^2)-\sqrt{2}C_s M_\phi^2 D_\rho(q_1^2)D_\rho(q_2^2)\right]$$

$$+\frac{32\pi^2 F_V^2 d_{123}^\star m_\eta^2}{3}\left[(5C_q D_\phi(q_1^2)D_\phi(q_2^2)-\sqrt{2}C_s D_\rho(q_1^2)D_\rho(q_2^2)\right]$$

$$-\frac{256\pi^2 F_V^2 d_2^\star}{3}\left[(5C_q\Delta_{\eta\pi}^2 D_\phi(q_1^2)D_\phi(q_2^2)+\sqrt{2}C_s\Delta_{2K\pi\eta}^2 D_\rho(q_1^2)D_\rho(q_2^2)\right]\Bigg\}.$$

- The η'-TFF can be obtained from $\mathcal{F}_{\eta\gamma^\star\gamma^\star}$ by making $C_q \to C_q'$, $C_s \to -C_s'$ and $m_\eta \to m_{\eta'}$.

- (Here we define $\Delta_{\eta\pi}^2 := m_\eta^2 - m_\pi^2$ and $\Delta_{2K\pi\eta}^2 := 2m_K^2 - m_\pi^2 - m_\eta^2$)

[13] AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)

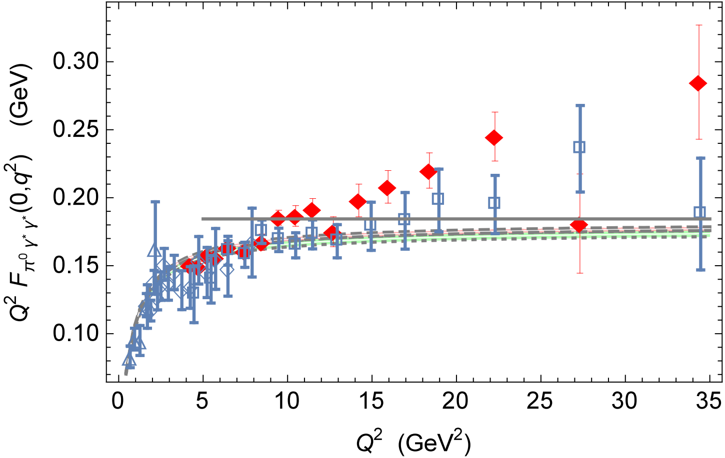
Fit to experimental TFF

- We fit $e_m^V, M_V, d_{123}^\star, d_2^\star$ and $\eta - \eta'$ mixing parameters to experimental determinations of $\mathcal{F}_{\pi\gamma\gamma^\star}$ and $\mathcal{F}_{\eta^{(\prime)}\gamma\gamma^\star}$.
- We avoid observables involving $q^2 > 0$ since radiative corrections might have a large effect.[14]
- BaBar π^0-TFF is at odds with the asymptitic QCD limit, with Belle data and $\eta^{(\prime)}$-TFF related through chiral symmetry.
- Neglecting BaBar π^0-TFF data reduces χ^2/dof from 150./101 $\rightarrow$ 69./84.
- Therefore, our best fit will exclude BaBar π^0-TFF.

[14] K. Kampf, J. Novotný & P. Sánchez-Puertas, PRD97 (2018)

$F_{\pi\gamma\gamma^\star}$

- After fitting we get[15].

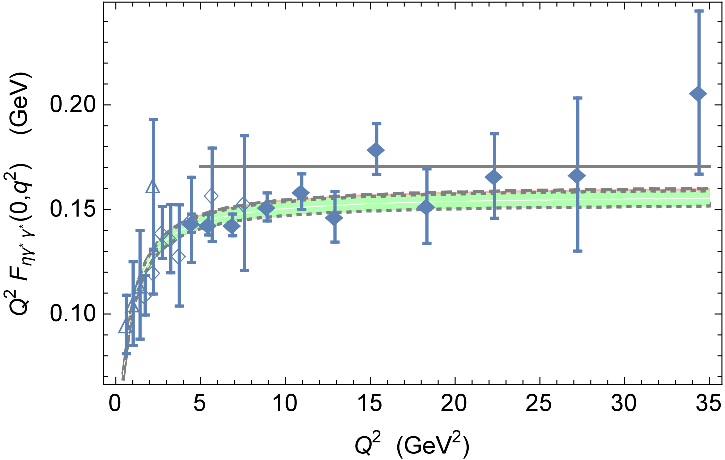


- BaBar data is shown in red.

[15] AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)

$\mathcal{F}_{\eta\gamma\gamma^\star}$ and $\mathcal{F}_{\eta'\gamma\gamma^\star}$

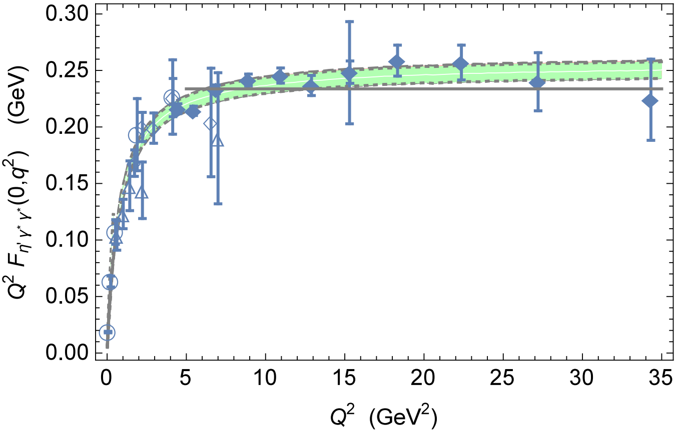
- Our fit for the η-TFF gives[16]



[16] AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)

$\mathcal{F}_{\eta\gamma\gamma^\star}$ and $\mathcal{F}_{\eta'\gamma\gamma^\star}$

- While for the η'-TFF gives[17]



[17] AG, P. Roig, JJ Sanz Cillero, JHEP 1806 (2018)

$a_\mu^{P,HLbL}$

- We get a total pseudoscalar exchange contribution of

$$a_\mu^{P,HLbL} = (8.47 \pm 0.16) \cdot 10^{-10}$$

- For TFFs in the chiral limit we get $a_\mu^{P,HLbL} = 8.27 \cdot 10^{-10}$.

- This shows that NNLO corrections, which will be suppressed by further powers of m_P^2, must be negligible.

- NLO effects from $1/N_C$ can be estimated from $\pi\pi$ and $K\overline{K}$ loops contribution to D_ρ: $(\Delta a_\mu^{P,HLbL})_{1/N_C} = \pm 0.09 \times 10^{-10}$.

- Our TFF $\sim 1/Q^4$ when $Q^2 \to \infty$ for doubly off-shell photon. A rough estimate of uncertainty is $(\Delta a_\mu^{P,LbL})_{\mathrm{asym}} =^{+0.5}_{-0.0} \cdot 10^{-10}$

$a_\mu^{P,HLbL}$

- Now we can compare our results with earlier results.

$a_\mu^{HLbL} \cdot 10^{10}$	Contribution
8.3 ± 1.2	M. Knecht and A. Nyffeler, PRD 65(2002)
8.5 ± 1.3	J. Bijnens, E. Palante and J. Prades, Phys.Lett.75(1995)
8.60 ± 0.25	P. Roig, AG and G. López Castro, PRD 89 (2014)
9.4 ± 0.5	P. Masjuan and P. Sánchez Puertas, PRD 95 (2017)
8.28 ± 0.34	H. Czyż, P. Kisza and S. Tracz, PRD 97 (2018)

- Our contribution gives

$$a_\mu^{P,HLbL} = (8.47 \pm 0.16_{\rm sta} \pm 0.09_{1/N_C}\,{}^{+0.5}_{-0.0}\,{}_{\rm asym}) \cdot 10^{-10}$$

Conclusions

- Our determination of the $a_\mu^{P,HLbL}$ has an improved theoretical accuracy with lower uncertainty compared with previous determinations.
- We found that BaBar π^0-TFF data is incompatible with measurements of $\eta^{(\prime)}$ form factors.
- Excluding fitting data in the $q^2 > 0$ region we avoid large uncertainties due to EM radiative corrections.
- We find that further chiral corrections to $\mathcal{F}_{P\gamma^\star\gamma^\star}$ must be negligible.

Back up

Short Distance constraints

- One finds by taking the limits

$$\lim_{Q^2\to\infty} \mathcal{F}_{\pi\gamma^\star\gamma^\star}(Q^2,0) \quad \text{and} \quad \lim_{Q^2\to\infty} \mathcal{F}_{\pi\gamma^\star\gamma^\star}(Q^2,Q^2),$$

- $@\mathcal{O}(m_\pi^0)$:

$$C_{22}^W = 0,\ c_{125} = 0,\ c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V},\ d_3 = \frac{c_{1256}}{\sqrt{2}}\frac{M_V}{F_V}$$

- $@\mathcal{O}(m_\pi^2)$:

$$\lambda_V = -\frac{32\pi^2 F_V}{N_C} C_7^W,\ c_{1235}^\star = \frac{N_C M_V}{4\sqrt{2}\pi^2 F_V}\left(\frac{e_m^V}{2} + \frac{M_V^2 \lambda_V}{F_V}\right)$$

- From $\mathcal{F}_{\eta\gamma^\star\gamma^\star}$:

$$C_8^W = 0,\ c_3 = \frac{c_{1235}}{8}$$

- From VVP Green's function: $C_7^W = \lambda_V = 0.$

$a_\mu^{P,HLbL}$

- We get for π^0

$$a_\mu^{\pi,LbL} = 5.81 \pm 0.09 \times 10^{-10}$$

- While we get for η

$$a_\mu^{\eta,LbL} = 1.51 \pm 0.06 \times 10^{-10}$$

- And for η'

$$a_\mu^{\eta',LbL} = 1.15 \pm 0.07 \times 10^{-10}$$

- Getting a total pseudoscalar exchange contribution of

$$\boxed{a_\mu^{P,HLbL} = 8.47 \pm 0.16 \times 10^{-10}}$$

Fitted parameters

	With π^0-BaBar	Without π^0-BaBar	Fixing M_V and e_m^V
$\mathcal{P}_1$	-0.2 ± 1.0	0.0 ± 1.0	0.0 ± 1.0
$\mathcal{P}_2$	0.5 ± 1.0	0.0 ± 0.5	0.0 ± 1.0
$\overline{d}_2$	$(-2.9 \pm 1.7) \cdot 10^{-2}$	$(-2.7 \pm 1.7) \cdot 10^{-2}$	$(-3 \pm 2) \cdot 10^{-2}$
$\overline{d}_{123}$	$(-2.5 \pm 1.5) \cdot 10^{-1}$	$(-2.3 \pm 1.5) \cdot 10^{-1}$	$(-3 \pm 2) \cdot 10^{-1}$
M_V	(799 ± 5) MeV	(791 ± 6) MeV	764.3 MeV †
e_m^V	-0.35 ± 0.10	-0.36 ± 0.10	-0.228 †
θ_8	$(-19.5 \pm 0.9)^\circ$	$(-19.5 \pm 0.9)^\circ$	$(-21.7 \pm 0.9)^\circ$
θ_0	$(-9.5 \pm 1.6)^\circ$	$(-9.5 \pm 1.6)^\circ$	$(-10.4 \pm 1.6)^\circ$
f_8	(118 ± 4) MeV	(118 ± 3) MeV	(118 ± 3) MeV
f_0	(108 ± 3) MeV	(107.5 ± 1.0) MeV	(107 ± 3) MeV
χ^2/dof	150./101	69./84	101./86

$\mathcal{P}_{1/2}$ are related to $\overline{d_{123}}$ and $\overline{d_2}$ through a rotation that reduces correlation between the two latter.

Asymptotic behavior

- We obtained the correct behavior for an on-shell photon,

$$\lim_{Q^2\to\infty} \mathcal{F}_{\pi\gamma^\star\gamma^\star}(Q^2,0) \approx \frac{2F}{Q^2}.$$

- The correct behavior for $\mathcal{F}_{\pi\gamma^\star\gamma^\star}(Q^2,Q^2)$ can be obtained considering another vector multiplet. In the chiral limit we get

$$\begin{aligned}
\mathcal{F}_{\pi^0\gamma^\star\gamma^\star}(q_1^2,q_2^2) \quad &= \quad \frac{-1}{12\pi^2 F\left(M_\rho^2-q_1^2\right)\left(M_\rho^2-q_2^2\right)\left(M_{\rho'}^2-q_1^2\right)\left(M_{\rho'}^2-q_2^2\right)} \\
\times\Big[&- q_1^2 q_2^2\left(N_C M_{\rho'}^4 - 48\pi^2 F^2 M_{\rho'}^2 + 4\pi^2 F^2\left(q_1^2+q_2^2\right)\right) \\
&+ N_C M_\rho^4 M_{\rho'}^4 - 8\pi^2 F^2 M_\rho^2\left(3\left(q_1^2+q_2^2\right)M_{\rho'}^2 - q_1^2 q_2^2\right) \\
&+ 64\pi^2 F_\rho^2 d_3^{(\rho,\rho)} M_\rho^2 q_1^2 q_2^2\left(1-\frac{M_{\rho'}^2}{M_\rho^2}\right)^2 \\
&- \frac{16\pi^2\sqrt{2}F_\rho c_{125}^{(\rho)}}{M_\rho} q_1^2 q_2^2\left(q_1^2-q_2^2\right)^2\left(1-\frac{M_{\rho'}^2}{M_\rho^2}\right)\Big].
\end{aligned}$$

Asymptotic behavior

- Only two parameters remain unconstrained after matching with high energy QCD behavior, which we choose c^{ρ}_{125} and d^{ρ}_{3}
- Since contributions from the second multiplet are considered subleading, and one constraint is

$$F_\rho \frac{c^{\rho}_{125}}{M_\rho} + F_{\rho'} \frac{c^{\rho'}_{125}}{M_{\rho'}} = 0$$

  we assume that $c^{\rho}_{125} = c^{\rho'}_{125} = 0$.
- For d_3 we use the SD constraint from previous analysis.

$$F_\rho^2 d_3^\rho = \frac{N_C M_\rho^2}{64\pi^2}$$

- Comparison is done in the chiral limit, using $M_\rho = 770$ MeV.

$1/N_C$ error

- A NLO effect from $1/N_C$ terms will be the intermediate $\pi\pi$ and $K\bar{K}$ contribution[18] to D_ρ.
- This gives

$$M_\rho^2 - q^2 \;\longrightarrow\; M_\rho^2 - q^2 + \frac{q^2 M_\rho^2}{96\pi^2 F_\pi^2}\left(A_\pi(q^2) + \frac{1}{2}A_K(q^2)\right),$$

where

$$A_P(q^2) \;=\; \ln\frac{m_P^2}{M_\rho^2} + 8\frac{m_P^2}{q^2} - \frac{5}{3} + \sigma_P^3(q^2)\ln\left(\frac{\sigma_P(q^2)+1}{\sigma_P(q^2)-1}\right),$$

and $\sigma_P(q^2) = \sqrt{1 - \frac{4m_P^2}{q^2}}$.

- Since for $a_\mu^{H,LbL}$ the photon momenta are $q^2 < 0$, D_ρ is real.

---

[18] D. Gómez-Dumm, A. Pich and J. Portolés, PRD62 (2000)

Beyond Standard Model (BSM) probe

- Precise measurements of a_ℓ make feasible the search of BSM effects.
- Contributions to BSM interactions, like chiral d=5 operator $\mathcal{O}_{d=5} = \frac{g}{\Lambda}\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi$ mixes helicities of ℓ.
- Helicity flips are allowed only for massive particles, so $\mathcal{O}_{d=5}$ must be suppressed by a factor $\sim \frac{gm_\ell}{\Lambda^2}$.
- If current discrepancy is from BSM contribution to a_μ,

$$\Lambda \approx \sqrt{g}\ 100\ \text{TeV}$$

Why not $\ell = \tau$?

- Since transition probability is squared modulus of the amplitude, BSM effects will be easier to detect with $\ell = \mu$

$$\left(\frac{m_\mu}{m_e}\right)^2 \sim 4 \times 10^4$$

- Therefore, BSM effects should be larger on a_τ. Nevertheless, τ_τ is so small that experimental results[19] are still compatible with $a_\tau = 0$.

$$\tau_\mu = 2.197 \times 10^{-6} s, \quad \tau_\tau = 2.906 \times 10^{-13} s \quad \Rightarrow \quad \frac{\tau_\tau}{\tau_\mu} \sim 10^{-7}$$

[19] K. Ackerstaff *et al.*, [OPAL Collab.] Phys.Lett.B431(1998)
M. Acciarri *et al.*, [L3 Collab.] Phys.Lett.B434(1998)
W. Lohmann, Nucl.Phys.B144(2005)

a_e vs a_μ precission

- Even though measurements of a_e are 2250 times more precise[20] a_μ is

$$\frac{1}{2250}\left(\frac{m_\mu}{m_e}\right)^2 \sim 19$$

times more sensitive to BSM contributions.

- Therefore, it would be more plausible to find such a deviation in the a_μ.

[20] R.S. Van Dyck *et al.*, PRL59(1987);
P.J. Mohr *et al.*, Rev.Mod.Phys.72(2000)

Electromagnetic current

- The way to compute a_μ is through the interaction Lagrangian

$$\mathcal{L}_{int}^{QED}(x) = -e\bar{\psi}(x)\gamma^\mu A_\mu(x)\psi(x),$$

- where $A = A^{QED} + A^{\text{ext}}$. A^{QED} will give the radiative corrections as that given by Schwinger and A^{ext} is a classic EM field.

- Through Gordon identity, the lepton current in momentum space can be written as

$$\tilde{j^\alpha} = (-ie)\bar{u}(p+q)\left[\gamma^\alpha F_E(q^2) + i\frac{\sigma^{\alpha\beta}q_\beta}{2m_\mu}F_M(q^2)\right]u(p),$$

- where $F_E(q^2)$ is called the Dirac (or electric charge) form factor and $F_M(q^2)$ is the Pauli (or magnetic) form factor.

Magnetic moment

- Then, $\vec{\mu}$ is the part interacting with the $\vec{B}$ from A^{ext}, $\vec{\mu} \cdot \vec{B}$.
- This gives

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{s},$$

- where

$$\boxed{g = 2[F_1(0) + F_2(0)].}$$

- By neglecting contributions from A_μ^{QED} one gets $F_1(0) = 1$ and $F_2(0) = 0$, recovering Dirac's result $g = 2$.
- Therefore, the $\vec{\mu} \cdot \vec{B}$ interaction is needed to measure a_μ.

How to measure a_μ?

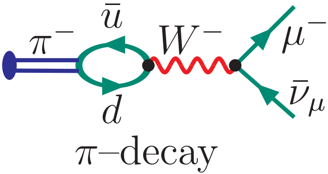
- If $\overrightarrow{B}$ is constant, the problem reduces to determining the helicity.
- However, one big issue arises. Muons are unstable!
- Thanks to maximal parity violation of weak interactions one can determine the helicity of the muon.
- To see this one needs to know how to generate muons.

The π decay

- Charged pions decay 99.99% of the time to muons

$$\mathcal{B}(\pi^{\pm} \to \mu^{\pm}\nu_\mu) \approx 99.99\%.$$

- Therefore, one can produce muons by first producing $\pi^{\pm}$, generated by hitting a fixed target with a proton beam.



.

$\star$

- The lepton current coupling to the weak gauge boson, W^α, is

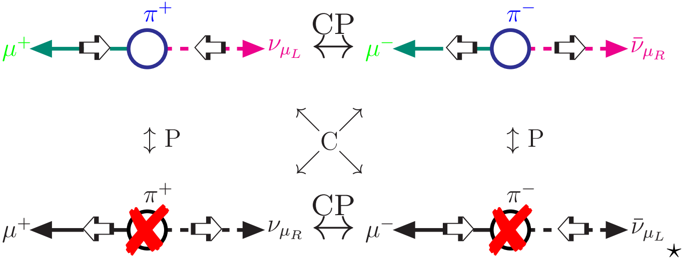
$$j^W_\alpha(x) = \bar{\psi}_{\nu L}(x)\gamma_\alpha \psi_{\mu L}(x),$$

- where $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ is a left eigenstate of helicity.

* Figure treacherously stolen from F. Jegerlehner & A. Nyffeler, Phys.Rep.477(2009).

Helicity of muons.

- This means that muons obtained from π decays have a determined helicity.



- From π^+ decays results right anti-muons, where from π^- decays results left muons.

⋆ Also treacherously taken from Jegerlehner & Nyffeler, Phys.Rep.477(2009).

Helicity of electrons

- The muon also decays through a weak gauge boson exchange.

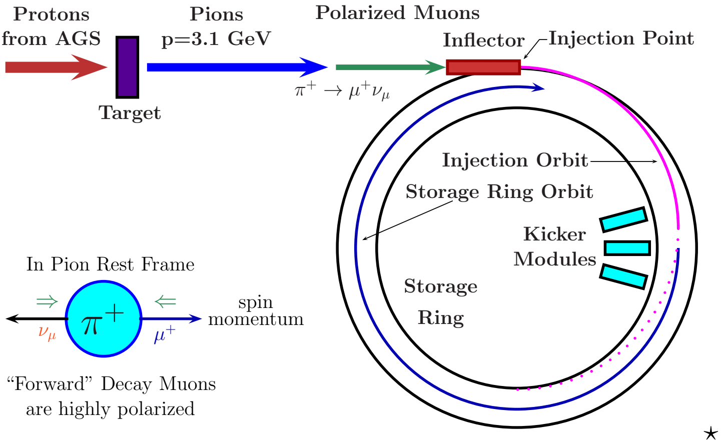
★

- This means that the helicity of the electron (positron) can also be determined.
- Therefore, in wherever direction the electron is ejected, it must be parallel (e^+) or antiparallel (e^-) to its momentum.
- An additional electric quadrupole field normal to the muon orbit is used to focus the beam.

★ Same as before, Jegerlehner and Nyffeler, Phys.Rep.33(2009)

Experimental summary

- To summarize, this is the experimental setup.

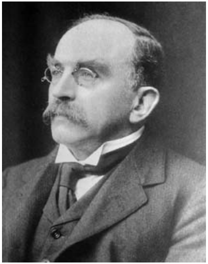


- All remaining is to determine the Larmor precession.

⋆ Same, F. Jegerlehner and A. Nyffeler, Phys.Rep.33(2009)

Who TF Larmor?♠

- The Larmor precession is defined as the precession of a magnetic moment about a magnetic field.



- The Larmor frequency in this case is

$$\vec{\omega} = -\frac{e}{m_\mu}\left[a_\mu \vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{v}\cdot\vec{B})\vec{v} + \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{E}\times\vec{v}\right].$$

♠ "Who is That Famous Larmor?"

It's magic?

- One can *magically disappear* the electric quadrupole field contribution.

Magic? Always believe it's not so

- It is done by choosing *the magic* Lorentz factor to be $\gamma^{\infty} = 29.3$, corresponding to a *magic* energy $E_\mu^{\infty} \approx 3.098$ GeV.
- $\overrightarrow{E}$ generates an oscillation in the beam direction and in $\overrightarrow{B}$ direction.
- The reason to disregard the contribution from $\overrightarrow{E}$ is to minimize $\overrightarrow{\omega}$. This will reduce the error for a_μ.

Resonance Chiral Theory RχT

- The relevant degrees of freedom are[21] the octet of the lightest pseudoscalar (π, K, η and η').

- The expansion parameter in this theory is $1/N_C$, and in large N_C the $U(1)_A$ broken symmetry is restored, that is the reason for taking η' at the same level as the other resonances.

[21] G. Ecker, J. Gasser A. Pich y E. De Rafael Nucl.Phys. B321(1989)

$F_{\pi\gamma\gamma}$ parameters

- RχT parameters can be found using short distance behavior of QCD, which predicts an asymptotic behavior of s^{-1} for this process.
- Thus, short distance relationships[22] ensure a convergent behavior

$$d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} + \frac{F^2}{8F_V^2} - \frac{4\sqrt{2}P_2}{F_V}; \qquad c_{125} = 0; \qquad d_{123} = \frac{1}{24};$$

$$F_V = \sqrt{3}F; \qquad c_{125} = 0; \qquad c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V}$$

[22] J. Sanz-Cillero and P. Roig, Phys.Rev.Lett.B733(2014)

Restored $U(1)_A$

- Within t'Hooft's large N_C, the anomaly term is suppressed by a factor $1/N_C$ with respecto to the rest of the QCD lagrangian

$$\frac{g^2}{8\pi^2}\frac{\theta}{N_C} TrF^{\mu\nu}\tilde{F}_{\mu\nu},$$

- Therefore in the limit $N_C \to \infty$ the $U(1)_A$ symmetry is restored.

Wess-Zumino-Witten

- A fundamental part of the analysis is the *WZW* term, wich is order p^4 in the chiral counting and describe intrinsic odd interactions [23].

$$Z[U,l,r] = -\frac{iN_C}{240\pi^2}\int_{M^5} d^5x\varepsilon^{ijklm}\langle\Sigma_i^L\Sigma_j^L\Sigma_k^L\Sigma_l^L\Sigma_m^L\rangle$$
$$-\frac{iN_C}{48\pi^2}\int d^4x\varepsilon_{\mu\nu\rho\sigma}(W(U,l,r)^{\mu\nu\rho\sigma} - W(\mathbf{1},l,r)^{\mu\nu\rho\sigma})$$
$$W(U,l,r)_{\mu\nu\rho\sigma} = \langle U\ell_\mu\ell_\nu\ell_\rho U^\dagger r_\sigma + \frac{1}{4}U\ell_\mu U^\dagger r_\nu U\ell_\rho U^\dagger r_\sigma + iU\partial_\mu\ell_\nu\ell_\rho U^\dagger r_\sigma$$
$$+ i\partial_\mu r_\nu U\ell_\rho U^\dagger r_\sigma - i\Sigma_\mu^L\ell_\nu U^\dagger r_\rho U\ell_\sigma + \Sigma_\mu^L U^\dagger\partial_\nu r_\rho U\ell_\sigma$$
$$-\Sigma_\mu^L\Sigma_\nu^L U^\dagger r_\rho U\ell_\sigma + \Sigma_\mu^L\ell_\nu\partial_\rho\ell_\sigma + \Sigma_\mu^L\partial_\nu\ell_\rho\ell_\sigma - i\Sigma_\mu^L\ell_\nu\ell_\rho\ell_\sigma$$
$$+\frac{1}{2}\Sigma_\mu^L\ell_\nu\Sigma_\rho^L\ell_\sigma - i\Sigma_\mu^L\Sigma_\nu^L\Sigma_\rho^L\ell_\sigma - (L\leftrightarrow R)\rangle,$$
$$\Sigma_\mu^L = U^\dagger\partial_\mu U, \Sigma_\mu^R = U\partial_\mu U^\dagger, \tag{1}$$

[23] J. Wess and B. Zumino Phys.Lett.37B(1971)
E. Witten, Nucl. Phys. B223 (1983)

Contribución de resonancias a las LEC de χPT a $\mathcal{O}(p^4)$

- El lagrangiano de interacción de las resonancias vectoriales es

$$\mathcal{L}(V) = \langle V_{\mu\nu} J^{\mu\nu} \rangle; \qquad J^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + i \frac{G_V}{2\sqrt{2}} [u^\mu, u^\nu]$$

- Con $f^\mu \nu_\pm = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$, donde

$$F_{R,L}^{\mu\nu} = \partial^\mu (r,\ell)^\nu - \partial^\nu (r,\ell)^\mu - i \left[(r,\ell)^\mu, (r,\ell)^\nu \right]$$

- siendo r y ℓ las corrientes vectoriales y axiales externas, respectivamente.
- y $u^\mu = i \left[u^\dagger \left(\partial^\mu - i r^\mu \right) u - u \left(\partial^\mu - i \ell^\mu \right) u^\dagger \right] = i u^\dagger D_\mu U u^\dagger$
- F_V y G_V son parámetros reales.

- Así, se encuentra que V debe cumplir una ecuación de constricción

$$\nabla^\alpha \nabla_\rho V^{\alpha\beta} - \nabla^\beta \nabla_\rho V^{\rho\alpha} + M_V^2 V^{\alpha\beta} = -2J^{\alpha\beta}$$

- Donde $\nabla_\mu R = \partial_\mu R + [\Gamma_\alpha, R]$ y

$$\Gamma_\alpha = \frac{1}{2}[u^\dagger(\partial_\alpha - ir_\alpha)u + u(\partial_\alpha - i\ell_\alpha)u^\dagger].$$

Al sustituir V y a órden p^4 se tiene que

$$L_1^V = \frac{G_V^2}{8M_V^2} \qquad L_2^V = 2L_1^V \qquad L_3^V = -6L_1^V$$

$$L_9^V = \frac{F_V G_V}{2M_V^2} \qquad L_{10}^V = -\frac{F_V^2}{4M_V^2}$$

- y de igual forma para las demás resonancias.