Fractal structure, power-law distribution and hadron spectrum

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XIII Quark Confinement and the Hadron Spectrum
Maynooth - Ireland
1 - 6 Aug., 2018
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QGP: QCD and its applications

Fractal structure, power-law distribution and hadron spectrum

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Scales in YM theory
Fractal structure of gauge fields

Fractal structure of gauge fields
R. Hagedorn: thermodynamical approach to HEC

exponential distributions of energy and momentum

exponential hadron mass spectrum

Hadron Resonance Gas models, conf./deconf. phase-transition

but disagrees from experimental data
Renormalization of gauge fields

Yang-Mills theory is renormalizable:

\[ \Gamma(p, m, g) = \lambda^{-D} \Gamma(p, \mu, \bar{g}) \]

F. Dyson, PR 75 (1949) 1736
M. Gell-Mann and F.E. Low, PR 95 (1954) 1300

Renormalization group equation:

\[ \left[ M \frac{\partial}{\partial M} + \beta \frac{\partial}{\partial \bar{g}} + \beta \frac{\partial}{\partial \mu} + d \right] \Gamma = 0 \]  

Callan-Symanzik Equation

C.G. Callan Jr., PRD 2 (1970) 1541

Effective coupling constant \( \bar{g} \)

Effective mass \( \mu \)
Renormalization of gauge fields

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F. Dyson, PR 75 (1949) 1736
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Renormalization group equation:
\[ \left[ M \frac{\partial}{\partial M} + \beta_g \frac{\partial}{\partial \bar{g}} + \beta_\mu \frac{\partial}{\partial \mu} + d \right] \Gamma = 0 \]

Callan-Symanzik Equation

C.G. Callan Jr., PRD 2 (1970) 1541

\( n=0 \) ———

\( n=1 \) ——— + 

\( n=2 \) ——— + 

Effective coupling constant \( \bar{g} \)

Effective mass \( \mu \)
Multiparticle production

Example of complex graphs in multiparticle production:

Summing up all diagrams $\rightarrow$ ideal gas of particles with different masses


Particle production is complex, not chaotic

K. Konishi, Phys. Scr. 19 (1979) 195

Hadron structure is complex

Too many complex graphs to be considered. Calculations limited to first leading orders or LQCD.
Including fractal structure in YM fields

At any scale: ideal gas of particles with different masses.

\[ Z = Tr < x|U(i\beta H, 0)|x > = \int Dx < x|e^{-\beta H}|x > \]
\[ Z = \prod_i \int dmd^3 p < x_i|e^{-\beta H_i}|x_i > \]
\[ Z = \prod_i \int dm\tilde{P}(m)d^3 p < x_{i,m}|e^{-\beta H_i}|x_{i,m} > \]

This partition function can be written as
\[ Z = \prod_i \int dm\tilde{P}(m)d^3 p < x_{i,m}|e^{-\beta H_i}|x_{i,m} > \]

\[ i \text{ and } \mu \text{ are the particle index and effective mass.} \]

Therefore: \[ Z = \prod_i \int dm\tilde{P}(m)e^{-\beta \epsilon_i}d^3 p_i, \quad \epsilon^2 = p^2 + m^2. \]
Including fractal structure in YM fields

\[ Z = \prod_i \int dm \tilde{P}(\mu)e^{-\beta\epsilon_i}d^3p_i, \quad \epsilon^2 = p^2 + \mu^2. \]

\[ P(U)dU = \prod_i AkT \tilde{P}(\mu)e^{-\beta\epsilon_i}d\mu d^3\pi, \]

is the probability to find the system with energy between \( U \) and \( U + dU \),

\[ U = \sum_i \epsilon_i, \quad \pi = p/(kT), \quad \mu = m/(kT). \]

Parent parton is also a parton \( \rightarrow P(U) \propto \tilde{P}(\mu) \).

Self-symmetry in gauge fields!

It can be show that \( P(\mu) \) must be such that:

\[ P(\epsilon) = A[1 + (q - 1)\frac{\epsilon}{kT}]^{-\frac{1}{q-1}} \]

\[ \frac{\epsilon}{kT} = \frac{\mu}{K}, \text{ with } K \text{ being the parton kinetic energy.} \]

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AD, PRD (2016)
Non extensivity in gauge field theory

\[ P(\varepsilon) = A[1 + (q - 1)\frac{\varepsilon}{kT}]^{-\frac{1}{q-1}} \]

Tsallis q-exponential function \( \rightarrow \) Tsallis Statistics

\[ \frac{\varepsilon}{kT} = \frac{\mu}{K}, \text{ with } K \text{ being the parton kinetic energy.} \]
\[ (q - 1) = \frac{2}{3N}(1 - \nu) \text{ and } \tau = (q - 1)NT \]

\( q \) and \( \tau \) are completely determined in terms of the fractal structure.

Suggest that at each vertex, momentum and effective masses are determined by the same scaled distribution

\[ \bar{g}^2 = \prod_i \left[1 + (q - 1)\frac{\varepsilon_i}{kT}\right]^{-\frac{1}{q-1}} \]

We can show that with this ansatz \( \beta \bar{g} \propto g^3 \), same behavior as in pQCD.
Comparison with experiments

Extended Hagedorn theory to non extensive statistics: AD, Physica A 391 (2012) 6380

use of Tsallis factor: \[ P(\varepsilon) = A\left[ 1 + (q - 1) \frac{\varepsilon}{kT} \right]^{-\frac{1}{q-1}} \]

L. Marques, E. Andrade-II, AD, PRD 87 (2013) 114022
Effective mass spectrum

\[ \bar{g}(m, \varepsilon, T) = \left[ \prod_{i=1}^{N'} \rho(m_i)[\tilde{P}(\varepsilon_i)]^\nu \right] \]

\[ \left[ M \frac{\partial}{\partial M} + \beta \bar{g} \frac{\partial}{\partial \bar{g}} + d \right] \Gamma = 0 \]

We can show that to satisfy CS Equation the mass spectrum must be given by:

\[ \rho(m) = \rho_0 \left[ 1 + (q - 1)m/M \right]^{1/(q-1)} \]

Such result was already obtained by extending Hagedorn’s Self-Consistent Thermodynamics to the non extensive case.
Effective mass spectrum and observed data

\[ \rho(m) = \rho_o \left[ 1 + (q - 1)m/M \right]^{1/(q-1)} \]

Obtained in Non Extensive Self-Consistent Thermodynamics, by a completely different approach


L. Marques, E. Andrade-II, AD, PRD 87 (2013) 114022
Summary of theory results

Scale invariance of gauge theory leads to

- fractal structure
- fractal dimension in multiparticle production
- Tsallis statistics
- non extensive self-consistent thermodynamics
Experimental verification

Scale invariance of gauge theory

leads to fractal structure

fractal dimension in multiparticle production

Tsallis statistics

non extensive self-consistent thermodynamics

\[ D \sim 0.65 \]
Experimental verification

Scale invariance of gauge theory leads to fractal structure.

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- Tsallis statistics
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Scales in YM theory

Fractal structure of gauge fields

Fractal structure of gauge fields

power-law distributions

non extensive mass spectrum
Applications

High energy collisions:


Hadron models:

P.H.G Cardoso; T.N. da Silva; AD; D.P. Menezes, EPJA 51 (2015) 155

Hadron mass spectrum:

L. Marques; E. Andrade-II; AD, Phys. Rev. D 2013, 87, 114022

Neutron stars:


LQCD:

AD PG 41 (2014) 055108

Non extensive statistics:

AD, Physica A 391 (2012) 6380
Conclusions:

Scale invariance in gauge fields leads to:

- Self-consistency and fractal structure
- Recursive calculations at any order
- Non extensive statistics
- Reconciles Hagedorn’s theory with QCD
- Agreement with experimental data

Thank you