Baryon masses and σ terms in SU(3) BChPT × 1/Nc

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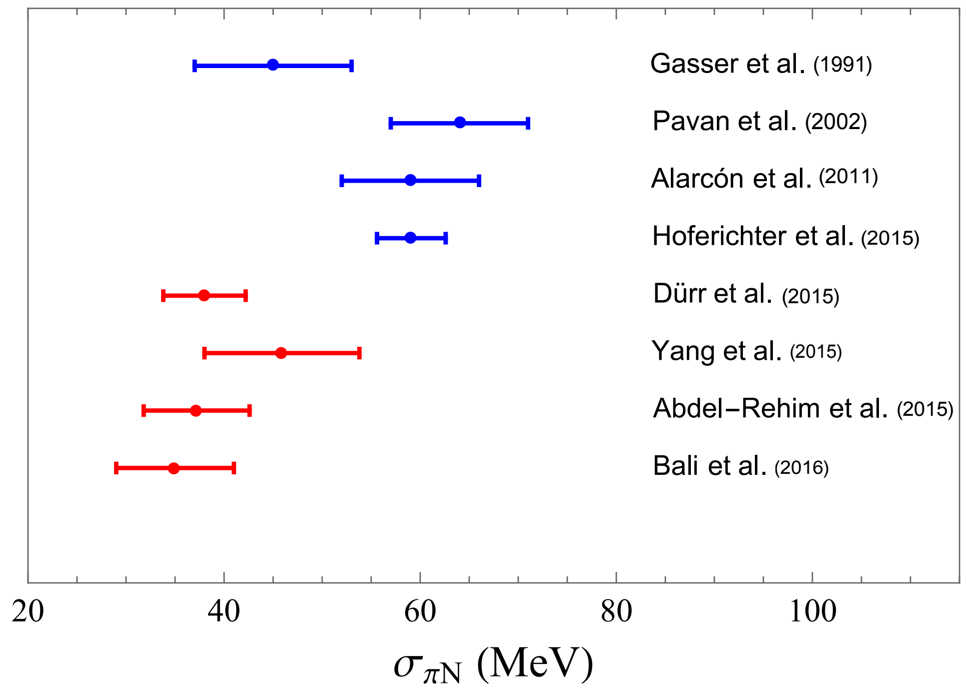
1 Motivation

1) The value of the pion-Nucleon sigma term ranges from 45 MeV to 64 MeV



$$\sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N}\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle$$

2) There is a long lasting "puzzle" associated with a combination of baryon masses (in SU(3)) in the iso-spin symmetric limit, to obtain the pion-Nucleon sigma term, assuming the contribution by strange quark mass to the nucleon mass is negligible (OZI).

3) The connection between the pion-Nucleon sigma term and size of the correction to the Gell-Mann-Okubo relation

Can one explain these from the ChPT point of view ?

2

Introduction to the combined approach : BChPT x 1/Nc expansion

Non relativistic version of the BChPT or HBChPT is based on the expansion in terms of the "baryon mass"

$$p_\mu = m_{\mathcal{B}} v_\mu + k_\mu \qquad v^\mu v_\mu = 1 \qquad v.k \ll m_{\mathcal{B}}$$

The issue of experiencing a slower rate of convergence compare to the Goldstone Boson Sector

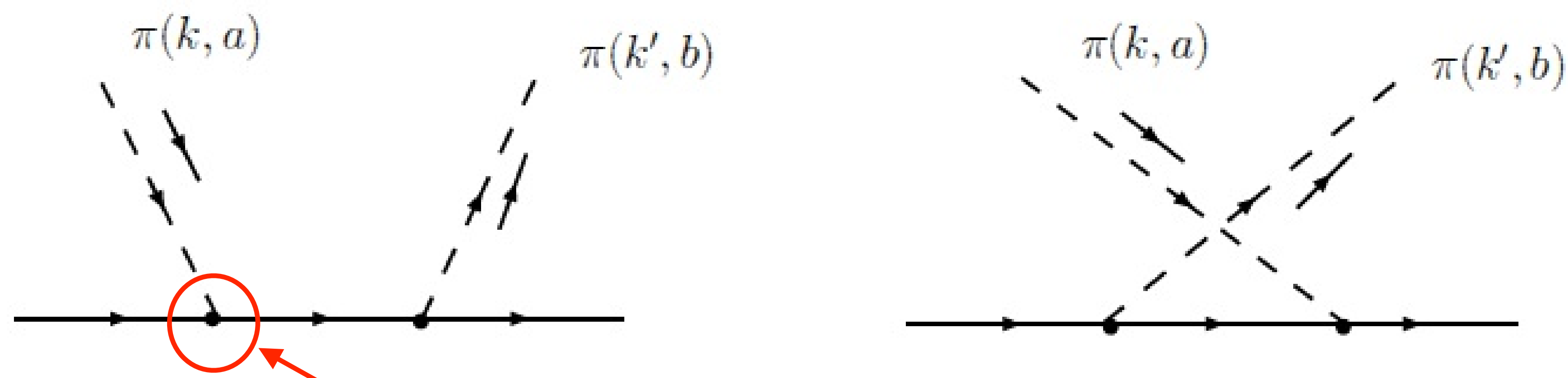
$$\frac{1}{p^2 - m_{\mathcal{B}}^2} \rightarrow \frac{1}{2m_{\mathcal{B}}} \frac{1}{(v.k)} + \mathcal{O}\left(1/m_{\mathcal{B}}^2\right)$$

Solution : Inclusion of the decuplet baryons in one-loop corrections to physical observables, has been showing a great improvement!

On the other hand, studying the baryons in the large Nc limit of QCD emerges a dynamical symmetry called "spin-flavor symmetry" which requires the possibility of having degenerate baryon multiplets of higher spin in the intermediate state/s.

2 Spin-flavor symmetry of Baryons in large Nc

Gervais & Sakita; Dashen & Manohar



$\mathcal{O}(\sqrt{N_c})$

$$\frac{\partial_\mu \pi^a}{f_\pi}\langle B'|\bar{q}\gamma^\mu\gamma_5 T^a q|B\rangle \equiv \frac{\partial_i \pi^a}{f_\pi} g_A N_c \langle B'|X^{ia}|B\rangle \Rightarrow \mathcal{O}(\sqrt{N_c})$$

Since, πN amplitude is $\mathcal{O}(N_c^0)$

This result is a tree level, so definitely we can apply it to the case of one loop

$$A = -i\frac{k^i k^j}{k_0}\frac{{N_c}^2 g^2}{f_\pi^2}[X^{ia}, X^{ib}] \Rightarrow [X^{ia}, X^{ib}] \leqslant \mathcal{O}(1/N_c)$$

Large N_c consistency condition
$[X_0^{ia}, X_0^{ib}] = 0$

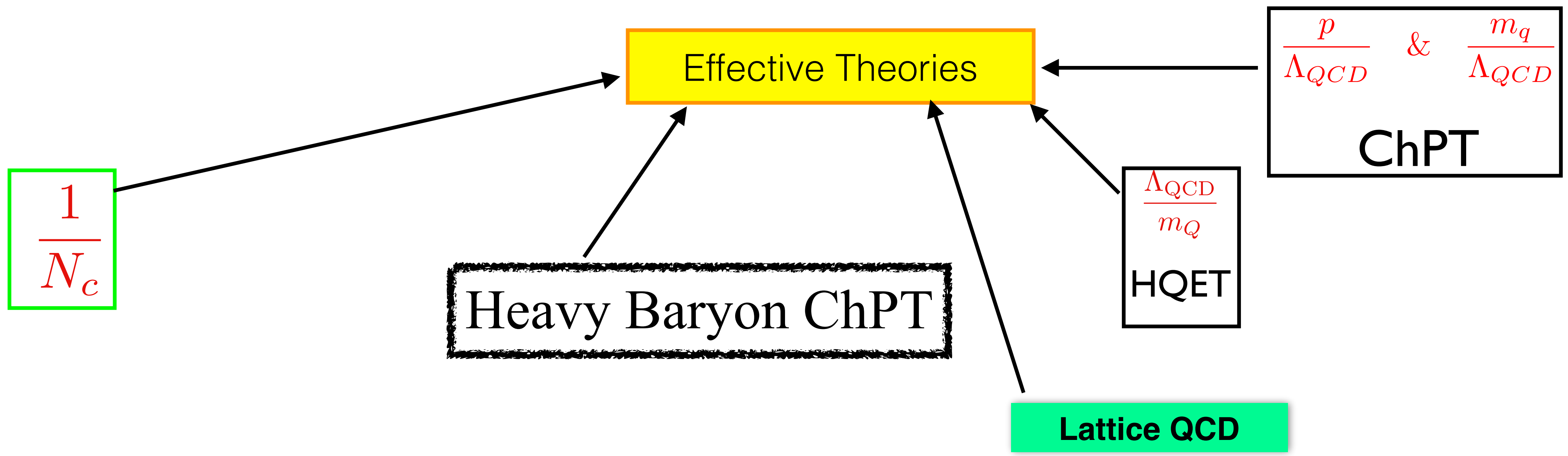
At large Nc, QCD has contracted spin-flavor symmetry $SU_c(2N_f)$ in baryon sector

This spin-flavor symmetry requires the existence of degenerate baryon multiplets with different spins (a dynamical symmetry) : leads to the consideration of both octet and decuplet contributions in the intermediate state

This symmetry is broken at sub-leading orders in 1/**Nc**

2 Introduction to the combined approach : BChPT x 1/Nc expansion (Continued…)

$$L(\text{Lagrangian}) = x^0 L_{LO} + x^1 L_{NLO} + x^2 L_{NNLO} + x^3 L_{NNNLO} + \ldots.$$



Spin-flavor Symmetry + Chiral Symmetry

Combined approach

Combining the HBChPT with 1/Nc provides a well behaved expansion in the low energy phenomenology

$$\xi - \text{expansion}: \quad 1/N_c = \mathcal{O}(p)$$

Introduction to the combined approach : BChPT x 1/Nc expansion (Continued...)

Leading order Lagrangian

$$\mathcal{L}_{\mathbf{B}}^{(1)} = \mathbf{B}^\dagger \left(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{\mathrm{HF}}}{N_c} \hat{\vec{S}}^2 + \frac{c_1}{2\Lambda} \hat{\chi}_+ \right) \mathbf{B}$$

Only the hyperfine mass splitting term breaks symmetry at $\mathcal{O}(\frac{1}{N_c})$

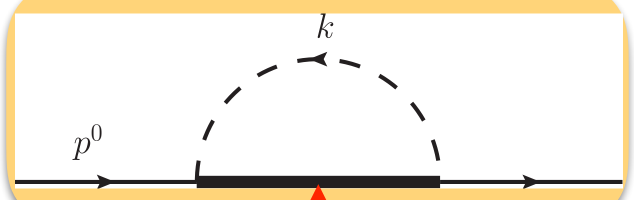
$$-\frac{C_{HF}}{N_c} \mathbf{B}^\dagger \hat{S}^2 \mathbf{B}$$

$$\begin{aligned}
\mathcal{L}_{\mathbf{B}}^{(2)} &= \mathbf{B}^\dagger \Big(-\frac{1}{2m} D^2 + \frac{c_2}{\Lambda} \chi_+^0 \frac{C_1^A}{N_c} u^{ia} S^i I^a + \frac{C_2^A}{N_c} \epsilon^{ijk} u^{ia} \{S^j, G^{ka}\} + \frac{1}{m} (\vec{B}_+^0 + \vec{B}_+^a I^a) \cdot \vec{S} \\
&+ \frac{1}{2m} (2(\kappa_0 \, \vec{B}_+^0 + \kappa_1 \, \vec{B}^a I^a) \cdot \vec{S} + \frac{6}{5} \kappa_2 \, B_+^{ia} G^{ia}) + \rho_0 \vec{E}_-^0 \cdot \vec{S} + \rho_1 E_-^{ia} G^{ia} \\
&+ i \frac{\tau_{1+}}{N_c} (u_0^a G^{ia} D_i + D_i u_0^a G^{ia}) + \frac{\tau_{1-}}{N_c} [D_i, u_0]^a G^{ia} \Big) \mathbf{B}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\mathbf{B}}^{(3)} = \mathbf{B}^\dagger \Big(& \frac{c_3}{N_c \Lambda^3} \hat{\chi}_+^2 + \frac{h_1 \Lambda}{N_c^3} \hat{S}^4 + \frac{h_2}{N_c^2 \Lambda} \hat{\chi}_+ \hat{S}^2 + \frac{h_3}{N_c \Lambda} \chi_+^0 \hat{S}^2 + \frac{h_4}{N_c \Lambda} \chi_+^a \{S^i, G^{ia}\} \\
& + \frac{C_3^A}{N_c^2} u^{ia} \{\hat{S}^2, G^{ia}\} + \frac{C_4^A}{N_c^2} u^{ia} S^i S^j G^{ja} \\
& + \frac{D_1^A}{\Lambda^2} \chi_+^0 u^{ia} G^{ia} + \frac{D_2^A}{\Lambda^2} \chi_+^a u^{ia} S^i + \frac{D_3^A(d)}{\Lambda^2} d^{abc} \chi_+^a u^{ib} G^{ic} + \frac{D_3^A(f)}{\Lambda^2} f^{abc} \chi_+^a u^{ib} G^{ic} \\
& + g_E [D_i, F_{+i0}] + \alpha_1 \frac{i}{N_c} \epsilon^{ijk} F_{+0i}^a G^{ia} D_k + \beta_1 \frac{i}{N_c} F_{-ij}^a G^{ia} D_j + \cdots \Big) \mathbf{B}
\end{aligned}$$

Introduction to the combined approach : BChPT x 1/Nc expansion (Continued...)

Chiral Symmetry + Spin-flavor Symmetry



$$= \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2 - M_\pi^2} \frac{i}{p^0 + k^0 - \underbrace{(m_{B'} - m_B)}_{\mathcal{O}(1/N_c)}} \times \text{vertex factors}$$

Intermediate Octet and Decuplet baryon contributions are included

ξ – expansion : $1/N_c = \mathcal{O}(p)$

$$\begin{aligned} I_{1-loop}(Q, M_\pi) &= \int \frac{d^dk}{(2\pi)^d} \frac{\vec{k}^2}{k^2 - M_\pi^2 + i\epsilon} \frac{1}{k^0 - Q + i\epsilon} \\ &= \frac{i}{16\pi^2} \Big\{ Q \left((3M_\pi^2 - 2Q^2)(\lambda_\epsilon - \log \frac{M_\pi^2}{\mu^2}) + (5M_\pi^2 - 4Q^2) \right) \\ &+ 2\pi (M_\pi^2 - Q^2)^{3/2} + 4(Q^2 - M_\pi^2)^{3/2} \tanh^{-1} \frac{Q}{\sqrt{Q^2 - M_\pi^2}} \Big\}, \end{aligned}$$

$$Q = \delta m_n - p^0, \ \lambda_\epsilon = \tfrac{1}{\epsilon} - \gamma + \log 4\pi$$

3

Baryon Masses to $\mathcal{O}(\xi^3)$ in SU(3)

$$\mathcal{L}_B = \mathbf{B}^\dagger \Big(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{HF}}{N_c}\hat{S}^2 - \frac{1}{2\Lambda} c_2 \hat{\chi}_+ + \frac{c_3}{N_c\,\Lambda^3}\hat{\chi}_+^2 + \frac{h_1}{N_c^3}\hat{S}^4 + \frac{h_2}{N_c^2\Lambda}\hat{\chi}_+\hat{S}^2 + \frac{h_3}{N_c\Lambda}\chi_+^0\hat{S}^2 + \frac{h_4}{N_c\,\Lambda}\chi_+^a\{S^i, G^{ia}\} + \alpha\hat{Q} + \beta\hat{Q}^2 \Big)\mathbf{B}$$

$$m_B = M_0 + \frac{C_{HF}}{N_c}\hat{S}^2 - \frac{c_1}{\Lambda}2B_0(\sqrt{3}m_8 Y + N_c m_0) - \frac{c_2}{\Lambda}4B_0 m_0 - \frac{c_3}{N_c\Lambda^3}\left(4B_0(\sqrt{3}m_8 Y + N_c m_0)\right)^2 - \frac{h_1}{N_c^2\Lambda}\hat{S}^4 - \frac{h_2}{N_c\Lambda}4B_0(\sqrt{3}m_8 Y + N_c m_0)\hat{S}^2 - \frac{h_3}{N_c\Lambda}4B_0 m_0\hat{S}^2 - \frac{h_4}{N_c\Lambda}\frac{4B_0 m_8}{\sqrt{3}}\left(3\hat{I}^2 - \hat{S}^2 - \frac{1}{12}N_c(N_c+6) + \frac{1}{2}(N_c+3)Y - \frac{3}{4}Y^2\right) + \delta m_B^{\text{loop}},$$

$$\hat{\chi}_+ = N_c\,\chi_+^0 + \tilde{\chi}_+$$
$$\chi_+^0 \to 4B_0 m^0$$
$$\tilde{\chi}_+^a \to 8B_0\delta^{a8}m^8$$
$$\hat{\chi}_+ \to 4B_0(m^8 T^8 + N_c m^0)$$

$$m^0 = (2\hat{m} + m_s)/3$$
$$m^8 = 2/\sqrt{3}(\hat{m} - m_s)$$

$$\delta m_B^{1-loop} = i\,\frac{\mathring{g}_A^2}{F_\pi^2}\,\frac{1}{d-1}\sum_n G^{ia}\mathcal{P}_n G^{ia}\, I_{1-loop}(\delta m_n - p^0, M_\pi)$$

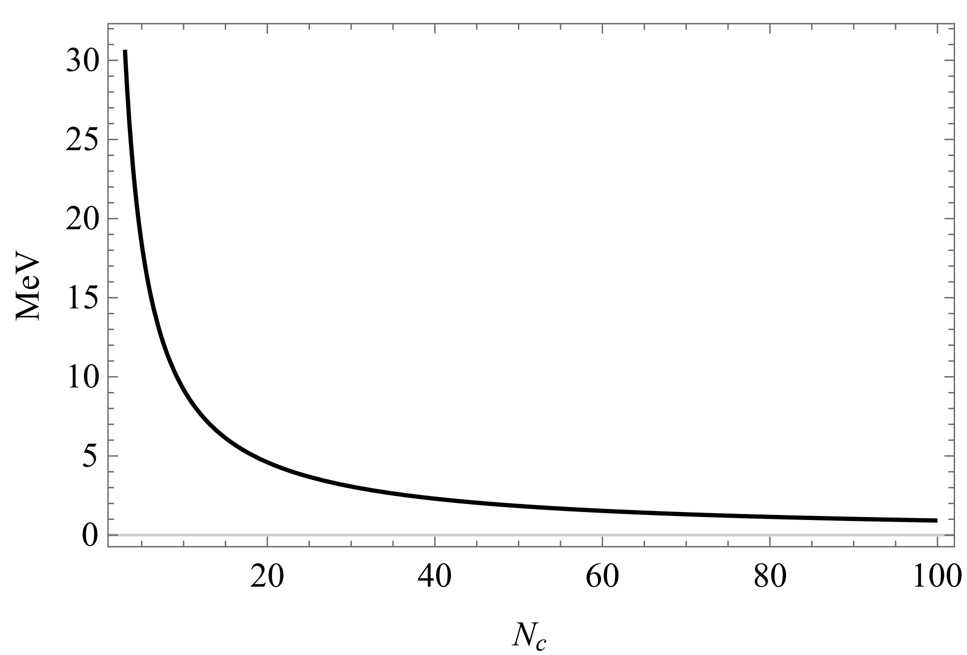
4 Violation of the GMO relation

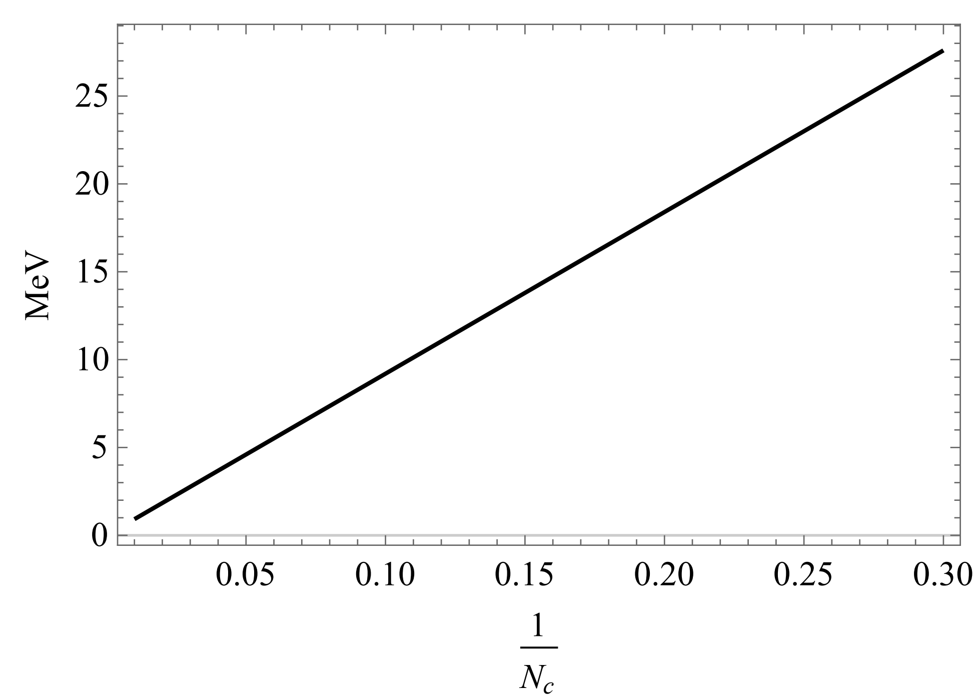
At tree level the GMO relation is exact for any Nc up to $\mathcal{O}(\xi^3)$

$$\Delta_{\mathrm{GMO}} = -\left(\frac{\overset{\circ}{g}_A}{4\pi F_\pi}\right)^2 \left(\frac{2\pi}{3}\left(M_K^3 - \frac{1}{4}M_\pi^3 - \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{4}M_\pi^2\right)^{\frac{3}{2}}\right)\right.$$
$$\left. + \frac{C_{\mathrm{HF}}}{2N_c}\left(4M_K^2 \log\left(\frac{4M_K^2 - M_\pi^2}{3M_K^2}\right) - M_\pi^2 \log\left(\frac{4M_K^2 - \frac{1}{3}M_\pi^2}{3M_\pi^2}\right)\right)\right)$$
$$+ \mathcal{O}(1/N_c^3).$$

The breaking / violation to the GMO relation is only coming through the loop corrections and it behaves like 1/Nc with Nc

GMO relation violation vs Nc





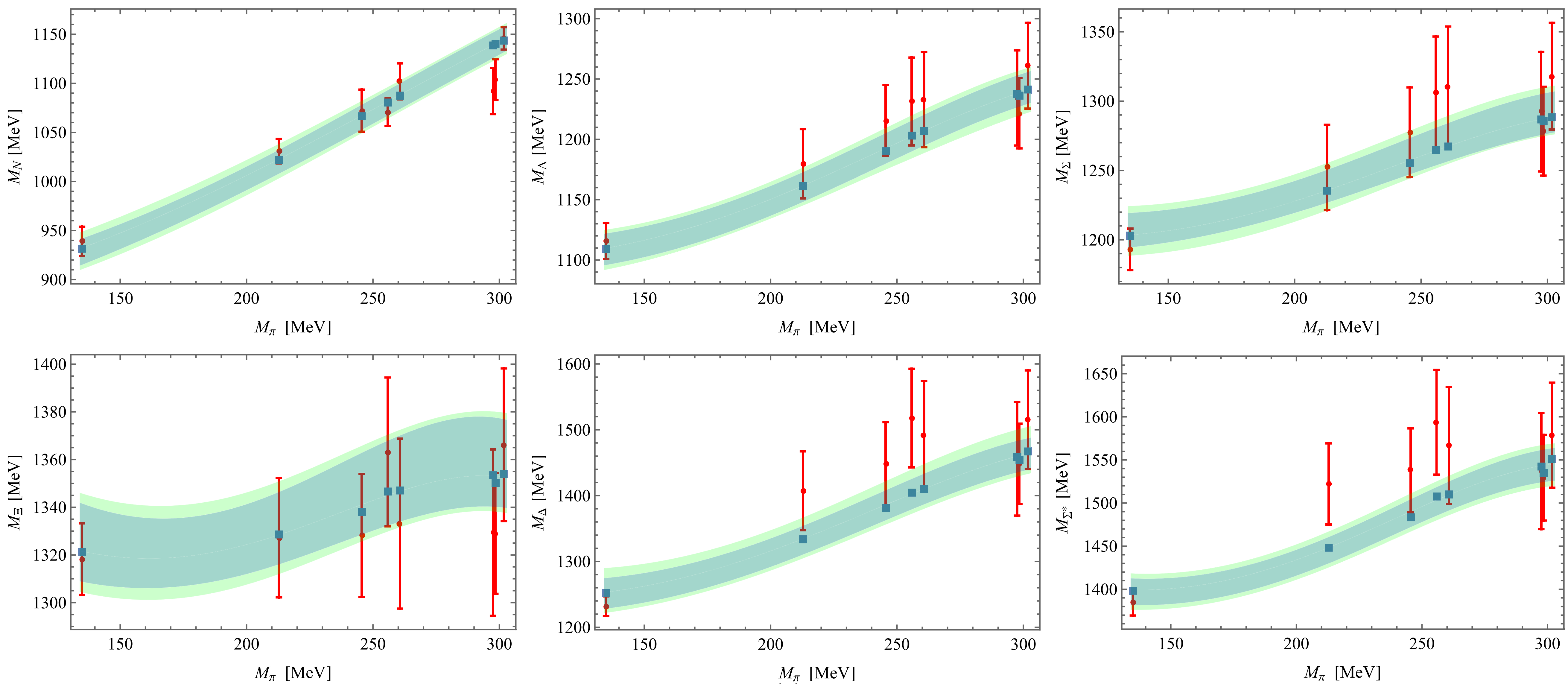
5 Fit Results to Baryon Masses

Fit results to Experimental & Lattice QCD masses

I. P. FERNANDO and J. L. GOITY — PHYS. REV. D **97,** 054010 (2018)

TABLE II. Results for LECs: the ratio $\overset{\circ}{g}_A/F_\pi = 0.0122$ MeV^{-1} is fixed by using Δ_{GMO}. The first row is the fit to LQCD octet and decuplet baryon masses [48] including results for $M_\pi \leq 303$ MeV (dof = 50), and second row is the fit including also the physical masses (dof = 58). Throughout the $\mu = \Lambda = m_\rho$.

χ^2_{dof}	m_0 [MeV]	C_{HF} [MeV]	c_1	c_2	h_2	h_3	h_4
0.47	221(26)	215(46)	−1.49(1)	−0.83(5)	0.03(3)	0.61(8)	0.59(1)
0.64	191(5)	242(20)	−1.47(1)	−0.99(3)	0.01(1)	0.73(3)	0.56(1)



LQCD baryon masses : C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou. *Phys. Rev.*, D90:074501, (2014)

6 Sigma Terms

Baryon masses and σ terms in SU(3) BChPT $\times$ 1/N_c

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Baryon matrix elements of scalar quark densities give us the information on the amount of baryon mass originates from the quark masses

Feynman-Hellman theorem

$$\sigma_i(B) = m_i \frac{\partial}{\partial m_i} m_B$$

Baryon mass dependencies on quark masses

$$\sigma_{\pi N} = \hat{\sigma} + 2\frac{\hat{m}}{m_s}\sigma_s$$

m_i indicates a quark mass

$$\sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N}\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle$$

$$\hat{\sigma} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$$

$$\sigma_s = \frac{m_s}{2m_N}\langle N|\bar{s}s|N\rangle$$

$$\sigma_{\pi N} \sim \hat{\sigma}$$

$$|\ \sigma_s\ | \lesssim 50 \text{ MeV}$$

6 Sigma Terms

A long lasting puzzle !

$$\sigma_{\pi N} = \hat{\sigma} + 2\frac{\hat{m}}{m_s}\sigma_s$$

$$\hat{\sigma} \equiv \sqrt{3}\frac{\hat{m}}{m_8}\sigma_8$$

$$\sigma_8 = \frac{1}{3}(2m_N - m_\Sigma - m_\Xi)$$

$$\hat{\sigma} = \underbrace{\frac{\hat{m}}{m_s - \hat{m}}}_{\Delta\hat{\sigma}}\overbrace{(m_\Xi + m_\Sigma - 2m_N)}^{\sim\ 26\ \text{MeV}}$$

$$m_3 = m_u - m_d$$

$$m_8 = \frac{1}{\sqrt{3}}(\hat{m} - m_s)$$

There is a (hidden) large correction ~ 44 MeV from non-analytic contributions from baryon self-energies

$$\sigma_8 = \frac{1}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$$

$$\Delta\sigma_8 \equiv \sigma_8 - \frac{1}{3}(2m_N - m_\Sigma - m_\Xi)$$

$$\Delta\sigma_8 = \sigma_8 - \frac{1}{9}\left(\frac{5N_c - 3}{2}m_N - (2N_c - 3)m_\Sigma - \frac{N_c + 3}{2}m_\Xi\right)$$

$$\Delta_{GMO} \equiv 3m_\Lambda + m_\Sigma - 2(m_N + m_\Xi) \sim 25\ \text{MeV}$$

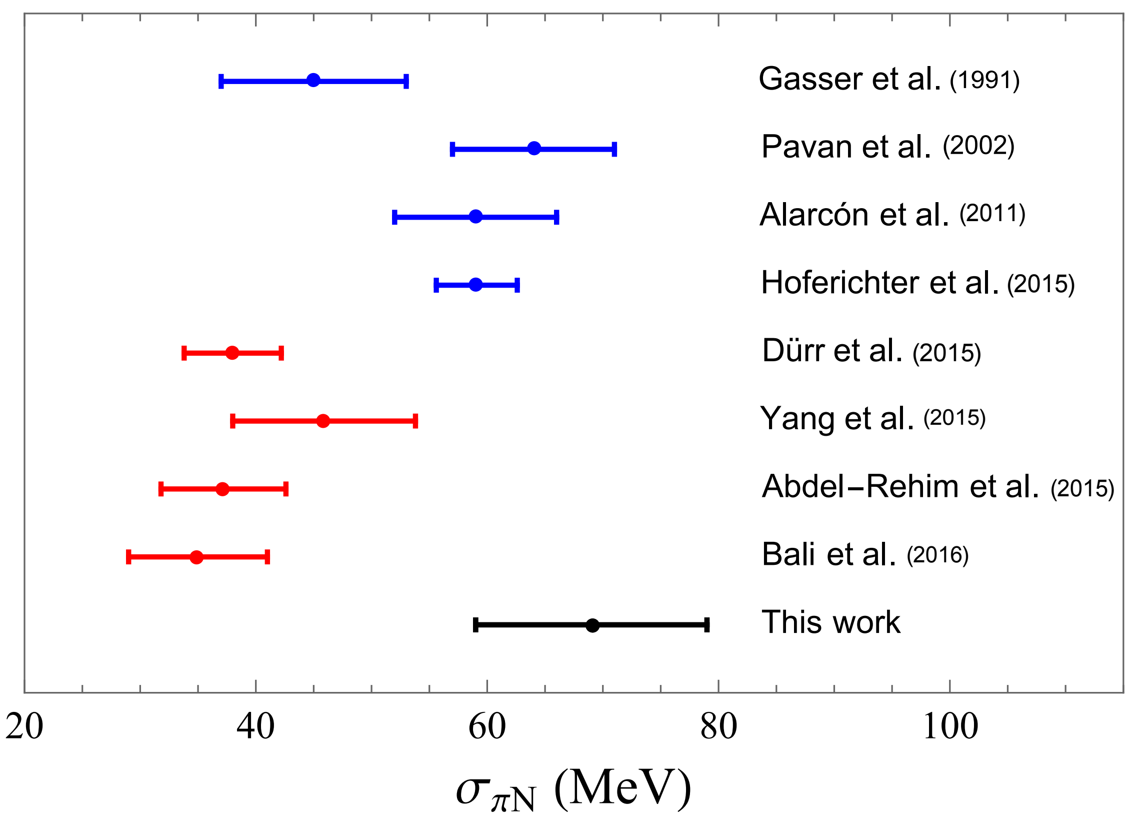
The dominant contributions to Δ_{GMO} and $\Delta\sigma_8$ are calculable non-analytic contributions: $\Delta\sigma_8/\Delta_{\rm GMO}$ (~ -13.5 for $N_c = 3$)

6 Sigma Terms (Results)

$$\sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N}\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle$$

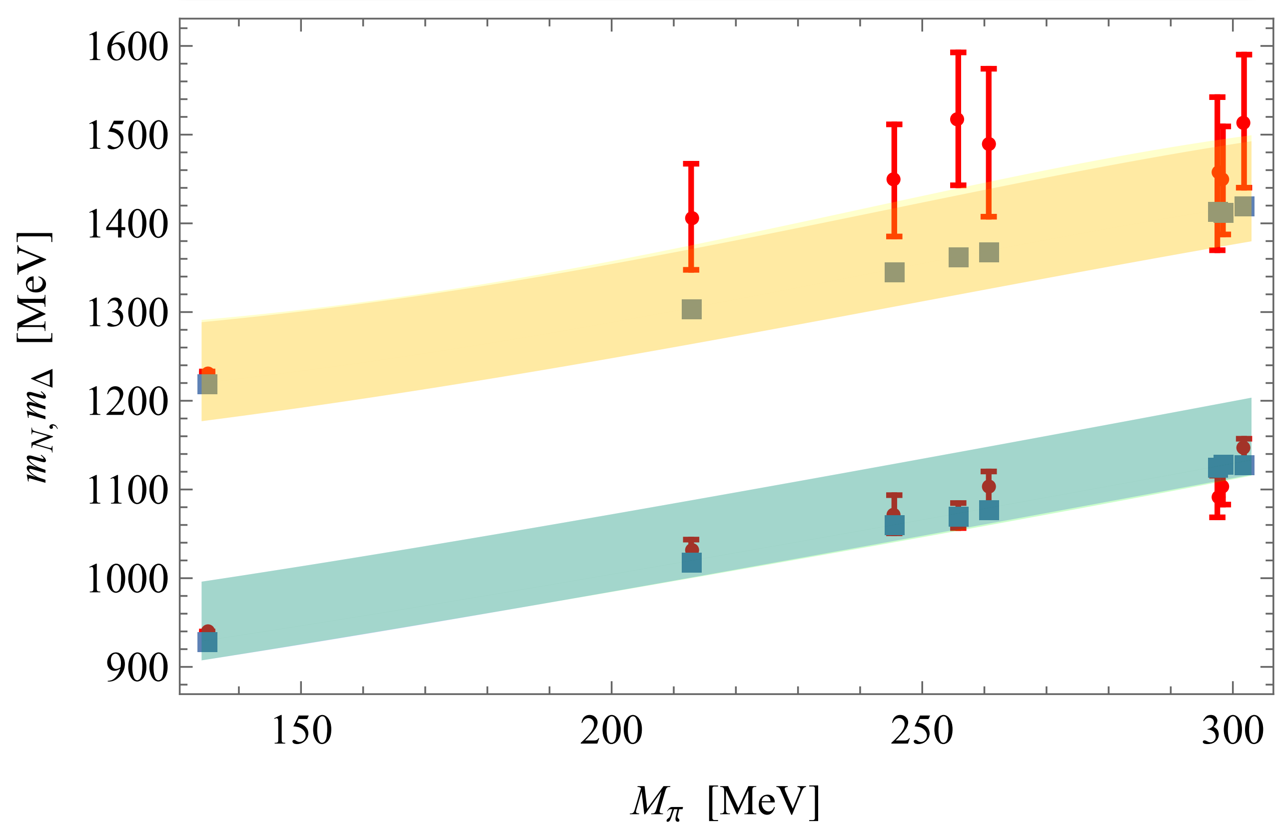
Fit	$\frac{\mathring{g}_A}{F_\pi}$ MeV^{-1}	$\frac{M_0}{N_c}$ MeV	C_{HF} MeV	c_1	c_2	h_2	h_3	h_4	α MeV	β MeV
1	0.0126(2)	364(1)	166(23)	−1.48(4)	0	0	0.67(9)	0.56(2)	−1.63(24)	2.16(22)
2	0.0126(3)	213(1)	179(20)	−1.49(4)	−1.02(5)	−0.018(20)	0.69(7)	0.56(2)	−1.62(24)	2.14(22)
3	0.0126*	262(30)	147(52)	−1.55(3)	−0.67(8)	0	0.64(3)	0.63(3)	−1.63*	2.14*

Fit	$\Delta^{\text{phys}}_{GMO}$ MeV	σ_8 MeV	$\Delta\sigma_8$ MeV	$\hat{\sigma}$ MeV	$\sigma_{\pi N}$ MeV	σ_s MeV	σ_3 MeV	$\sigma_{u+d}(p-n)$ MeV
1	25.6(1.1)	−583(24)	−382(13)	70(3)(6)	–	–	−1.0(3)	−1.6(6)
2	25.5(1.5)	−582(55)	−381(20)	70(7)(6)	69(8)(6)	−3(32)	−1.0(4)	−1.6(8)
3	25.8*	−615(80)	−384(2)	74(1)(6)	65(15)(6)	−121(15)	–	–



summary of the determinations of $\sigma_{\pi N}$ from πN scattering

LQCD data from : ALEXANDROU et al. (**PRD 90, 074501 (2014)**)



N and Δ masses: Fit 2 of Table

$$\sigma_{\pi N} = 69(8)(6)\ \text{MeV}$$

7 Observations

- The value of $\mathring{g}_A/F_\pi$ can be fixed by Δ_{GMO}, and it is consistent with the other calculations.
- Octet baryons in the intermediate states contribute 43% to Δ_{GMO} and 33% to $\Delta\sigma_8$.
- One can realize that this is a well behaved expansion by considering the contribution to the baryon mass from each LEC.
- Δ_{GMO} and $\Delta\sigma_8$ can be determined only by $\mathring{g}_A/F_\pi$, C_{HF} and the meson masses, whereas the ratio $\Delta\sigma_8/\Delta_{\text{GMO}}$ doesn't depend on $\mathring{g}_A/F_\pi$.
- Fit 2 is compatible with Fit 1: implies that the chiral extrapolation of the LQCD to the physical case is consistent.
- LQCD baryon masses have an issue of describing the hyperfine mass shifts between the octet and decuplet.
- Both $\hat{\sigma}$ and $\sigma_{\pi N}$ has mild dependence on M_K.
- Determination of σ_s was not precise because the LQCD results are at approximately fixed m_s.
- Our result for $\sigma_{\pi N}$ is consistent with the larger values obtained from $\pi - N$ scattering analyses.
- Iso spin breaking sigma terms σ_3 and $\sigma_{(u+d)}$ were estimated.
- With the information we have we can determine the contribution of Nucleon mass due to the mass difference of m_{u-d} and therefore m_{Proton} and m_{Neutron} difference.

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Observations (Continued...)

The discussion can be extended to the rest of the σ terms for the different baryons and their various relations (Tree level)

$$\sigma_{Nm_s} = \frac{m_s}{8\hat{m}}(-4(N_c-1)\sigma_{N\hat{m}} + (N_c+3)\sigma_{\Lambda\hat{m}} + 3(N_c-1)\sigma_{\Sigma\hat{m}})$$
$$\sigma_{\Lambda m_s} = \frac{m_s}{8\hat{m}}(-4(N_c-3)\sigma_{N\hat{m}} + (N_c-5)\sigma_{\Lambda\hat{m}} + 3(N_c-1)\sigma_{\Sigma\hat{m}})$$
$$\sigma_{\Sigma m_s} = \frac{m_s}{8\hat{m}}(-4(N_c-3)\sigma_{N\hat{m}} + (N_c+3)\sigma_{\Lambda\hat{m}} + (3N_c-11)\sigma_{\Sigma\hat{m}})$$
$$\sigma_{\Delta m_s} = \frac{m_s}{8\hat{m}}(-4(N_c-1)\sigma_{\Delta\hat{m}} - 5(N_c-3)(\sigma_{\Lambda\hat{m}} - \sigma_{\Sigma\hat{m}}) + 4N_c\sigma_{\Sigma^*\hat{m}})$$
$$\sigma_{\Sigma^* m_s} = \frac{m_s}{8\hat{m}}(-(N_c-3)(4\sigma_{\Delta\hat{m}} + 5\sigma_{\Lambda\hat{m}} - 5\sigma_{\Sigma\hat{m}}) + 4(N_c-2)\sigma_{\Sigma^*\hat{m}}).$$

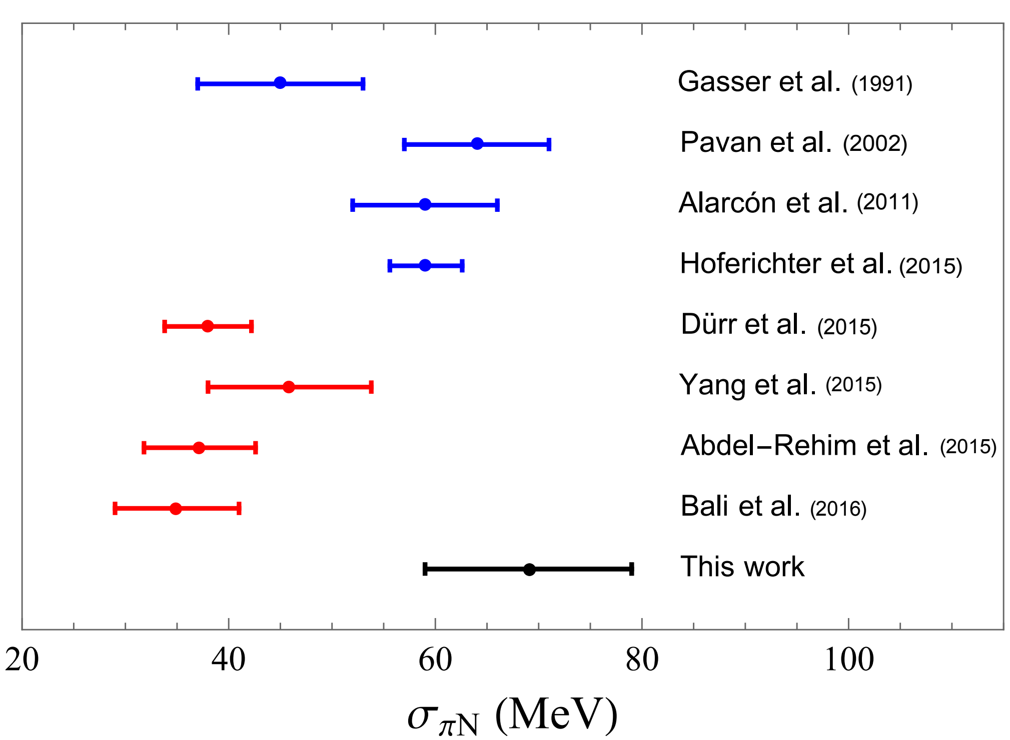
I.P. Fernando, J.L. Goity, Phys. Rev. D 97 (2018) 054010, arXiv:1712.01672.

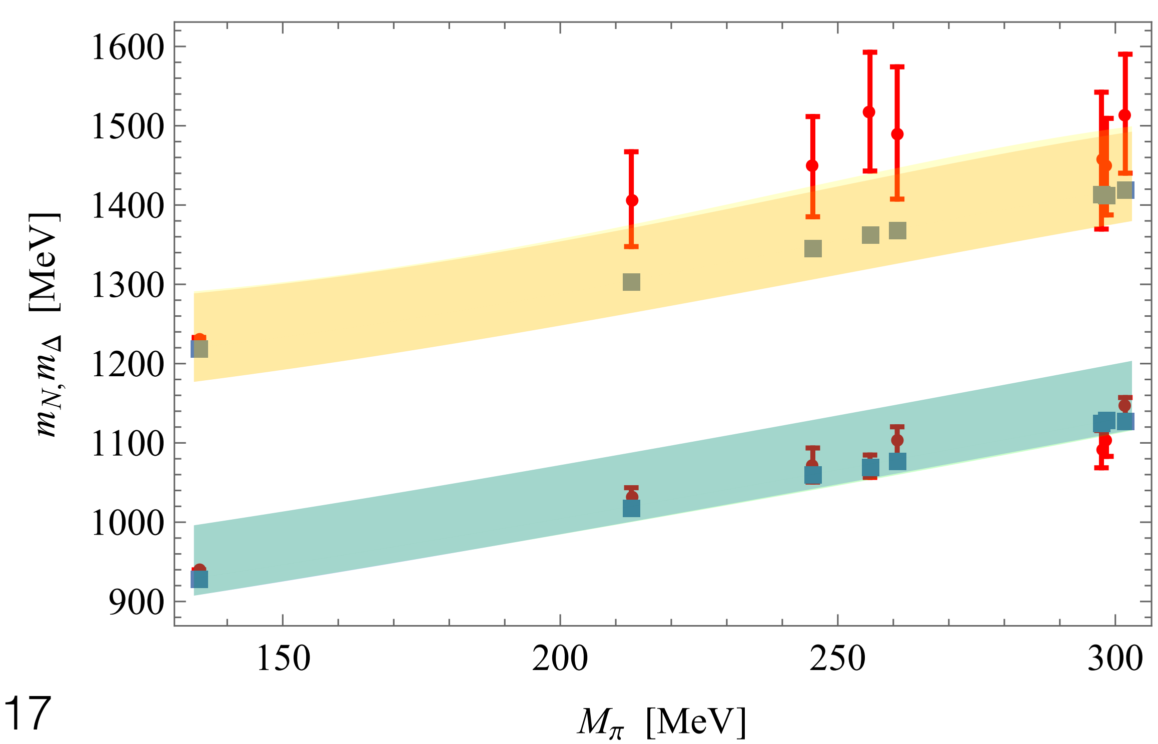
The LO axial charge can be obtained by the fits to axial currents from LQCD, which is shown to have a value lower than 20% of the physical value.

More applications…..

8 **Summary**

- The σ terms of nucleons were calculated using SU(3) BChPT × 1/Nc
- Our value for sigma Pi-N is in agreement with similar determinations in calculations that included the decuplet baryons as explicit degrees of freedom
- The "σ term puzzle" is understood as the result of large non-analytic contributions to the mass combination, while the higher order corrections to the σ terms have natural magnitude.
- The intermediate spin 3/2 baryons play an important role in enhancing $\hat{\sigma}$ and thus $\sigma_{\pi N}$
- The analysis carried out here shows that there is compatibility in the description of *GMO* and the nucleon σ terms
- The value of $\sigma_{\pi N} = 69 \pm 10$ MeV obtained here from fitting to Physical & LQCD baryon masses agrees with the more recent results from πN analyses





Thank you