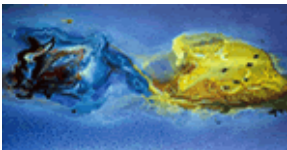


Dynamical spin effects in the pseudoscalar meson octet within holographic light-front QCD

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XIIIth Quark confinement and the Hadron Spectrum
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Overview

- 1 The light-front holographic Schrödinger Equation
- 2 Dynamical spin effects
- 3 Predicting observables
- 4 Conclusions

M. Ahmady, C. Mondal, R. Sandapen, PRD (2018). arXiv: 1805.08911

The holographic light-front Schrödinger Equation

Brodsky & de Téramond (PRL, 2009)

Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 2015)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

- $\zeta^2 = x(1-x)b_{\perp}^2$: key light-front variable for bound states.
- $x = k^+/P^+$: light-front momentum fraction of quark.
- b_{\perp} : transverse distance between quark and antiquark.
- U_{eff} : effective (includes coupling to higher Fock sectors) $q\bar{q}$ potential.

Meson light-front wavefunction :

$$\Psi(x, \zeta, \varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} f(x) e^{iL\varphi}$$

Assumptions behind the holographic LFSE

- The holographic light-front SE is derived in light-front QCD with
- No quantum loops (no Λ_{QCD})
 - Massless quarks ($m_f \rightarrow 0$)
- } semiclassical approximation
- No mass scale \rightarrow Conformal invariance \rightarrow Gravity dual in anti-de Sitter space

Brodsky, de Téramond, Dosch & Erlich (Phys. Rep. 2015)

Maldacena: AdS=CFT

A unique confinement potential

$$U_{\text{eff}}(\zeta) = \overbrace{\kappa^4 \zeta^2}^{\text{Conformal SB}} + \overbrace{2\kappa^2(J-1)}^{\text{AdS/QCD}}$$

- Mass scale κ emerges as the strength of unique harmonic oscillator potential while the semiclassical QCD action retains conformal symmetry: dAFF mechanism.
- AdS/QCD mapping : $\zeta \leftrightarrow z_5$ (z_5 is 5th dimension of AdS) gives the spin term $2\kappa^2(J-1)$ in potential and fixes $f(x) = \sqrt{x(1-x)}$ in wavefunction.

dAFF: de Alfaro, Furbini & Furlan (1976)

dAFF in LFQCD: Brodsky, de Téramond & Dosch (Phys. Lett. 2013)

Massless pseudoscalars

Solving the holographic LF Schrödinger Equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\text{eff}}(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

with

$$U_{\text{eff}}(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$$

gives

$$M^2 = (4n + 2L + 2)\kappa^2 + 2\kappa^2(J-1)$$

- Predicts Regge trajectories with massless ground states ($n = L = J = 0$).
- Identify with the Goldstone bosons (π, K, η) of spontaneous chiral symmetry breaking in QCD.

Fixing the fundamental AdS/QCD scale

Regge slopes global fits

Reference	Fit	κ [MeV]
Brodsky et al. [Phys. Rep. 15]	Mesons	540–590
Brodsky et al. [IJMD, 16]	Mesons & Baryons	523

Universal $\kappa \sim 500$ MeV successfully predicts

- Diffractive ρ production Forshaw & Sandapen (PRL, 2012)
- Diffractive ϕ production Ahmady, Sandapen, Sharma (PRD, 2016)
- Λ_{QCD} to 5-loops in $\overline{\text{MS}}$ Brodsky, Deur, de Téramond (J.Phys, 2017)

Restoring dependence on quark masses

- In momentum space, 'complete' invariant mass of $q\bar{q}$ pair

$$M_{q\bar{q}}^2 = \frac{\mathbf{k}_\perp^2}{x(1-x)} \rightarrow \frac{\mathbf{k}_\perp^2 + m_f^2}{x(1-x)}$$

Brodsky, de Téramond, Subnuclear Series Proc. (2009)

- Pseudoscalar meson wavefunction becomes

$$\Psi^P(x, \zeta^2) = \mathcal{N} \sqrt{x(1-x)} \exp\left[-\frac{\kappa^2 \zeta^2}{2}\right] \exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]$$

- m_f are effective quark masses (because of coupling of valence sector to higher Fock sectors).

Restoring dependence on quark spins

- In semiclassical approximation :

$$\Psi^P(x, \zeta^2, \lambda, \lambda') = \Psi^P(x, \zeta^2) \times S_{\lambda, \lambda'}^{\text{non dynamical}}$$

with

$$S_{\lambda, \lambda'}^{\text{non dynamical}} = \lambda \delta_{\lambda, -\lambda'}$$

- Normalization

$$\sum_{\lambda, \lambda'} \int d^2\mathbf{b} dx |\Psi^\pi(x, \zeta^2, \lambda, \lambda')|^2 = 1$$

Dynamical spin effects

$$\Psi(x, \mathbf{k}_\perp) \rightarrow \Psi(x, \mathbf{k}_\perp, \lambda, \lambda') = \Psi(x, \mathbf{k}_\perp) \times S_{\lambda\lambda'}(x, \mathbf{k}_\perp)$$

- For pseudoscalar meson, we use

$$S_{\lambda\lambda'}^{\text{dynamical}}(x, \mathbf{k}_\perp) = \frac{\bar{v}_{\lambda'}(x, \mathbf{k}_\perp)}{\sqrt{1-x}} \left[\overbrace{\frac{M_P^2}{2P^+} \gamma^+ \gamma^5}^{\text{from } (P \cdot \gamma) \gamma^5} + BM_P \gamma^5 \right] \frac{u_\lambda(x, \mathbf{k}_\perp)}{\sqrt{x}}$$

so that

$$S_{\lambda\lambda'}^{\text{dynamical}}(x, \mathbf{k}_\perp)|_{B=0} \rightarrow S_{\lambda\lambda'}^{\text{non dynamical}} = \frac{1}{\sqrt{2}} \lambda \delta_{\lambda, -\lambda'}$$

- We call B the dynamical spin parameter:
 - ① $B \neq 0$: dynamical spin effects.
 - ② $B \gg 1$: maximal dynamical spin effects.

AdS/QCD scale, quark masses and mixing angle

To generate predictions for all observables, we use

- the universal AdS/QCD scale: $\kappa = (523 \pm 24)$ MeV.
- constituent quark masses: $m_{u/d} = (330 \pm 30)$ MeV, $m_s = (500 \pm 30)$ MeV.
- $\theta = (-14.1 \pm 2.8)^\circ$ for η - η' mixing in singlet-octet flavour basis:

$$|\eta_1\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$$

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$$

$$|\eta\rangle = \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle$$

$$|\eta'\rangle = \sin\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle$$

Charge radii

$$\sqrt{\langle r_P^2 \rangle} = \left[\frac{3}{2} \int dx d^2 \mathbf{b}_\perp [b_\perp (1-x)]^2 |\Psi_P(x, \mathbf{b}_\perp)|^2 \right]^{1/2}$$

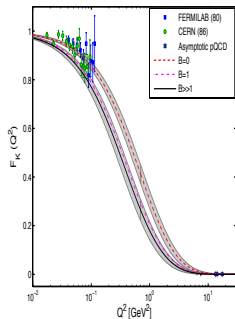
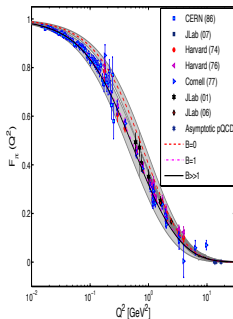
- π^\pm : maximal ($B \gg 1$)
dynamical spin preferred
- K^\pm : no ($B = 0$) dynamical
spin preferred

P	B	$\sqrt{\langle r_P^2 \rangle}_{\text{Th.}}$ [fm]	$\sqrt{\langle r_P^2 \rangle}_{\text{Exp.}}$ [fm]
π^\pm	0	0.543 ± 0.022	0.672 ± 0.008
	1	0.673 ± 0.034	
	$\gg 1$	0.684 ± 0.035	
K^\pm	0	0.615 ± 0.038	0.560 ± 0.031
	1	0.778 ± 0.065	
	$\gg 1$	0.815 ± 0.070	

Spacelike EM form factors

Drell & Yan (PRL, 70); West (PRL, 70)

$$F_{EM}(Q^2) = 2\pi \int dx db_{\perp} b_{\perp} J_0[(1-x)b_{\perp}Q] |\Psi^P(x, \mathbf{b}_{\perp})|^2$$



π^{\pm} : maximal ($B \gg 1$) dynamical spin. K^{\pm} : no ($B = 0$) dynamical spin.

Decay constants

$$\langle 0 | \bar{\Psi} \gamma^\mu \gamma_5 \Psi | P \rangle = f_P p^\mu$$

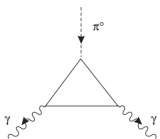
$$f_P = 2 \sqrt{\frac{N_c}{\pi}} \int dx \left\{ ((x(1-x)M_\pi^2) + Bm_f M_\pi) \frac{\Psi^\pi(x, \zeta)}{x(1-x)} \right\} \Big|_{\zeta=0}$$

- π^\pm : maximal ($B \gg 1$)
dynamical spin preferred
- K^\pm : no ($B = 0$) dynamical
spin preferred

P	B	$f_P^{\text{Th.}}$ [MeV]	$f_P^{\text{Exp.}}$ [MeV]
π^\pm	0	162 ± 8	$130 \pm 0.04 \pm 0.2$
	1	138 ± 5	
	$\gg 1$	135 ± 6	
K^\pm	0	156 ± 8	156 ± 0.5
	1	142 ± 7	
	$\gg 1$	135 ± 6	

Radiative decay widths

- Using Adler-Bell-Jackiw anomaly relations



- π^0 : maximal ($B \gg 1$) dynamical spin favoured
- η : no ($B = 0$) dynamical spin preferred
- η' : maximal ($B \gg 1$) dynamical spin preferred

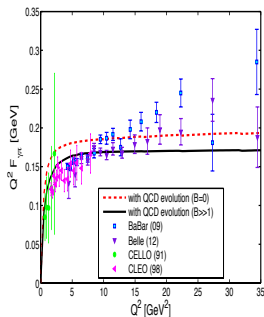
P	B_l	B_s	$\Gamma_{P \rightarrow 2\gamma}^{\text{Th.}} [\text{KeV}]$	$\Gamma_{P \rightarrow 2\gamma}^{\text{Exp.}} [\text{KeV}]$
π^0	0	-	$(5.62 \pm 0.57) \times 10^{-3}$	$(7.82 \pm 0.22) \times 10^{-3}$
	1	-	$(7.72 \pm 0.62) \times 10^{-3}$	
	$\gg 1$	-	$(8.13 \pm 0.68) \times 10^{-3}$	
η	0	0	0.542 ± 0.082	0.516 ± 0.018
	1	0	0.600 ± 0.056	
	1	1	0.622 ± 0.055	
	$\gg 1$	0	0.663 ± 0.061	
	$\gg 1$	$\gg 1$	0.710 ± 0.059	
η'	0	0	3.51 ± 0.48	4.28 ± 0.19
	1	0	3.88 ± 0.49	
	1	1	3.94 ± 0.49	
	$\gg 1$	0	4.51 ± 0.56	
	$\gg 1$	$\gg 1$	4.73 ± 0.57	

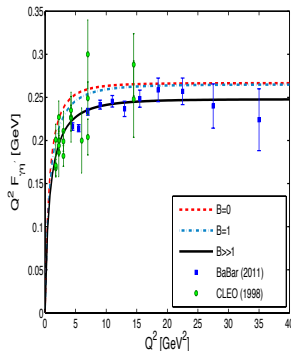
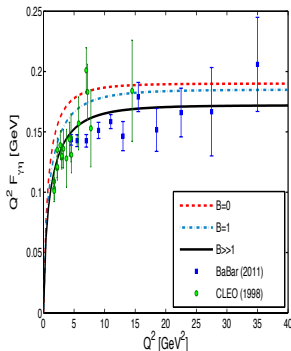
$\pi \rightarrow \gamma$ transition form factor

Lepage & Brodsky, PRD (1980)

$$F_{\gamma P}(Q^2) = \frac{\sqrt{2}}{3} f_P \int_0^1 dx \overbrace{\frac{\varphi_P(x, xQ)}{Q^2 x}}^{\text{meson DA}}$$

- Meson DA is obtained from holographic wavefunction
- Maximal ($B \gg 1$) dynamical spin preferred for π^0



$\eta^{(\prime)} \rightarrow \gamma$ transition form factors

Maximal ($B \gg 1$) dynamical spin effects preferred for both η and η'

Conclusions

Dynamical spin effects are

- maximal in pion but negligible in kaon.
- perhaps important in $\eta-\eta'$ system.
- more correlated to the quark flavour content of the pseudoscalar mesons rather than to their masses.

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