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How to find the glueball among the f_0s with QCD counting rules

A model-independent method to ascertain the leading valence composition of a hadron: to measure the energy dependence of its production cross section in a fixed angle interval. By the QCD Brodsky-Farrar counting rules, the descent steepness at high energy depends on the hadron's leading quark and gluon composition.

A reaction that could help classify the f_0 mesons with an easily reconstructible final state:

$\boxed{\text{exclusive } e^-e^+ \to \phi + f_0}$

Some f_0s may have a glueball gg component in their wavefunction; this will dominate at high energy over higher twist $q\bar{q}$ components (which must be in a p-wave) or hybrid/tetraquark components (because of the higher number of particles in the final state). We discuss the prospects for Belle II.

QCD (Brodsky-Farrar) counting rules

✓ High Energy + fixed scattering angle: ALL SCALES LARGE (>>2GeV) (the cross section will fall with a power of s)

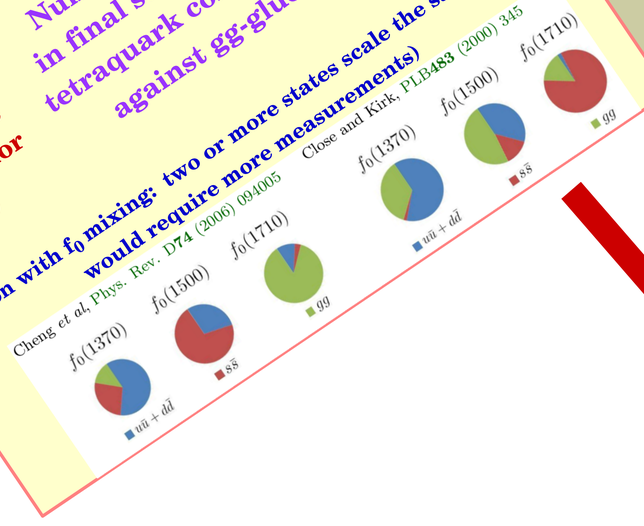
$$\frac{d\sigma}{dt} \propto f(\theta) \times \frac{s^2}{s^{n_i+n_f+L}}$$

Number of pointlike particles in initial state (e.g. n_i=2 for e^-e^+)

Orbital angular momentum: crucial to suppress f_0 $q\bar{q}$ wfs as $J^{PC}=0^{++}$ with $P=(-1)^{L+1}$ requires L=1

Number of pointlike particles in final state: crucial to suppress tetraquark configurations (n_f=4+2) against gg-glueball (n_f=2+2)

(Caution with f_0 mixing: two or more states scale the same way → would require more measurements)



Stanley J. Brodsky and Glennys R. Farrar, Phys. Rev. D 11, 1309 (1975)
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S. J. Brodsky, Chueng-Ryong Ji and G. Peter Lepage, Phys. Rev. Lett 51, 83 (1983)
S. J. Brodsky and G.P. Lepage, Phys. Rev. D 24, 2848 (1981)
D. Amati, L Caneschi and R. Jengo, Nuovo Cimento LVIII, 783 (1968)
V. A. Matveev, R. M. Muradian, and A.N. Tavkhelidze, Lett. Nuovo Cimento 7, 719 (1973)

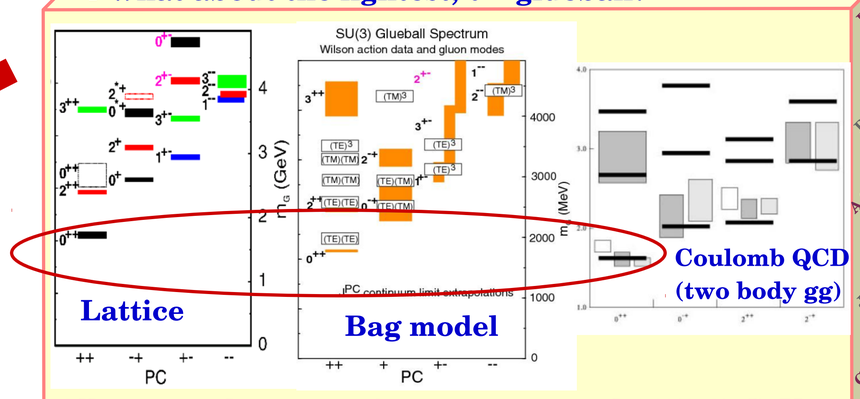
How to distinguish glueballs from other mesons?

✗ Not J^{PC} exotic: 0^{++}, 2^{++}, 0^{-+}... lowest states

✗ Not flavor exotic: isospin 0

✓ 2^{++} at 2 GeV (because of pomeron & lattice)

➔ What about the lightest, 0^{++} glueball?



All approaches agree in a mass around 1.7(0.2) GeV

LORE: "J/ψ decays are gluon rich, a good place to search for the glueball"

DUBIOUS: At 1-2 GeV interactions are strong and Watson's final state theorem renders same spectrum

Felipe J. Llanes-Estrada + Cotanch, Bicudo, Ribeiro, Szczepaniak Nucl.Phys. A710 (2002) 45-54
Biagio Lucini, "Glueballs from lattice" arXiv: 1401.1494 (proc. of Science)
A.Szczepaniak, E. S. Swanson, C.-R. Ji, and S. R. Cotanch, Phys. Rev. Lett.76 2011 (1996);
J. Kuti, Nucl. Phys. Proc. Suppl. 73 (1999) 72
C. Morningstar and M. Peardon, Phys. Rev. D60, 034509 (1999)
F. Buisseret, V. Mathieu, C. Semay, Phys. Rev. D81 034004 (2010)

J. V. Bennett, PhD Dissertation, Indiana University 2014 gives the spectrum of $J/\psi \to \gamma\pi\pi$ that shows the various f_0 and f_2 mesons

BaBar and Belle measured $e^-e^+ \to \phi f_0(980)$
S. Cheng-Pin, Y. Chang-Zheng, Chinese Phys. C 34 1045 (2010)

The first gives us the spectrum at $s=(3\text{ GeV})^2$; the second, the normalization of the cross-section pinned at $f_0(980)$

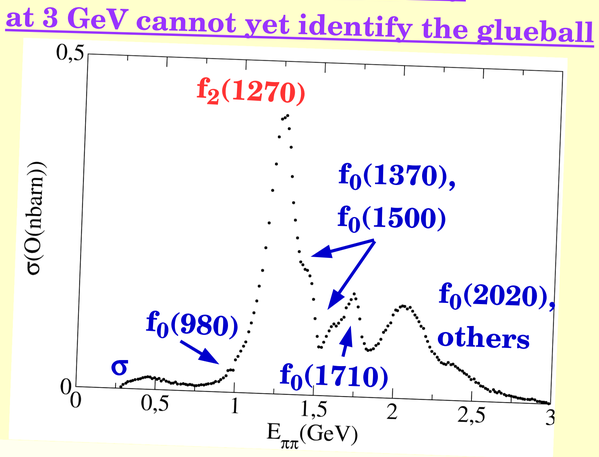
Reasonable assumption: quark-antiquark ideal mixing, $\phi = |s\bar{s}\rangle$

Lower the power by one unit of s

$$\sigma|_{\text{barrel}} = 4|\mathbf{p}_\phi||\mathbf{p}_{f_0}| \int_0^{\cos\theta_{\min}} d\cos\theta \, \frac{d\sigma}{dt}$$

✓ Fixed θ interval (central barrel)

$e^-e^+ \to \phi\, f_0$ as case study: at 3 GeV cannot yet identify the glueball



$$\sigma(f_0 = |gg\rangle + \ldots) \sim \frac{\text{constant}}{s^3}$$

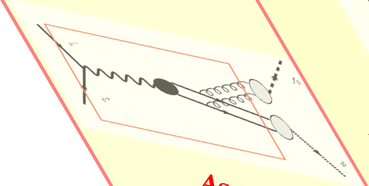
$$\sigma(f_0 = |q\bar{q}\rangle_{L=1} + \ldots) \sim \frac{\text{constant}}{s^4}$$

$$\sigma(f_0 = |q\bar{q}q\bar{q}\rangle_{s-\text{wave}} + \ldots) \sim \frac{\text{constant}}{s^5}$$

S.J. Brosdky, R.F. Lebed and V.E. Lyubovitskij, 1712.08853 find that the hadron counting can be relevant for other kinematic regions such as forward Vector Meson Production (not all scales are large)

Typical model based on $\phi \to \gamma f_0$ from D. Black, M. Harada and J. Schechter, Phys. Rev. D73 054017 (2006)

With pointlike final particles, n_i+n_f=4



As opposed to composite mesons, n_i+n_f = 6 (here 2 leptons, 2 quarks, 2 gluons)

In contrast, hadron models may fall less steeply (and must fail at high Q^2)

$$\mathcal{L}_{\phi\phi f_0} = \frac{\beta}{2} f_0 (\partial_\mu\phi_\nu - \partial_\nu\phi_\mu)(\partial^\mu\phi^\nu - \partial^\nu\phi^\mu)$$

Used e.g. with Vector Meson Dominance

$$\frac{d\sigma}{dt} = \frac{1}{s^2} \times \frac{\pi^2\alpha^3\tilde{g}^2 f_\pi^4 \beta^2 \sin^2\theta}{72 m_\phi^2}$$

✗ It does not distinguish inner structure: easy to rule out comparing different f_0s as all behave the same way here, σ ~ 1/s

Separately: an additional interesting experiment at Belle II is radiative $e^-e^+ \to \gamma f_0$

*Smaller by α_{EM}

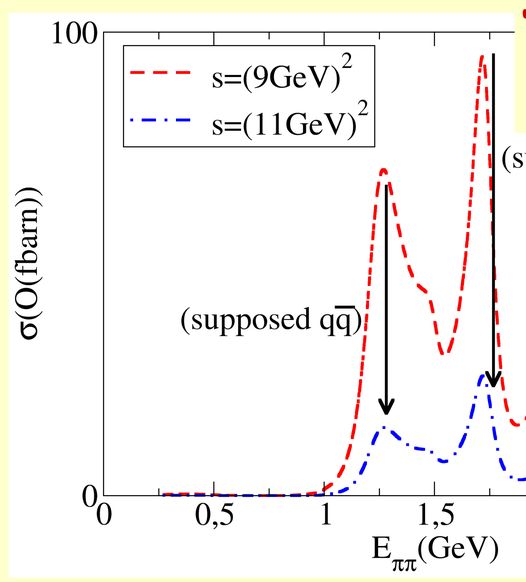
*Larger by one power of s with composite φ (but not with pointlike mesons)

Estimates for an experimental search at Belle II

$\frac{\sigma(11\text{GeV})}{\sigma(9\text{GeV})} \to$	$\sim \frac{1}{3.4}$
gg	$\sim \frac{1}{5}$
$q\bar{q}$: $L=1$	$\sim \frac{1}{7.5}$
$q\bar{q}q\bar{q}$: s − wave	

✓ Glueball component falls the slowest

✓ Measure σ to 25% precision to distinguish



✓ Whichever is the glueball dominates the ππ spectrum in $e^-e^+ \to \phi f_0 \to \phi\pi\pi$

✓ Belle II will get 5 abarn^{-1}/yr but mostly on Υ resonances

✓ With 1 ab^{-1} off-peak & σ=70fb@ 9 GeV, expect 70 000 events (20 000 @11 GeV)