Studying running coupling effects on the numerical solutions of the JIMWLK equation

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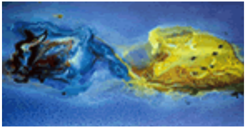
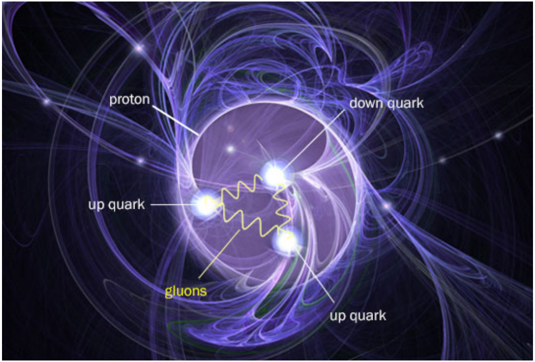


XIIIth Quark Confinement and the Hadron Spectrum, August 5, 2018

Hadrons' internal structure

Standard Model of elementary particles: electrons, muons, quarks, gluons, photons, $W^{\pm}$, Z, Higgs, ...



Credit: *Brookhaven National Lab website*

Experiment: HERA, LHC, BNL, JLab, SLAC, Fermilab, Electron-Ion Collider study hadron structure functions and try to discover the origin of mass,

Theory: Quantum Chromodynamics (QCD) is the theory describing the

Proton structure function

Large x

Lattice QCD is able to provide x-dependent structure functions, such as the PDFs. The method seems to be applicable for large $x > 0.1$. Reaching smaller values of x would require lattices large lattices.

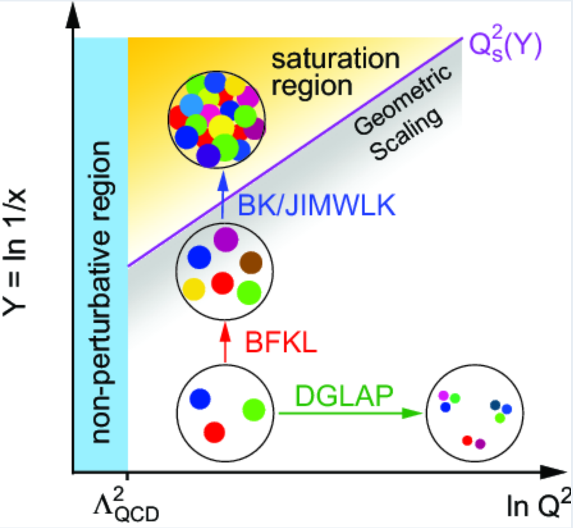
Small x

At small x where gluons dominate one introduces CGC - effective theory which can be used among other applications to describe the structure of hadrons. The basic equations are B-JIMWLK and BK. In the dilute region - i.e. x is moderate and k_t is large enough one can formally neglect nonlinearities and use BFKL. The initial conditions can be obtained for instance from McLerran-Venugopalan model.

Check

As for finding the signatures of nonlinearities of CGC type one first needs to fit total cross section like F2 to extract the parameters of the initial condition. Before this can be achieved, several systematic effects need to be taken into account.

Proton structure function

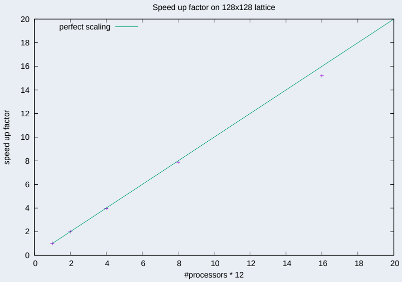


Fit to the HERA F2 data: Mäntysaari, Schenke, arxiv:1806.06783

Parallel code

Following the works by Rummukainen, Weigert, Lappi, Mäntysaari, Marquet, Petreska, Roiesnel we wrote a highly parallel implementation of the stochastic approach to the JIMWLK equation:

- Fourier acceleration: as many parts as possible in momentum space
- parallel FFTW library
- MPI parallelization in one dimension
- openmp parallelization of the volume loops



Initial condition

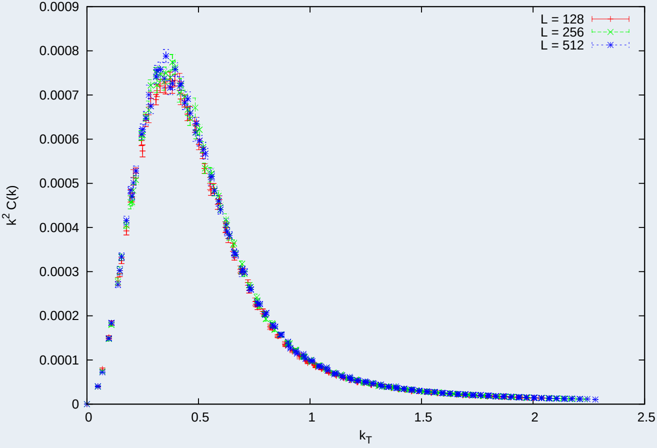
McLerran-Venugopalan model

We generate the initial condition following Rummukainen and Weigert:

dipole correlation function	$C(\mathbf{x}-\mathbf{y}) = \langle \mathrm{Tr} U_{\mathbf{x}}^{\dagger} U_{\mathbf{y}} \rangle$
light-cone Y-M equations	$U_{\mathbf{x}} = \exp\left(-i\frac{g\rho(\mathbf{x})}{\nabla^2}\right)$
random gaussian sources	$\langle \rho^a(\mathbf{x})\rho^b(\mathbf{y})\rangle = \delta^{a,b}\delta(\mathbf{x}-\mathbf{y})g^2\mu^2$
lattice discretization	$\langle \rho_k^a(\mathbf{x})\rho_l^b(\mathbf{y})\rangle = \delta^{a,b}\delta^{k,l}\delta(\mathbf{x}-\mathbf{y})\frac{g^2\mu^2}{N_y}$

Initial condition

Example



Langevin equation

We implement the stochastic process following Lappi, Mäntysaari (Eur. Phys. J. C 73 (2013) 2307), Marquet, Petreska and Roiesnel (JHEP 1610 (2016) 065)

$$
\begin{aligned}
U_{\mathbf{x}}(s+\delta s) = \exp\left[-\sqrt{\delta s}\sum_{\mathbf{y}} U_{\mathbf{y}}(s)\left(\vec{K}(\mathbf{x}-\mathbf{y})\vec{\xi}(\mathbf{y})\right)U^{\dagger}_{\mathbf{y}}(s)\right]\times \\
\times U_{\mathbf{x}}(s)\times \\
\times \exp\left[\sqrt{\delta s}\sum_{\mathbf{y}} \vec{K}(\mathbf{x}-\mathbf{y})\vec{\xi}(\mathbf{y})\right]
\end{aligned}
$$

where

$$
s = \frac{\alpha_s}{\pi^2} y, \qquad y = \ln \frac{x_0}{x_2}
$$

The noise vectors ξ are uncorrelated, gaussian random variables with $\sigma = 1$ variance.

JIMWLK equation as a stochastic process

Including the running coupling

The running of the coupling can be included in the "square root" prescription

$$U_{\mathbf{x}}(s+\delta s) = \exp\left[-\sqrt{\delta y}\sum_{\mathbf{y}} U_{\mathbf{y}}(s)\Big(\alpha_s(|x-y|)\vec{K}(\mathbf{x}-\mathbf{y})\vec{\xi}(\mathbf{y})\Big)U^{\dagger}_{\mathbf{y}}(s)\right]\times$$
$$\times U_{\mathbf{x}}(s)\times$$
$$\times \exp\left[\sqrt{\delta y}\sum_{\mathbf{y}} \alpha_s(|x-y|)\vec{K}(\mathbf{x}-\mathbf{y})\vec{\xi}(\mathbf{y})\right]$$

Including the running coupling

Alternatively the running of the coupling can be included as a modification of the ξ vectors (Lappi, Mäntysaari, Eur. Phys. J. C (2013) 73)

$$U_{\mathbf{x}}(s+\delta s) = \exp\left[-\sqrt{\delta s}\sum_{\mathbf{y}} U_{\mathbf{y}}(s)\left(\vec{K}(x-y)\vec{\eta}(\mathbf{y})\right)U^{\dagger}_{\mathbf{y}}(s)\right]\times$$
$$\times\, U_{\mathbf{x}}(s)\times$$
$$\times \exp\left[\sqrt{\delta s}\sum_{\mathbf{y}} \vec{K}(x-y)\vec{\eta}(\mathbf{y})\right]$$

where now

$$\langle \eta^{a,i}_{\mathbf{x}} \eta^{b,j}_{\mathbf{y}} \rangle = \delta^{a,b}\delta^{i,j}\int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}\alpha_s(\mathbf{k})$$

Adding the running coupling to the JIMWLK equation

Including the running coupling

Two possibilities:

- momentum space

$$\langle \eta^{a,i}_{\mathbf{p}} \eta^{b,j}_{\mathbf{q}} \rangle = \delta^{a,b} \delta^{i,j} \delta(\mathbf{q} - \mathbf{p}) \alpha_s(\mathbf{p})$$

  $\Rightarrow$ diagonal in momentum space: for each $\mathbf{p}$ generate uncorrelated gaussian variable with variance $\sigma = \alpha_s(\mathbf{p})$

- position space

$$\langle \eta^{a,i}_{\mathbf{x}} \eta^{b,j}_{\mathbf{y}} \rangle = \delta^{a,b} \delta^{i,j} \alpha_s(\mathbf{x} - \mathbf{y})$$

  $\Rightarrow$ correlated random variables for each $\mathbf{x}$ and $\mathbf{y}$: generate the correlation matrix Σ, use Cholesky decomposition to get A such that $AA^T = \Sigma$, transform uncorrelated gaussian variables: $\eta = A\xi$

Adding the running coupling to the JIMWLK equation

$\alpha_s(\mathbf{k})$ and $\alpha_s(\mathbf{r})$

$$\alpha_s(\mathbf{k}) = \frac{4\pi}{\beta \ln\left\{\left[\left(\frac{\mu_0^2}{\Lambda_{\mathrm{QCD}}^2}\right)^{\frac{1}{c}} + \left(\frac{\mathbf{k}^2}{\Lambda_{\mathrm{QCD}}^2}\right)^{\frac{1}{c}}\right]^c\right\}}$$

$$\alpha_s(\mathbf{r}) = \frac{4\pi}{\beta \ln\left\{\left[\left(\frac{\mu_0^2}{\Lambda_{\mathrm{QCD}}^2}\right)^{\frac{1}{c}} + \left(\frac{4e^{-2\gamma_E}}{\mathbf{r}^2\Lambda_{\mathrm{QCD}}^2}\right)^{\frac{1}{c}}\right]^c\right\}}$$

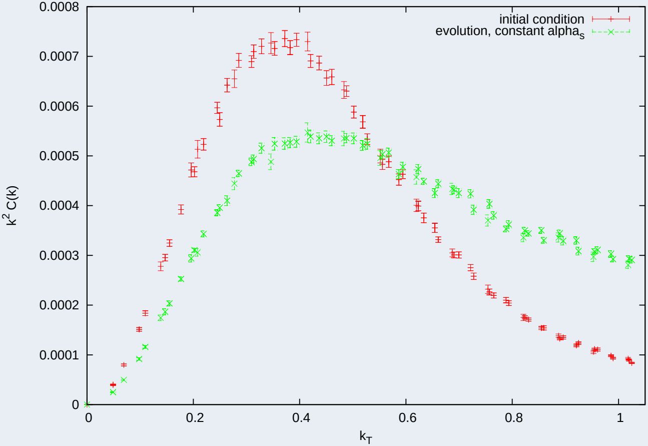
Following Lappi and Mäntysaari we use (Eur. Phys. J. C (2013) 73):

- $\beta = 11 - 2N_f/3$, $N_f = 3$,
- $c = 0.2$, $\mu_0 L = 15$, $\Lambda_{\mathrm{QCD}} L = 6$
- the running coupling freezes at the value $\alpha_0 = 0.76$
- we regularize $r = 0$ case by setting $\alpha_s(0) = 0.0001$

They have shown that both definitions give compatible results for the BK equation using the "square root" prescription.

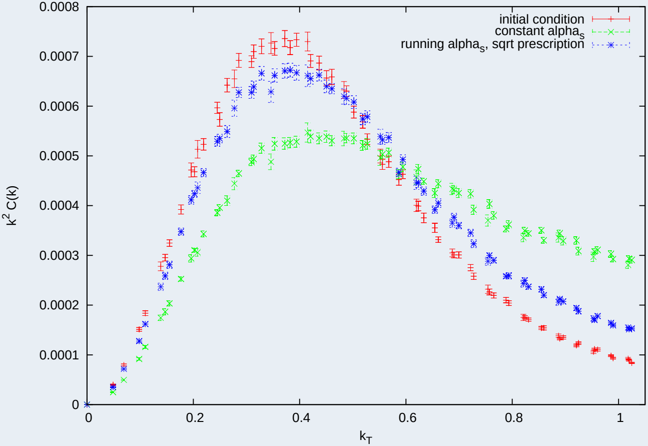
Adding the running couling to the JIMWLK equation

Evolution with constant coupling constant

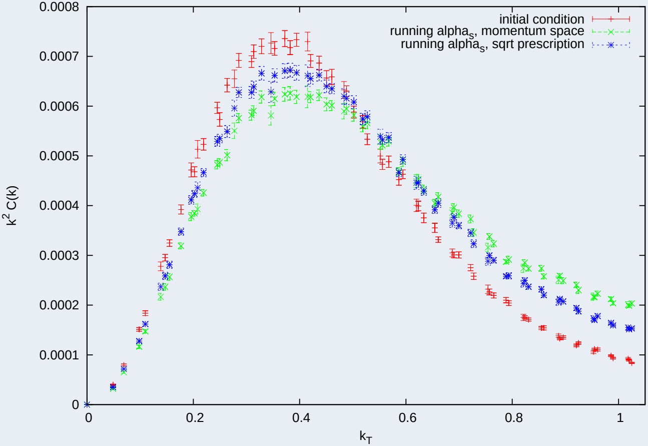


Adding the running couling to the JIMWLK equation

Constant coupling vs. running coupling in momentum space

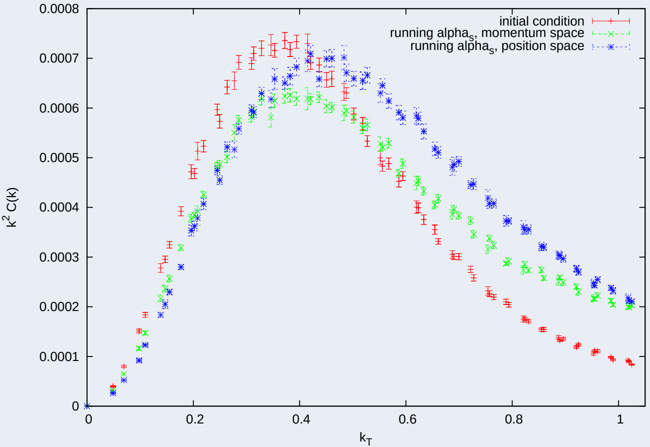


"Square root" vs. Lappi's prescription



Adding the running couling to the JIMWLK equation

Running coupling in momentum vs. position space



Connection with observations at NLO BK (Ducloue et al., arXiv:1807.04971) ?

Conclusions

Summary

- we have implemented the numerical framework for solving the JIMWLK equation expressed as a Langevin equation
- we implemented both the "square root" and Lappi's prescription to include the effects of the running coupling
- both prescriptions can be implemented either in position or momentum space
- we find that for the same evolution parameters, each prescription gives different evolution speeds

Outlook

- study other remaining systematics
- reconstruct cross-section from the correlation function
- implement and perform the fit to experimental data

Fourier transforms

$$\langle \eta(\mathbf{x})\eta(\mathbf{y})\rangle = \delta(\mathbf{x}-\mathbf{y})$$

$$\eta(\mathbf{p}) = \eta(\mathbf{p})\eta(\mathbf{p}) = \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}(\mathbf{x}+\mathbf{y})}\eta(\mathbf{x})\eta(\mathbf{y})$$

$$C(\mathbf{z}) = \sum_{\mathbf{p}} e^{-i\mathbf{p}\mathbf{z}} C(\mathbf{p}) =$$

$$= \sum_{\mathbf{p}}\sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}(\mathbf{z}-\mathbf{x}-\mathbf{y})}\eta(\mathbf{x})\eta(\mathbf{y}) =$$

$$= \sum_{\mathbf{x},\mathbf{y}} \delta(\mathbf{z}-\mathbf{x}-\mathbf{y})\eta(\mathbf{x})\eta(\mathbf{y}) =$$

$$= \sum_{\mathbf{x}} \eta(\mathbf{x})\eta(\mathbf{z}-\mathbf{x})$$

$$\langle C(\mathbf{z})\rangle = \sum_{\mathbf{x}} \langle \eta(\mathbf{x})\eta(\mathbf{z}-\mathbf{x})\rangle = \sum_{\mathbf{x}} \delta(2\mathbf{x}-\mathbf{z})$$