Pion structure from lattice QCD

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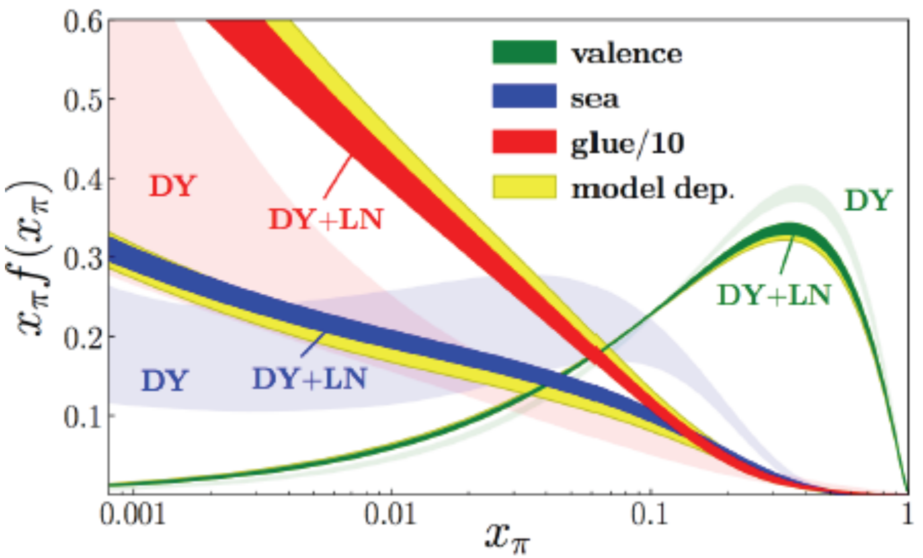
Aim: determine the valence quark distribution of pion $q(x) = q_u(x) - q_d(x)$ from lattice QCD using large momentum effective theory (LaMET)

X. Ji '2013

No disconnected Diagrams !



Barry et al (JAM), 2018

In collaboration with:

T. Izubuchi, L. Jin, K. Kallidonis, N. Karthik, S. Mukherjee, C. Shugert, S. Syritsyn

Quark Confinement and Hadron spectrum, August 1-6,2018

From parton distributions to quasi parton distribution

X. Ji '2013

calculable in perturbation theory

$$\tilde{q}(x,\Lambda,P_z)=\int_{-1}^{+1}\frac{dy}{|y|}C(\frac{x}{y},\mu,\Lambda,P_z)q(y,\mu)+\mathcal{O}(M_\pi^2/P_z^2,\Lambda_{QCD}^2/P_z^2)$$

$$\tilde{q}(z,P_z,\Lambda)=\langle P|\psi(z)\gamma_z\exp(-ig\int_0^z dz' A_z(z'))\psi(0)|P\rangle$$

calculable on the lattice, $\Lambda\sim 1/a$

$$\tilde{q}(x,P_z,\Lambda)=\int\frac{dz}{2\pi}e^{ixP_z z}\tilde{q}(z,P_z,\Lambda)$$

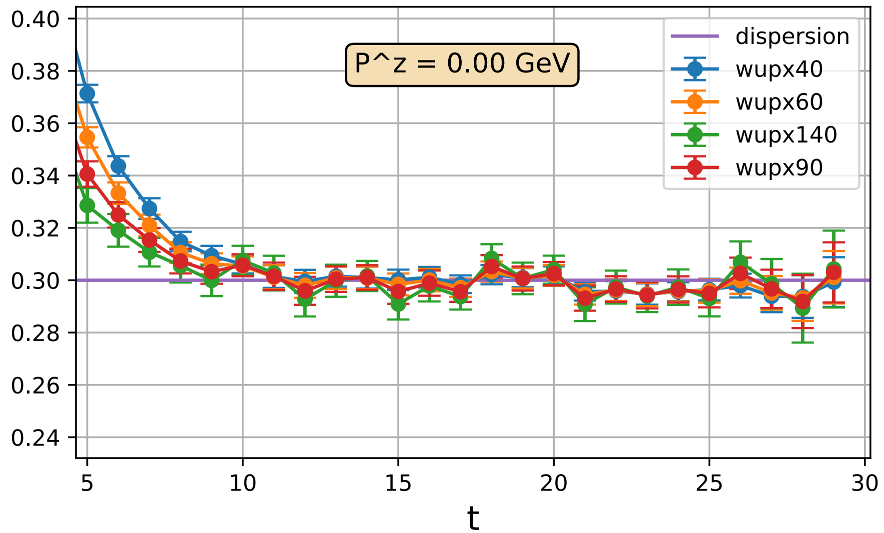
Challenges for lattice calculations:

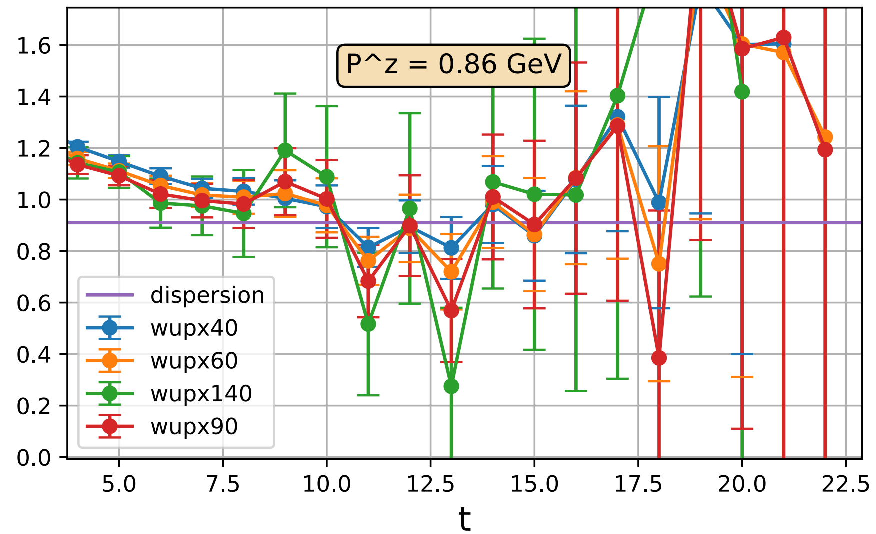
- Small lattice spacings to make perturbative matching possible
  $\Rightarrow$ HYP smeared Wilson valence quarks on HISQ sea:
  2+1 flavor HISQ, $48^3\times 64$, $a=0.06$ fm, $m_\pi^{val}=300$ MeV,
  $m_\pi^{sea}=161$ MeV
- Need large P_z which makes the correlators noisy
  $\Rightarrow$ momentum smearing
- Need to match the lattice scheme to $\overline{MS}$ scheme
  $\Rightarrow$ use RI/MOM as an intermediate renormalization scheme

Calculating correlators at large momenta

Gaussian sources via Wuppertal smearing for the moving pion, Güsken et al, 1989

effective masses (50 configs, 32 sources)





AMA with 32 sloppy propagators
Izubuchi, Shintani, 2011

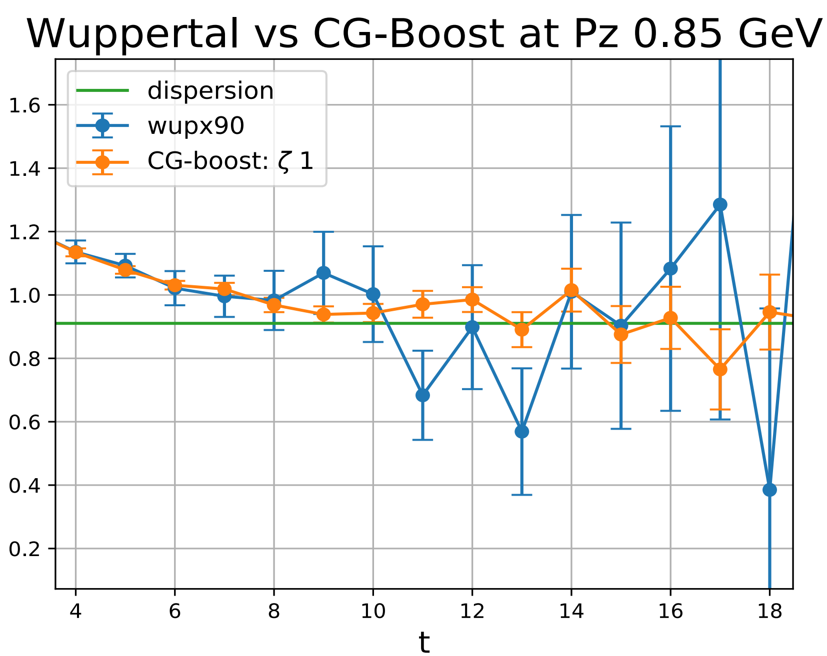
Wuppertal smearing works fine for $P_z = 0$ and the ground energy is approached for $t = 8$, but for effective masses are too noisy already for $P_z = 0.86$ GeV

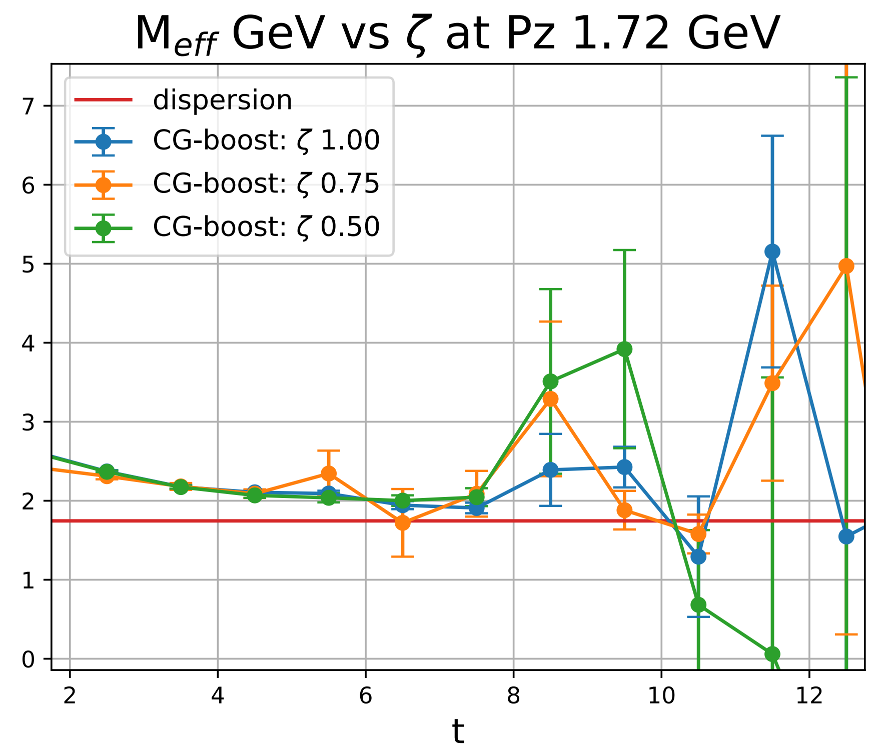
Calculating correlators at large momenta (cont'd)

Modify the Gaussian smearing such that quarks inside the source have non-zero momenta $k_z = \zeta P_z$, Bali 2016

We use momentum boosted Coulomb gauge fixed Gaussian sources (cheaper computationally)

effective masses (50 configs, 32 sources)





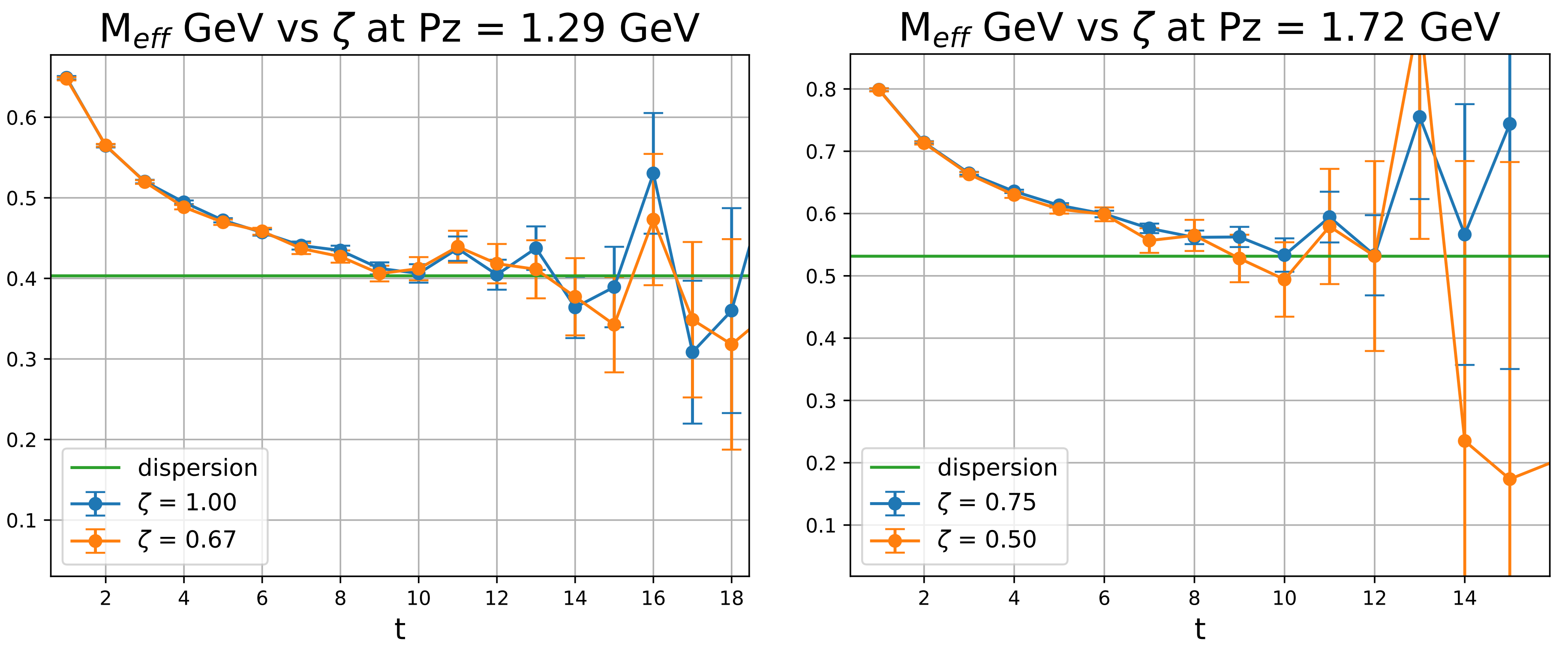
$\zeta = 0.75 - 1$ is needed for a good signal

Calculating correlators at large momenta (cont'd)

Modify the Gaussian smearing such that quarks inside the source have non-zero momenta $k_z = \zeta P_z$, Bali 2016

We use momentum boosted Coulomb gauge fixed Gaussian sources (cheaper computationally)

effective masses (168 configs, 32 sources)



$\zeta = 0.75 - 1$ is needed for a good signal

Renormalization and matching of qPDF

Workflow:

$$q_b(z, P_z, a) \to q_R(z, P_z, p_z^R, \mu_R) \to q_R(x, P_z, p_z^R, \mu_R)$$

Perform matching from RI/MOM to $\overline{MS}$
Stewart, Zhao, 2017

$$\tilde{q}(x, P_z, p_z^R, \mu_R) = \int_{-1}^{+1} \frac{dy}{|y|} C^{RI/MOM}\left(\frac{x}{y}, \frac{yP_z}{\mu}, \frac{p_z^R}{P_z}, \frac{\mu_R}{p_z^R}\right) q(y, \mu)$$

For Wilson fermions γ_z operator mixes with unit operator $\Rightarrow$ to be considered in the renormalization
Constantinou, Panagopoulos, 2017, Alexandrou et al, 2017, Chen et al, 2017

Non-perturbative RI/MOM: fix Landau gauge on the lattice, calculate the amputated 3-point function on quark states at momentum p and require that for $p_z = p_z^R$ and $p_z^2 + p_T^2 = \mu_R^2$ it is equal to the tree level result
Stewart, Zhao, 2017, Chen et al 2017

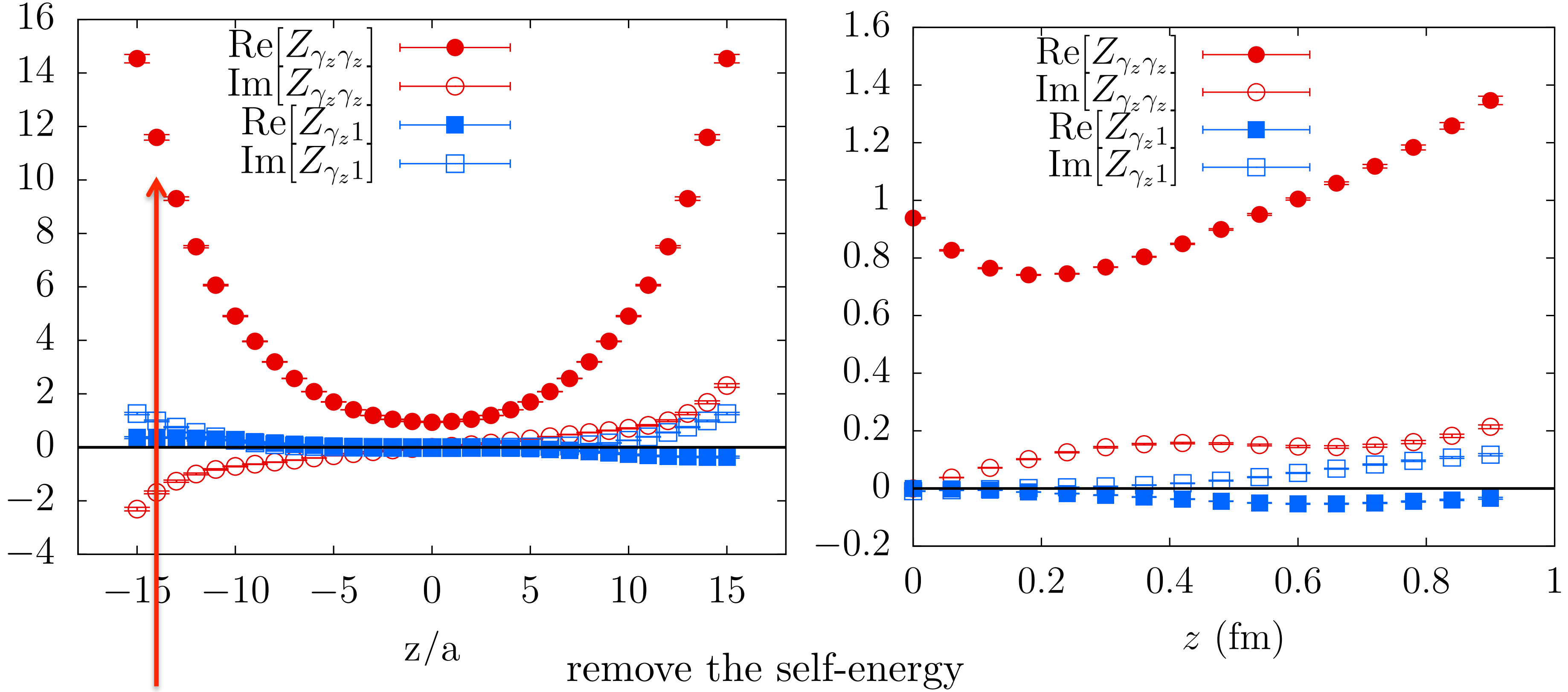
$$\mathrm{Tr}\left(\not{p}\, \Gamma^R_{\gamma_z}(p)\right)\Big|_{p=p^R} = 12 p_z^R e^{i p_z^R z}, \quad \mathrm{Tr}\left(\Gamma^R_{\gamma_z}(p)\right)\Big|_{p=p^R} = 0$$

$$\mathrm{Tr}\left(\not{p}\, \Gamma^R_1(p)\right)\Big|_{p=p^R} = 0, \quad \mathrm{Tr}\left(\Gamma^R_1(p)\right)\Big|_{p=p^R} = 12 p_z^R e^{i p_z^R z}$$

$\Rightarrow Z_{\gamma_z \gamma_z}, Z_{\gamma_z 1}$

Renormalization constants in non-perturbative RI/MOM

$$p_z^R = 1.28 \text{ GeV}, \; p_T^R = 1.48 \text{ GeV}$$



linear divergent self-energy

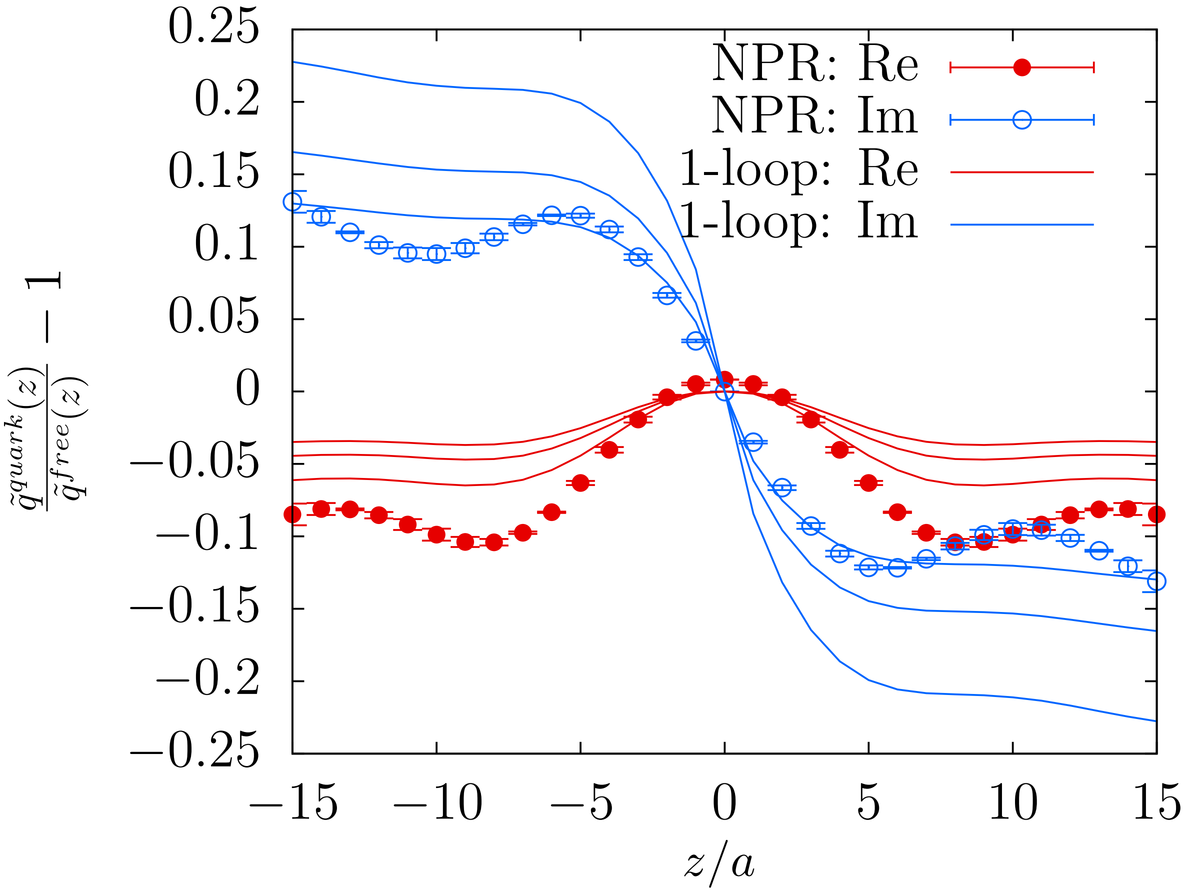
remove the self-energy divergence using the known renormalization factors of Polyakov loops TUMQCD, 2016-2018

renormalization constant close to 1 for γ_z operator and small mixing

How well the 1-loop result describes the quark 3-pt function ?

Non-perturbative RI/MOM renormalization removes some of the lattice artifacts and the self-energy divergence ⇒ the renormalized quark 3-pt function should agree to some extent with 1-loop result of Stewart and Zhao, 2017

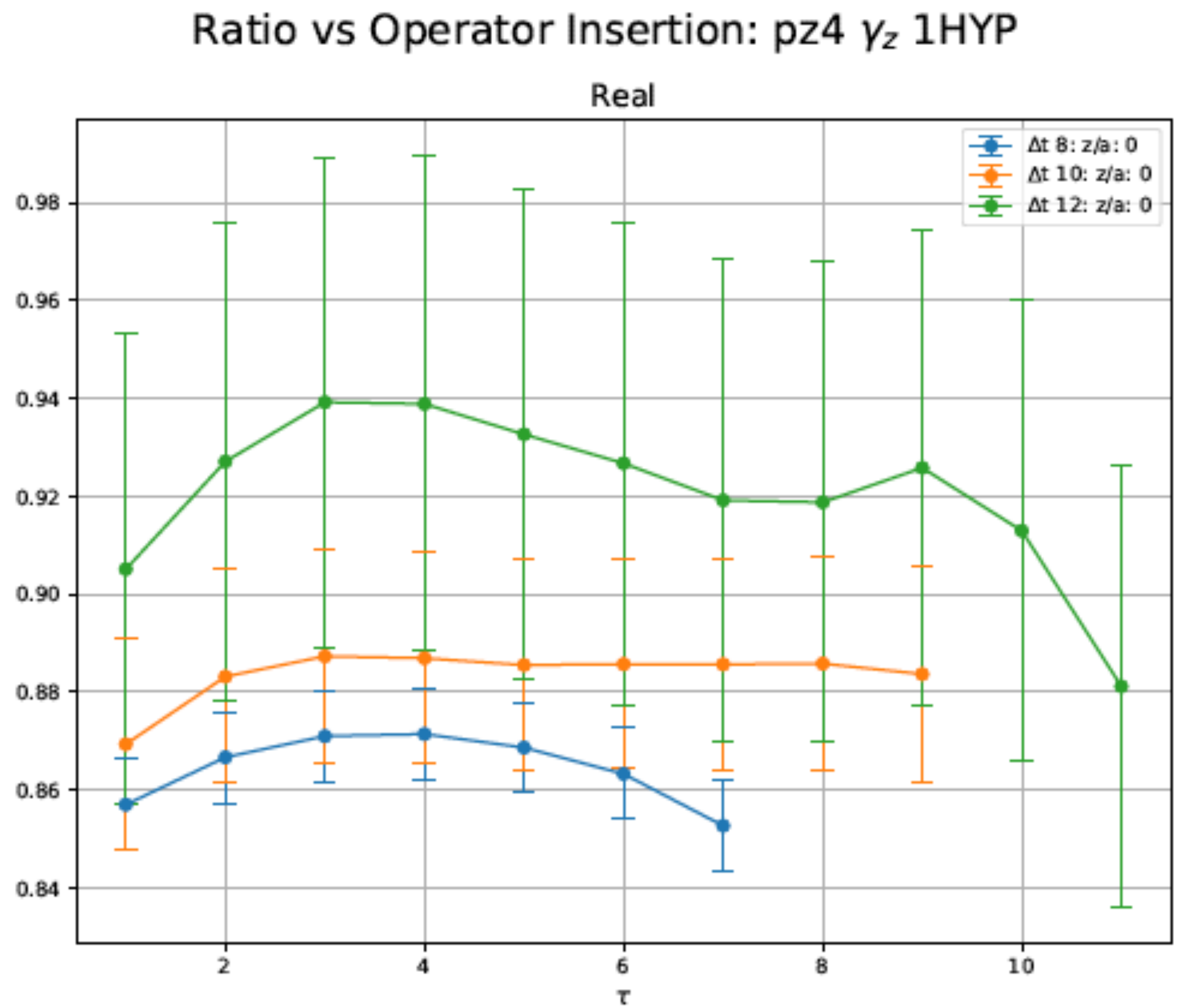
$p_z^R = 1.28$ GeV, $p_T^R = 1.48$ GeV $\quad\quad$ $p_z = 1.28$ GeV, $p_T = 2.22$ GeV



Lines:
$\alpha_s(\mu = \mu_R/2)$
$\alpha_s(\mu = \mu_R)$
$\alpha_s(\mu = 2\mu_R)$

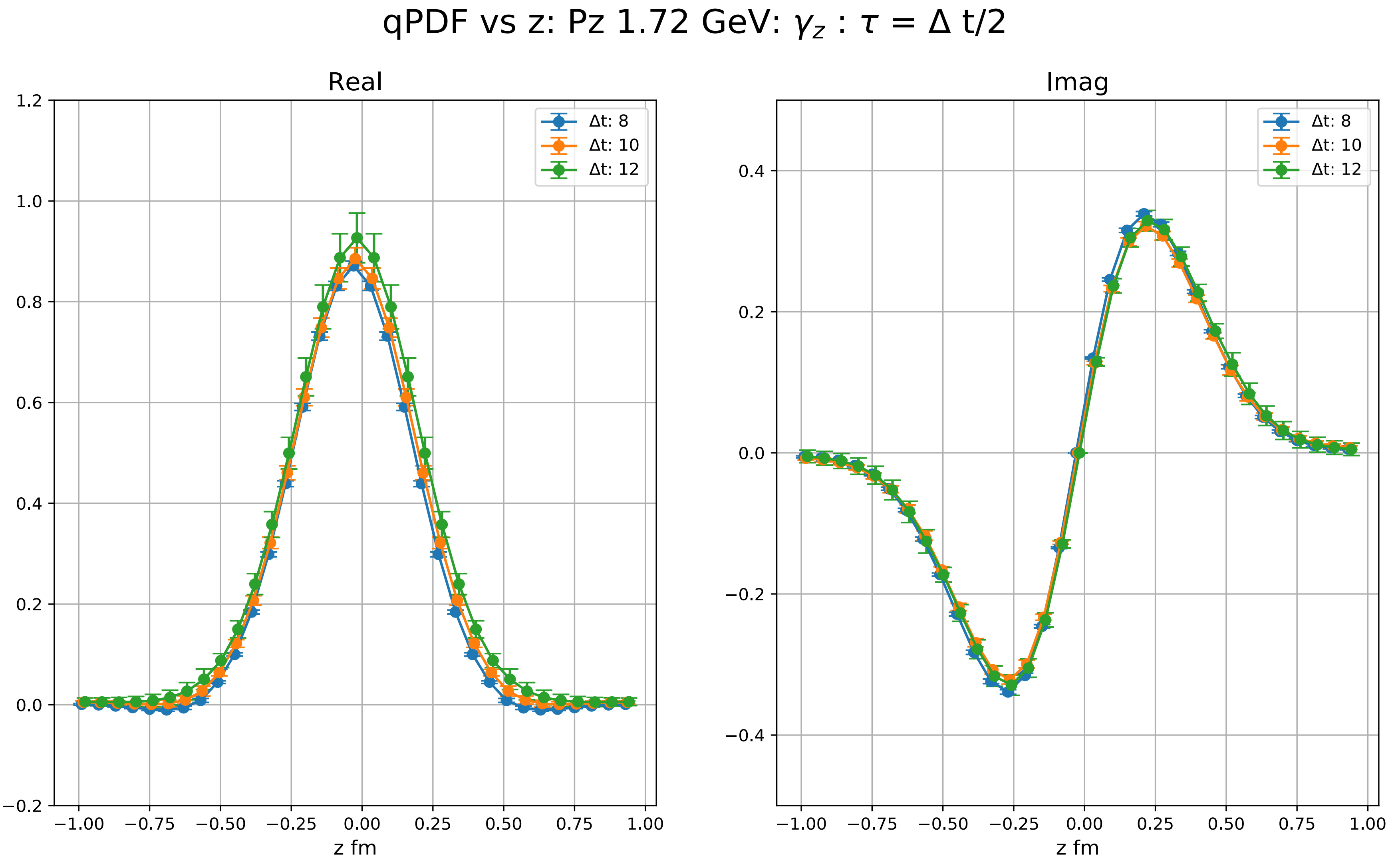
$a = 0.06$ fm

Pion 3-point functions at different momenta



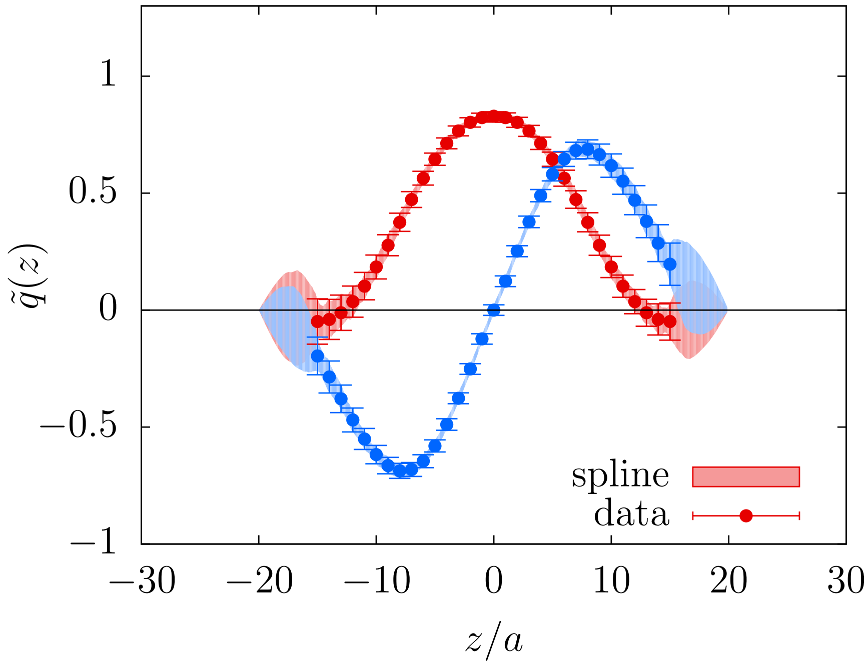
it is sufficient to take $\tau = \Delta t/2$

Pion 3-point functions at different momenta (cont'd)



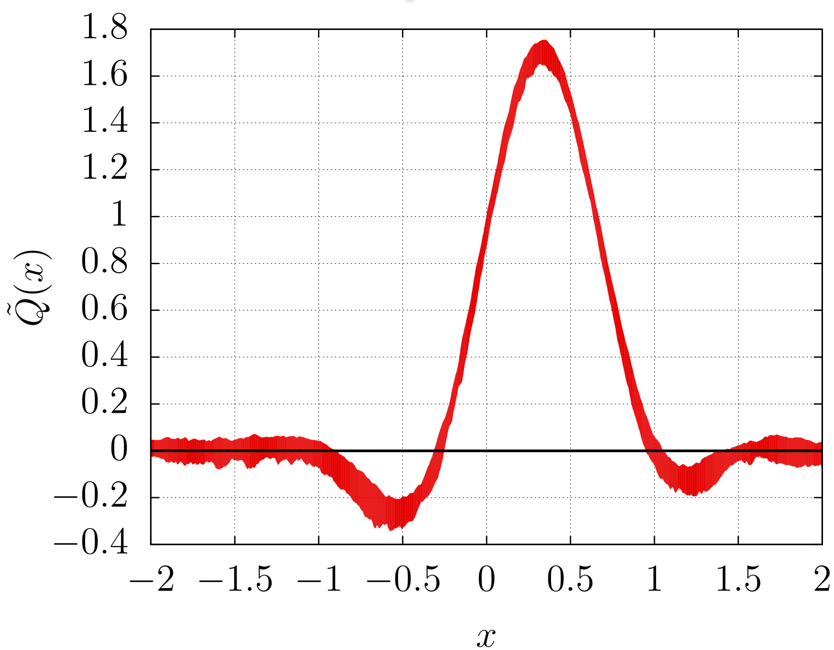
$\Delta t = 10$ and $\Delta t = 12$ results agree within errors
$\rightarrow$ take $\Delta t = 10$ results for extracting qPDF.

From coordinate space qPDF to momentum space qPDF



Need information about qPDF for $z/a > 16$ not available on the lattice. Set $\tilde{q}(z, P_z) = 0$ at $z_{max}/a = 20$ and perform interpolation

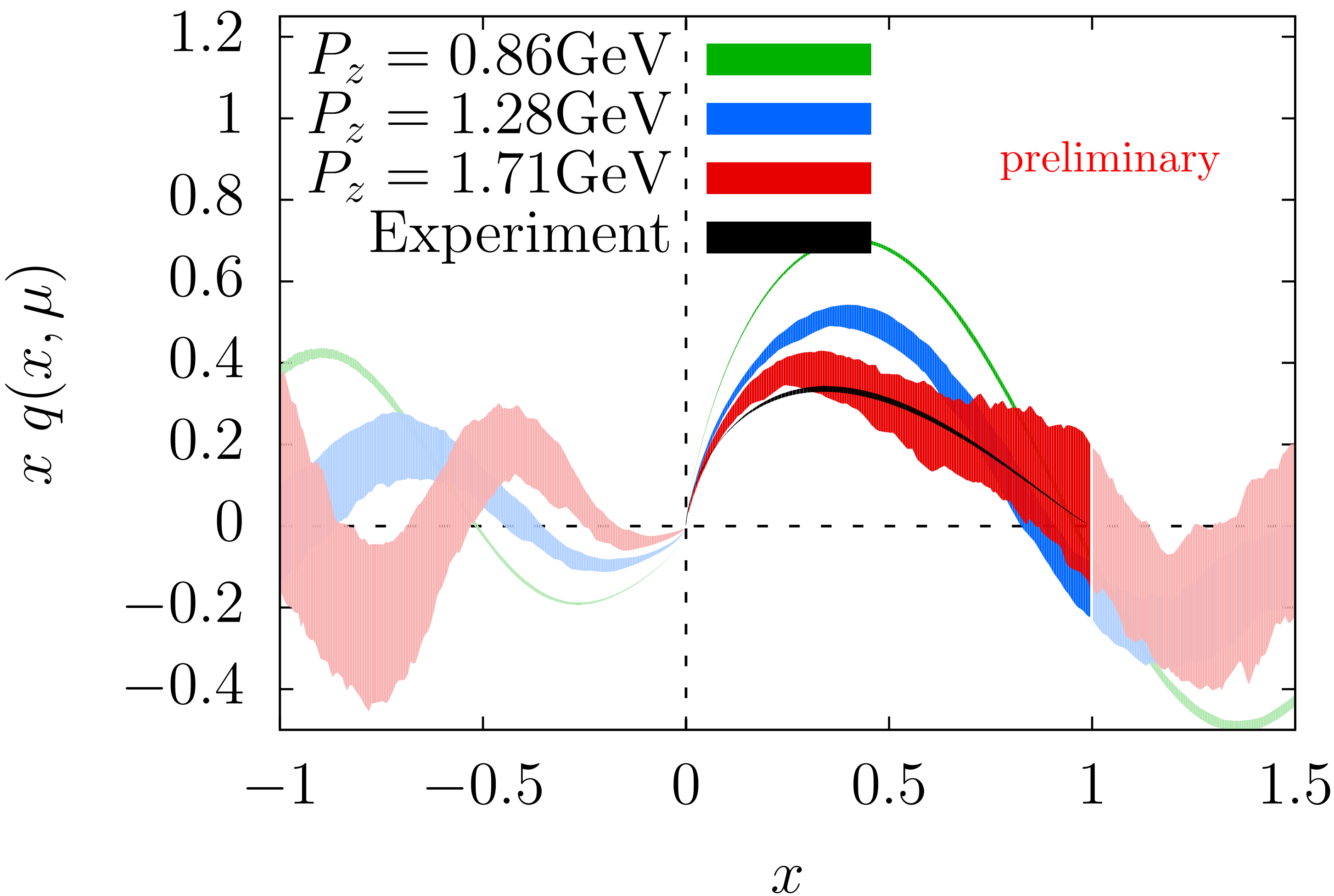
Fourier transformation



Results are insensitive to z_{max} if $z_{max} \geq 0$

Pion PDF and comparison to extraction from experiments

$p_z^R = 1.28$ GeV, $p_T^R = 1.48$ GeV $\qquad \mu = 3.2$ GeV



comparison to pion PDF from data on πA DY and neutron electro-production at HERA: Barry et al (JAM), 2018

Pion PDF and comparison to extraction from experiments

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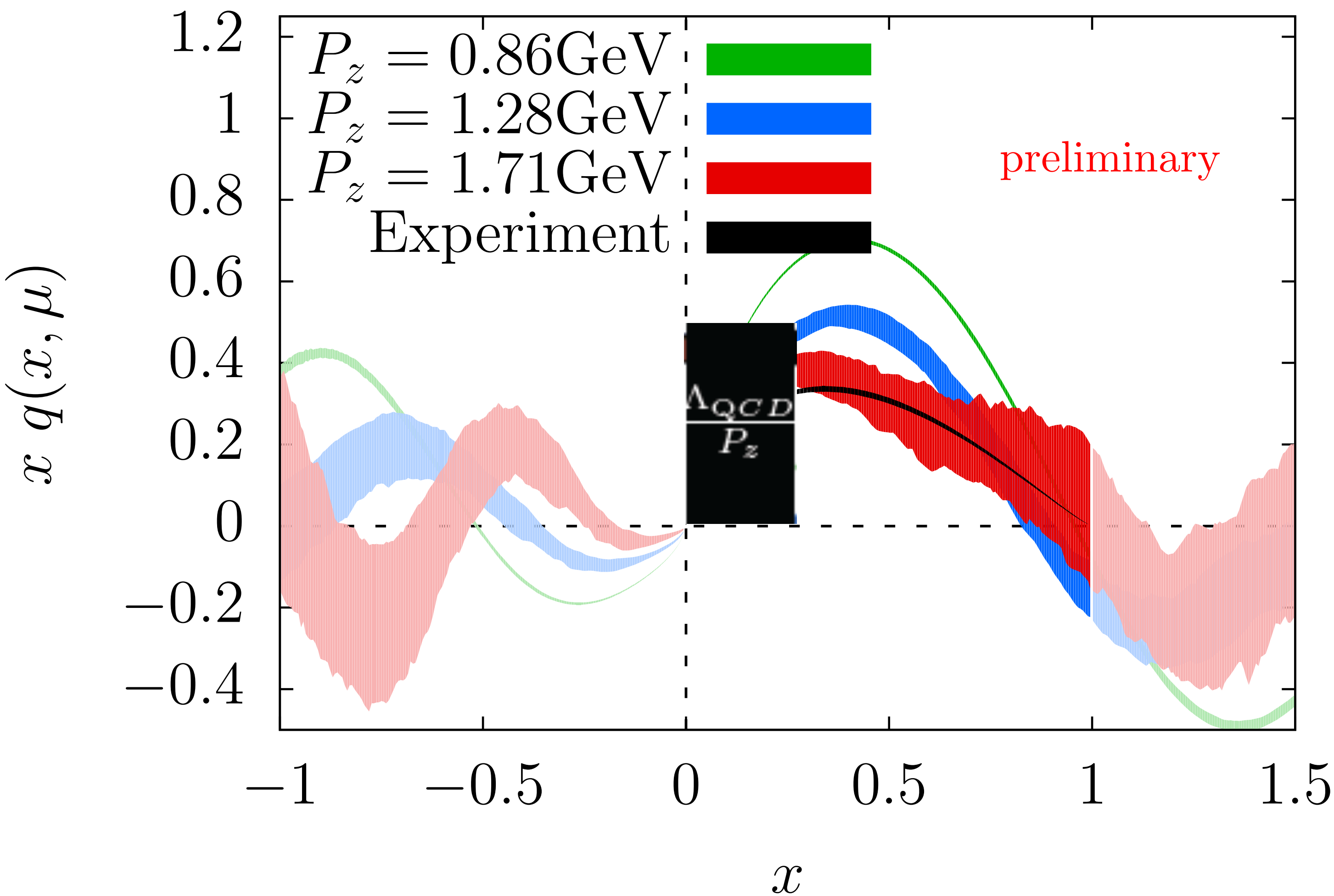
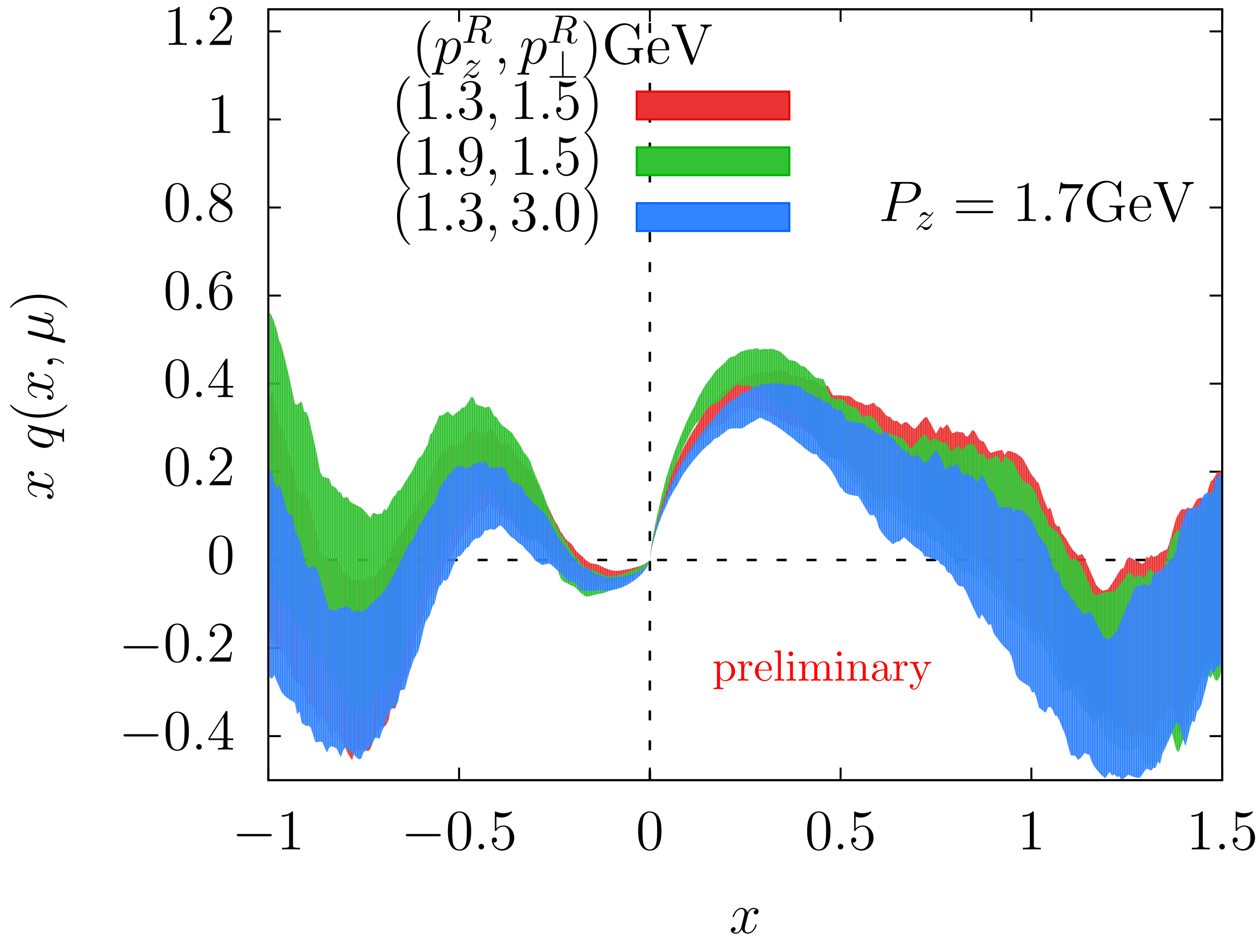


comparison to pion PDF from data on πA DY and neutron electro-production at HERA: Barry et al (JAM), 2018

Dependence of Pion PDF on RI/MOM scales



Summary

- The valence quark distribution have been calculated on fine lattice ($a = 0.06$ fm) using partially quenched calculations (HYP smeared Wilson on HISQ)
- Non-perturbative RI/MOM renormalization has been implemented and it seems to follow the expectations from perturbation theory $\Rightarrow$ matching from qPDF to PDF is meaningful
- The lattice result on valence pion PDF agree with phenomenological determination for large momenta, $P_z = 1.72$ GeV within (large) errors
- Obtaining a good signal is challenging, especially at large z