Quark-photon vertex from lattice QCD in Landau gauge

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Calculation for hadron physics

Lattice QCD

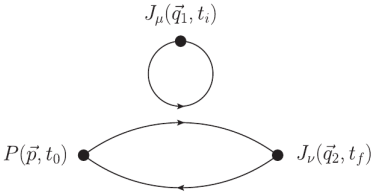
- currently preferred tool to provide theoretical estimates
- can reach now physical point on large and fine lattices
- full control over systematic error, hard/expensive in practice

Example: transition form factor $\pi^0 \to \gamma^* \gamma^*$

- allows study of evolution between non-perturbative and perturbative QCD
- transition matrix

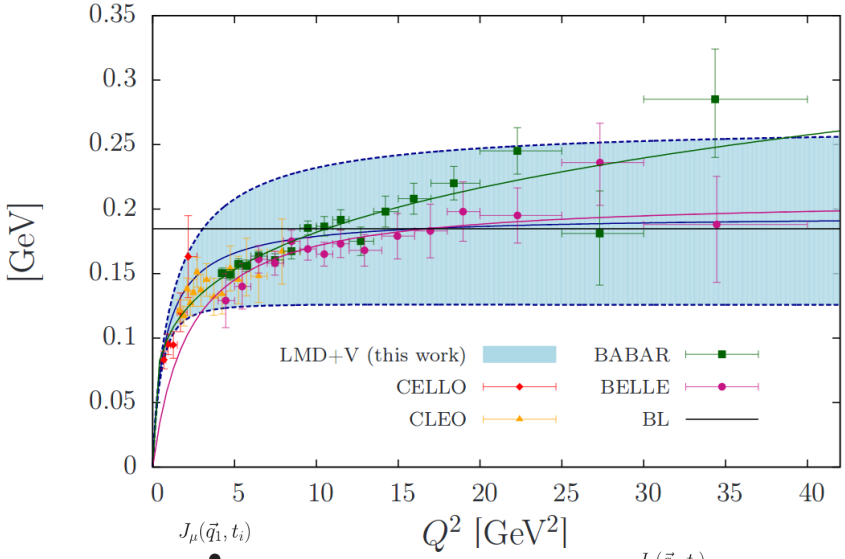
$$\Lambda_{\mu\nu} = e^2 \frac{F(Q^2, Q'^2)}{4\pi^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} Q'^\alpha Q^\beta$$

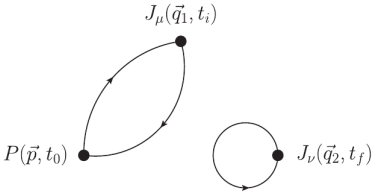
- large-Q^2 limit of $F(Q^2, 0)$ known [Lepage&Brodsky, Efremov&Radyushkin, 1980]
- onset of limit questioned (Fac.scale 1 vs. 10-100 GeV^2)

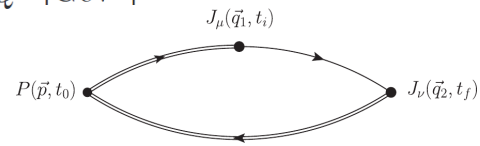


Lattice QCD calculation

[Géradin et al PRD94(2016)074507]







Calculation for hadron physics

Functional methods

- bound-state / Dyson-Schwinger equations / Functional Renormalization group
- input: nonperturbative n-point functions in a gauge
- pros/cons different to lattice (self-consistent truncation of infinite system)

Example: transition form factor $\pi^0 \to \gamma^* \gamma^*$

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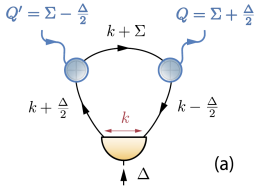
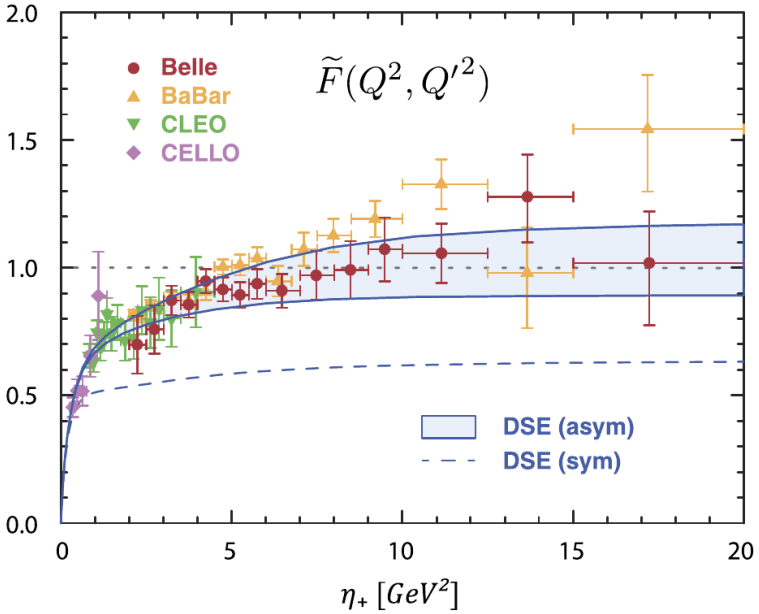
BSE / DSE calculation

[Eichmann et al PLB774(2017)425]





BSE/DSE calculation

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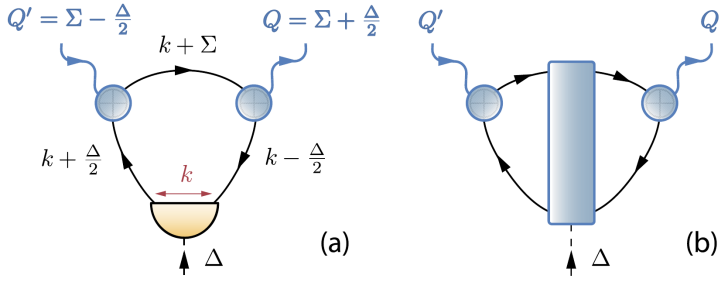


(Figure from Eichmann et al., PLB774(2017)425)

Rainbow-ladder truncation: diagram (a) remains

$$\Lambda_{\mu\nu} = 2e^2\,\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4} S(k_+)\,\Gamma_\pi(k,\Delta)\,S(k_-)\,\Gamma_\mu(k_-,k_+ + \Sigma)\,S(k+\Sigma)\,\Gamma_\nu(k+\Sigma,k_+)$$

Requires nonperturbative tensor structure (i.e. form factors) of

- quark propagator (quarks DSE | lattice data)

$$S(p) = Z(p^2)/(i\not{p} + M(p^2))$$

- pion bound-state amplitude (truncated BSE | lattice: work in progress)

$$\Gamma_\pi(k,\Delta) = (f_1 + f_2 i\Delta + f_3 k\cdot\Delta i\not{k} + f_4[\not{k},\not{\Delta}]\gamma_5)$$

- quark-photon vertex (inhom. quark-photon BSE | first lattice data)

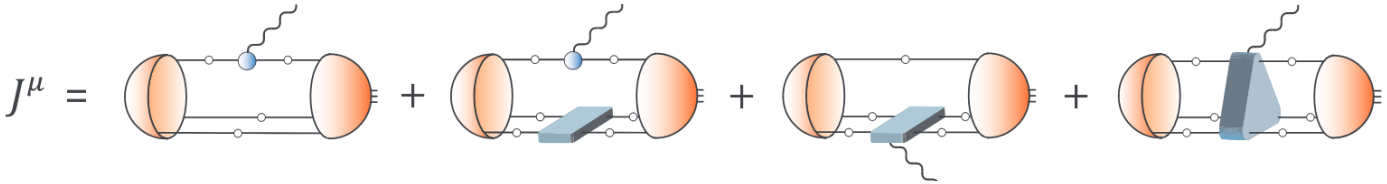
$$\Gamma_\mu(k,Q) = i\gamma_\mu\lambda_1 + 2k_\mu[i\not{k}\,\lambda_2 + \lambda_3] + \sum_{j=1,\ldots,8} i\tau_j\,T_\mu^{(j)}(k,Q)$$

$k \equiv (k_1 + k_2)/2, \quad Q \equiv k_2 - k_1$

Electromagnetic properties from bound-state amplitude

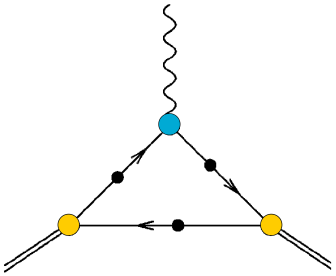
Nucleon electromagnetic current

(G. Eichmann, PRD84 (2011) 014014)



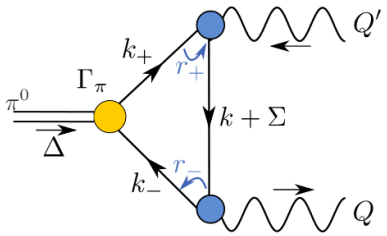
Pion form factor

(Maris, Tandy (2000), impulse approximation)



Meson transition form factor $\pi^0 \to \gamma^* \gamma^*$

(E. Weil et al. (2017), impulse approximation)



Require

- nonperturbative quark propagators (quarks DSE | lattice data)
- BS amplitudes Γ (truncated BSE | lattice: work in progress)
- Quark-photon vertex (quark-photon BSE | first lattice data $\to$ this talk)

Hadron vacuum polarization

Quark-Photon-Vertex

- contains information about QCD contributions to $a_\mu = \frac{1}{2}(g_\mu - 2)$
- LbL and HVP

$$a_\mu^{HVP} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \left[-e^2 \Pi_R \left(\frac{x^2 m_\mu^2}{1-x} \right) \right]$$

HVP: Fourier transform of hadronic part of vector-vector current correlator

$$\Pi^{\mu\nu}(Q) = \int d^4x \, e^{iQ\cdot x} \langle 0 | \, \mathrm{T} j^\mu(x) j^\nu(0) \, | 0 \rangle = \left(Q^2 \delta_{\mu\nu} + Q_\mu Q_\nu \right) \Pi(Q^2)$$

$$= \mathrm{Tr} \int \frac{d^4 k}{(2\pi)^4} \, Z_2 \, i\gamma^\mu \, S(k_+) \, \Gamma^\nu(k, Q) \, S(k_-)$$

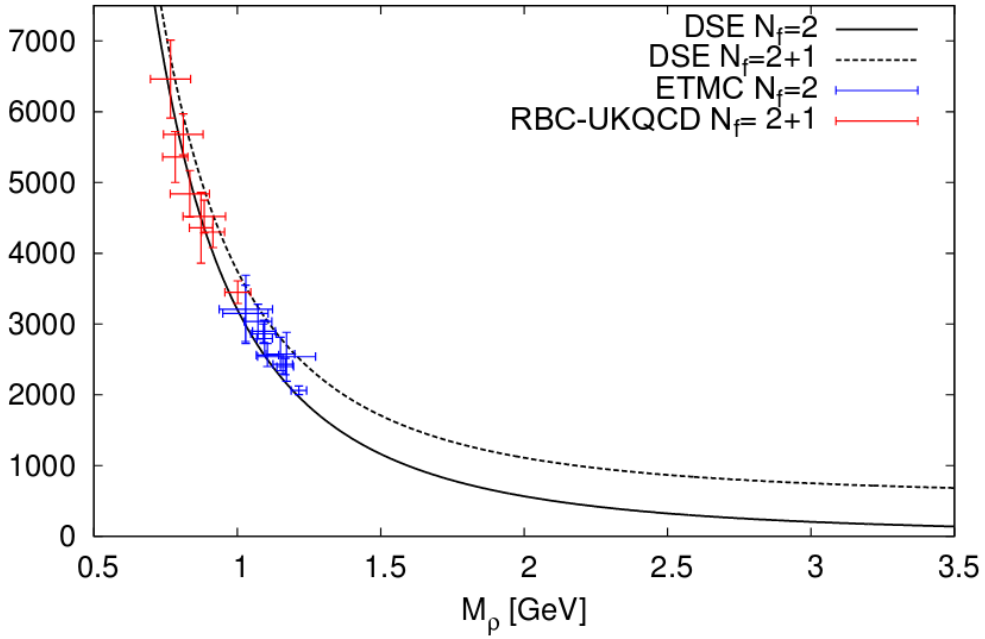
Requires nonperturbative

- quark propagator (quarks DSE | lattice data)
- Quark-photon vertex (quark-photon BSE | first lattice data)

Hadron vacuum polarization

DSE vs. lattice (2011)

$$a_\mu^{HVP} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\left[-e^2\Pi_R\left(\frac{x^2 m_\mu^2}{1-x}\right)\right]$$

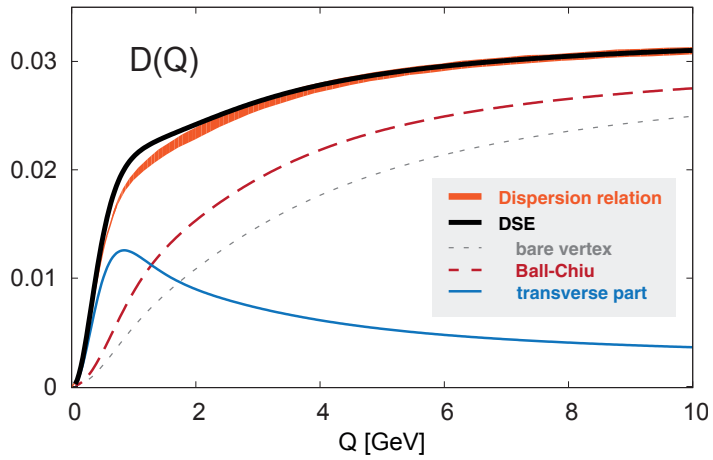


DSE value: $a_\mu^{HVP} = 676\ldots744 \times 10^{-10}$

[Goecke et al. (2012)]

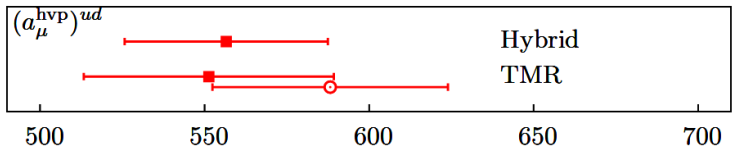
Adler function(2011)

$$D(Q^2) = Q^2\frac{d}{dQ^2}\Pi(Q^2)$$



Latest lattice $N_f = 2$ estimate:

[Mainz: Della Morte et al. (2017)]



ETMC (2018) : $a_\mu^{HVP}(ud) = 619 \pm 23 \times 10^{-10}$

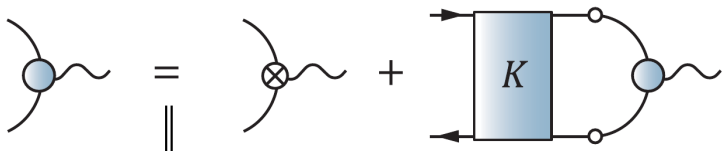
Quark-Photon (γ_μ) vertex as solution of BS equation

QCD dressing of tensor structure

- key to any BS/DSE calculation of electromagnetic properties of hadrons
  (elastic and transition form factors, HVP, ...)

$$\Gamma_\mu(k,Q) = \underbrace{i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3]}_{\Gamma_\mu^{\mathrm{BC}}} + \sum_{j=1}^{8} i\tau_j\, T_\mu^{(j)}(k,Q)$$

- Γ_μ satisfies <u>inhomogeneous BS equation</u> and Γ_μ^{BC} vector WT identity



$$Q_\mu\Gamma_\mu = S^{-1}(k_-) - S^{-1}(k_+)$$

$$\Gamma_\mu(k,Q) = Z_2\gamma_\mu - Z_2^2\frac{4}{3}\int_q [S(q+Q/2)\,\Gamma_\mu(q,Q)\,S(q-Q/2)]\,K(k-q)$$

- In practice: "rainbow-ladder" truncated quark DSE and kernel ...systematic error?

$$K_{\rho\sigma,\alpha\beta}(k) = G(k^2)\,T^k_{\mu\nu}\gamma_\mu^{\alpha\rho}\gamma_\nu^{\sigma\beta} \qquad T^k_{\mu\nu}G(k^2)\ldots\text{eff. gluon propagator}$$

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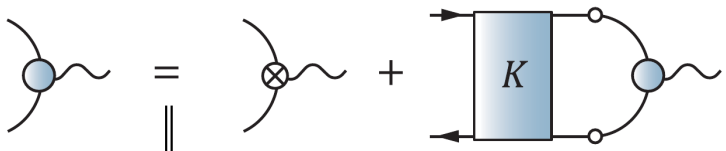

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Quark-Photon (γ_μ) vertex from lattice QCD

Monte Carlo averages: quark-γ_μ bilinear and propagator in Landau gauge
(Inverse of D_U via momentum sources, numerically demanding)

$$(1) \qquad G_\mu(k,Q) = \sum_{x,y,z} e^{ik_+(x-z)} e^{ik_-(z-y)} \left\langle [D_U^{-1}]_{xz} \gamma_\mu [D_U^{-1}]_{zx} \right\rangle_U$$

$$(2) \qquad S(k_\pm) = \sum_{x,y} e^{ik_\pm(x-y)} \left\langle [D_U^{-1}]_{xy} \right\rangle_U \qquad \text{with} \quad k_\pm = k \pm \frac{Q}{2}$$

Vertex from amputated 3-point function

$$\Gamma_\mu(k,Q) = S^{-1}\left(k - \frac{Q}{2}\right) G_\mu(k,Q)\, S^{-1}\left(k + \frac{Q}{2}\right)$$

Form factors from solving

$$\Gamma_\mu(k,Q) = i\gamma_\mu \lambda_1 + 2k^\mu [i\not{k}\, \lambda_2 + \lambda_3] \; + \; \sum_{j=1}^{8} i\tau_j \, T_\mu^{(j)}(k,Q)$$

Parameters of our gauge field ensembles ($N_f = 2$)

Lattice action

- Wilson gauge action
- Wilson clover fermions
- Landau gauge
  (after thermalization)

Can study:

- quark mass dependence
- discret./ volume effects

β	κ	$L_s^3 \times L_t$	a [fm]	m_b [MeV]	m_π [MeV]
5.20	0.13584	$32^3 \times 64$	0.08	14	411
5.20	0.13596	$32^3 \times 64$	0.08	6	280
5.29	0.13620	$32^3 \times 64$	0.07	17	422
5.29	0.13632	$32^3 \times 64$	0.07	8	295
5.29	0.13632	$64^3 \times 64$	0.07	8	290
5.29	0.13640	$64^3 \times 64$	0.07	2	150
5.40	0.13647	$32^3 \times 64$	0.06	18	426
5.40	0.13660	$48^3 \times 64$	0.06	7	260

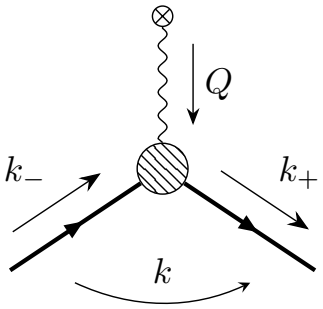
Consider:

- local vector current: $\bar{\psi}_x \gamma_\mu \psi_x$
- twisted boundary condition for quarks

Acknowledgements

- $N_f = 2$ configurations provided by RQCD collaboration (Regensburg)
- Gauge-fixing and calculation of propagators at the HLRN, LRZ and FSU Jena

Quark-Photon vertex: discrete lattice momenta



Momenta quarks: $k_\pm = k \pm Q/2$

- twisted b.c. for quarks: $\psi(x + \hat{\mu} L_\mu) = e^{i\theta_\mu}\psi(x)$

$$ak_\mu = \frac{2\pi n_\mu}{L_\mu} + \frac{\theta_\mu}{L_\mu} \equiv \frac{2\pi}{L_\mu}\left(n_\mu + \frac{\tau_\mu}{2}\right) \qquad \text{twist angle:} \quad \theta_\mu = \pi\tau_\mu/L_\mu$$

- **symmetric** setup (preferred but no access to $\lambda_2, \lambda_3, f_5, f_7$)

$$Q^2 = k_-^2 = k_+^2, \qquad n^- = n\,(1,1,0,0) + (\tau,\tau,0,0)\,, \qquad z \equiv \frac{k\cdot Q}{|k||Q|}$$

$$z = 0 \qquad n^+ = n\,(0,1,1,0) + (0,\tau,\tau,0)$$

- **asymmetric** setup

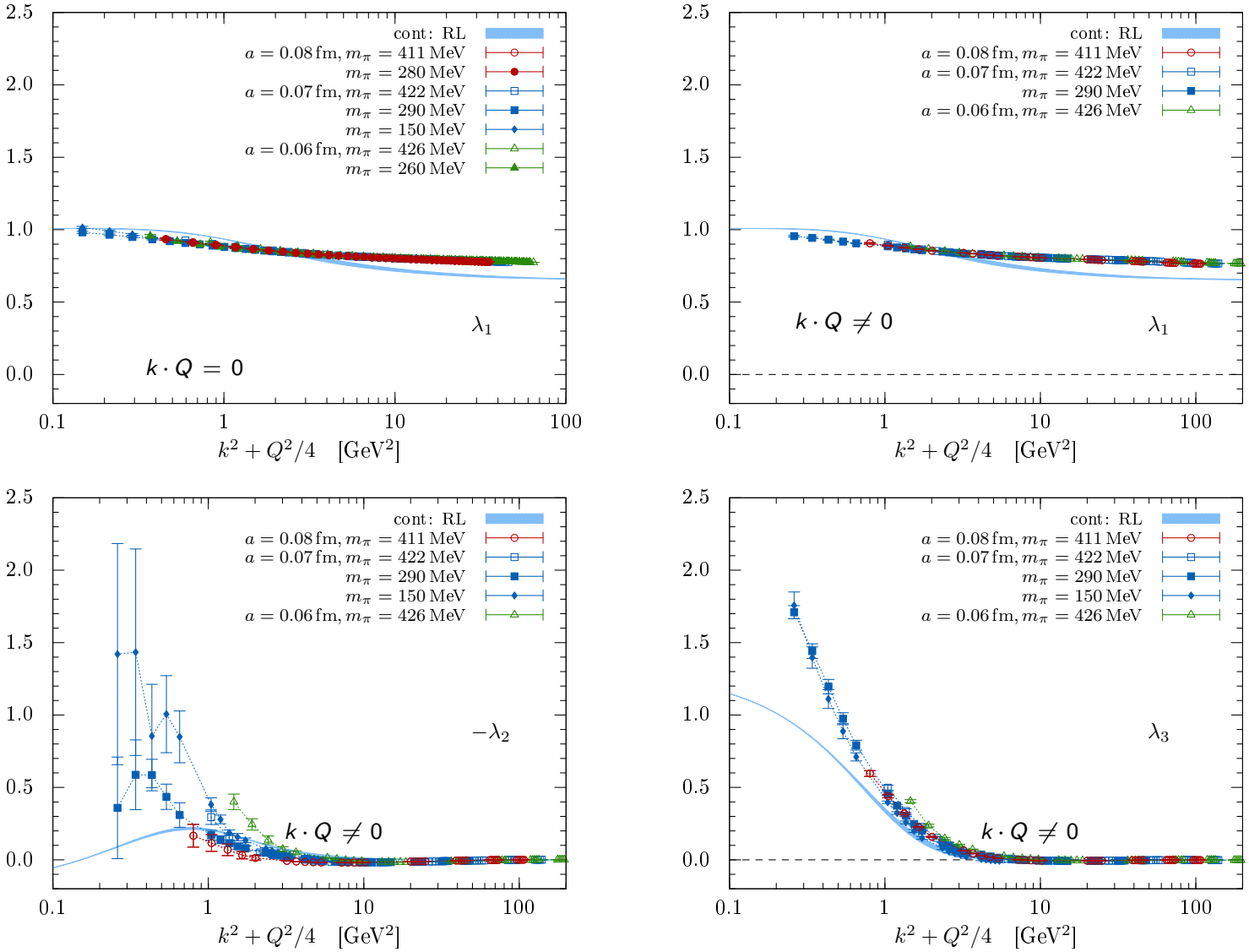
$$Q^2 = k_-^2 > k_+^2, \qquad n^- = n\,(2,1,0,0) + (2\tau,\tau,0,0)$$

$$z = \frac{1}{\sqrt{5}} \qquad n^+ = n\,(0,1,1,0) + (0,\tau,\tau,0)$$

- discrete values: $n = 1, 2, \ldots, L_s/4$ and twists $\tau = 0, 0.4, 0.8, 1.2$ and 1.6
(gives smooth interpolation)

Lattice results $\Gamma_\mu = i\gamma_\mu\lambda_1 + 2k^\mu[i\not{k}\,\lambda_2 + \lambda_3] + \sum_{j=1}^{8} i\tau_j T_\mu^{(j)}(k,Q)$

(preliminary) vs. continuum (rainbow-ladder)



(Rainbow-ladder results is from G. Eichmann)

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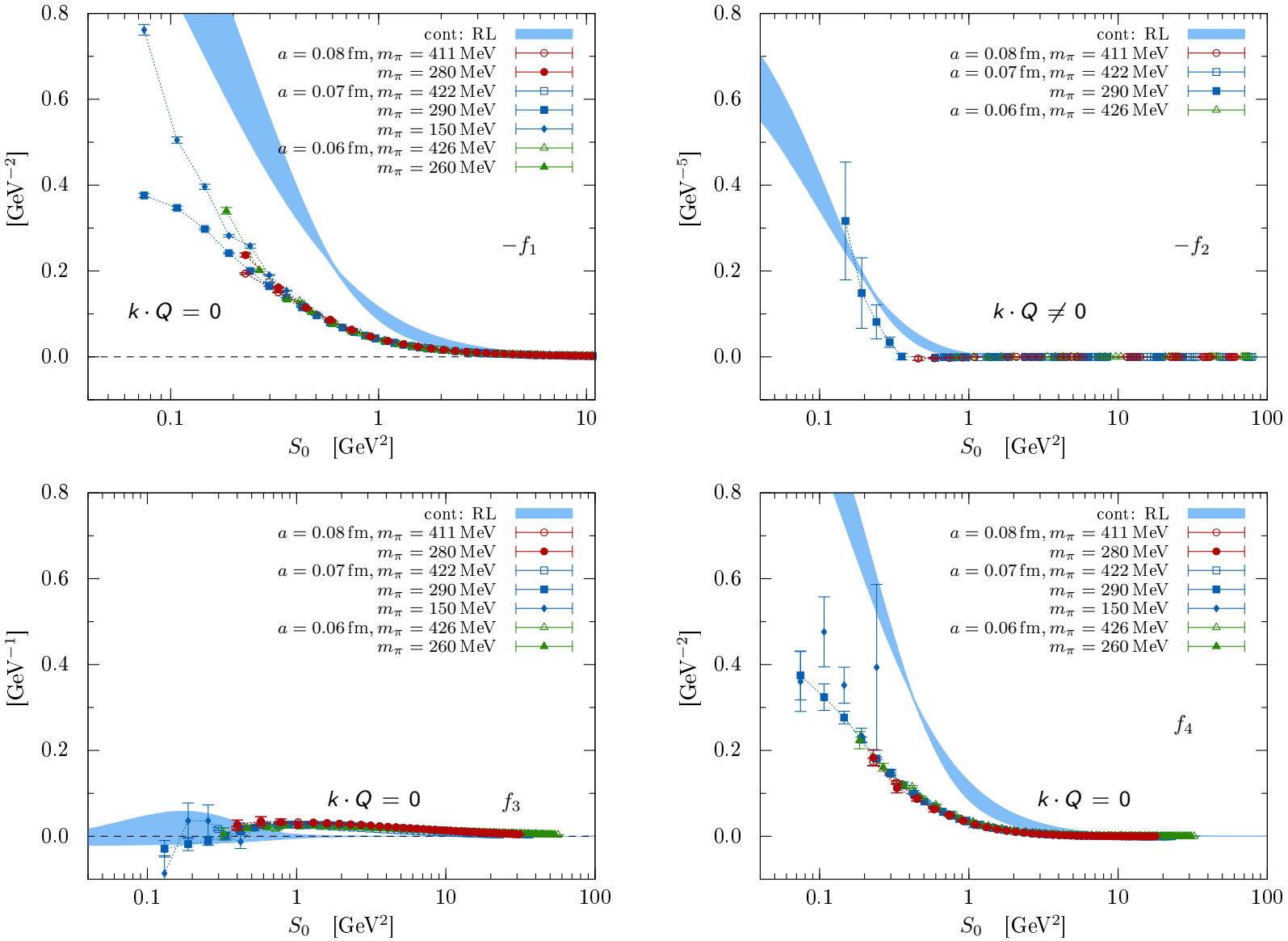

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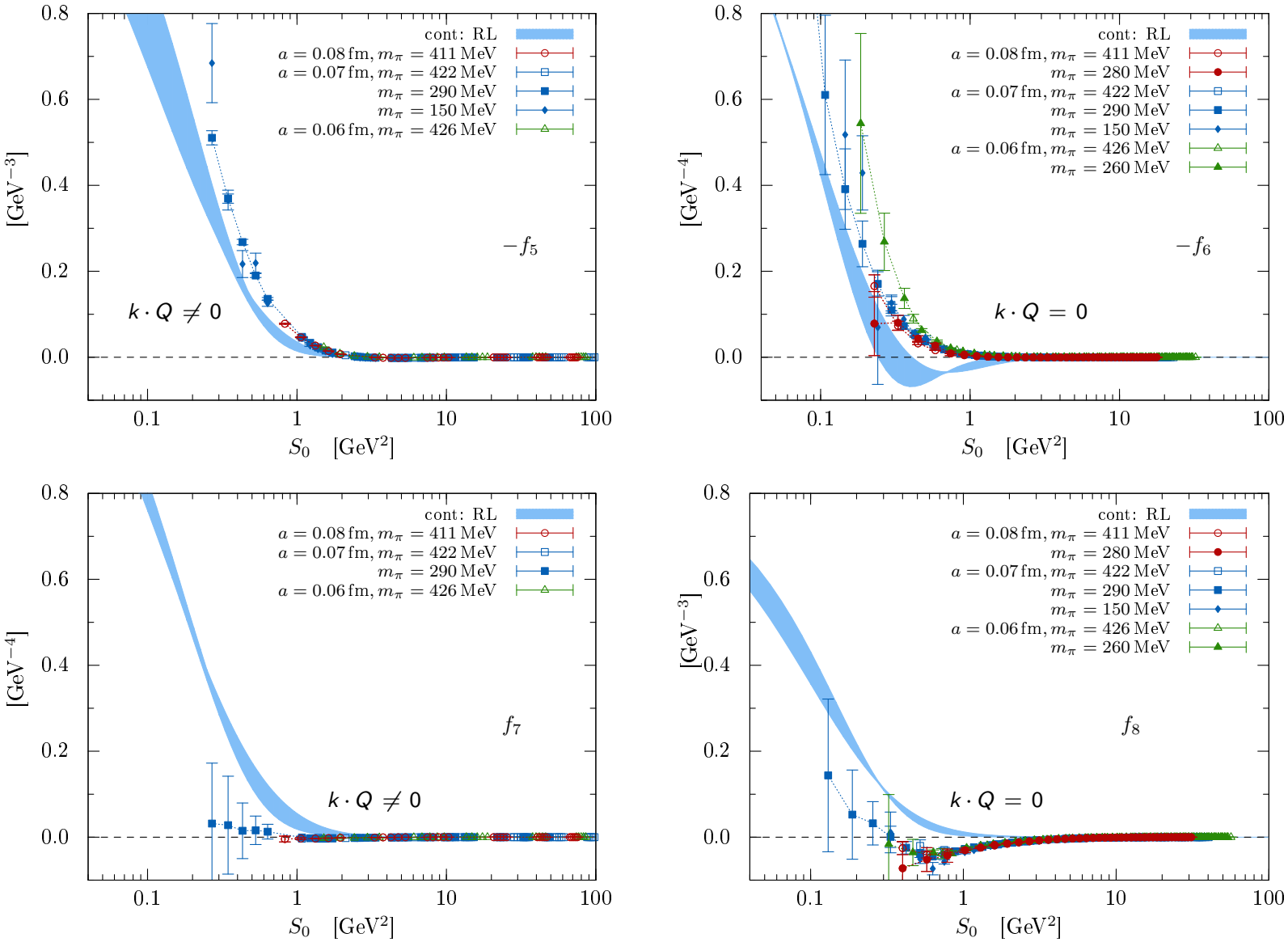


(Rainbow-ladder results is from G. Eichmann)

Lattice results: Summary

First data (ever)

- volume, discretization effects smaller than deviations to continuum
- give good hints for improving currently used truncations

Future: high precision

- lattice implementation should be O(a)-improved
- work out lattice corrections (nothing available yet)
- improve statistics at low momentum
- data may then serve as direct input (interpolation of points, connecting UV and IR)

Remark on future O(a)-improvement $\Longrightarrow$

Vector WT identity and the lattice

Continuum: vector Ward-Takahashi identity (vWTI)

$$Q_\mu G_\mu(k,Q) = S(k_-) - S(k_+) \qquad \text{with} \quad G_\mu = S(k_-)\,\Gamma_\mu(k,Q)\,S(k_+)$$

Lattice: (Wilson fermions)

- 3-point function:

$$G_\mu(k,Q) = \sum_{x,y,z} e^{ik_-(x-z)} e^{-ik_+(y-z)} \left\langle \psi_x \tilde{V}_\mu(z)\, \bar{\psi}_y \right\rangle$$

- point-split vector current satisfies WTI [Karsten/Smit (1981)]

$$\tilde{V}^f_\mu(z) = \frac{1}{2}\left(\bar{\psi}_z[\gamma_\mu - 1] U_{x\mu} \frac{\lambda^f}{2} \psi_{z+a\hat{\mu}} + \bar{\psi}_{z+a\hat{\mu}}[\gamma_\mu + 1] U^\dagger_{x\mu} \frac{\lambda^f}{2} \psi_z \right)$$

$$= V^f_\mu(z) + aK_\mu(z) \;\equiv\; Z_V(g_0^2)\, V^f_\mu(z) + O(a)$$

We used (so far)

- local vector current $V^f_\mu(z) = \bar{\psi}_z \gamma_\mu \frac{\lambda^f}{2} \psi_z$
- plain Wilson quark propagator

Vector WT identity and the lattice

Lattice: vector Ward-Takahashi identity (vWTI)

$$Q_\mu Z_V(g_0^2) G_\mu(k,Q) = S(k_-) - S(k_+) + O(a,k,Q)$$

Lattice correction

- applied $Z_V(g_0^2, m_q = 0)$ to data, should fix prefactor (in chiral limit) (values from RQCD collaboration)

Calculate $\lambda_1(k,Q)$ in two ways

1. from vertex data

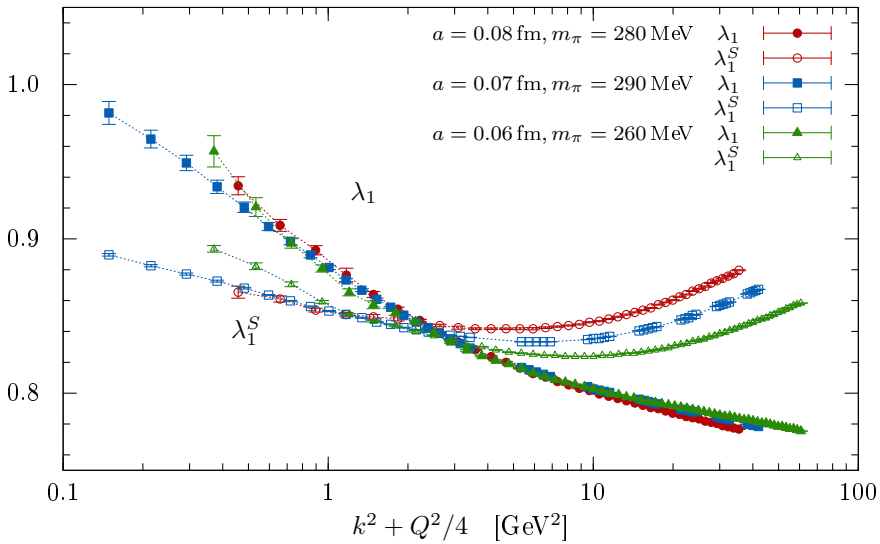
$$\Gamma_\mu = S^{-1}(k_-)\, G_\mu(k,Q)\, S^{-1}(k_+) \quad = i\gamma_\mu \lambda_1 + 2k^\mu [i\not{k}\,\lambda_2 + \lambda_3] \; + \sum_{j=1}^{8} i\tau_j T_\mu^{(j)}(k,Q)$$

2. from data for quark propagator S

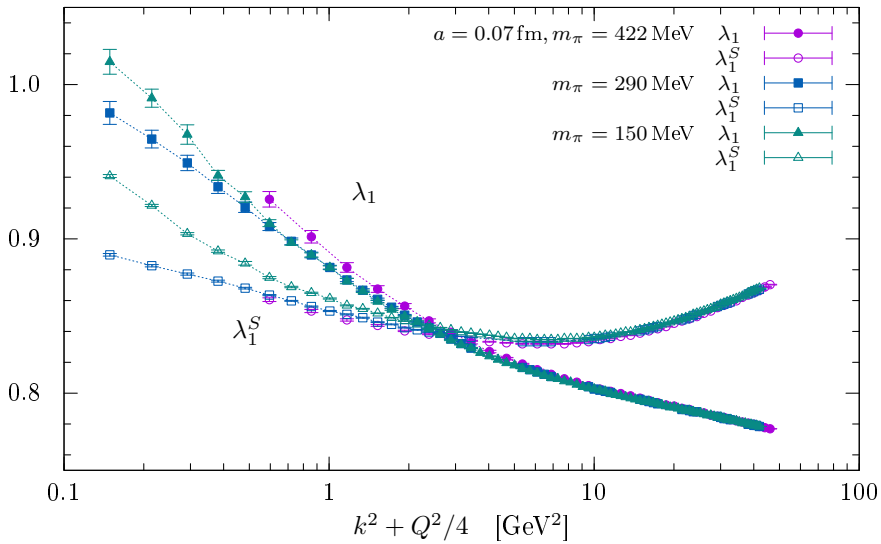
$$\lambda_1^S = \frac{A(k_+^2) + A(k_-^2)}{2} \qquad \text{where} \quad S^{-1}(p) = i\gamma_\mu \bar{p}_\mu A(p) + B(p)$$

Vector WT identity and the lattice: λ_1 vs. λ_1^S

($\approx$ same m_b, $a = 0.08 \ldots 0.06\,\mathrm{fm}$)



(same a, $m_b = 2 \ldots 17\,\mathrm{MeV}$)



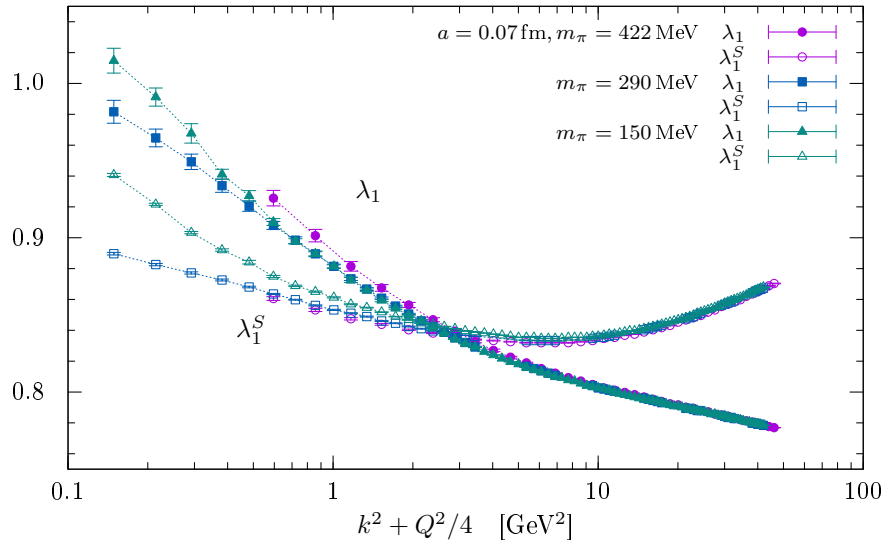
- deviations grow with m_b and momentum $k_\pm^2 = k^2 + Q^2/4 + k{\cdot}Q$
- surprised about deviations at small $k_\pm^2$
- effects larger for λ_1^S

$O(a)$-improved Wilson quark propagator

Wilson quark propagator

- essential for vertices / RI'(S)MOM renormalization
- defacto standard: no improvement
- clover improves on-shell quantities
- extra efforts for off-shell quantities

(same a, $m_b = 2 \ldots 17\,\mathrm{MeV}$)



First data for O(a)-improved propagator

(to appear Oliveira, Silva, Skullerud, *A.S.*, Williams, 2018)

$$S_{\mathrm{rot}}(x,y) = (1 + 2b_q am) \left\langle L_x(U)\, M_{xy}^{-1}(U)\, R_y(U) \right\rangle_U$$

- M is Wilson clover-fermion matrix
- left/right rotation: $L_x(U) \equiv \left[1 - c_q a \overrightarrow{\not{D}}_x(U)\right], \quad R_y(U) \equiv \left[1 + c_q a \overleftarrow{\not{D}}_y(U)\right]$
- has smaller $O(a^2)$ effects than other definitions, see e.g. QCDSF (2001)

$O(a)$-improved Wilson quark propagator

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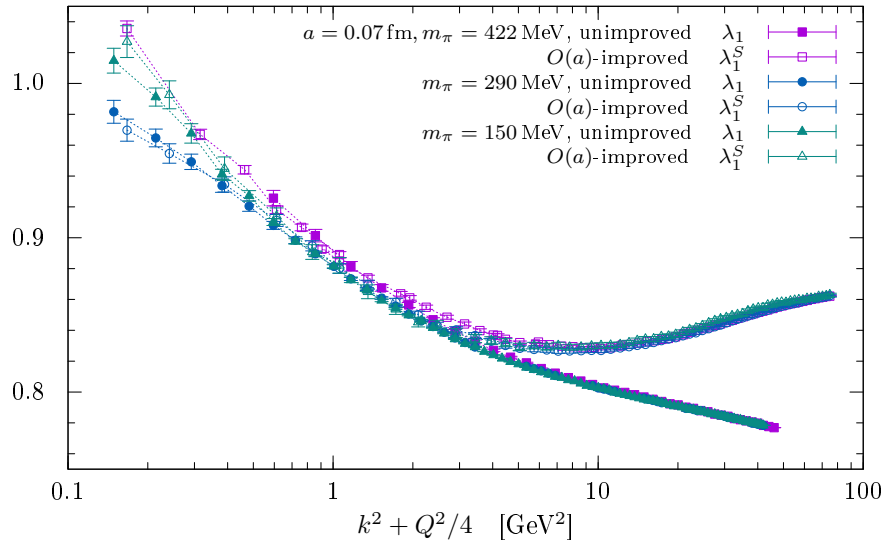

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$$S_{\mathrm{rot}}(x,y) = (1 + 2b_q am) \left\langle L_x(U)\, M_{xy}^{-1}(U)\, R_y(U) \right\rangle_U$$

Improves WTI!

- M is Wilson clover-fermion matrix
- left/right rotation: $L_x(U) \equiv \left[1 - c_q a \overrightarrow{\not{D}}_x(U)\right], \quad R_y(U) \equiv \left[1 + c_q a \overleftarrow{\not{D}}_y(U)\right]$
- has smaller $O(a^2)$ effects than other definitions, see e.g. QCDSF (2001)

Lattice results: Summary

First data (ever)

- Volume, discretization effects smaller than deviations to continuum
- give good hints for improving currently used truncations

Future: high precision

- lattice implementation should be O(a)-improved
- work out lattice corrections (nothing available yet)
- improve statistics at low momentum
- Data may then serve as direct input (interpolation of points, connecting UV and IR)

Future O(a)-improvement

- Should use point-split current (straightforward)
- use mentioned O(a)-improved also for quark bilinears

Thank you for your attention!