

# Meson spectroscopy and scattering from lattice QCD

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UNIVERSITY OF  
CAMBRIDGE

hadspec

## Excited spectroscopy

Most hadrons appear as resonances in scattering of lighter hadrons.

All but the simplest couple to final states (or intermediate states) containing hadrons with non-zero spin.

This talk: first-principles lattice QCD calculations of

- $\rho\pi$  isospin-2 scattering
- exotic-flavour charm channels (tetraquarks)

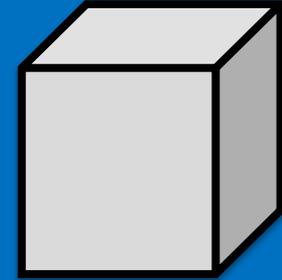
[For other recent Hadron Spectrum Collaboration work see talks by Jo Dudek and Raúl Briceño on Friday.]

# Excited lattice QCD spectroscopy

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \langle 0 | \underline{\mathcal{O}_i(t)} \underline{\mathcal{O}_j^\dagger(0)} | 0 \rangle$$

Use many different interpolating operators

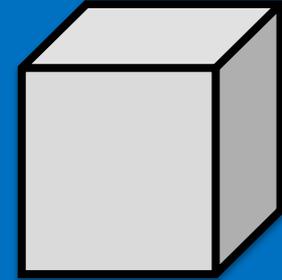


# Excited lattice QCD spectroscopy

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**Lüscher method** (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering  $t$ -matrix** (any number of 2-hadron channels).

**Elastic scattering**: 1-to-1 correspondence (ignoring partial-wave mixing).  
But **in general under-constrained problem** (determinant equ. at each  $E_{\text{cm}}$ )  
→ parameterize  $E_{\text{cm}}$  dependence of  $t$ -matrix and fit  $\{E_{\text{lat}}\}$  to  $\{E_{\text{param}}\}$   
Consider many different parameterizations (e.g.  $K$ -matrix, eff. range, B.W.)

Talk by Max Hansen this morning (includes progress for  $> 2$  hadrons)

Recent review: Briceño, Dudek, Young [arXiv:1706.06223]

## $\rho\pi$ isospin-2 scattering

[Woss, CT, Dudek, Edwards,  
Wilson, JHEP 1807, 043 (2018)]

Different partial waves with same  $J^P$  can mix dynamically

E.g.  $J^P = 1^+ (^3S_1, ^3D_1)$ ,  $J^P = 2^- (^3P_2, ^3F_2)$

$$\mathbf{t} = \begin{bmatrix} t(^3S_1| ^3S_1) & t(^3S_1| ^3D_1) \\ t(^3S_1| ^3D_1) & t(^3D_1| ^3D_1) \end{bmatrix}$$

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Finite-volume lattice QCD calculations: reduced symmetry  
→ additional 'mixing'

[0,0,0]	$T_1^+$
$J^+ (^3\ell_J)$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$
	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$
	$4^+ (^3G_4)$

Calc. at  $SU(3)_F$  symmetric point ( $m_u=m_d=m_s$ )

$$m_\pi \approx 700 \text{ MeV}$$

$$m_\rho \approx 1020 \text{ MeV (stable)}$$

Large bases of  $\rho\pi$   $SU(3)_F$  '**meson-meson**' operators:

$$\rho\pi(\vec{P}) = \sum_{\hat{p}_1, \hat{p}_2} \mathcal{C}(\vec{P}, \vec{p}_1, \vec{p}_2; \lambda_1) \rho_{\lambda_1}(\vec{p}_1) \pi(\vec{p}_2)$$

'optimised'  $\rho$  and  $\pi$  ops  $\sim \bar{\psi}\Gamma D \dots \psi$

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'optimised'  $\rho$  and  $\pi$  ops  $\sim \bar{\psi}\Gamma D \dots \psi$

One lattice spacing

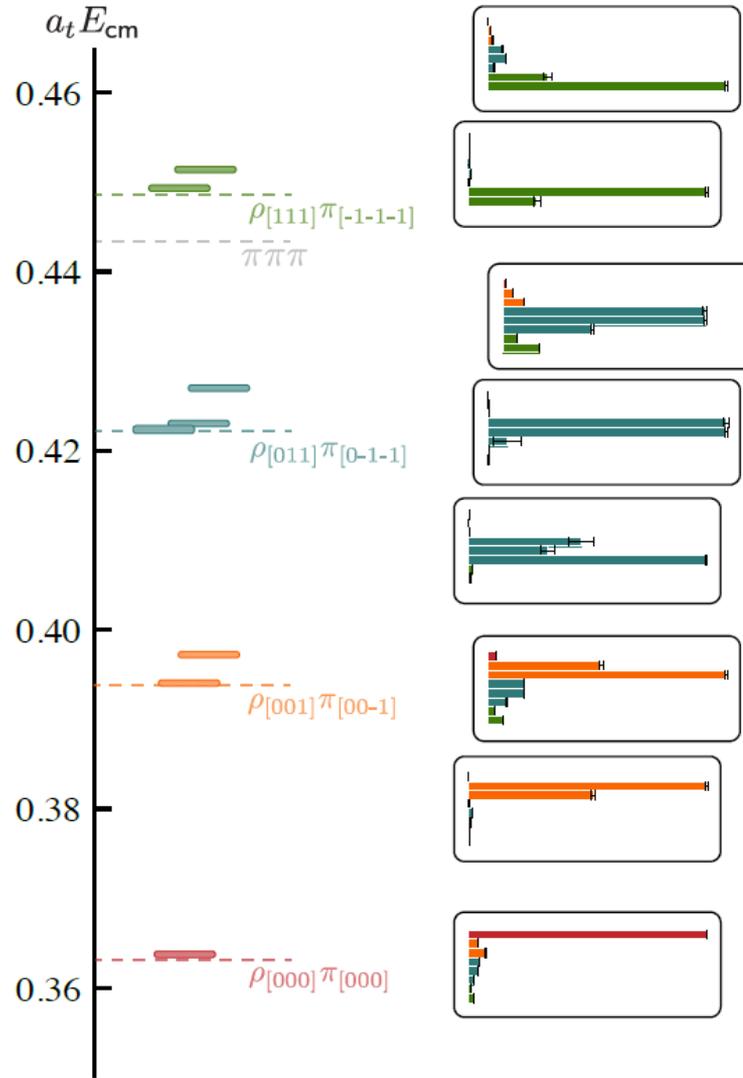
2 volumes ( $\approx 2, 3$  fm)

# $\rho\pi$ isospin-2 scattering – spectra

[JHEP 1807, 043 (2018)]

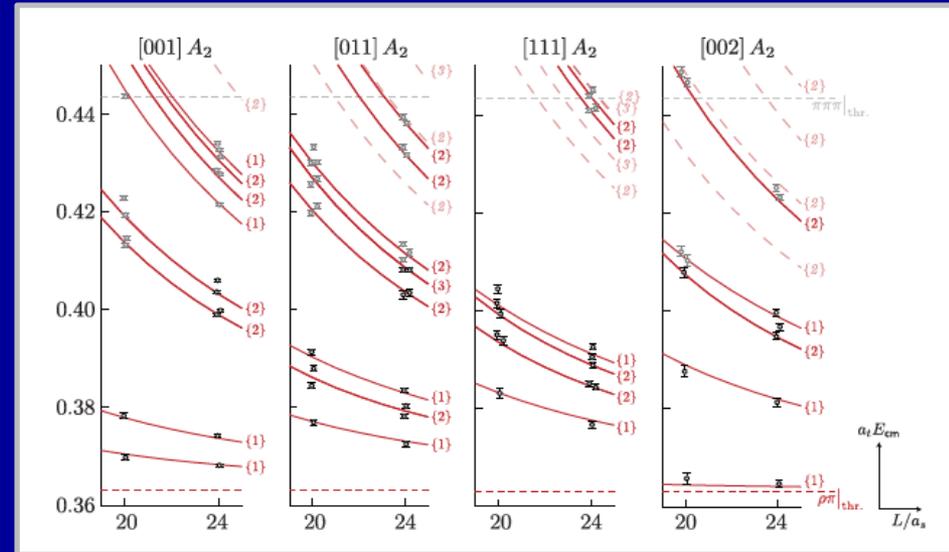
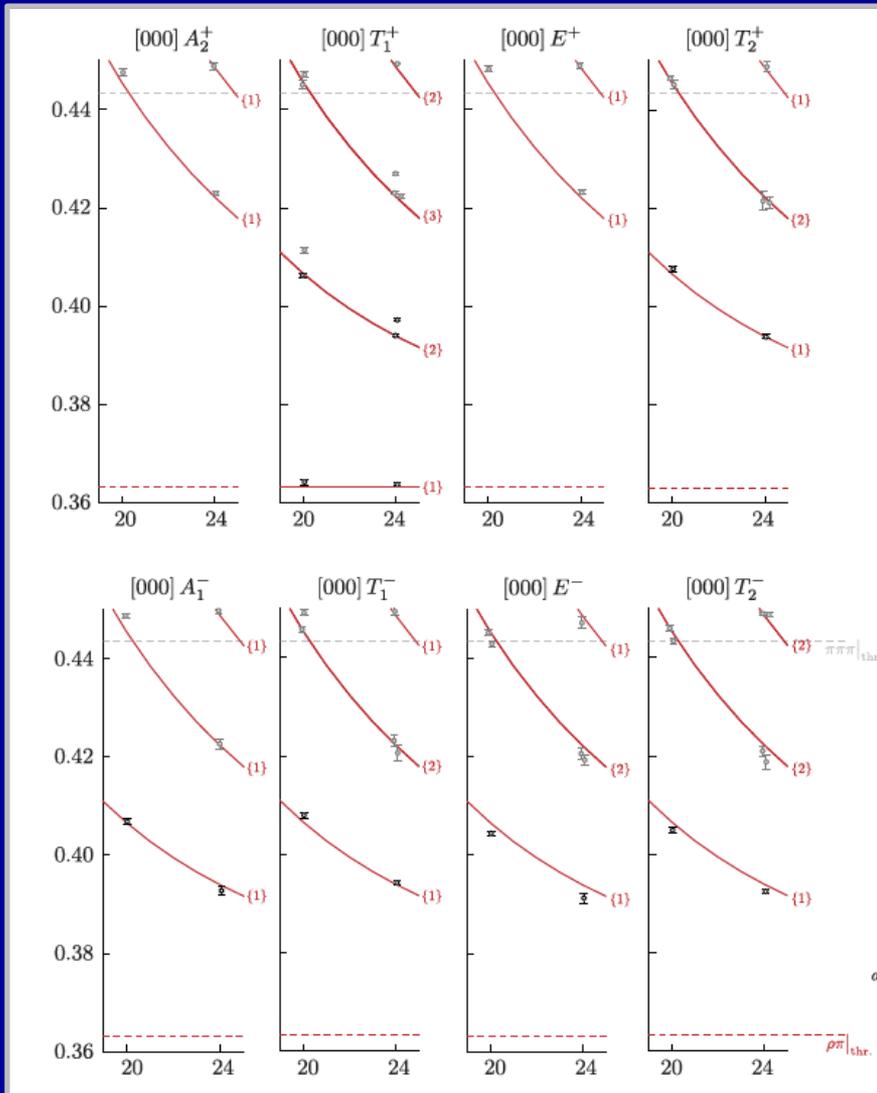
$[000] T_1^+ (1^+, 3^+, \dots)$

ops	
	$\rho_{[000]}\pi_{[000]}$
{2}	$\rho_{[001]}\pi_{[00-1]}$
{3}	$\rho_{[011]}\pi_{[0-1-1]}$
{2}	$\rho_{[111]}\pi_{[-1-1-1]}$



# $\rho\pi$ isospin-2 scattering – spectra

[JHEP 1807, 043 (2018)]

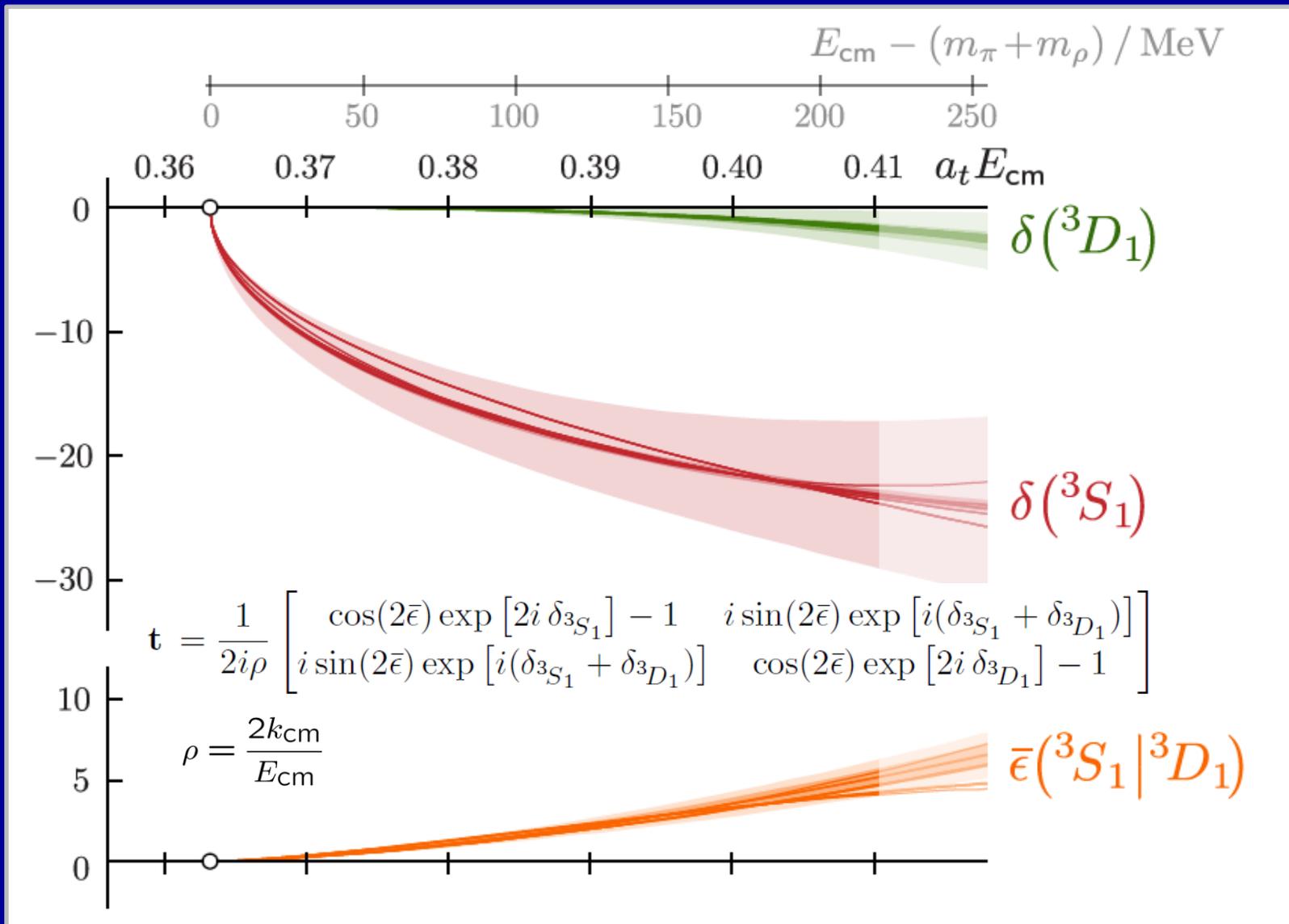


+ others

Used 141 energy levels for  $\ell = 0, 1, 2$

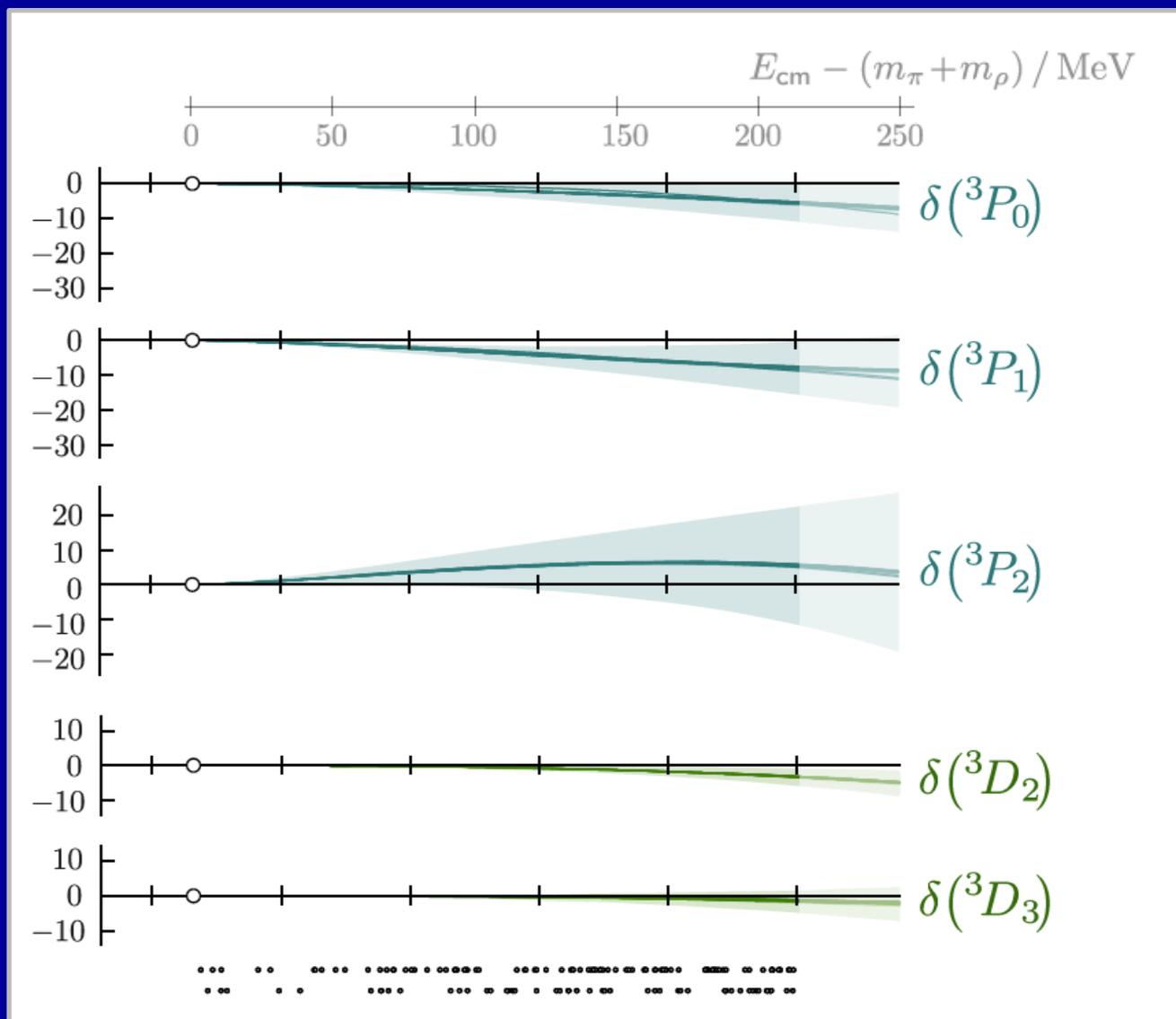
# $\rho\pi$ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]



# $\rho\pi$ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]



Compute spectra in some **exotic-flavour** charm channels

Use a range of ‘**meson-meson**’ operators,  
and ‘**tetraquark**’ (diquark-antidiquark) operators,

$$\sim \sum_{a,d} C_{ad} \left[ c_{abc} q_a^T \Gamma_1 q_b \right] \left[ c_{def} \bar{q}_e \Gamma_2 \bar{q}_f^T \right]$$

1 volume ( $\approx 2$  fm),  $m_\pi \approx 391$  MeV

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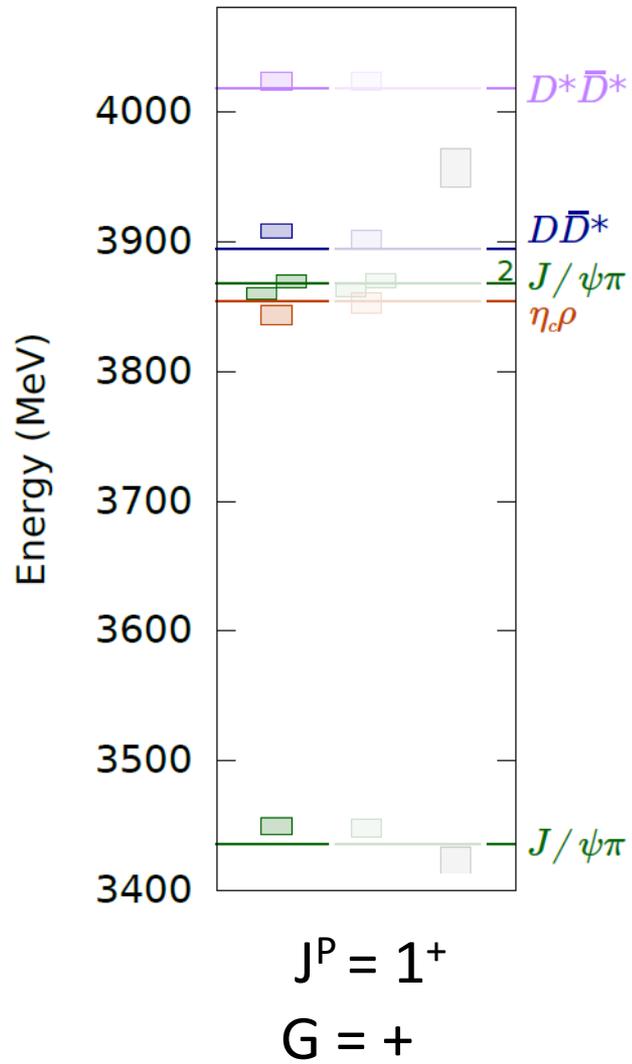
1 volume ( $\approx 2$  fm),  $m_\pi \approx 391$  MeV

Previous LQCD work on charm tetraquarks or relevant channels:

- Ikeda *et al* [PL B729, 85 (2014); PRL 117, 242001 (2016)]
- Prelovsek *et al* [PR D91, 014504 (2015)]
- Guerrieri *et al* [PoS (LATTICE2014) 106]
- Padmanath *et al* [PR D92, 034501 (2015)]
- Chen *et al* [PR D89, 094506 (2014); PR D92, 054507 (2015)]

# Hidden-charm $I=1$ ( $c\bar{c}l\bar{l}$ )

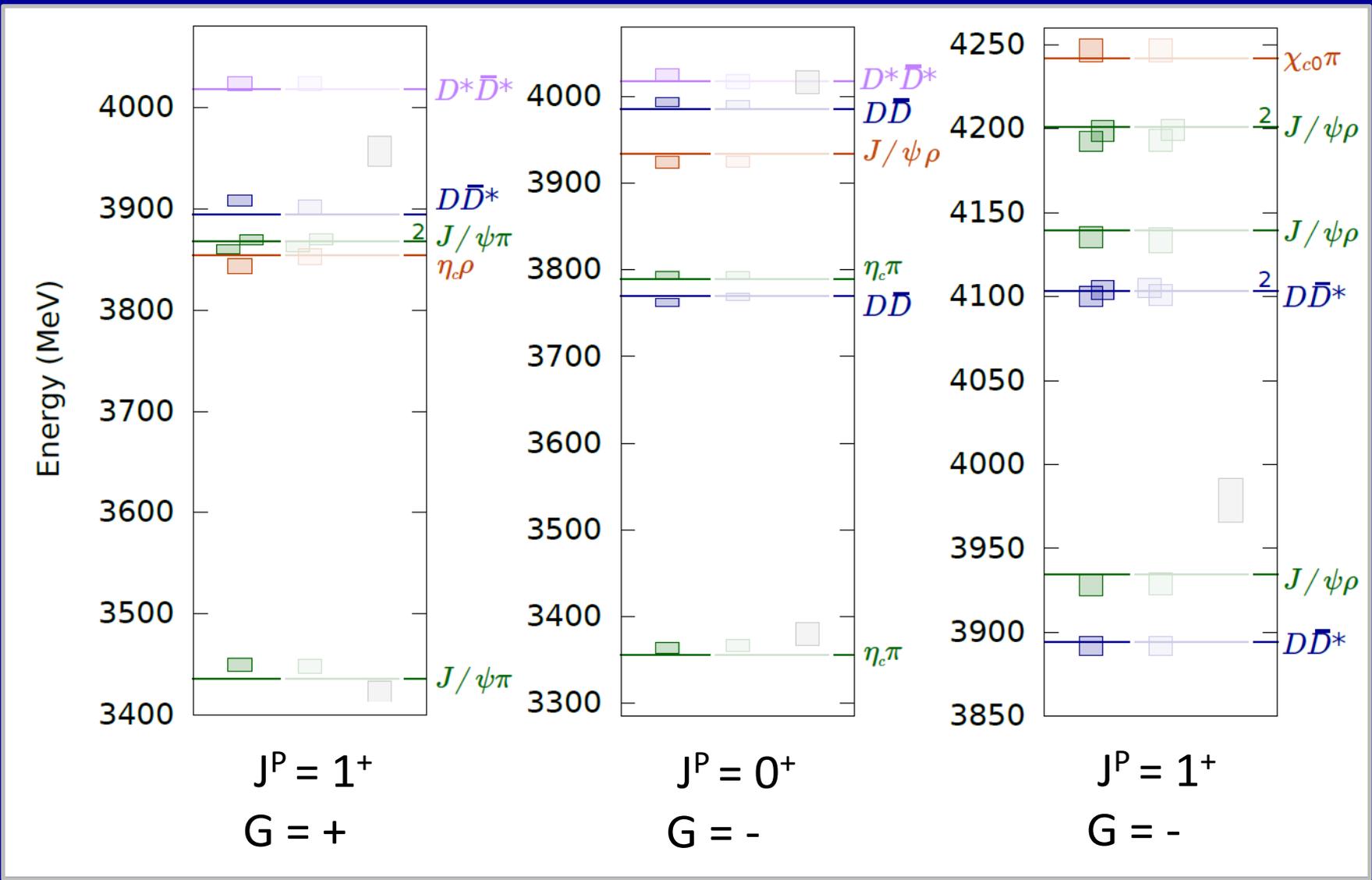
[JHEP 1711, 033 (2017)]





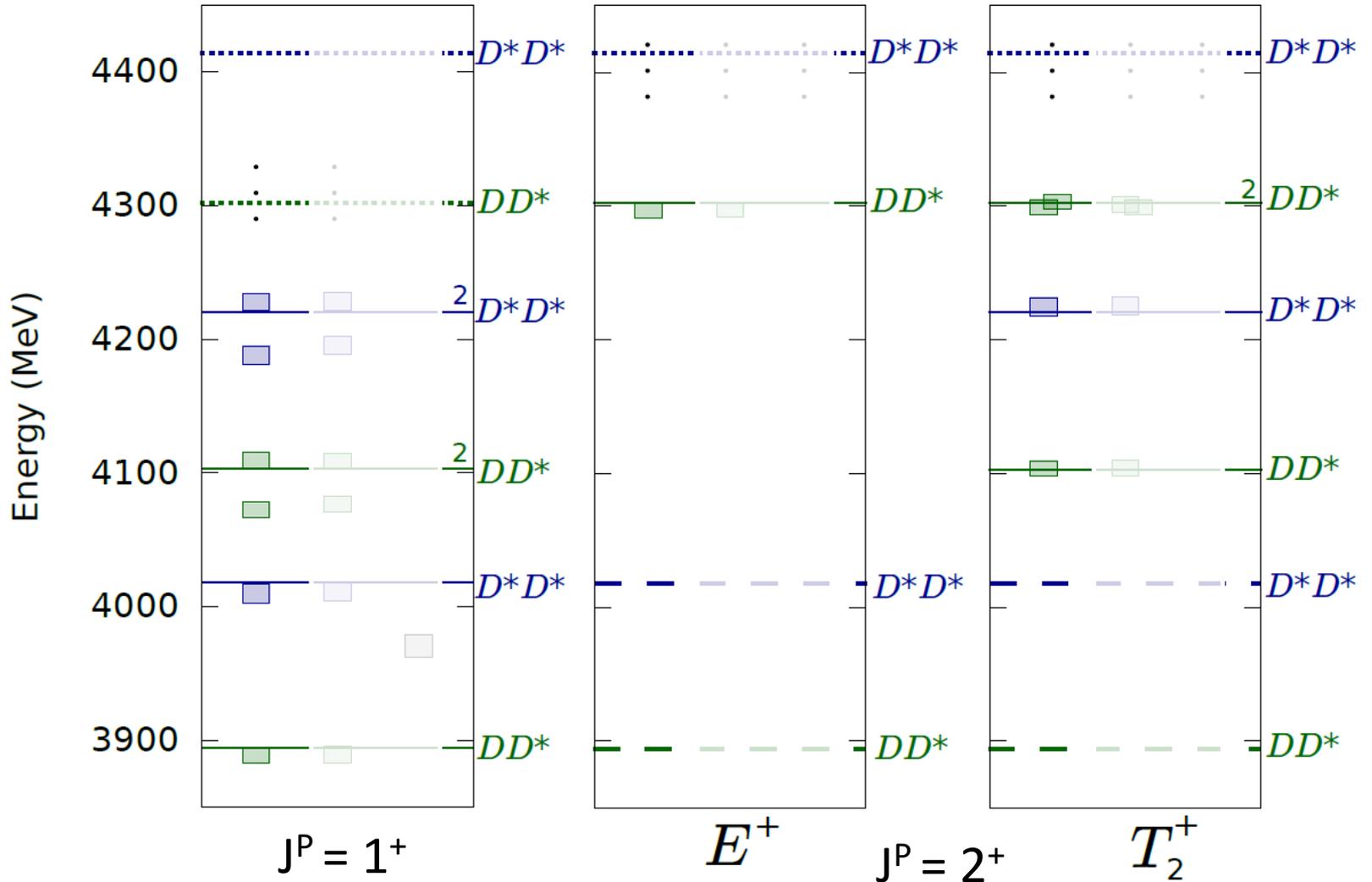
# Hidden-charm $I=1$ ( $c\bar{c}l\bar{l}$ )

[JHEP 1711, 033 (2017)]



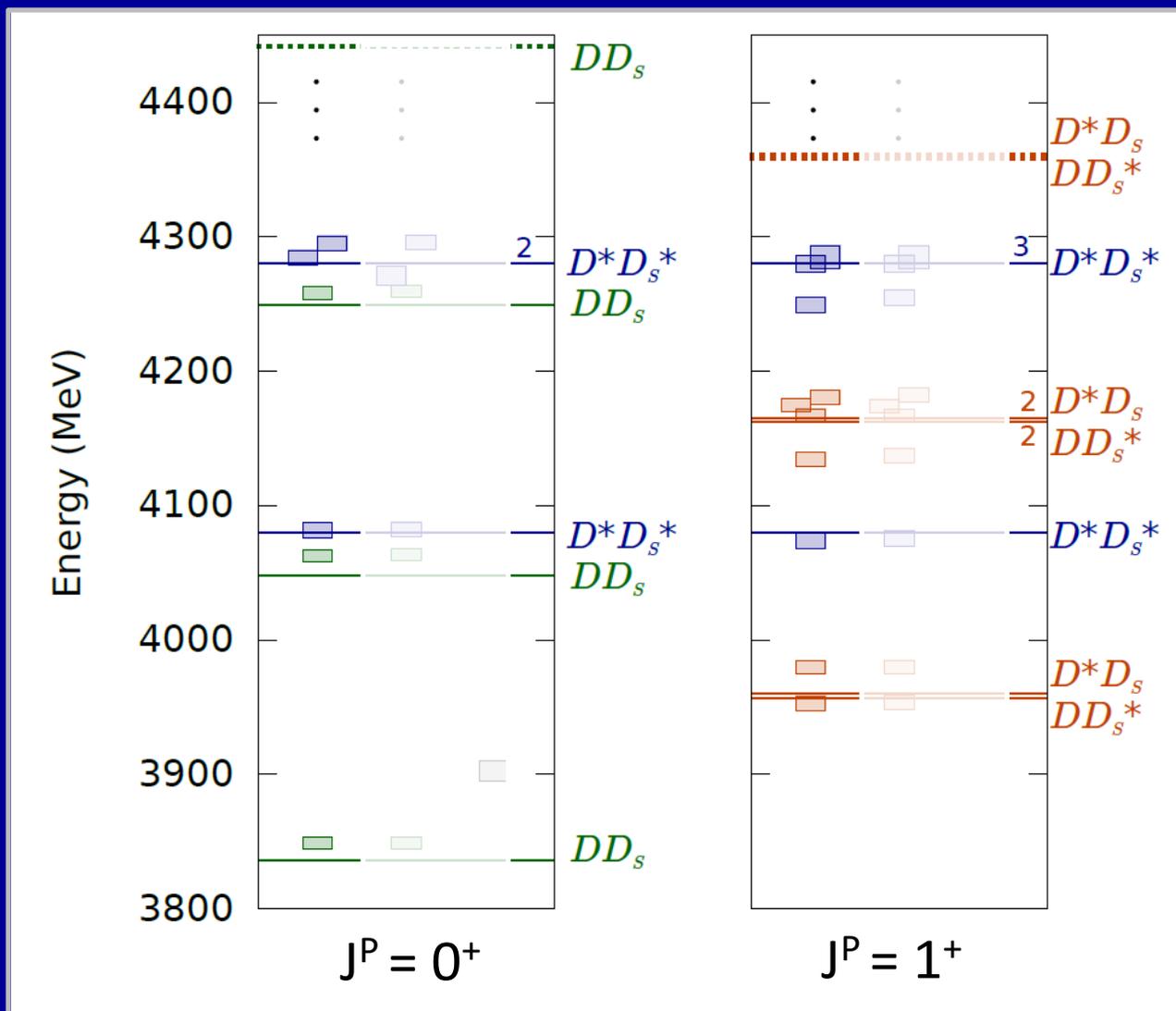
# Doubly-charmed $I=0$ ( $cc\bar{l}\bar{l}$ )

[JHEP 1711, 033 (2017)]



# Doubly-charmed $I=\frac{1}{2}$ ( $cc\bar{l}\bar{s}$ )

[JHEP 1711, 033 (2017)]



## Summary

- Significant progress in LQCD calculations of meson-meson scattering, resonances, etc.
- Examples:
  - $\rho\pi$  isospin-2 scattering –  $^3S_1, ^3D_1$  mixing
  - exotic-flavour charm channels (tetraquarks)
- Work in progress on other channels and different  $m_\pi$
- Ongoing work on formalism (e.g. 3-hadron scattering)

Jefferson Lab and surroundings, USA:

Raúl Briceño<sup>1</sup>, Jozef Dudek<sup>2</sup>, Robert Edwards, Bálint Joó,  
David Richards, Frank Winter, Bipasha Chakraborty  
(<sup>1</sup> and Old Dominion University, <sup>2</sup> and William & Mary)  
*W&M: Christopher Johnson, Archana Radhakrishnan*

Trinity College Dublin, Ireland:

Michael Peardon, Sinéad Ryan, David Wilson,  
*Cian O'Hara, David Tims*

University of Cambridge, UK:

CT, Graham Moir, *Gavin Cheung, Antoni Woss*

Tata Institute, India:

Nilmani Mathur

[www.hadspec.org](http://www.hadspec.org)



Extra slides



$$\lim_{k_{\text{cm}} \rightarrow 0} k_{\text{cm}}^{2\ell+1} \cot [\delta_{3\ell_J}] = a({}^3\ell_J | {}^3\ell_J)^{-1}$$

$$a({}^3S_1 | {}^3S_1) = (-5.44 \pm 0.10 \pm 0.88) \cdot a_t$$

$$a({}^3P_0 | {}^3P_0) = (-132 \pm 14 \pm 104) \cdot a_t^3$$

$$a({}^3P_1 | {}^3P_1) = (-303 \pm 12 \pm 114) \cdot a_t^3$$

$$a({}^3P_2 | {}^3P_2) = (502 \pm 14 \pm 362) \cdot a_t^3$$

$$a({}^3D_2 | {}^3D_2) = (-8950 \pm 1050 \pm 5330) \cdot a_t^5$$

$$a({}^3D_3 | {}^3D_3) = (-4320 \pm 1310 \pm 7270) \cdot a_t^5$$

$$m_\pi a({}^3S_1 | {}^3S_1) = (-0.80 \pm 0.01 \pm 0.13)$$

$$m_\pi^3 a({}^3P_0 | {}^3P_0) = (-0.43 \pm 0.05 \pm 0.34)$$

$$m_\pi^3 a({}^3P_1 | {}^3P_1) = (-0.98 \pm 0.04 \pm 0.37)$$

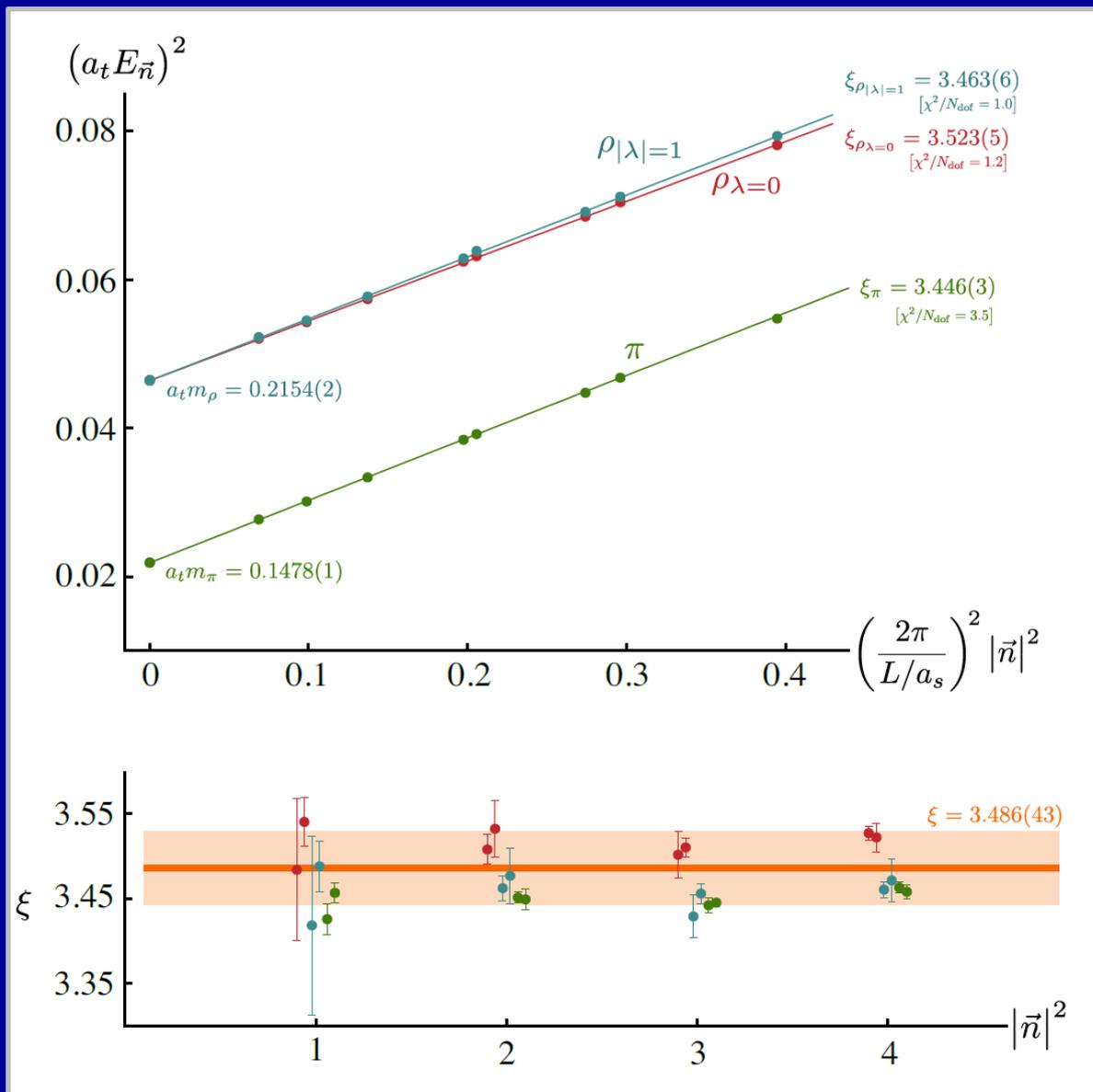
$$m_\pi^3 a({}^3P_2 | {}^3P_2) = (1.62 \pm 0.05 \pm 1.17)$$

$$m_\pi^5 a({}^3D_2 | {}^3D_2) = (-0.63 \pm 0.07 \pm 0.38)$$

$$m_\pi^5 a({}^3D_3 | {}^3D_3) = (-0.30 \pm 0.09 \pm 0.51).$$

# $\rho\pi$ isospin-2 scattering

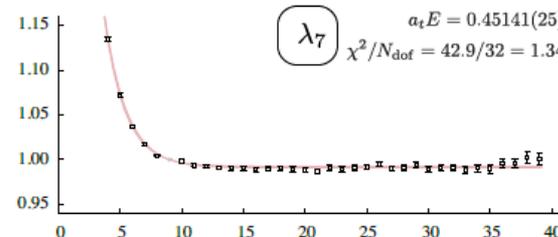
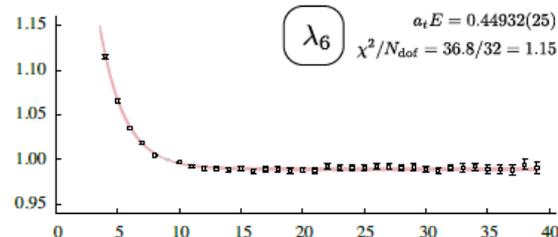
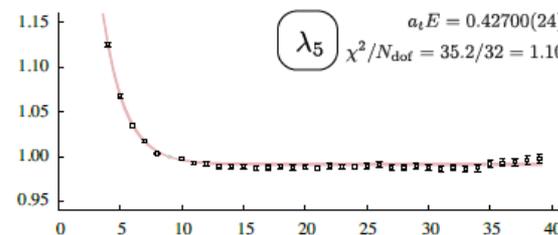
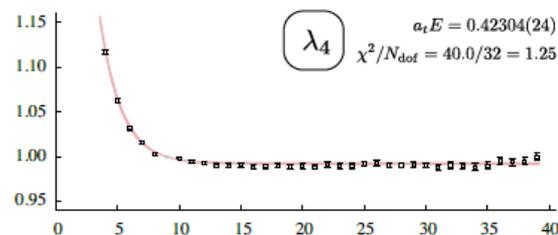
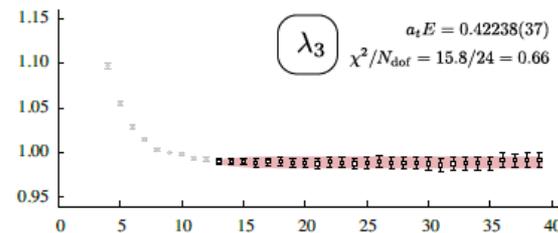
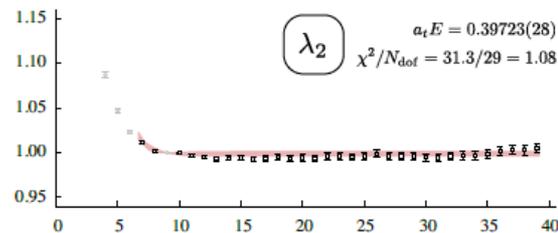
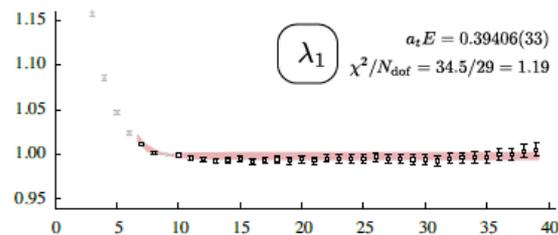
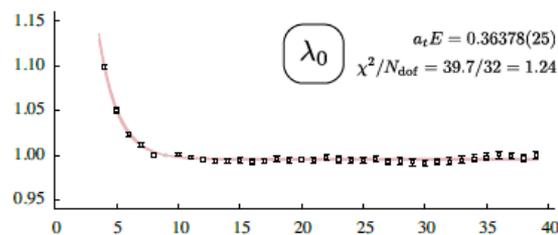
[JHEP 1807, 043 (2018)]



# $\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

[000]  $T_1^+$



$$\mathbf{t} = \begin{bmatrix} t(^3S_1|^3S_1) & t(^3S_1|^3D_1) \\ t(^3S_1|^3D_1) & t(^3D_1|^3D_1) \end{bmatrix}$$

$$\begin{aligned} \mathbf{t} &= \begin{bmatrix} t(^3S_1|^3S_1) & t(^3S_1|^3D_1) \\ t(^3S_1|^3D_1) & t(^3D_1|^3D_1) \end{bmatrix} \\ &= \frac{1}{2i\rho} \begin{bmatrix} \cos(2\bar{\epsilon}) \exp [2i \delta_{3S_1}] - 1 & i \sin(2\bar{\epsilon}) \exp [i(\delta_{3S_1} + \delta_{3D_1})] \\ i \sin(2\bar{\epsilon}) \exp [i(\delta_{3S_1} + \delta_{3D_1})] & \cos(2\bar{\epsilon}) \exp [2i \delta_{3D_1}] - 1 \end{bmatrix} \end{aligned}$$

$$\mathbf{t} = \begin{bmatrix} t(^3S_1|^3S_1) & t(^3S_1|^3D_1) & 0 \\ t(^3S_1|^3D_1) & t(^3D_1|^3D_1) & 0 \\ 0 & 0 & t(^3D_3|^3D_3) \end{bmatrix}$$

$$K_{\ell_J, \ell'_J}(s) = \sum_{n \geq 0}^{N({}^3\ell_J | {}^3\ell'_J)} c_n({}^3\ell_J | {}^3\ell'_J) s^n$$

$$k_{\text{cm}}^{2\ell+1} \cot(\delta_{3\ell_J}) = \frac{1}{a({}^3\ell_J | {}^3\ell_J)} + \frac{1}{2} r({}^3\ell_J | {}^3\ell_J) k_{\text{cm}}^2 + \mathcal{O}(k_{\text{cm}}^4)$$

$[00n] \Lambda$	$A_1$	$A_2$	$E$
		$0^- \left( {}^3P_0 \right)$	
		$1^+ \left( \begin{matrix} {}^3S_1 \\ {}^3D_1 \end{matrix} \right)$	$1^+ \left( \begin{matrix} {}^3S_1 \\ {}^3D_1 \end{matrix} \right)$
	$1^- \left( {}^3P_1 \right)$		$1^- \left( {}^3P_1 \right)$
	$2^+ \left( {}^3D_2 \right)$		$2^+ \left( {}^3D_2 \right)$
$J^P({}^3\ell_J)$		$2^- \left( \begin{matrix} {}^3P_2 \\ {}^3F_2 \end{matrix} \right)$	$2^- \left( \begin{matrix} {}^3P_2 \\ {}^3F_2 \end{matrix} \right)$
		$3^+ \left( \begin{matrix} {}^3D_3 \\ {}^3G_3 \end{matrix} \right)$	$3^+ \left( \begin{matrix} {}^3D_3 \\ {}^3G_3 \end{matrix} \right)$ <sub>[2]</sub>
	$3^- \left( {}^3F_3 \right)$		$3^- \left( {}^3F_3 \right)$ <sub>[2]</sub>
	$4^- \left( \begin{matrix} {}^3F_4 \\ {}^3H_4 \end{matrix} \right)$	$4^- \left( \begin{matrix} {}^3F_4 \\ {}^3H_4 \end{matrix} \right)$ <sub>[2]</sub>	$4^- \left( \begin{matrix} {}^3F_4 \\ {}^3H_4 \end{matrix} \right)$ <sub>[2]</sub>

# $\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$$\begin{aligned}\rho[000] &\rightarrow T_1^-, & \pi[000] &\rightarrow A_1^- \\ [000] T_1^- \times [000] A_1^- &\rightarrow [000] T_1^+\end{aligned}$$

$$1 T_1^+$$

# $\rho\pi$ isospin-2 scattering

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$$\begin{aligned}\rho[000] &\rightarrow T_1^-, \quad \pi[000] \rightarrow A_1^- \\ [000] T_1^- \times [000] A_1^- &\rightarrow [000] T_1^+\end{aligned}$$

1  $T_1^+$

$$\begin{aligned}\rho[001] \lambda=0 &\rightarrow A_1, \quad \lambda=\pm 1 \rightarrow E_2, \quad \pi[001] \rightarrow A_2 \\ [001] A_1 \times [001] A_2 &\rightarrow [000] T_1^+, A_1^-, E^- \\ [001] E_2 \times [001] A_2 &\rightarrow [000] T_1^+, T_2^+, T_1^-, T_2^-\end{aligned}$$

2  $T_1^+$

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$$\begin{aligned}\rho[000] &\rightarrow T_1^-, \quad \pi[000] \rightarrow A_1^- \\ [000] T_1^- \times [000] A_1^- &\rightarrow [000] T_1^+\end{aligned}$$

1  $T_1^+$

$$\begin{aligned}\rho[001] \lambda=0 &\rightarrow A_1, \quad \lambda=\pm 1 \rightarrow E_2, \quad \pi[001] \rightarrow A_2 \\ [001] A_1 \times [001] A_2 &\rightarrow [000] T_1^+, A_1^-, E^- \\ [001] E_2 \times [001] A_2 &\rightarrow [000] T_1^+, T_2^+, T_1^-, T_2^-\end{aligned}$$

2  $T_1^+$

$$\begin{aligned}\rho[011] \lambda=0 &\rightarrow A_1, \quad \lambda=\pm 1 \rightarrow B_1, B_2, \quad \pi[011] \rightarrow A_2 \\ [011] A_1 \times [011] A_2 &\rightarrow [000] T_1^+, \dots \\ [011] B_1 \times [011] A_2 &\rightarrow [000] T_1^+, \dots \\ [011] B_2 \times [011] A_2 &\rightarrow [000] T_1^+, \dots\end{aligned}$$

3  $T_1^+$

# $\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$$\rho[000] \rightarrow T_1^-, \quad \pi[000] \rightarrow A_1^-$$
$$[000] T_1^- \times [000] A_1^- \rightarrow [000] T_1^+$$

1  $T_1^+$

$$\rho[001] \lambda=0 \rightarrow A_1, \quad \lambda=\pm 1 \rightarrow E_2, \quad \pi[001] \rightarrow A_2$$
$$[001] A_1 \times [001] A_2 \rightarrow [000] T_1^+, A_1^-, E^-$$
$$[001] E_2 \times [001] A_2 \rightarrow [000] T_1^+, T_2^+, T_1^-, T_2^-$$

2  $T_1^+$

$$\rho[011] \lambda=0 \rightarrow A_1, \quad \lambda=\pm 1 \rightarrow B_1, B_2, \quad \pi[011] \rightarrow A_2$$
$$[011] A_1 \times [011] A_2 \rightarrow [000] T_1^+, \dots$$
$$[011] B_1 \times [011] A_2 \rightarrow [000] T_1^+, \dots$$
$$[011] B_2 \times [011] A_2 \rightarrow [000] T_1^+, \dots$$

3  $T_1^+$

Overall non-zero momentum

$$[001] A_1 \times [011] A_2 \rightarrow [001] E_2, \dots$$
$$[001] E_2 \times [011] A_2 \rightarrow [001] E_2, E_2, \dots$$

3  $E_2$

$\Lambda^+$	$A_1^+$	$A_2^+$	$T_1^+$	$E^+$	$T_2^+$
$J^+(^3\ell_J)$			$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$2^+ (^3D_2)$	$2^+ (^3D_2)$
		$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$		$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$
	$4^+ (^3G_4)$		$4^+ (^3G_4)$	$4^+ (^3G_4)$	$4^+ (^3G_4)$
$\Lambda^-$	$A_1^-$	$A_2^-$	$T_1^-$	$E^-$	$T_2^-$
$J^-(^3\ell_J)$	$0^- (^3P_0)$				
			$1^- (^3P_1)$		
		$3^- (^3F_3)$	$3^- (^3F_3)$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$
	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$		$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$

$[00n] \Lambda$	$A_1$	$A_2$	$E$
$J^P(^3\ell_J)$		$0^- (^3P_0)$	
		$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$
	$1^- (^3P_1)$		$1^- (^3P_1)$
	$2^+ (^3D_2)$		$2^+ (^3D_2)$
		$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$
		$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$ [2]
	$3^- (^3F_3)$		$3^- (^3F_3)$ [2]
	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$ [2]	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$ [2]

$[0nn] \Lambda$	$A_1$	$A_2$	$B_1$	$B_2$
$J^P(^3\ell_J)$		$0^- (^3P_0)$		
		$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$
	$1^- (^3P_1)$		$1^- (^3P_1)$	$1^- (^3P_1)$
	$2^+ (^3D_2)_{[2]}$	$2^+ (^3D_2)$	$2^+ (^3D_2)$	$2^+ (^3D_2)$
	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}_{[2]}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$
	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}_{[2]}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}_{[2]}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}_{[2]}$
	$3^- (^3F_3)_{[2]}$	$3^- (^3F_3)$	$3^- (^3F_3)_{[2]}$	$3^- (^3F_3)_{[2]}$
	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[3]}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$

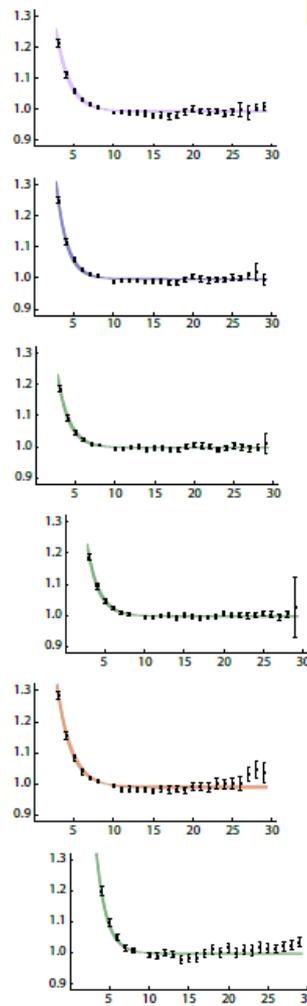
# $\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$[000]T_1^+$	$[001]A_2$	$[011]A_2$	$[111]A_2$	$[002]A_2$
$\rho_{[000]}\pi_{[000]}$	$\rho_{[001]}\pi_{[000]}$	$\rho_{[011]}\pi_{[000]}$	$\rho_{[111]}\pi_{[000]}$	$\rho_{[001]}\pi_{[001]}$
$\{2\} \rho_{[001]}\pi_{[00-1]}$	$\rho_{[000]}\pi_{[001]}$	$\{2\} \rho_{[001]}\pi_{[010]}$	$\{2\} \rho_{[011]}\pi_{[100]}$	$\rho_{[002]}\pi_{[000]}$
$\{3\} \rho_{[011]}\pi_{[0-1-1]}$	$\{2\} \rho_{[011]}\pi_{[0-10]}$	$\rho_{[000]}\pi_{[011]}$	$\{2\} \rho_{[100]}\pi_{[011]}$	$\{2\} \rho_{[011]}\pi_{[0-11]}$
$\{2\} \rho_{[111]}\pi_{[-1-1-1]}$	$\{2\} \rho_{[010]}\pi_{[0-11]}$	$\{2\} \rho_{[111]}\pi_{[-100]}$	$\rho_{[000]}\pi_{[111]}$	$\rho_{[000]}\pi_{[002]}$
	$\rho_{[002]}\pi_{[00-1]}$	$\{3\} \rho_{[110]}\pi_{[-101]}$	$\{2\} \rho_{[112]}\pi_{[00-1]}$	$\{2\} \rho_{[012]}\pi_{[0-10]}$
	$\{2\} \rho_{[111]}\pi_{[-1-10]}$	$\{2\} \rho_{[100]}\pi_{[-111]}$	$\{3\} \rho_{[012]}\pi_{[10-1]}$	$\{2\} \rho_{[111]}\pi_{[-1-11]}$
	$\{2\} \rho_{[110]}\pi_{[-1-11]}$	$\{2\} \rho_{[012]}\pi_{[00-1]}$	$\{2\} \rho_{[002]}\pi_{[11-1]}$	$\{2\} \rho_{[010]}\pi_{[0-12]}$
	$\rho_{[00-1]}\pi_{[002]}$	$\{2\} \rho_{[002]}\pi_{[01-1]}$	$\{2\} \rho_{[11-1]}\pi_{[002]}$	$\{2\} \rho_{[112]}\pi_{[-1-10]}$
	$\{2\} \rho_{[012]}\pi_{[0-1-1]}$	$\{2\} \rho_{[01-1]}\pi_{[002]}$	$\{3\} \rho_{[01-1]}\pi_{[102]}$	$\{2\} \rho_{[-1-10]}\pi_{[112]}$
		$\{2\} \rho_{[00-1]}\pi_{[012]}$	$\{2\} \rho_{[00-1]}\pi_{[112]}$	
		$\{3\} \rho_{[112]}\pi_{[-10-1]}$		
8 ops.	12 ops.	15 ops.	10 ops.	7 ops.

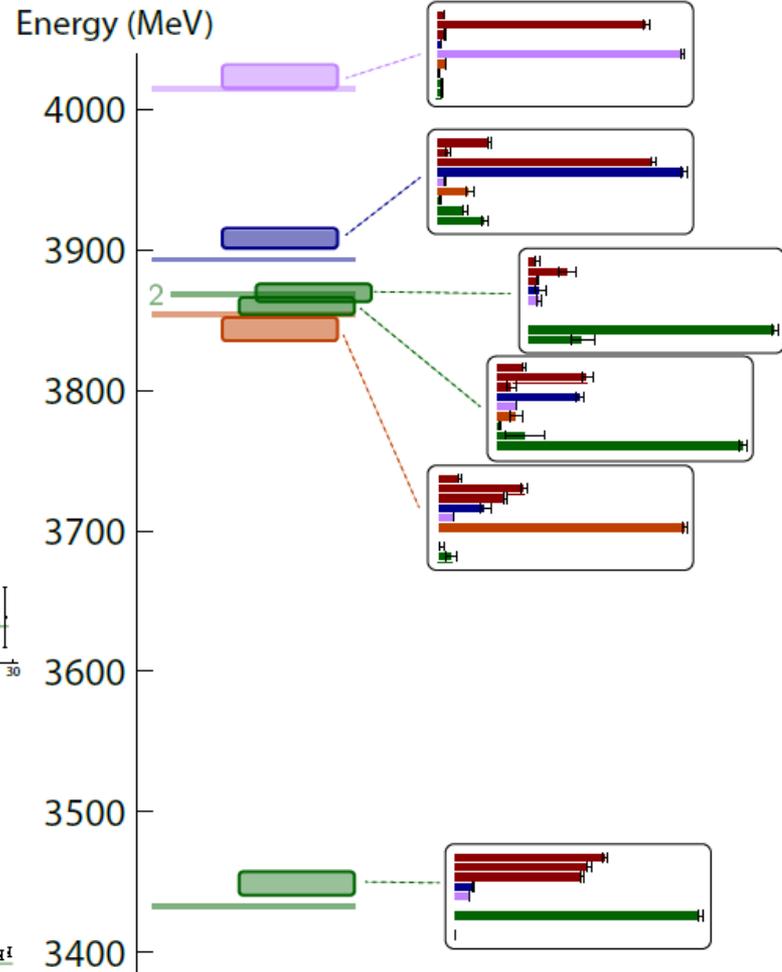


# Hidden-charm $I=1 (c\bar{c}l\bar{l})$



$$\lambda^n(t, t_0) e^{E_n(t-t_0)}$$

$\uparrow$   
 $t/a_t$



$J^P = 1^+$   
 $G = +$

- $\delta_{3,3}^{\psi_0} \bar{\delta}_{3,3}^{\psi_1}$
- $\delta_{3,3}^{\rho} \bar{\delta}_{3,3}^{\pi}$
- $\delta_{3,3}^{\rho} \bar{\delta}_{3,3}^{\pi}$
- $D_{A_1}^{[000]} \bar{D}_{T_1}^{[000]}$
- $D_{T_1}^{[000]} \bar{D}_{T_1}^{[000]}$
- $\eta_{cA_1} \rho_{T_1}^{[000]}$
- $J/\psi_{T_1}^{[000]} \pi_{A_1}^{[000]}$
- $J/\psi_{A_1}^{[100]} \pi_{A_2}^{[100]}$
- $J/\psi_{E_2}^{[100]} \pi_{A_2}^{[100]}$

[JHEP 1711, 033 (2017)]

# Doubly-charmed $I=0$ ( $cc\bar{l}\bar{l}$ )

[JHEP 1711, 033 (2017)]

