Disconnected contributions to the hadronic part of the muon anomalous magnetic moment

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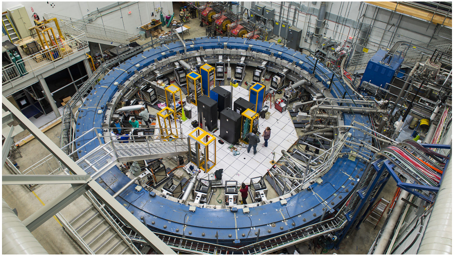
The anomalous magnetic moment of muon

- We only understand roughly 5% of the energy in the Universe.
- However, there are very few experiments which disagree with the prediction of the standard model.
- The magnetic moment of the muon a_μ governs the interaction of it with a magnetic field. There is a long standing deviation between the theory and experiment for a_μ.

This talk is about a lattice QCD calculation of some of the hadronic contribution to a_μ.

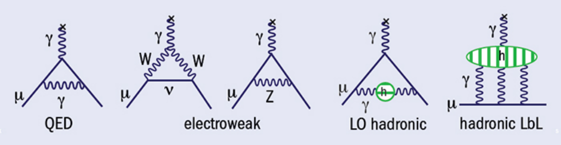


The 2017 PDG review quotes:

$$a_\mu^{\rm expt} - a_\mu^{\rm SM} = 268(63)(43)10^{-11}$$

where the first error is from the last experiment at Brookhaven. A 3.5σ deviation! Is this caused by BSM physics (but no BSM has seen at the LHC) or are the QCD contributions not under control?

- There is a new experiment at Fermilab.
- This talk is about the leading order (LO) **hadronic** contributions.



Current state of leading order hadronic contributions

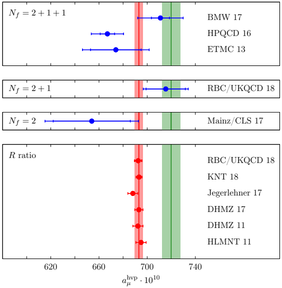


Figure: Plot from 1807.09370 Harvey B. Meyer, Hartmut Wittig.

- The difference between the red (theory) and green (derived from expt.) section "could be" caused by new BSM particles.
- Small errors are good! Large errors are bad!

Motivation

The goal of calculating the leading order hadronic contribution to the muon anomalous magnetic moment using lattice QCD, with an error under 1%, requires:

- Computation of disconnected diagrams.
- The reduction of the statistical errors in the "large" time part of the vector correlator.
- isospin violating terms (see 1710.11212)
- non-perturbative QED + hadronic

This talk is about computing the disconnected contributions. This is part of a much bigger calculation by the FNAL/HPQCD/MILC collaborations to improve the result from the HPQCD collaboration.

Some background to lattice QCD

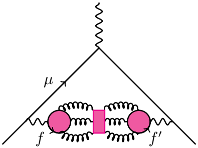
To solve QCD for bound state properties "all" that is required is to compute

$$c_{ii}(t) = \frac{1}{Z} \int du \int d\psi \overline{d\psi} V(t)_i V(0)_i^\dagger e^{-S_F - S_G}$$

The path integral is regulated by the introduction of a space-time lattice.

- The integral is computed in Euclidean space using Monte Carlo techniques on the computer.
- The Monte Carlo process produces an ensemble of gauge configurations (" snapshot of the vacuum.")
- Our lattice version of the Dirac operator is the Highly Improved Staggered Quark (HISQ).
- Taste singlet vector (one link) operator measured.

Disconnected diagrams



- Disconnected diagrams are OZI suppressed, but they still need to be calculated. (The disconnected diagrams in this calculation are zero in the SU(3) limit).
- The main difference between the calculation of the masses of the ω meson and ρ meson are disconnected diagrams.

The executive summary is that we are computing the trace of the inverse of a matrix (with order 20×10^6) inside a Monte Carlo lattice QCD calculation.

Computing with noise

$$\langle \eta_i^\dagger \eta_j \rangle = \delta_{ij}$$

The matrix equation is solved for the noise vector η_i for i running from 1 to the number of samples N_{samp}.

$$M\chi_i = \eta_i$$

The trace of the inverse of the M matrix, at time t, is estimated from

$$\begin{aligned} \text{trace}(M^{-1})_t &\approx \frac{1}{N_{samp}} \sum_{i=1}^{N_{samp}} (\eta_i^\dagger \chi_i)_t \\ &\approx \frac{1}{N_{samp}} \sum_{i=1}^{N_{samp}} (\eta_i^\dagger M^{-1} \eta_i)_t \end{aligned}$$

Algorithmic improvements

It is expensive to reduce the error

$$\text{error} \sim \frac{A}{(N_{samp})^{1/2}}$$

where N_{samp} is the number of stochastic sources.

- Choice of noise Z_2 versus Gaussian
- Variance reduction (eg. hopping parameter)
- dilution/partition
- distillation
- Use eigenvalues, eigenvectors
- hierarchical probing (1611.01193)
- Use a fast inverter, such as Multigrid

There are **many** choices of different algorithms, so much tuning is required.

Disconnected diagrams from eigenvalues

The vector space can be broken down into the space spanned by the first N_{eig} eigenvectors and the orthogonal space to that.

$$P_L + P_H = 1$$

$$\text{Trace}\left(\Gamma M^{-1}\right) = \text{Trace}\left(P_L \Gamma M^{-1}\right) + \text{Trace}\left(P_H \Gamma M^{-1}\right)$$

- Use the ARPACK eigensolver library with polynomial acceleration to estimate up to 2000 eigenvectors (eigenvalues μ_i).

$$\text{Trace}\left(P_L \Gamma M^{-1}\right) = \sum_{i=1}^{N_{eig}} \left(\langle v_i, \Gamma v_i \rangle \left(\frac{1}{\mu_i}\right) \right)$$

- Use random sources to estimate $\text{Trace}\left(P_H \Gamma M^{-1}\right)$

Correlators from loops

- Form a correlator from two loops separated by t in time.

$$d(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \mathcal{V}_j(t+t')\mathcal{V}_j(t')$$

where $\mathcal{V}_j(t)$ is vector loop with component i at time t and V is the space-time volume.
Vector current in SU(3) limit.

$$\mathcal{V}_j = \frac{1}{3}(\mathcal{V}_j^{u/d} - \mathcal{V}_j^s)$$

This really helps reduce the errors.

Computing $a_\mu^{HVP(LO)DISC}$

Method introduced by Mainz group and used by RBC/UKQCD collaboration.

$$a_\mu^{HVP(LO)DISC} = \sum_{t=0}^{\infty} w(t) d(t)$$

where $d(t)$is the strange-light disconnected correlator, measured in the simulation. Also some analysis with `https://github.com/gplepage/g2tools`

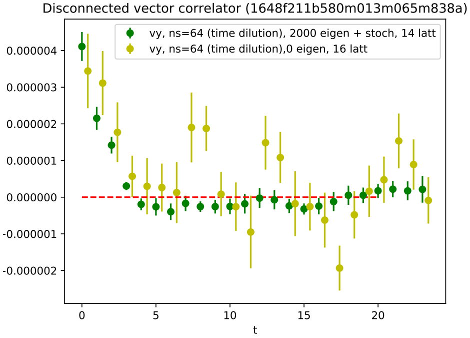
$$w(t) = 4\alpha^2 \int_0^\infty dq^2 f(q^2) \left(\frac{\cos(qt) - 1}{q^2} + \frac{1}{2} t^2 \right)$$

where $f(q^2)$ is a kinematic factor derived by Blum and α is the QED coupling.

$$a_\mu^{HVP(LO)DISC}(T) \approx \sum_{t=0}^{T} w(t) d(t) + \text{corrections}$$

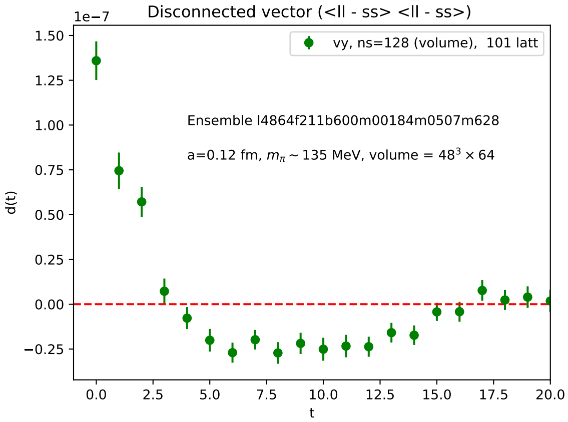
Preliminary measurement tests

- Low statistics analysis to try to optimize the measurements.
- Ensemble parameters: a=0.15 fm, pion mass 300 MeV, $16^3 \times 48$ volume.
- Correlator has similar shape to that published by RBC/UKQCD.



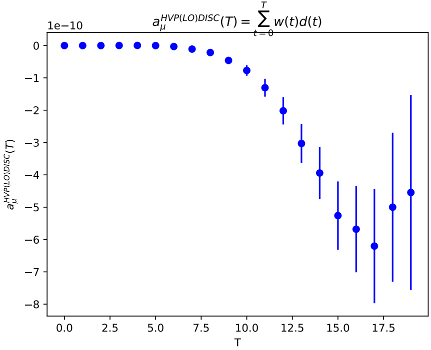
Preliminary results at the physical point

- We are running an ensemble with a $= 0.12$ fm, $m_\pi \sim 135$ MeV, with volume $32^3 \times 64$.

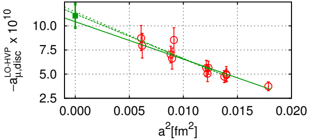


Preliminary results at the physical point

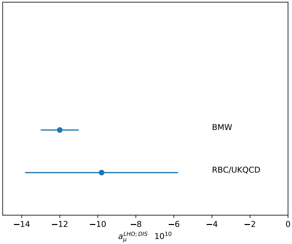
- Renormalization $Z_V = 1.0/0.852(2)$ (1511.07382)
- Need higher statistics, but the numbers for $a_\mu^{HVP(LO)DISC}(T)$ looks reasonable for this ensemble.



From BMW arXiv:1711.04980 also used staggered fermions.



- Strong lattice spacing dependence. (UKQCD/RBC also found strong quark mass dependence).



Conclusions

- I have described a technical lattice QCD calculation to reduce the error on $a_\mu^{HVP(LO)}$. Ongoing runs to reduce errors.

Option O

- In 2021 the FNAL experiment reports reduced experimental errors on a_μ^{HVP} and the central value doesn't change.
- If the hadronic contribution are reliably calculated, the the deviation between experiment and standard model is 7.5σ and this gives us the first indication of the scale of new physics.

Option P

- The new FNAL experimental result is consistent with the standard model. Or the standard model predictions move towards the experimental numbers.

Both options require reduced errors from lattice QCD calculations.