Towards the NNNLO pressure of cold and dense QCD

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3 Double-log at NNNLO

4 Conclusions

Introduction

Motivation

- Goal: Describe how strongly interacting matter behaves under different conditions
  - Corresponds to computing the QCD pressure $p\left(\{\mu_i\}_i;\{m_i\}_i;T,\ldots\right)=T\log\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\exp(-S)/V$

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- Plenty of applications: Early universe, heavy-ion collisions, neutron stars...
  - I'll concentrate on the last one: Cold & Dense Matter, $p\left(\{\mu_i\}_i;\{m_i\}_i;T,\ldots\right)\rightarrow p\left(\mu_B\right)$

Methodology – Problems With a Large Density

- Computing the pressure at large density presents challenges
  - Lattice methods are non-perturbative and use first-principles theory, but suffer from the sign problem

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- Computing the pressure at large density presents challenges
  - Lattice methods are non-perturbative and use first-principles theory, but suffer from the sign problem
  - Traditional EFTs (eg. χPT) are applicable at lower densities, but suffer from inaccuracies at larger densities, and aren't based on first principles
  - No suitable DR-framework

Methodology – Using pQCD at Large Density

- For the purposes of this talk, focus is on standard perturbation theory
  - pQCD is based on first principles[1], expansion in α_s is possible at asymptotically large μ_B

[1]...We do use some HTL, but only to get rid of terms that are of higher order in the coupling itself

Methodology – Using pQCD at Large Density

- For the purposes of this talk, focus is on standard perturbation theory
  - pQCD is based on first principles[1], expansion in α_s is possible at asymptotically large μ_B
  - Convergence is bad at physical densities, high-order corrections are required (hence NNNLO)
  - Suffers from IR-ambiguities, resummation needed

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Comparison With Large-T QCD

- In systems like the early universe & heavy-ion collisions, μ_B often fairly small
- Some of the problems plaguing large μ_B disappear
  - Lattice methods work if the sign problem is at most mild or can be circumvented
  - pQCD admits a dimensionally reduced framework when T/μ_B is large and is understood better
- Plenty of accurate results and experiments!

NNLO Logarithm

Thermal pQCD History Lesson pt. I

$$
\begin{aligned}
p\ =\ & c_0 + c_1\alpha_s + c_{2,1}\alpha_s^2 \log\alpha_s + c_{2,0}\alpha_s^2 \\
& + c_{3,2}\alpha_s^3 \log^2\alpha_s + c_{3,1}\alpha_s^3 \log\alpha_s + c_{3,0}\alpha_s^3 + \dots
\end{aligned}
$$

- Simplest case for dense matter: massless quarks and zero temperature
- c_0 , c_1 easy to compute
- NNLO factors $c_{2,1}$, $c_{2,0}$ calculated already in the 70s (!) [2]

[2] Freedman & McLerran, Phys. Rev. D 16 1977

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- c_0 , c_1 easy to compute
- NNLO factors $c_{2,1}$, $c_{2,0}$ calculated already in the 70s (!) [2]
- Assumptions later relaxed to e.g. $m \neq 0$ [3] and small but finite T [4], but no previous NNNLO calculations

[2] Freedman & McLerran, Phys. Rev. D 16 1977

[3] Kurkela et al., hep-ph/0912.1856

[4] Kurkela & Vuorinen, hep-ph/1603.00750

Thermal pQCD History Lesson pt. II

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- Compare with "hot" QCD (large-T limit)
- Partial NNNLO results: $c_{3,2} = 0$ and $c_{3,1}$ is known [5]

[5] Kajantie et al. hep-ph/0211321

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- Compare with "hot" QCD (large-T limit)
- Partial NNNLO results: $c_{3,2} = 0$ and $c_{3,1}$ is known [5]
- Theory fundamentally different:
  - A 3d DR description is available
  - Notably, the theory has a *magnetic scale*, $\alpha_s T$, which is absent at $T = 0$
  - As a consequence, the "hard" contribution $c_{3,0}$ *cannot* be computed in pure perturbation theory!

[5] Kajantie et al. hep-ph/0211321

Scale Hierarchy at Finite μ_B & $T = 0$

- Naïve diagrammatic expansion valid close to vacuum, where the fields are not screened

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  - Energetic excitations $\rightarrow$ medium effects suppressed
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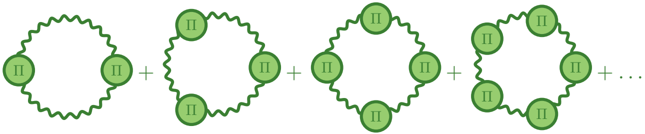
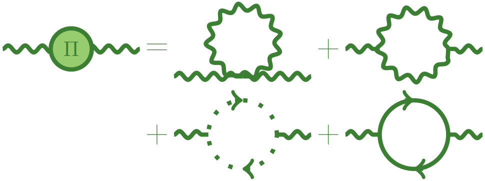
  $\rightarrow$ Resummation of diagram classes required
- In the intermediate region, when ratios of scales are involved, nonanalytic terms $\mathcal{O}(\alpha_s^m \log^n \alpha_s)$ are generated from resummed diagrams

- Hard Scale $P \sim \mu_B$:
  - Energetic excitations
    $\rightarrow$ medium effects suppressed
  - Standard Feynman graph expansion OK
- Semisoft Scale $\alpha_s^{1/2}\mu_B \ll P \ll \mu_B$:
  - "Somewhat" energetic excitations
    $\rightarrow$ medium effects present
  - Corrections to naïve diagrams not suppressed enough, but approximations possible
- Soft Scale $P \sim \alpha_s^{1/2}\mu_B$:
  - Low-energy excitations
    $\rightarrow$ medium effects significant
  - Resum IR-sensitive objects (gluons) to all orders

1-loop Ring Sum

- First logs at $\mathcal{O}(\alpha_s^2)$: The gluonic ring sum





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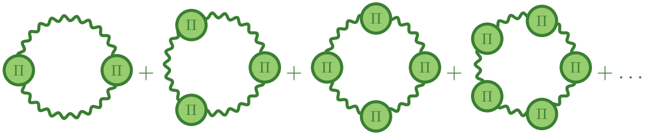

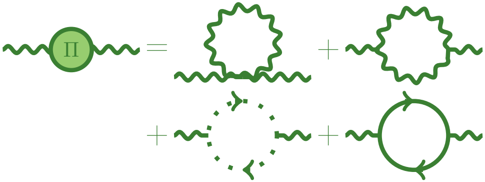

- Well-known: Contributes a non-analytic term at NNLO: $c_{2,1} \neq 0$

HTL Approximation

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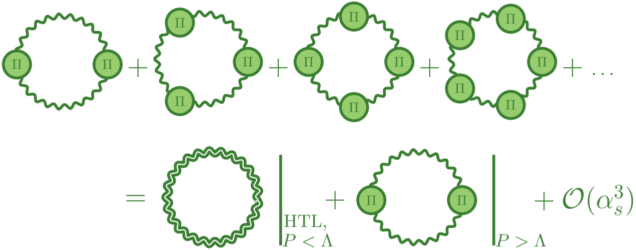
- IR DoFs admit an effective description, and only they require resummation!
- Hard Thermal Loops[6] suitable since hard modes dominate the self-energies

- Physical motivation: Accounts for the medium

[6] Hard Dense Loops, really, but abbreviated HTL here

Approximating the Ring Sum

- Applying the HTL approximation leads to the following:



- The logarithms, that is, the semisoft region, can be taken to come from the UV-limit of the first term or the IR-limit of the second one, we choose the first
- Using HTL gives the correct $c_{2,1}$: There is a Logarithmic enhancement at NNLO from the semisoft modes

HTL Propagators & m_E

- After scalarisation, transversal and longitudinal gluons:

$$G_T(P) = \frac{-1}{P^2 + \frac{m_E^2}{d-1} - \frac{P^2}{p^2}\Pi_{\rm HTL}(P)}, \; G_L(P) = \frac{1}{p^2 + \Pi_{\rm HTL}(P)}$$

- HTL structure is given by the function

$$\Pi_{\rm HTL}(P) = m_E^2 \left(1 - \int_{S^{d-1}} \Omega_{\mathbf{v}} \frac{iP_0}{iP_0 - \mathbf{p}\cdot\mathbf{v}} \right)$$

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- Without quark masses, the only scale is the effective mass

$$m_E^2 \equiv \text{Tr}\Pi \overset{T\to 0}{\sim} \alpha_s \mu^2$$

Double-log at NNNLO

Starting Point

- The next order is $\mathcal{O}(\alpha_s^3)$, NNNLO. How do we get there?
- Calculating everything up to $c_{3,0}$ is daunting, but...

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- ...At $\mathcal{O}(\alpha_s^2)$ the ring sum contribution was separate from everything else
- $\implies$ Try to compute the non-analytic terms $c_{3,1}, c_{3,2}$ first

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- ...At $\mathcal{O}(\alpha_s^2)$ the ring sum contribution was separate from everything else
- $\Longrightarrow$ Try to compute the non-analytic terms $c_{3,1}, c_{3,2}$ first
    - Again, non-hard modes require establishing a suitable resummation scheme
    - Analogous with hot QCD development
- Computation of $c_{3,2}$ is complete [7]

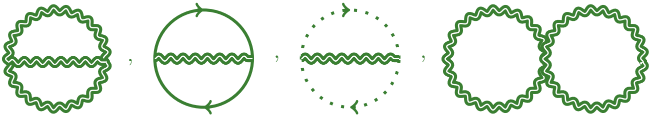
[7] Gorda, Kurkela, Romatschke, MS, Vuorinen, hep-ph/1807.04120

Finding the Logarithms

- There are now generally double logarithms $c_{3,2} \neq 0$, and single logarithms $c_{3,1} \neq 0$.
- Two sources of logs:
  - Corrections to the one-loop ring sum that require non-HTL corrections ($c_{3,1}$ only)
  - Two-loop resummations analogous to the NNLO case ($c_{3,1}$ and $c_{3,2}$, focus on these)

Constructing the Diagrams

- We can again split between naïvely loop-expanded hard contributions and resummed soft contributions, and we choose to compute the logarithm as the UV-limit of the soft contribution like in the NNNLO case
- There are four classes of IR-sensitive two-loop diagrams. They must be resummed by dressing the gluonic lines:



Double Logarithm From the Two-Loop Ring Sums

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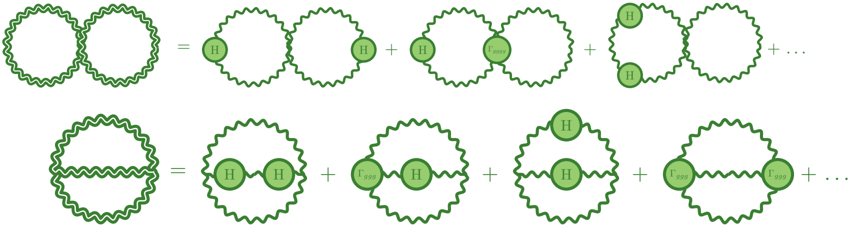
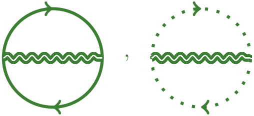


- For dimensional reasons, in fact only the "double-blob insertions" pictured above contribute

Single Logarithm From the Two-Loop Ring Sums

- The remaining resummed diagrams contribute only at $\mathcal{O}(\alpha_s^3 \log \alpha_s)$:



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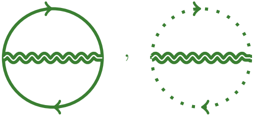

- They, and the subleading logs from the gluonic diagrams, can be handled with a similar expansion with the following prescription:
  - Gluons IR-sensitive $\implies$ resummed, but HTL-approximated
  - Fermions IR-insensitive $\implies$ not resummed, but loop-corrected

Wrap-up of the NNNLO Logarithms

- Double log $c_{3,2}$ was reasonably simple to extract
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- One needs two resummed gluon lines, so only the first two diagram classes contribute
- The HTL corrections to the ggg and $gggg$ vertices are required
- For the leading log only one-loop (HTL) self-energies and vertices are required...
  - ...But for the subleading log $c_{3,1}$ this will not be enough
  - ...And more diagram classes need to be considered
  - There is some effort involved in making sure that double counting etc. is avoided

Our Calculation of $c_{3,2}$

- Relevant HTL diagrams appear in the literature [8]
- One can extract the coefficient of the double-log (but not the constant under the log) from the semisoft regime
- We ended up with lengthy angular integrals and obtained the $\mathcal{O}(\alpha_s^3 \log^2 \alpha_s)$-term...

[8] Andersen et al., hep-ph/0205085

The Result

- ...Which is surprisingly simple:

$$c_{3,2}\alpha_s^3 \log^2 \alpha_s = -\frac{11}{48}\frac{N_c d_A}{(2\pi)^3}\alpha_s m_\infty^4 \log^2 \alpha_s, \quad m_\infty^2 = \frac{N_f}{9\pi}\mu_B^2 \alpha_s$$

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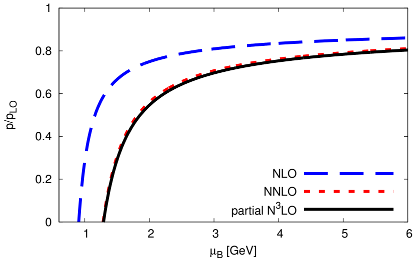
- No renormalisation scale dependence $\implies$ error bands still large ...
- ...But nothing blows up $\implies$ seems like pQCD still works!



m_∞-trick

- Physical intuition: In the semisoft region transverse gluons massive with mass m_∞^2, longitudinal gluons massless
- Makes sense, because this is the UV limit of HTL, and the semisoft regime is the UV limit of the soft contribution (or the IR limit of the hard contribution)

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- Makes sense, because this is the UV limit of HTL, and the semisoft regime is the UV limit of the soft contribution (or the IR limit of the hard contribution)
- This is a sufficient approximation for the $\log^2 \alpha_s$! Can remove complicated Π_{HTL} leaving just a constant mass
- Much easier to compute, we verified that it gives the same $\log^2 \alpha_s$

Conclusions

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- Resummation in the IR-sector allowed us to go beyond the well-understood NNLO order of dense pQCD
- $\mathcal{O}(\alpha_s^3 \log^2 \alpha_s)$ done!
- $\mathcal{O}(\alpha_s^3 \log \alpha_s)$ in progress, will require eg. higher order HTL-corrections, but might help constrain the error bands more
- The full $\mathcal{O}(\alpha_s^3)$ might also be possible (??)
- Resummation schemes, small-T corrections etc. also under consideration...