

Charmed baryon spectrum in Lattice QCD

a work in progress

K. Utku Can

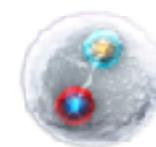
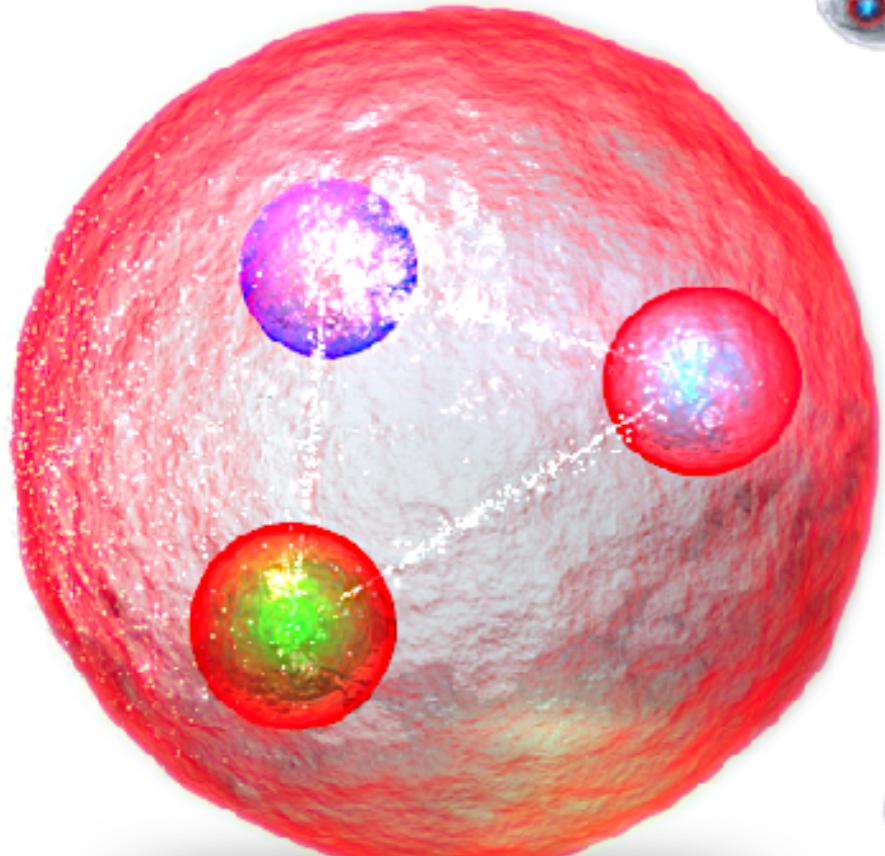
RIKEN

in collaboration with:

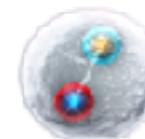
G. Erkol (Özyegin U.), H. Bahtiyar (MSGSU),
M. Oka (ASRC-JAEA), P. Gubler (ASRC-JAEA), T. T. Takahashi (Gunma Coll. Of Tech.)

Confinement'18, 1-6 August 2018
Maynooth University, Ireland

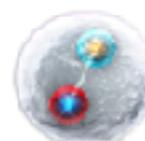
Outline



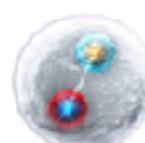
Introduction



Simulation Details

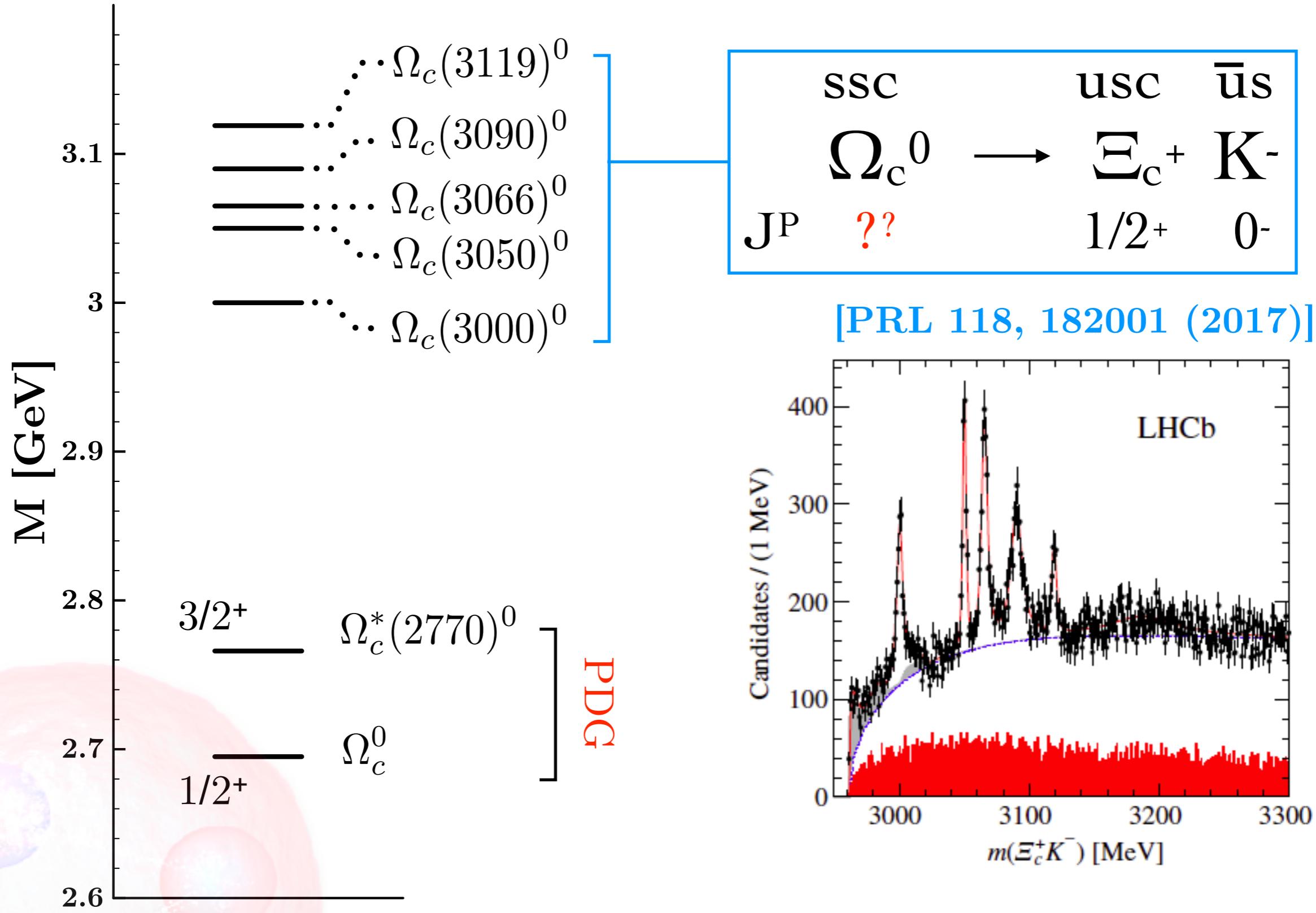


Results



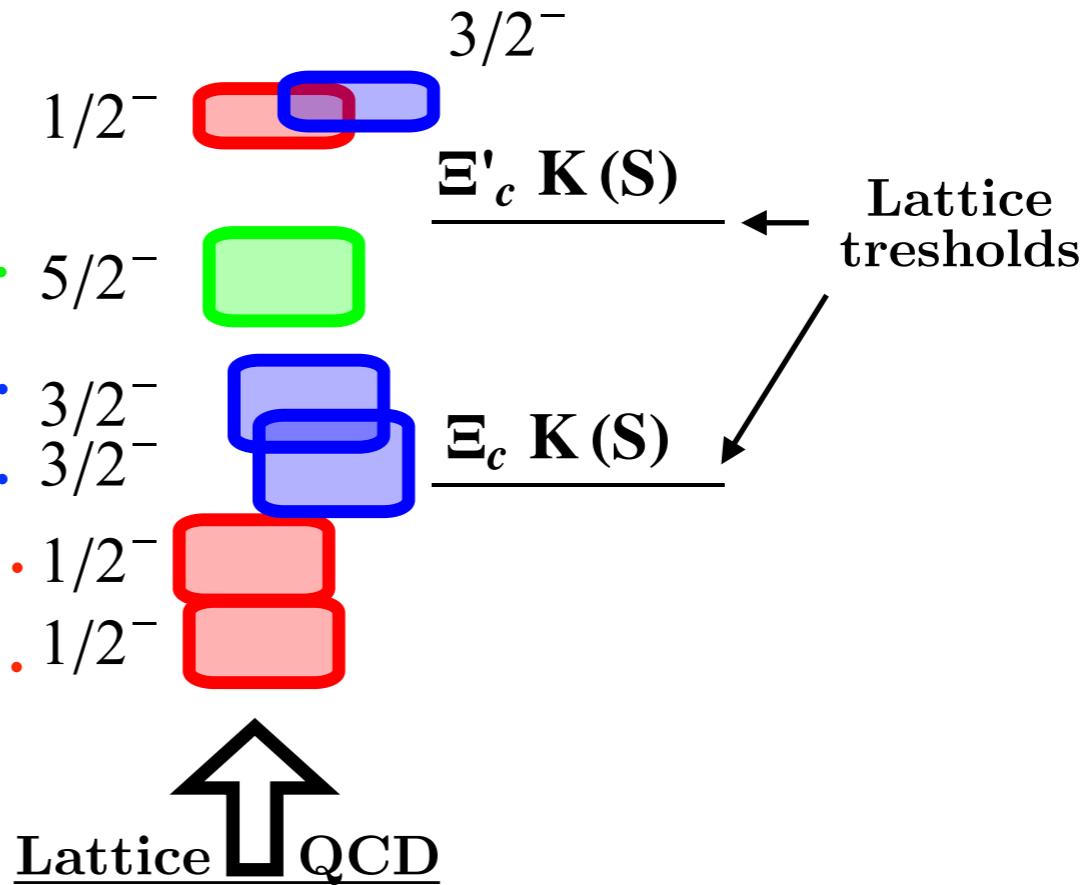
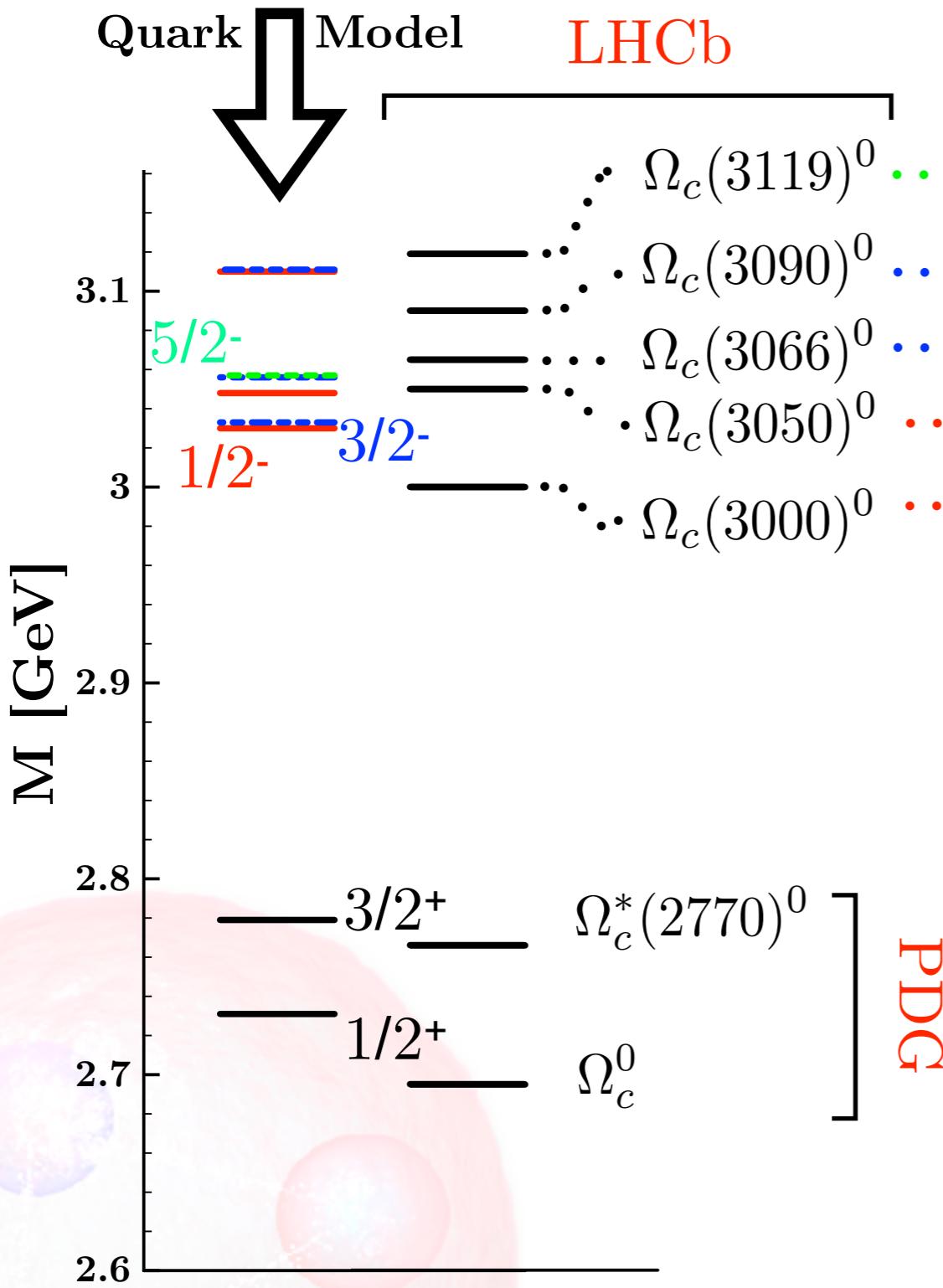
Summary

Motivation



Motivation

Yoshida et al. PRD92, 114029 (2015)



- Padmanath et.al PRL 119, 042001 2017
- $16^3 \times 128$, $N_f = 2+1$ Flavor
- $a_t \sim 0.035$ fm, $m_\pi \sim 391$ MeV

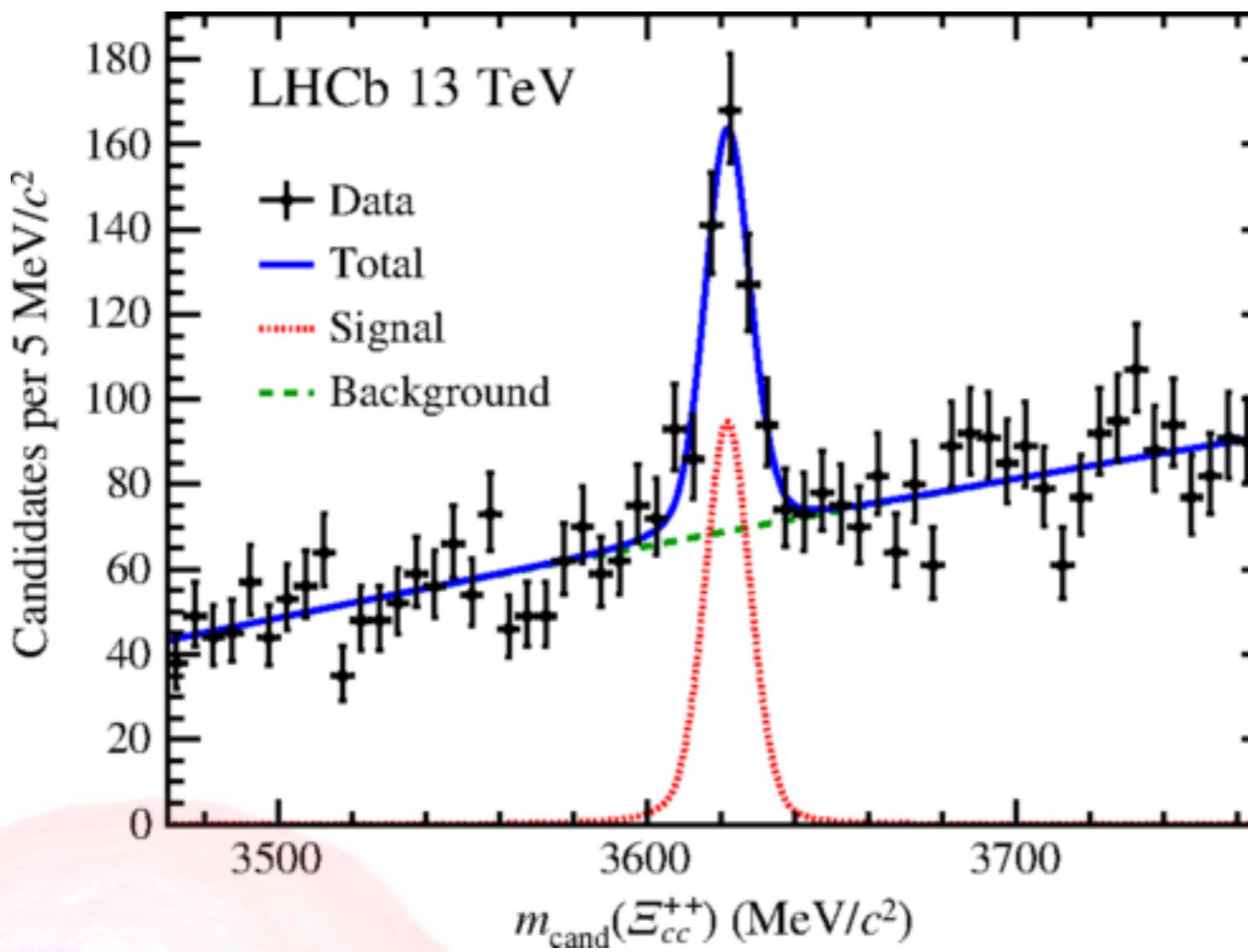
Chiral quark-soliton model

- Hyun-Chul Kim et.al PRD96 014009 (2017)
- Ω_c (3000) — $J^P=1/2^-$, Ω_c (3050) — $J^P=1/2^+$
- Ω_c (3066) — $J^P=1/2^-$, Ω_c (3090) — $J^P=3/2^-$
- Ω_c (3119) — $J^P=3/2^+$

Motivation

Ξ_{cc}

R. Aaij et al. (LHCb Collaboration)
[Phys. Rev. Lett. 119, 112001]



$$m_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14 \text{ MeV}$$

(stat.) (syst.) (Λ_c)

SELEX results

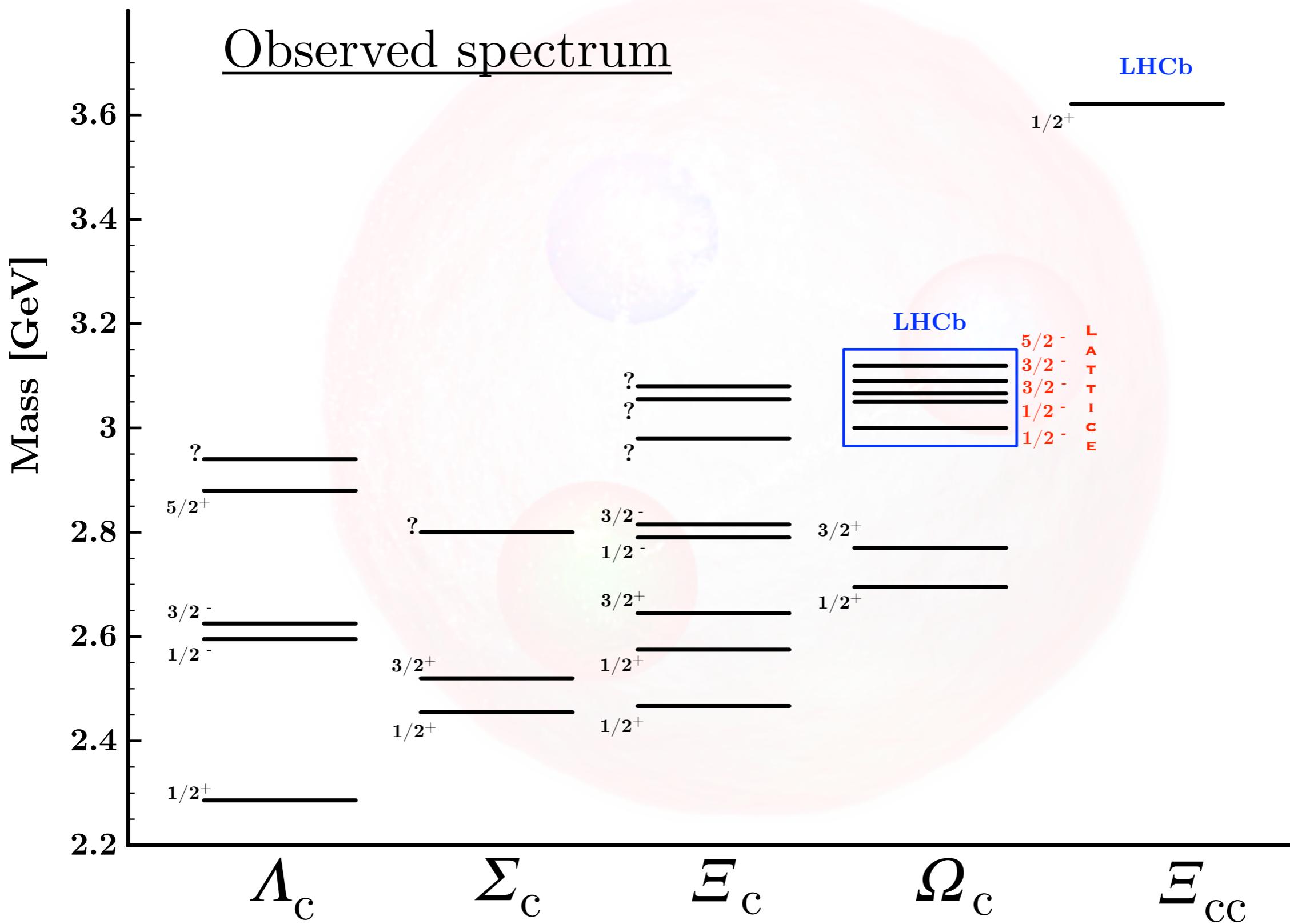
[Phys. Rev. Lett. 89, 112001 (2002)]
[Phys. Lett. B 628, 18 (2005)]

$$m_{\Xi_{cc}^{+}} = 3519 \pm 2 \text{ MeV}$$

$$m_{\Xi_{cc}^{+}}^{\text{LHCb}} - m_{\Xi_{cc}^{+}}^{\text{SELEX}} \sim 100 \text{ MeV}$$

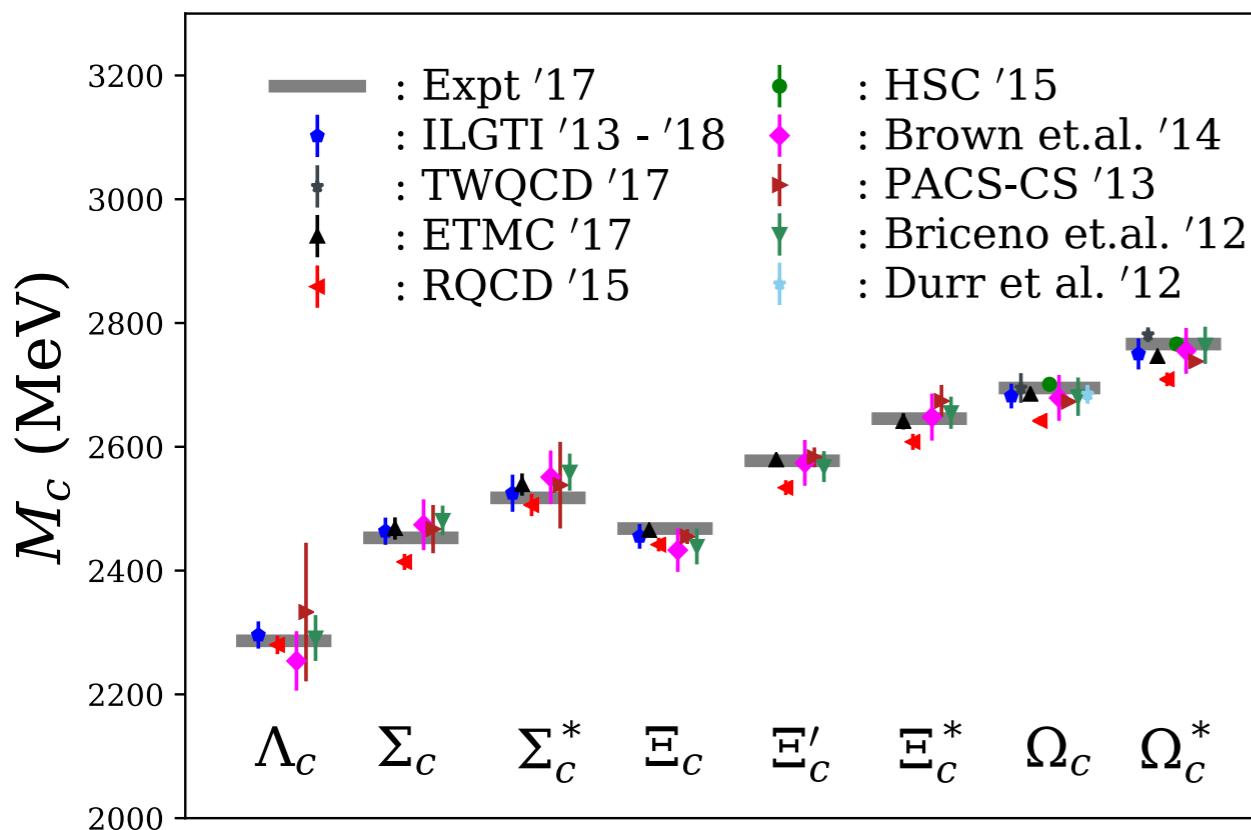
- LHCb and SELEX setups are different
- LHCb doesn't exclude SELEX results
- LQCD can provide an answer

Charmed baryons



Ground states

Figures from: Padmanath's plenary [Wed 11:00am]



● [ILGTI'13-'18]: Nf=2+1+1, Overlap, $a=0.12-0.06$ fm, $m_{\pi} \sim 300$ MeV

● [TWQCD'17]: Nf=2+1+1, Domain wall, $a = 0.063$ fm, $m_{\pi} \sim 280$ MeV

● [ETMC'17]: Nf=2, TwM+Clover, $a = 0.094$ fm, $m_{\pi} \sim 130$ MeV

● [RQCD'15]: Nf=2+1, Clover, $a = 0.075$ fm, $m_{\pi} = 259-460$ MeV

● [HSC'15]: Nf=2+1, Clover, at ~ 0.035 fm, $m_{\pi} \sim 390$ MeV

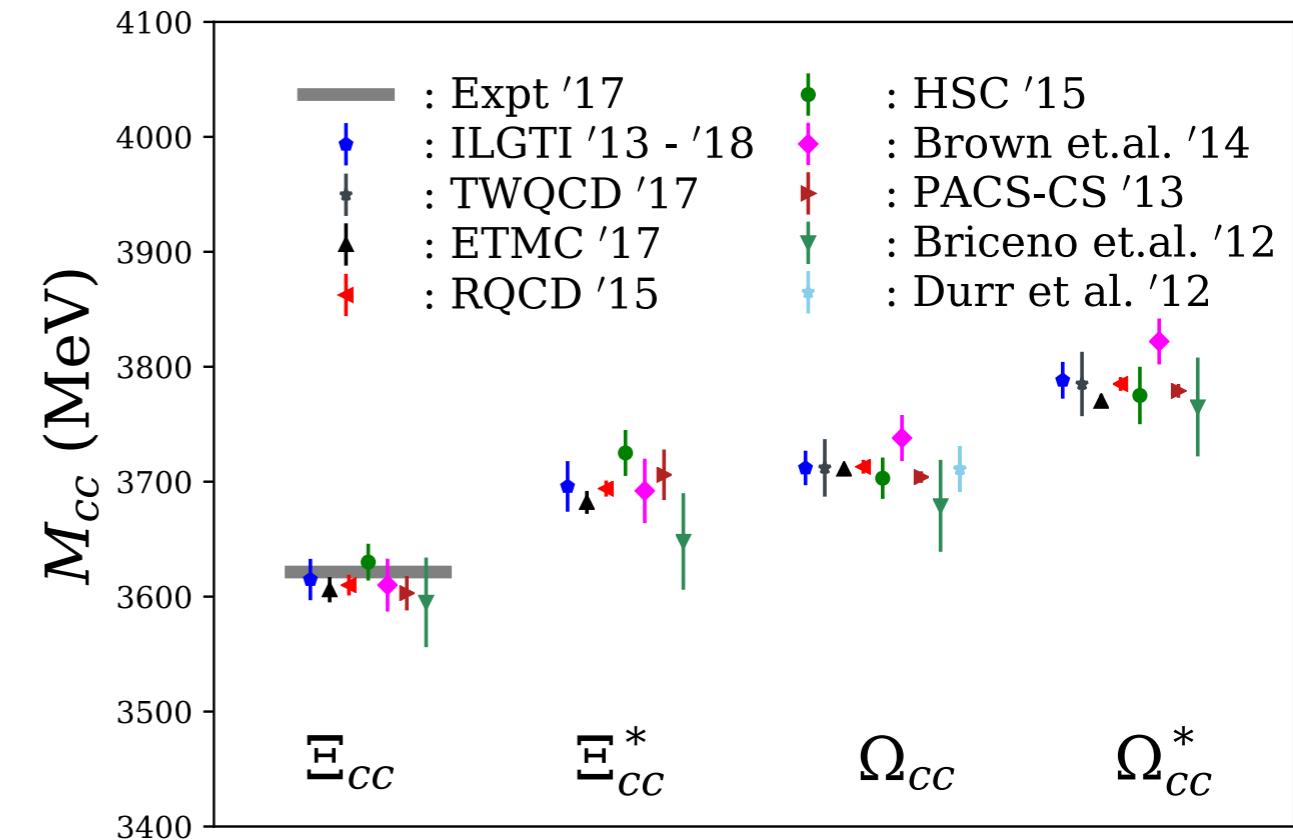
[PRD91 094502 \(2015\)](#), and [HSC'14] **triply charmed baryons** [PRD90 074504 \(2014\)](#)

● [Brown'14]: Nf=2+1, RHQA, $a = 0.085 - 0.114$ fm, $m_{\pi} = 227 - 419$ MeV

● [PACS-CS'13]: Nf=2+1, RHQA, $a \sim 0.09$ fm, $m_{\pi} \sim 135$ MeV

● [Briceno'12]: Nf=2+1+1, RHQA, $a = 0.06-0.12$ fm, $m_{\pi} = 220, 310$ MeV

● [Durr'12]: Nf=2, Brillouin, $a = 0.07$ fm, $m_{\pi} \sim 280$ MeV



[Basak et al, PoS\(LATTICE 2013\) 243](#),
[Mathur et al. arXiv:1807.00174](#)

[Y.-C. Chen et al., PLB767 193 \(2017\)](#)

[Alexandrou et al. PRD96 034511 \(2017\)](#)

[Pérez-Rubio et al., PRD92 034504 \(2015\)](#)

[Padmanath et al, PRL119 042001 \(2017\)](#),

[PRD91 094502 \(2015\)](#), and [HSC'14] **triply charmed baryons** [PRD90 074504 \(2014\)](#)

[Brown et al., PRD90 094507 \(2014\)](#)

[Namekawa et al., PRD87 094512 \(2013\)](#)

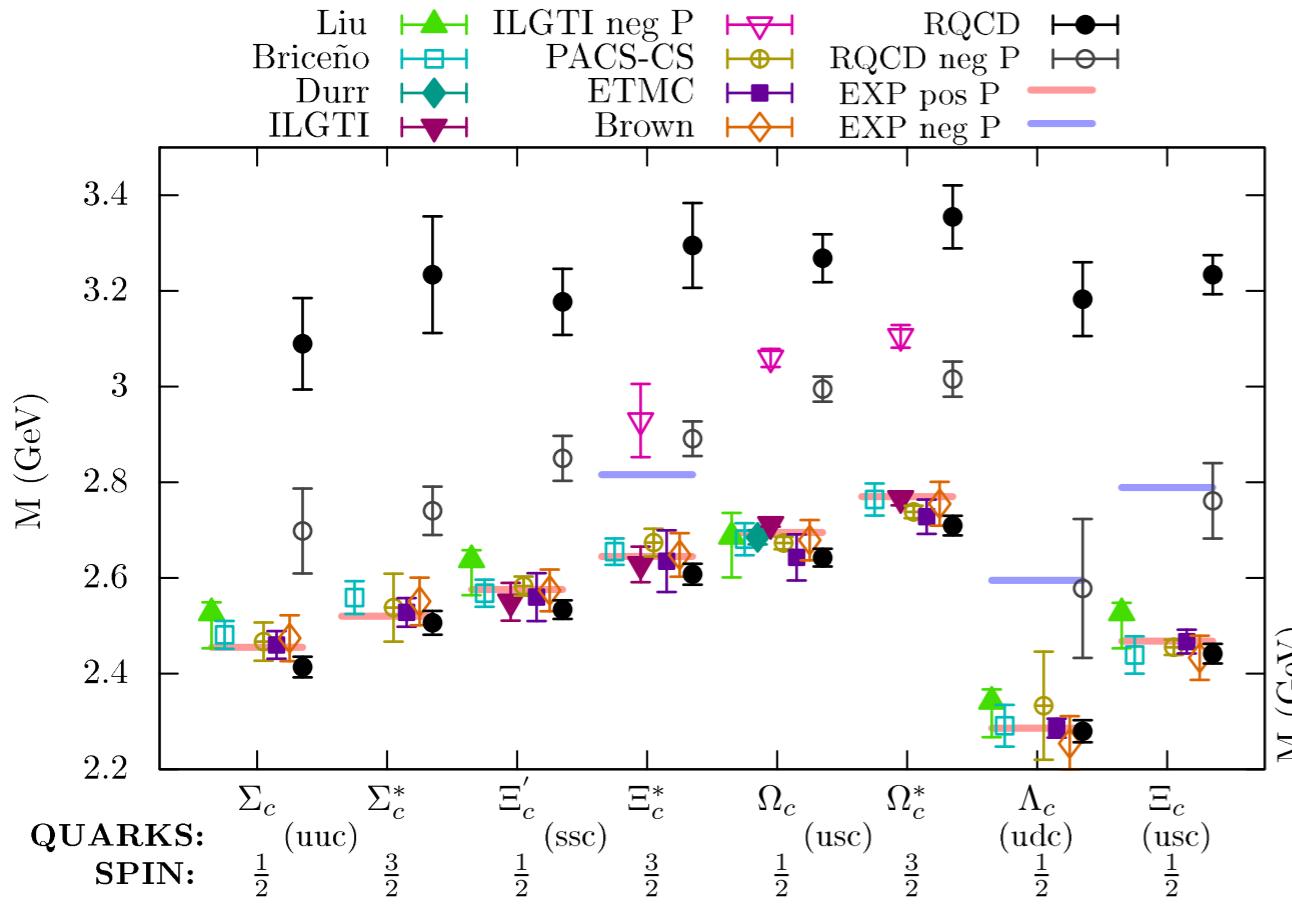
[Briceno et al., PRD86 094504 \(2012\)](#)

[Durr et al. PRD86 114514 \(2012\)](#)

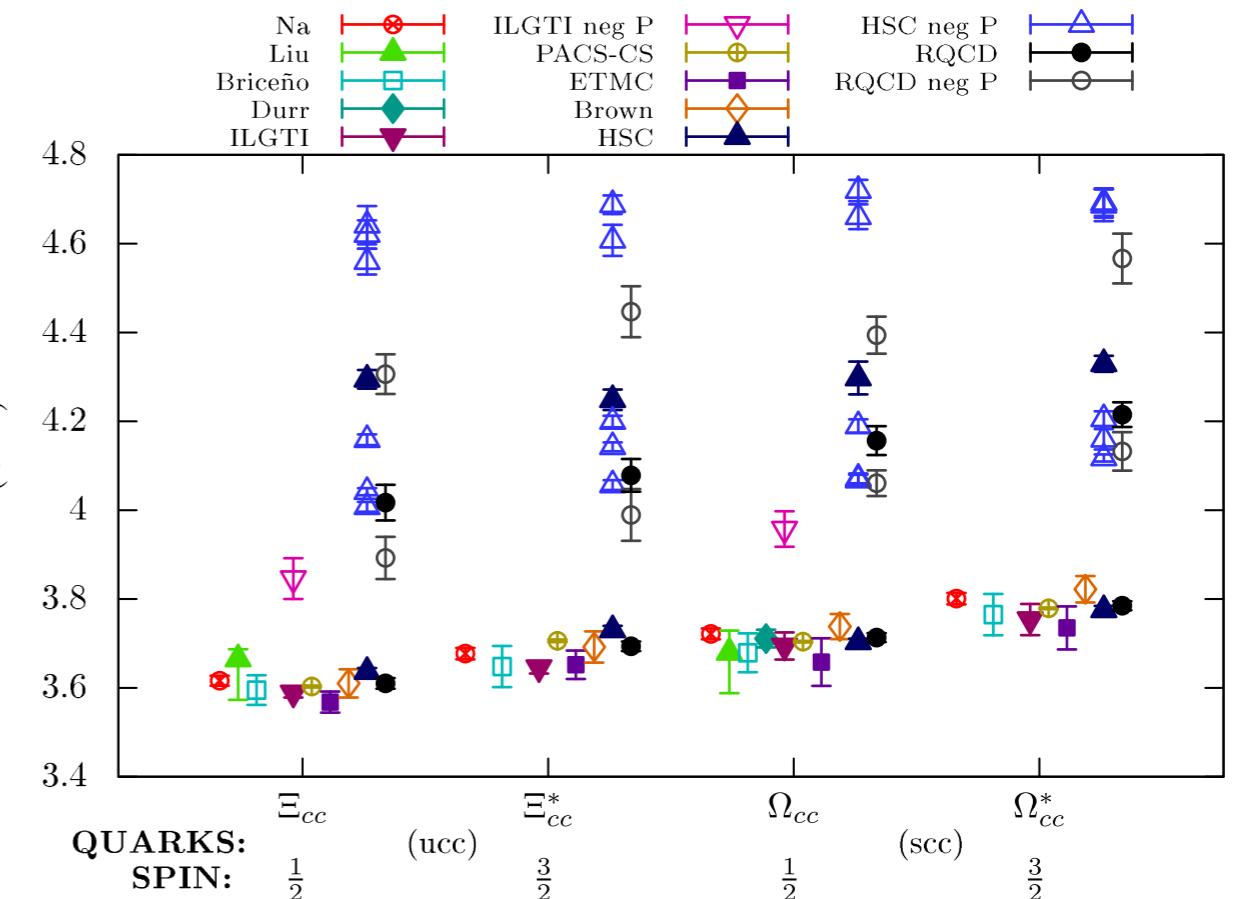
Excited states

Figures from: Pérez-Rubio et al. (RQCD), PRD92 034504 (2015)
figure references in paper

Singly Charmed



Doubly Charmed



Lattice QCD

ab initio
non-perturbative method

Two key equations:

$$\lim_{T \rightarrow \infty} \langle \hat{O}_2(t) \hat{O}_1(0) \rangle_T = \sum_h \langle 0 | \hat{O}_2 | h \rangle \langle h | \hat{O}_1 | 0 \rangle e^{-E_h t}$$

Hadron d.o.f.

$$\langle \hat{O}_2(t) \hat{O}_1(0) \rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} O_2[\Psi(\vec{x}, t)] O_1[\Psi(\vec{x}, 0)]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}}$$

Quark-gluon d.o.f.

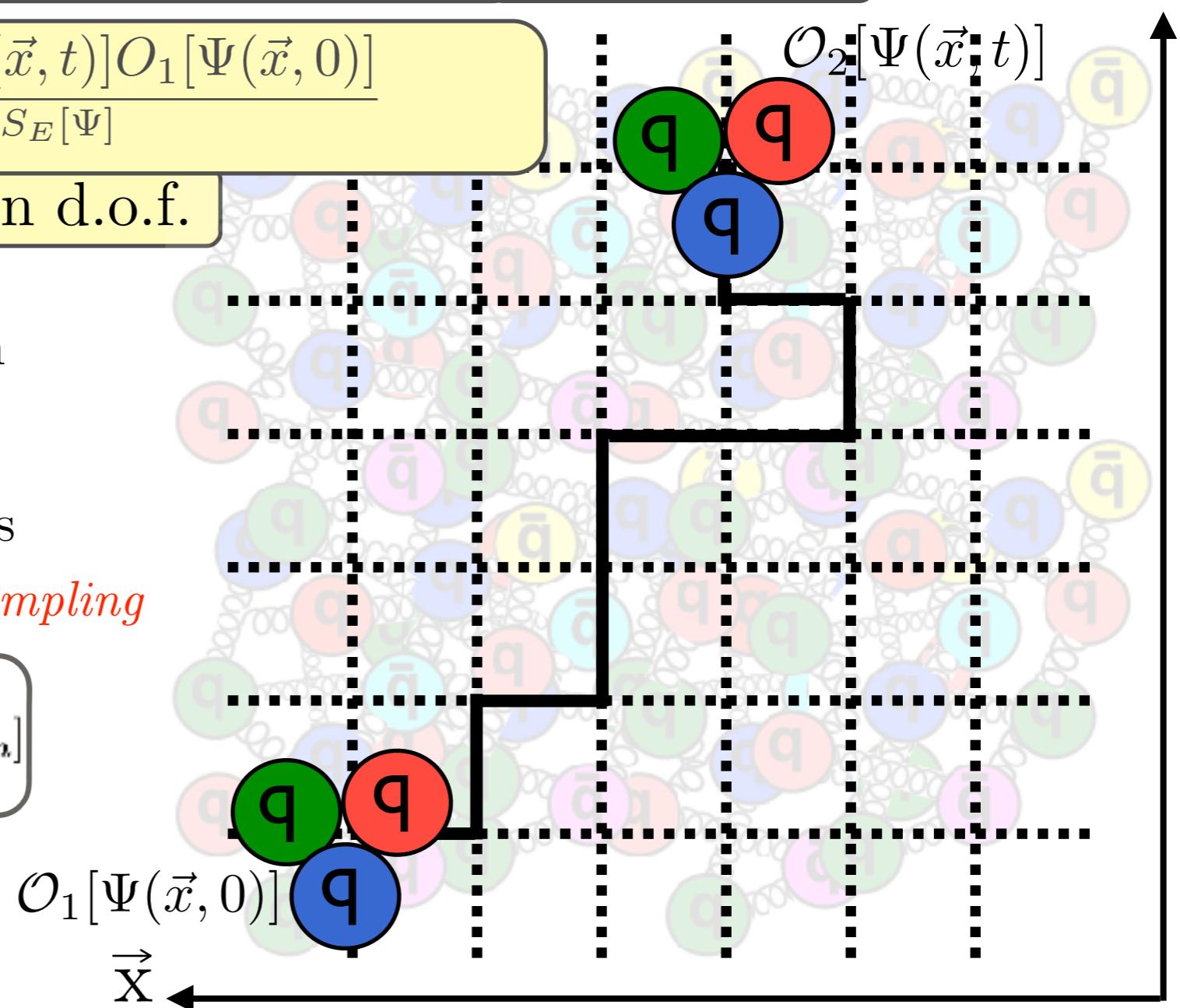
- Discretise the space-time continuum
- “Measure” quantities by computers

- Path integral has infinite dimensions

Tools of the stat. physics: *Importance Sampling*

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} \mathcal{O}[\Psi]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}} = \lim_{N \rightarrow \infty} \sum_{n=1}^N \mathcal{O}[U_n]$$

- e^{-S} acts as the QCD background

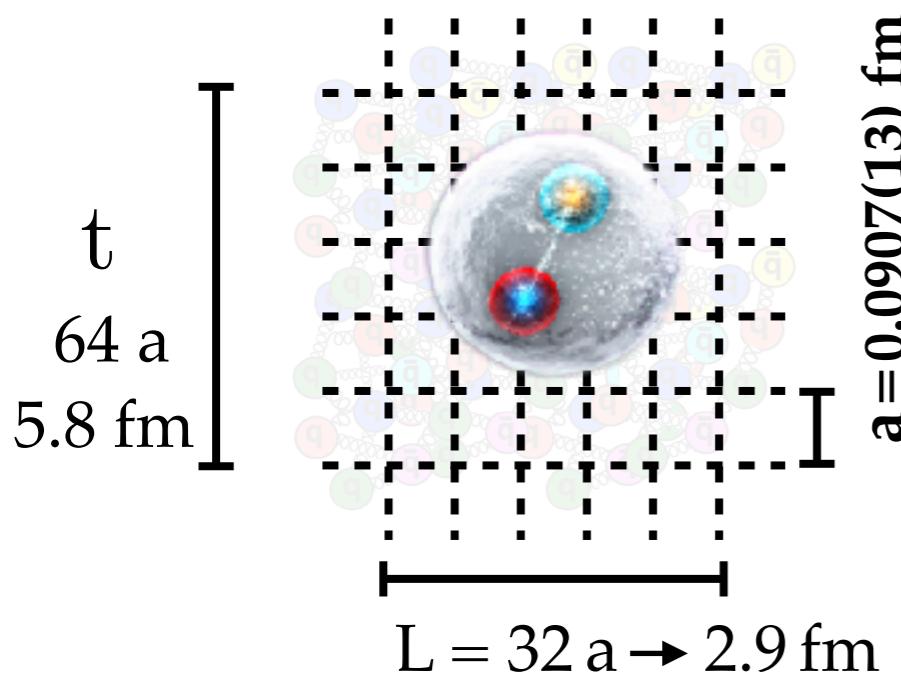


Simulation Details

PACS-CS, $32^3 \times 64$, 2+1 flavor (u/d+s) [Phys. Rev. D79 \(034503\)](#)

 $\kappa_{ud} = 0.13781$  $\kappa_s = 0.13640$

$m_\pi \sim 156$ MeV



-  $L = 32a \sim 2.9$ fm, $T = 64a \sim 5.8$ fm
-  $\beta=1.9$, **$a=0.0907(13)$ fm**, $a^{-1}=2.176(31)$ GeV
-  Gauge action: Iwasaki action,
-  Fermion action (u, d, s): O(a)-improved Wilson (Clover)
-  $\kappa_{ud} = 0.13781$ (sea & valance), $\kappa_s = 0.13640$ (sea)
-  **$m_\pi \sim 156$ MeV almost physical**, $m_\pi L \sim 2.3$
-  $\kappa_s = 0.13665$ (physical), re-tuned by Adelaide group: [PRL108 112001](#)
-  **Tsukuba RHQA for charm quark (physical)**

	M _{latt}	PDG	
η_c	2.984(2)	2.983	GeV
J/ψ	3.099(4)	3.096	GeV
1S	3.071(3)	3.068	GeV
ΔE_{hyp}	115(4)	113	MeV

-  Shell source - Point/Shell sink (details later)

Interpolating fields

$$\lim_{T \rightarrow \infty} \langle \hat{O}_2(t) \hat{O}_1(0) \rangle_T = \sum_h \langle 0 | \hat{O}_2 | h \rangle \langle h | \hat{O}_1 | 0 \rangle e^{-E_h t}$$

- $J = 1/2$ Σ_c , Ξ_{cc} , Ω_c and Ω_{cc} - $SU(4)$ type

$$\chi(x) = \epsilon^{abc} [Q^{Ta}(x) C \Gamma_1 q^b(x)] \Gamma_2 q^c(x)$$

- Three different interpolating operators with different $[\Gamma_1, \Gamma_2]$ choices

$$[\Gamma_1, \Gamma_2] = \{\gamma_5, 1\}$$

$$\chi_1(x) = \epsilon^{abc} [Q^{Ta}(x) C \gamma_5 q^b(x)] q^c(x)$$

$$[\Gamma_1, \Gamma_2] = [1, \gamma_5]$$

$$\chi_2(x) = \epsilon^{abc} [Q^{Ta}(x) C q^b(x)] \gamma_5 q^c(x)$$

$$[\Gamma_1, \Gamma_2] = [\gamma_5 \gamma_4, 1]$$

$$\chi_4(x) = \epsilon^{abc} [Q^{Ta}(x) C \gamma_5 \gamma_4 q^b(x)] q^c(x)$$

$J^P=3/2$ inspired operator

- Form 2x2 or 3x3 correlation function matrices with these operators (fixed smearing)

Interpolating fields

 $J=1/2 \Lambda_c$ - $SU(4)$ type

$$\begin{aligned}\chi(x) = & \frac{1}{\sqrt{6}}\epsilon^{abc} \left[2 \left(u^T a(x) C \Gamma_1 d^b(x) \right) \Gamma_2 Q^c(x) \right. \\ & \left. + \left(u^T a(x) C \Gamma_1 Q^b(x) \right) \Gamma_2 d^c(x) - \left(d^T a(x) C \Gamma_1 Q^b(x) \right) \Gamma_2 u^c(x) \right]\end{aligned}$$

 $J=1/2 \Xi_c$ and Ξ'_c - $SU(4)$ type

$$\chi_{\Xi'_c}(x) = \frac{1}{\sqrt{2}}\varepsilon_{abc} \left[\left(\ell_a^T(x) (C \Gamma_1) c_b(x) \right) \Gamma_2 s_c(x) + \left(s_a^T(x) (C \Gamma_1) c_b(x) \right) \Gamma_2 \ell_c(x) \right]$$

$$\begin{aligned}\chi_{\Xi_c}(x) = & \frac{1}{\sqrt{6}}\varepsilon_{abc} \left[2 \left(s_a^T(x) (C \Gamma_1) \ell_b(x) \right) \Gamma_2 c_c(x) + \left(s_a^T(x) (C \Gamma_1) c_b(x) \right) \Gamma_2 \ell_c(x) \right. \\ & \left. - \left(\ell_a^T(x) (C \Gamma_1) c_b(x) \right) \Gamma_2 s_c(x) \right]\end{aligned}$$

 $J=3/2$ interpolating fields, Δ^+ baryon like

$$\chi_\mu(x) = \frac{1}{\sqrt{3}}\varepsilon^{ijk} \left\{ 2[q_1^T i(x) C \gamma_\mu q_2^j(x)] q_3^k(x) + [q_1^T i(x) C \gamma_\mu q_3^j(x)] q_2^k(x) \right\}$$

$$\begin{aligned}\{q_1, q_2, q_3\} = & \quad \{u, u, c\} - \Sigma_c^*, \{u, s, c\} - \Xi_c^*, \{u, c, c\} - \Xi_{cc}^* \\ & \{s, s, c\} - \Omega_c^*, \{s, c, c\} - \Omega_{cc}^*, \{c, c, c\} - \Omega_{ccc}\end{aligned}$$

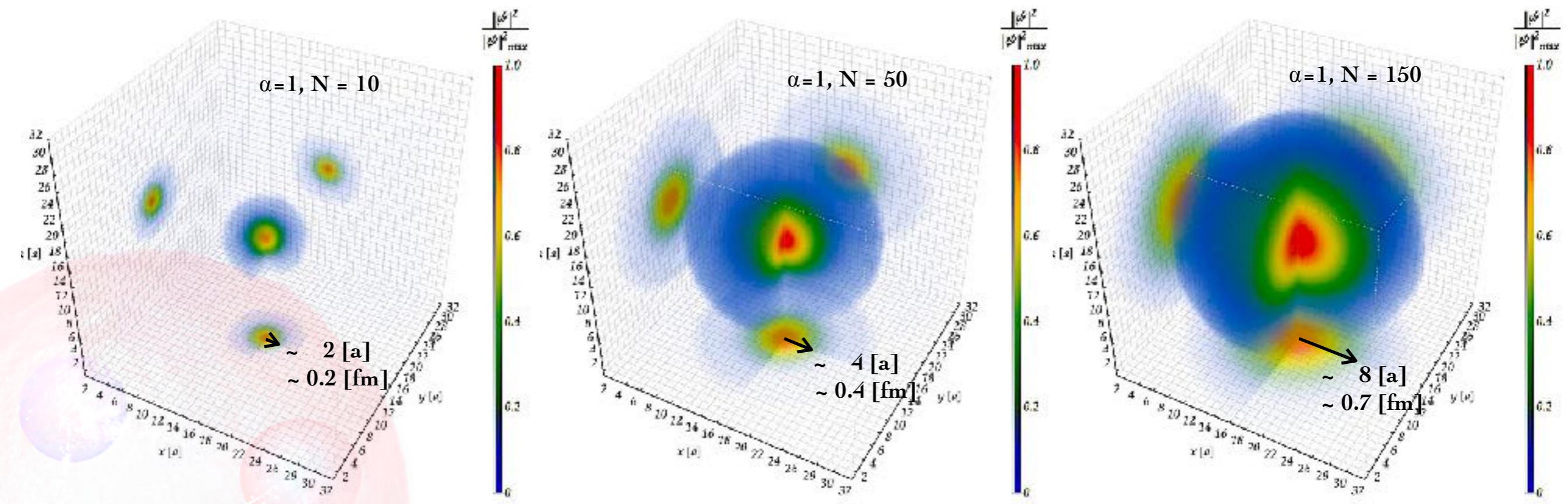
Gaussian Smearing

$$\psi(x, t) = \left(1 + \alpha \sum_{i=1}^3 [U(x, i)\delta_{x+\hat{i}, y} + U^\dagger(x - \hat{i}, i)\delta_{x-\hat{i}, y}] \right)^N \delta_{y,0}$$

$$r_{\text{rms}} = \langle r^2 \rangle^{(1/2)} = \left[\frac{\int d^3x |\vec{x}|^2 \psi^*(\vec{x}, t) \psi(\vec{x}, t)}{\int d^3x \psi^*(\vec{x}, t) \psi(\vec{x}, t)} \right]^{1/2}$$

 Gaussian Smeared source and sink:

$N = [10, 50, 150]$, $r_{\text{rms}} \sim [0.2, 0.4, 0.7] \text{ fm}$, $\sim [2, 4, 8] a$



Variational Method

- Two-point correlation function matrix Parity projector

$$C_{ij}^{\pm}(t) = P^{\pm} \langle \chi_i(t) \bar{\chi}_j(0) \rangle \quad P^{\pm} = (1 \pm \gamma_0)/2$$

$$C(t) = \begin{pmatrix} \langle \chi_1(t) \bar{\chi}_1(0) \rangle & \langle \chi_1(t) \bar{\chi}_2(0) \rangle & \dots \\ \langle \chi_2(t) \bar{\chi}_1(0) \rangle & \langle \chi_2(t) \bar{\chi}_2(0) \rangle & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

- Spectral decomposition can be written as

$$C_{ij}^{\pm}(t) = \sum_{\alpha=1}^N \nu_i^{\alpha*} \nu_j^{\alpha} e^{-tE_{\alpha}^{\pm}} \quad \nu_i^{\alpha}: \text{Overlap factors}$$

Variational Method

- Translate into a Generalized Eigenvalue Problem and solve

$$C(t)\psi_R^\alpha(t) = \lambda^\alpha(t, t_0)C(t_0)\psi_R^\alpha(t)$$

t_0 : Normalization time

$$C(t_0)^{-1}C(t)\psi_R^\alpha(t) = \lambda^\alpha(t, t_0)\psi_R^\alpha(t)$$

λ^α : Eigenvalues

ψ^α : Eigenvectors

- Diagonalize $C(t)$ using Right and Left $\psi^\alpha(T)$

$$C^\alpha(t) \equiv \psi_L^\alpha(T)C(t)\psi_R^\beta(T)\delta_{\alpha\beta} = \delta_{\alpha\beta} Z^\alpha \bar{Z}^\beta e^{-m_\alpha t}$$

mass/energy

Overlap of the superposed interpolating fields

- Usual effective mass method follows to extract masses

$$M_{\text{eff}}^\alpha(t) = \ln \left(\frac{C^\alpha(t)}{C^\alpha(t+1)} \right)$$

Variational Method

- Two-point correlation function matrix

$$C_{ij}^{\pm}(t) = P^{\pm} \langle \chi_i(t) \bar{\chi}_j(0) \rangle$$

Parity projector

$$P^{\pm} = (1 \pm \gamma_0)/2$$

3 x Interpolating Operators

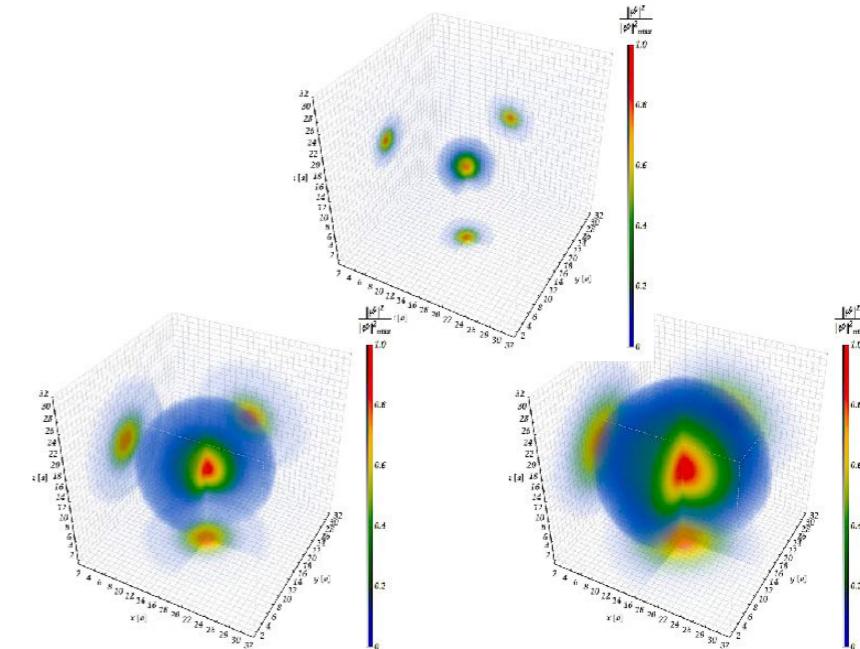
$$\chi_1(x) = \epsilon^{abc} [Q^{Ta}(x) C \gamma_5 q^b(x)] q^c(x)$$

$$\chi_2(x) = \epsilon^{abc} [Q^{Ta}(x) C q^b(x)] \gamma_5 q^c(x)$$

$$\chi_4(x) = \epsilon^{abc} [Q^{Ta}(x) C \gamma_5 \gamma_4 q^b(x)] q^c(x)$$

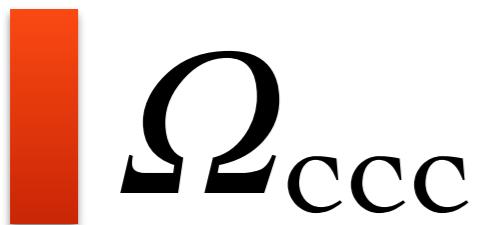
+

3 x Different Smearings

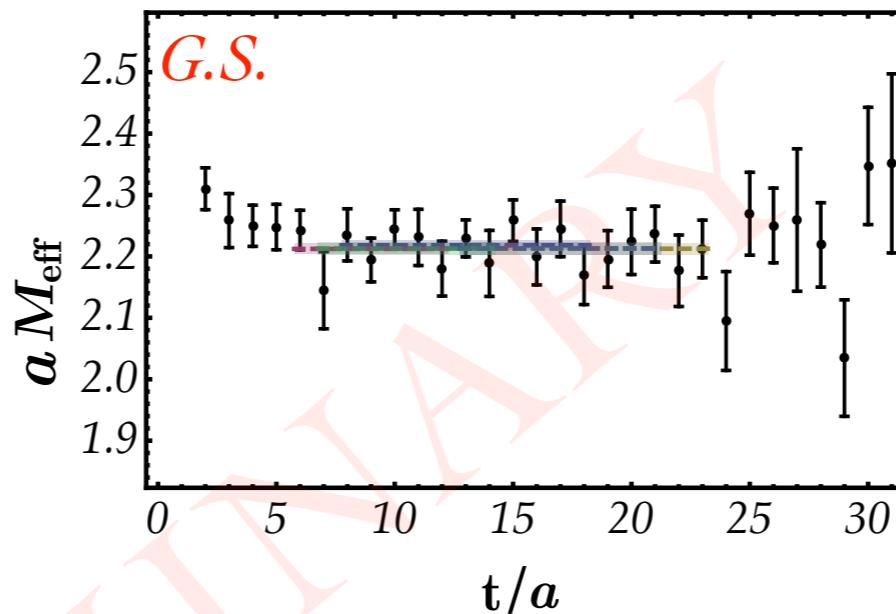
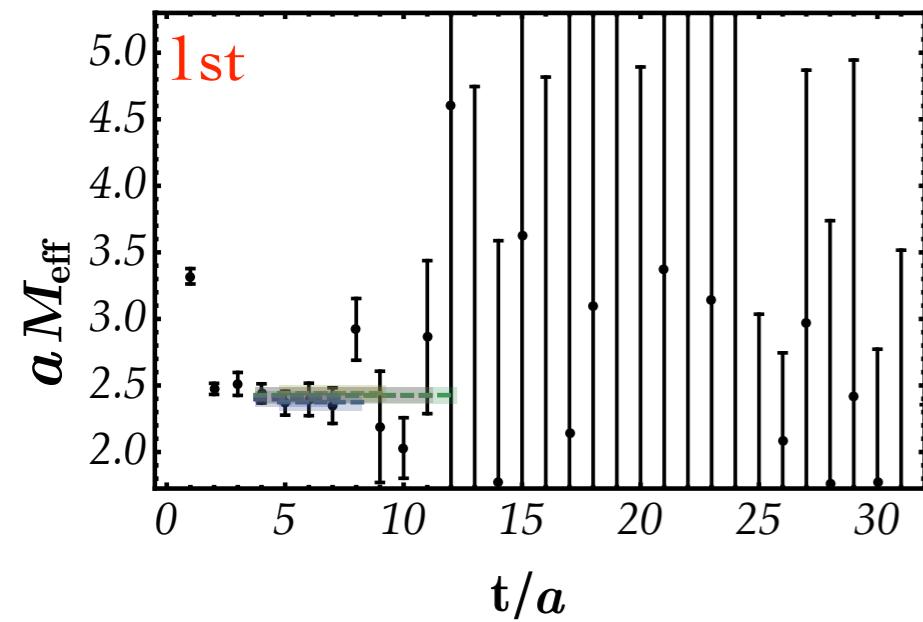


- Wisdom from Pérez-Rubio et al. (RQCD), PRD92 034504 (2015)

- Singly charmed baryons less sensitive to charm quark smearing
- Fix charm quark's smearing, *perform variation over light or strange*
- Doubly charmed baryons less sensitive to light or strange quark smearing
- Fix light or strange's smearing, *perform variation over charm*



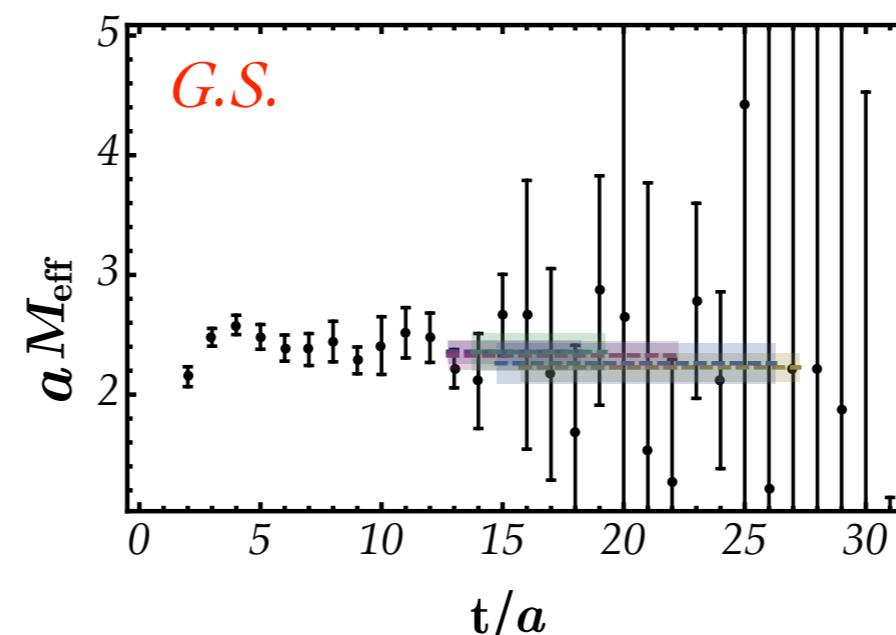
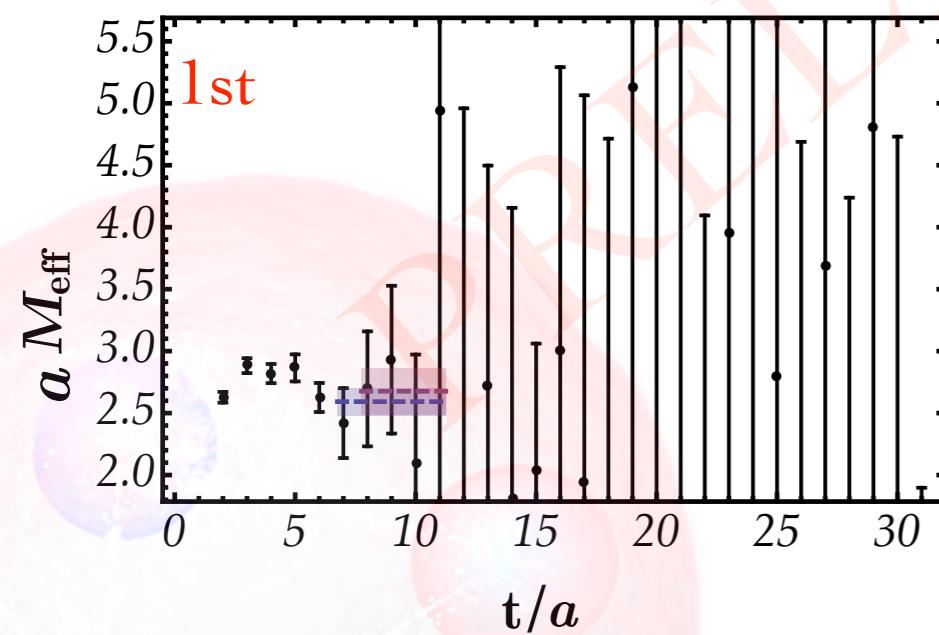
#meas ~ 145

 $JP = 3/2^+$ 

	GS	1st
srcS	50,150	50,150
snkS	10,50	10,50
[ti,tf]	8,18	4,11
Mass [GeV]	4.826(17)	5.492(130)

$\Delta M = 0.666 \text{ GeV}$ our
 $\Delta M = 0.554 \text{ GeV}$ HSC'14

PRD90 074504 (2014)

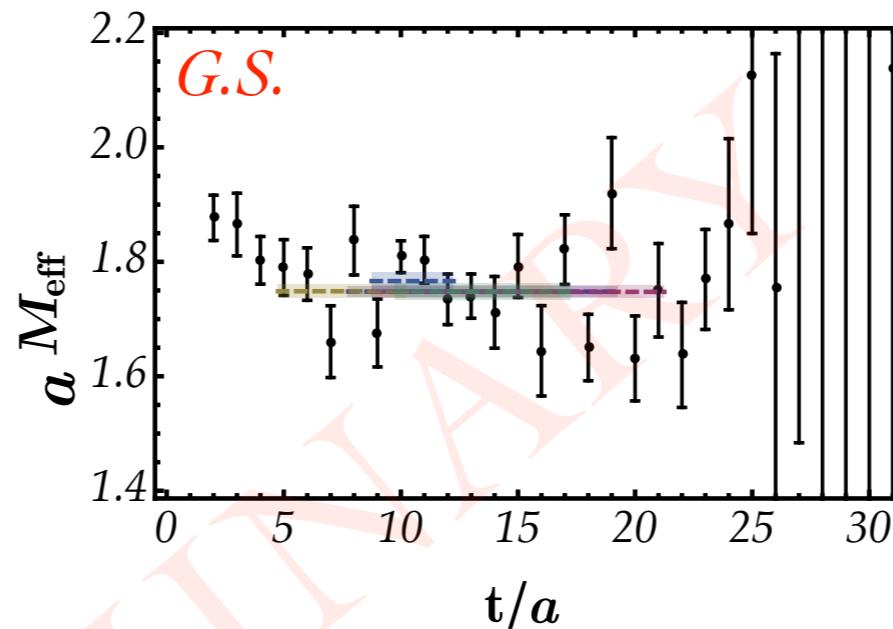
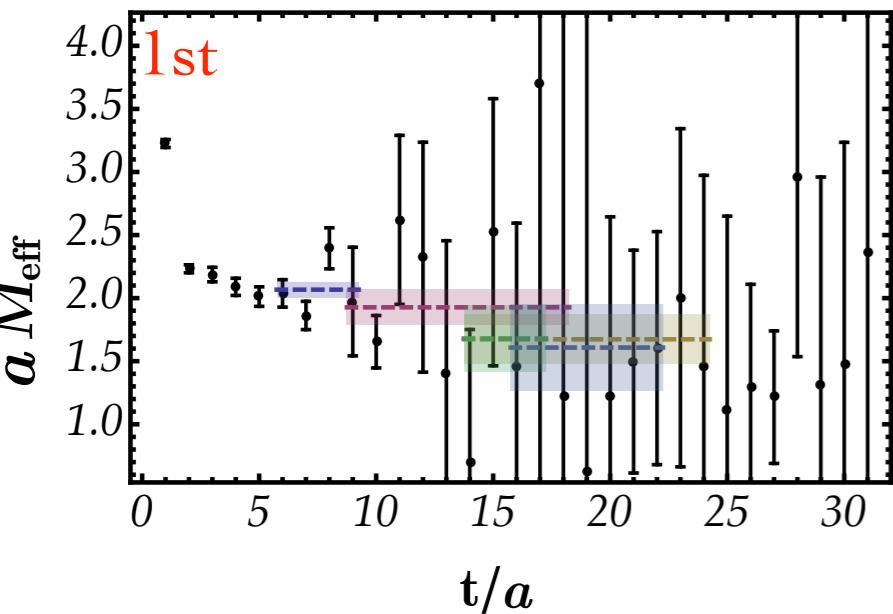
 $JP = 3/2^-$ 

	GS	1st
srcS	10,50	10,50
snkS	10,50	10,50
[ti,tf]	13,18	7,11
Mass [GeV]	5.131(215)	5.640(237)

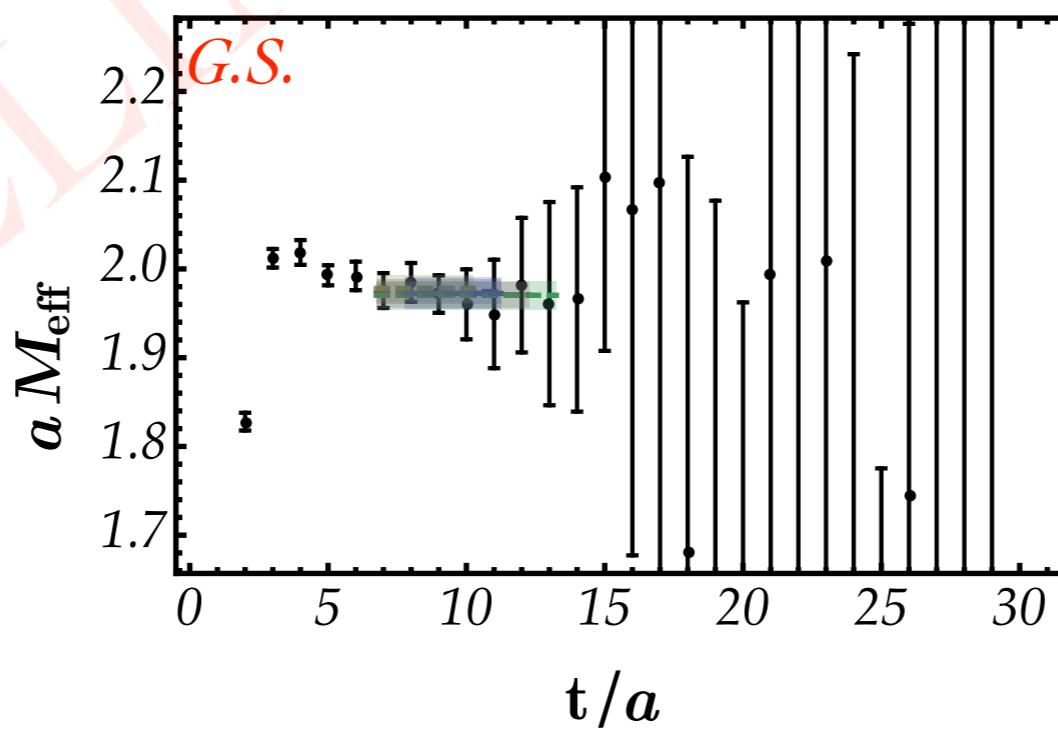
$\Delta M = 0.509 \text{ GeV}$ our
 $\Delta M = 0.538 \text{ GeV}$ HSC'14

 Ω^* CC

$JP = 3/2^+$



$JP = 3/2^-$



#meas ~ 145

	GS	1st
srcS	10,150	10,150
snkS	10,50	10,50
[ti,tf]	8,19	6,9
Mass [GeV]	3.804(20)	4.498(135)

$\Delta M = 0.694 \text{ GeV}$ our
 $\Delta M = 0.430 \text{ GeV}$ RQCD'15

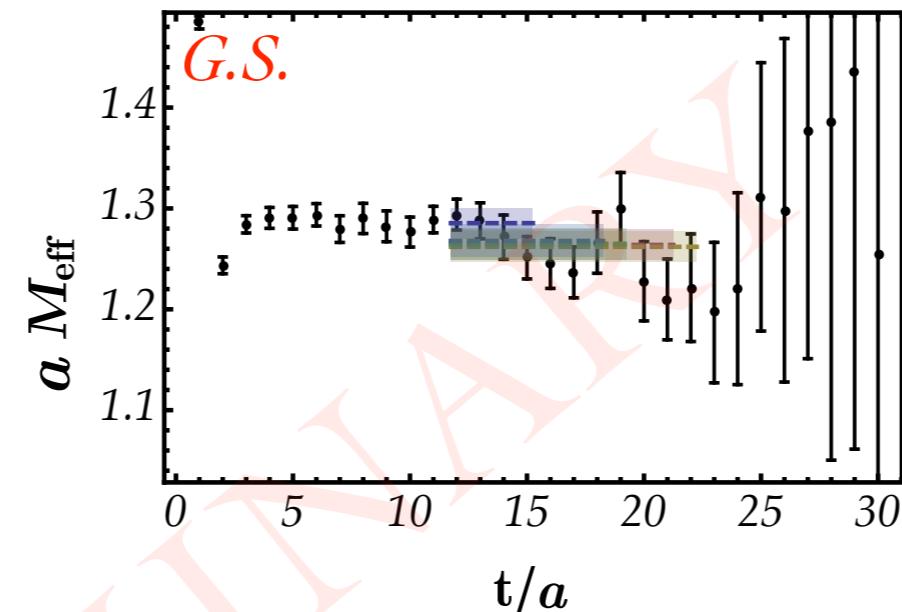
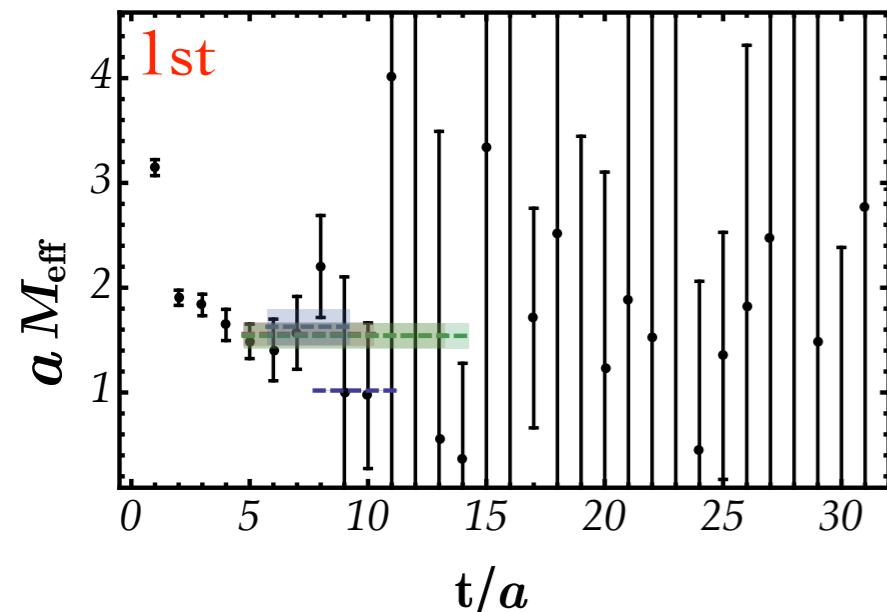
	GS	1st
srcS	50	-
snkS	0	-
[ti,tf]	7,13	-
Mass [GeV]	4.287(35)	-

$M = 4.132(42)(43) \text{ GeV}$
RQCD'15

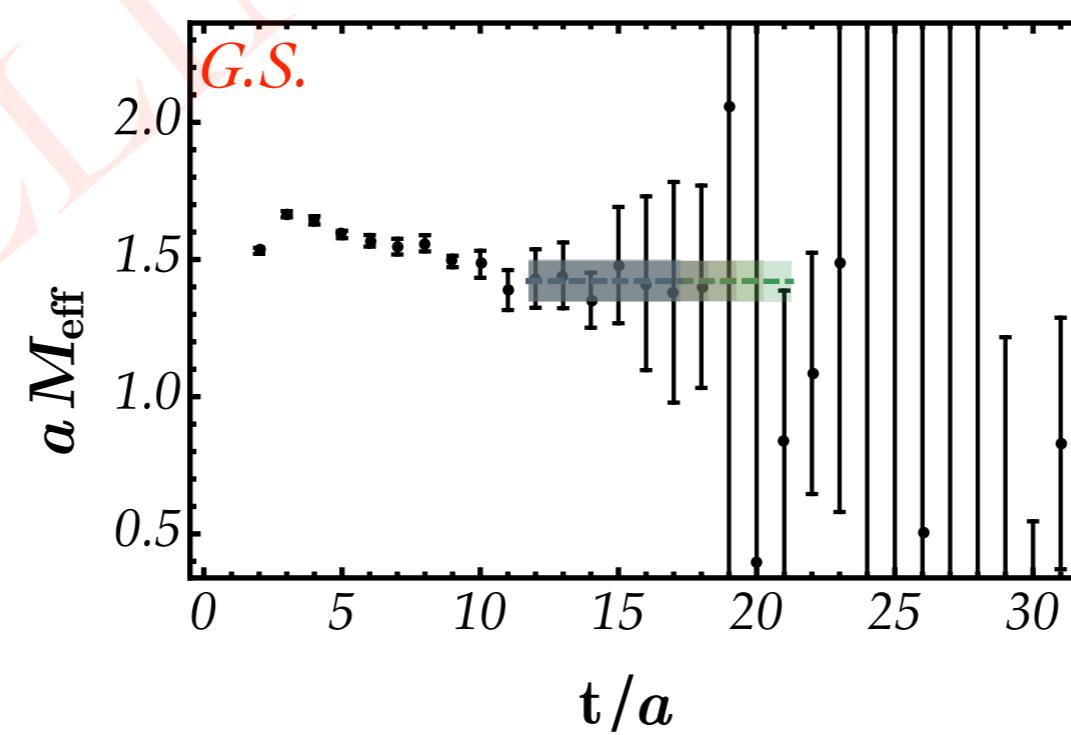
 Ω^* C

#meas ~ 145

$JP = 3/2^+$



$JP = 3/2^-$



	GS	1st
srcS	150	50,150
snkS	0	10,150
[ti,tf]	12,19	5,10
Mass [GeV]	2.753(35)	3.390(242)

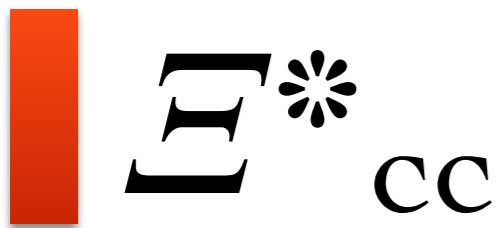
$\Delta M = 0.637 \text{ GeV}$ our

$\Delta M = 0.646 \text{ GeV}$ RQCD'15

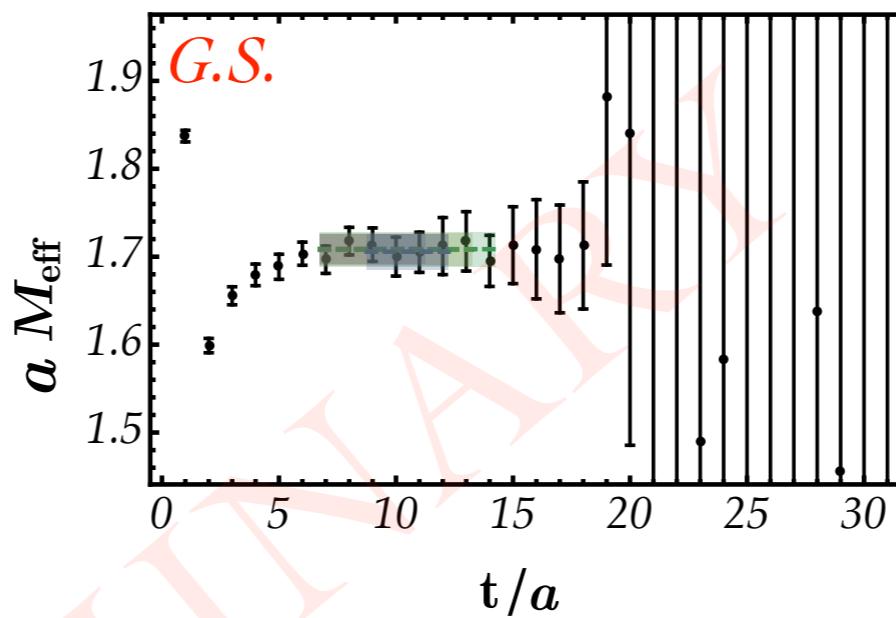
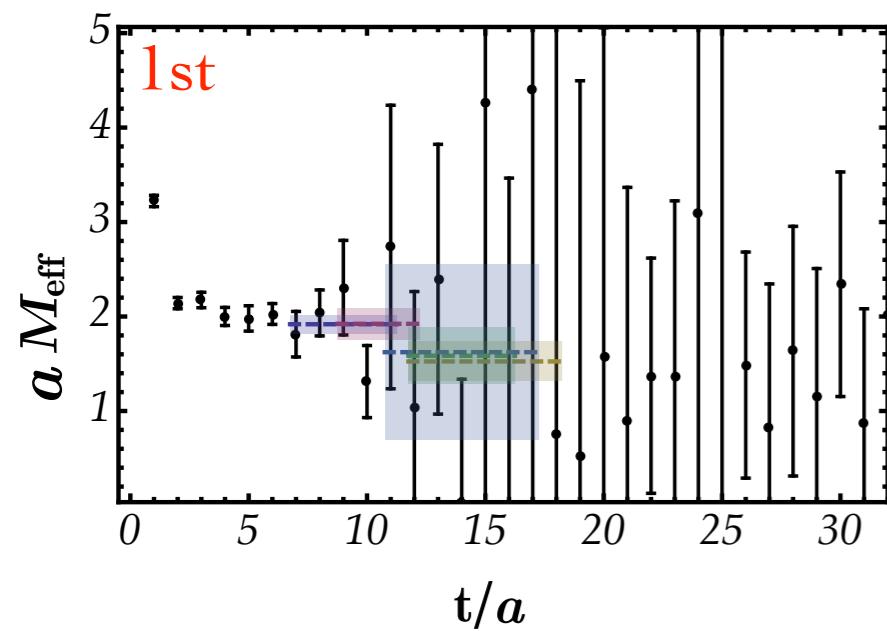
	GS	1st
srcS	50	-
snkS	0	-
[ti,tf]	12,18	-
Mass [GeV]	3.092(157)	-

$M = 3.016(32)(37) \text{ GeV}$ RQCD'15

$M = 3.066 \text{ GeV}$ LHCb'17 (?)

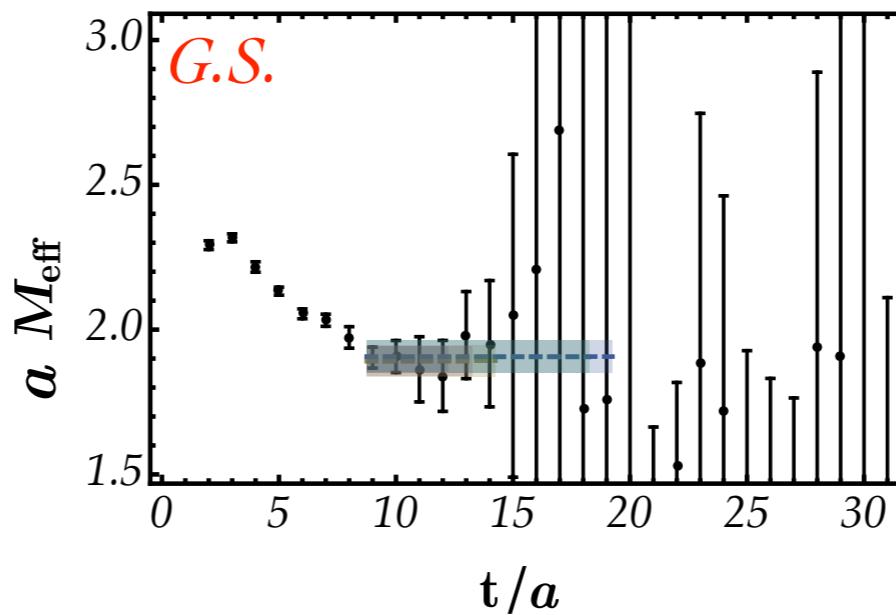
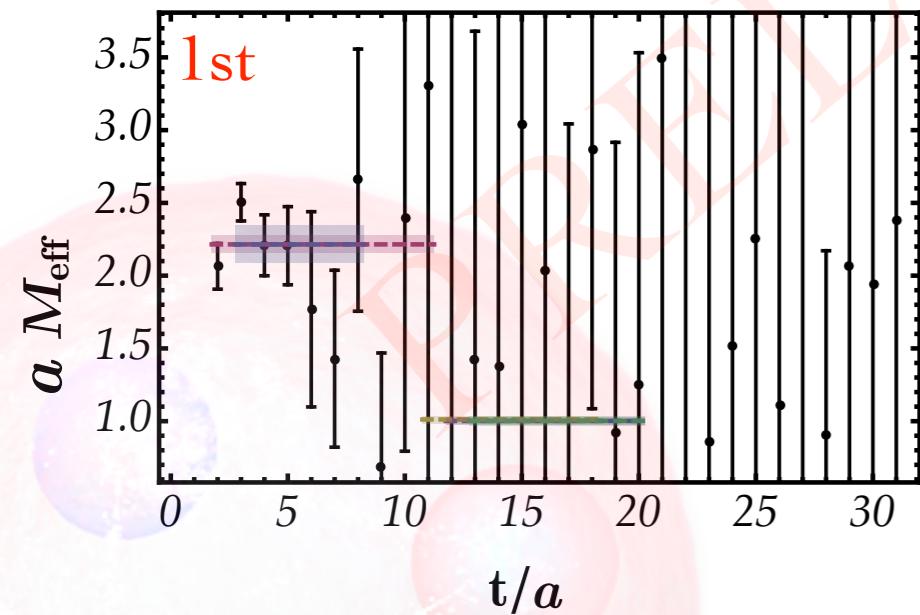


#meas ~ 100

 $JP = 3/2^+$ 

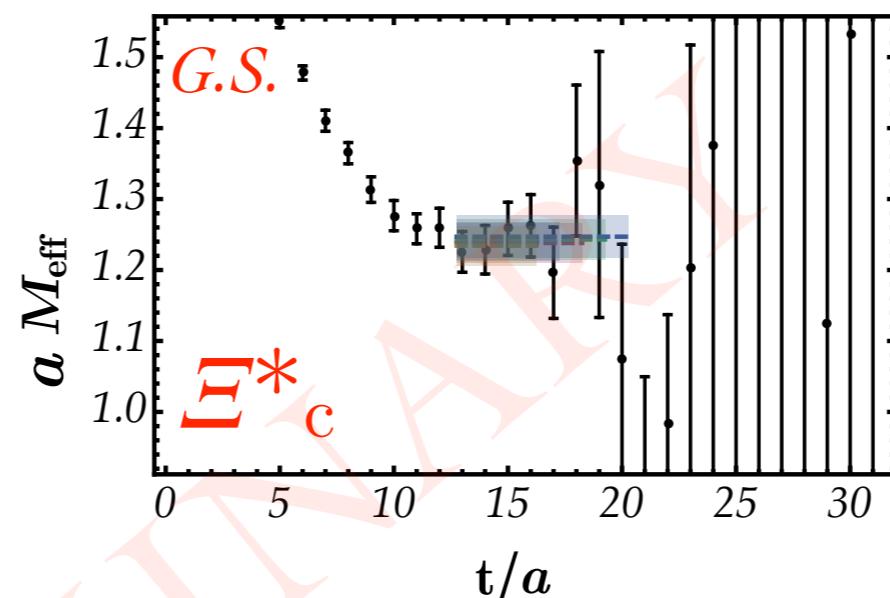
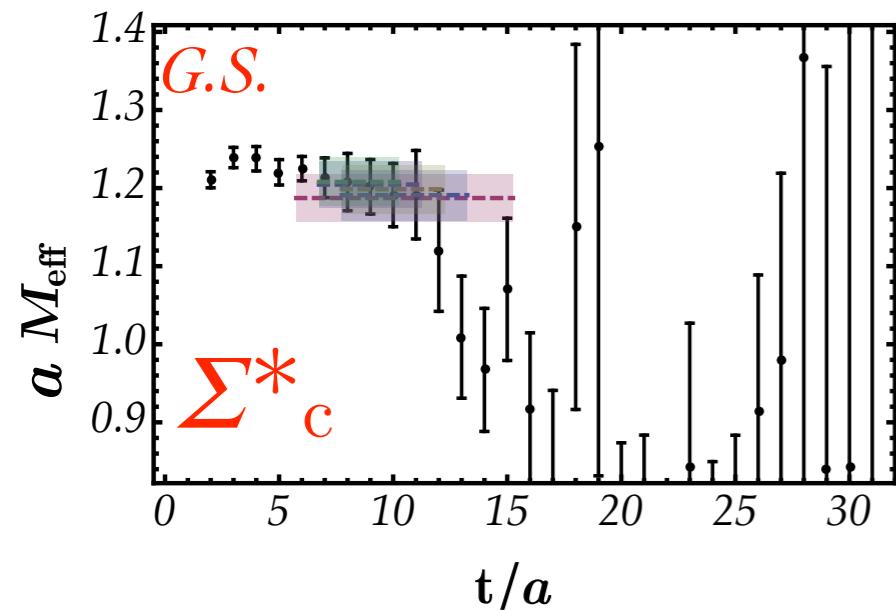
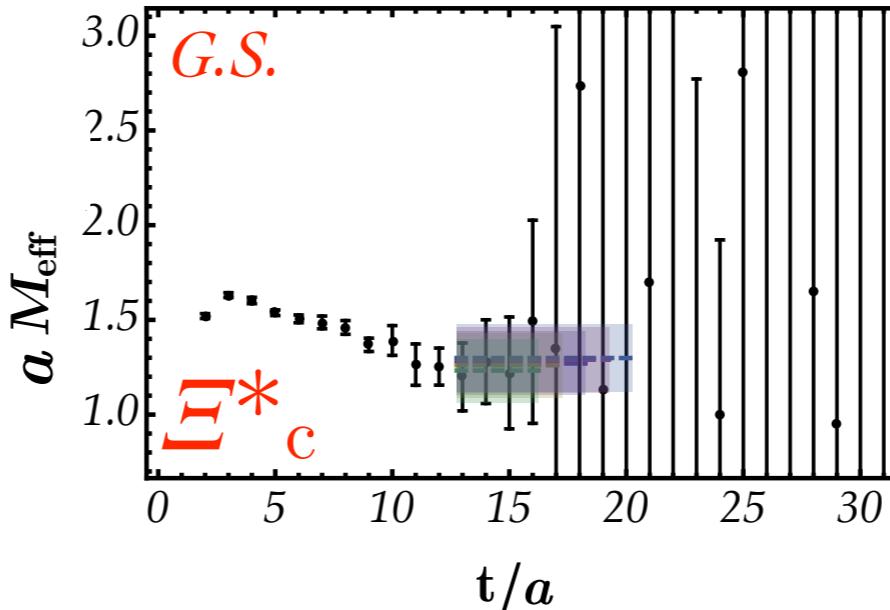
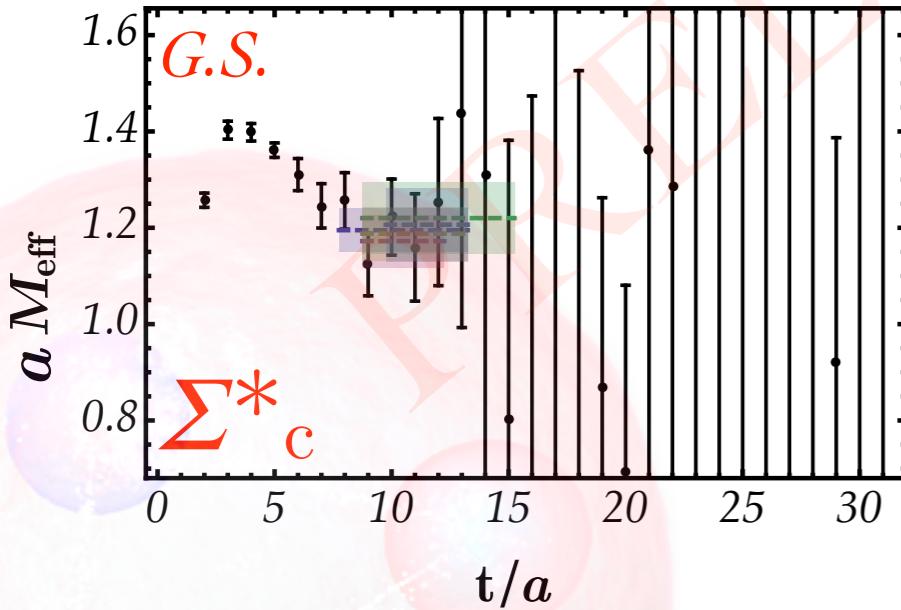
	GS	1st
srcS	150	10,50
snkS	0	10,150
[ti,tf]	9,12	7,11
Mass [GeV]	3.713(44)	4.174(209)

$\Delta M = 0.461 \text{ GeV}$ our
 $\Delta M = 0.384 \text{ GeV}$ RQCD'15

 $JP = 3/2^-$ 

	GS	1st
srcS	10	10.50
snkS	0	50,150
[ti,tf]	9,14	3,8
Mass [GeV]	4.120(118)	4.820(281)

#meas ~ 100

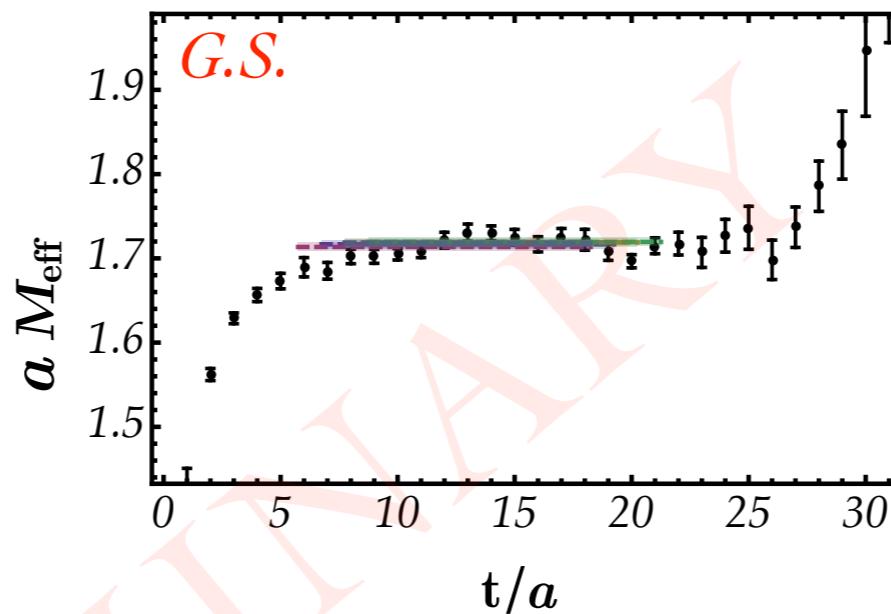
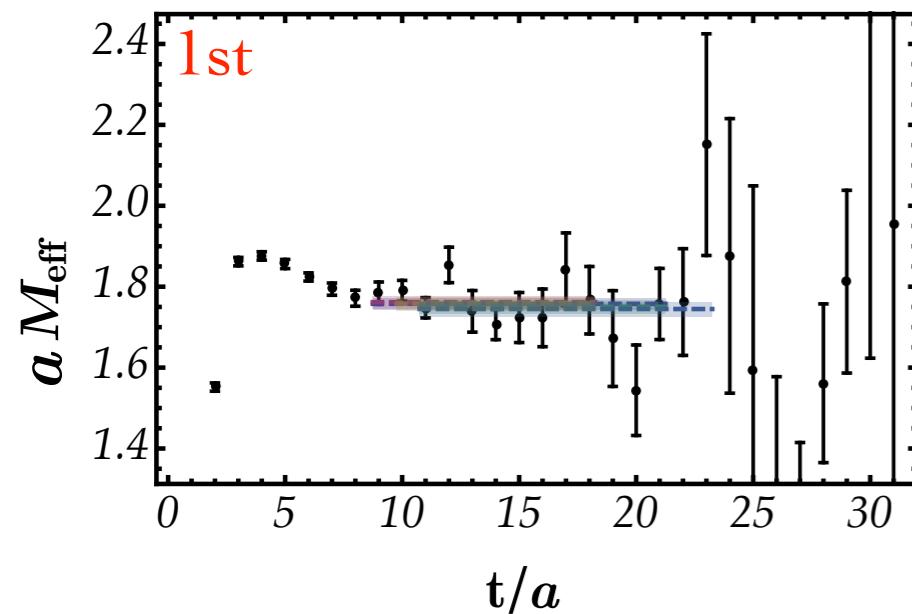
 $JP = 3/2^+$  $JP = 3/2^-$ 

	GS - Σ^*_c	GS - Ξ^*_c
srcS	150	10
snkS	0	0
[ti,tf]	9,12	13,19
Mass [GeV]	2.623(68)	2.705(63)

	GS - Σ^*_c	GS - Ξ^*_c
srcS	150	50
snkS	0	0
[ti,tf]	9,12	13,16
Mass [GeV]	2.600(98)	2.629(50)

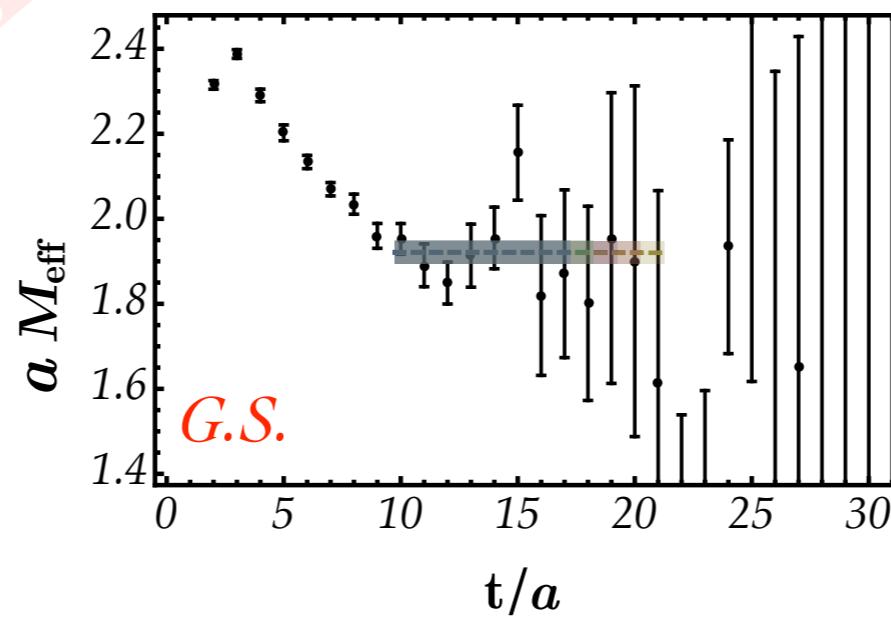
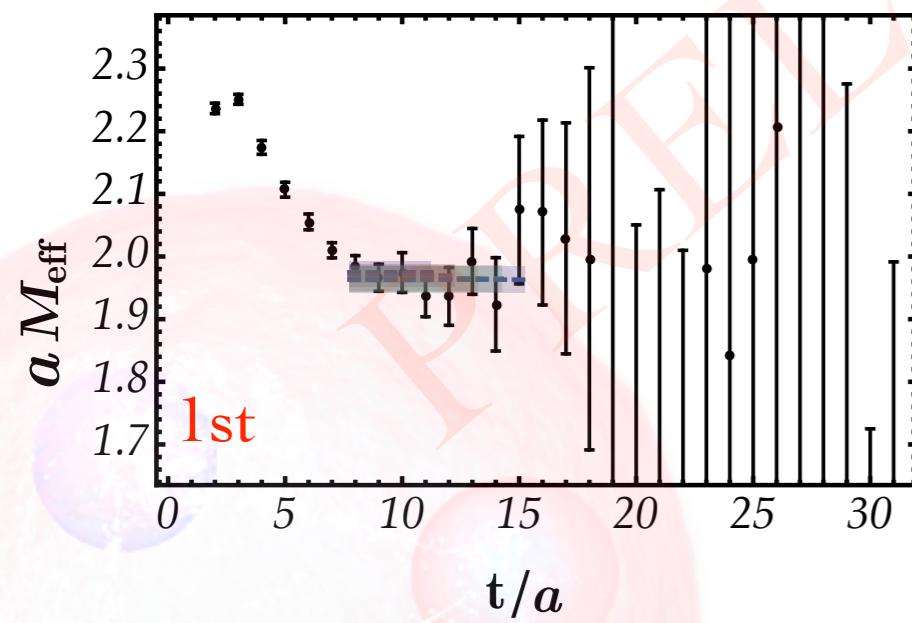
Ω_{CC}

$JP = 1/2^+$



#meas ~ 145

$JP = 1/2^-$



	GS	1st
srcG	1_g5, g5_1	1_g5, g5_1
snkG	1_g5, g5_1	1_g5, g5_1
srcS	150	150
[ti,tf]	9,21	8,20
Mass [GeV]	3.741(11)	3.825(33)

$\Delta M = 0.084 \text{ GeV}$ our
 $\Delta M = 0.444 \text{ GeV}$ RQCD'15

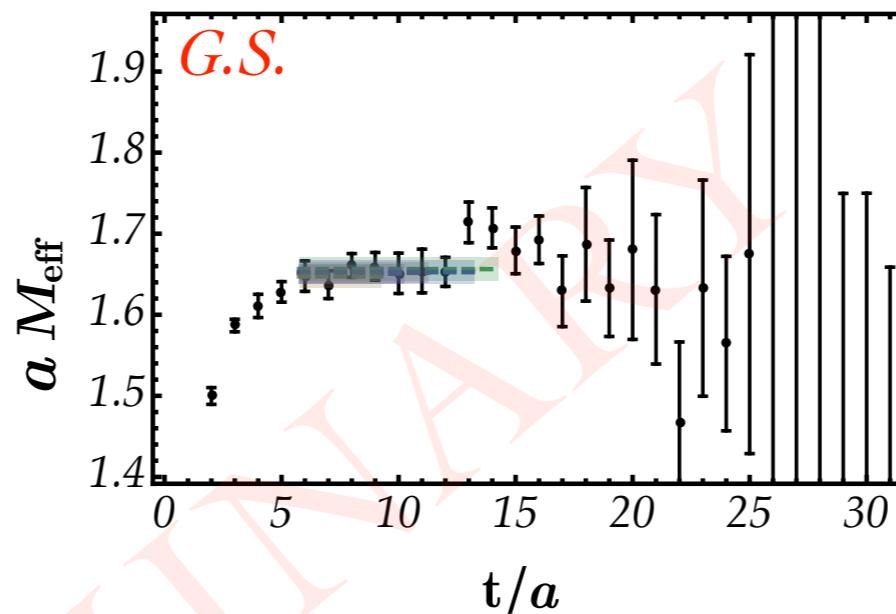
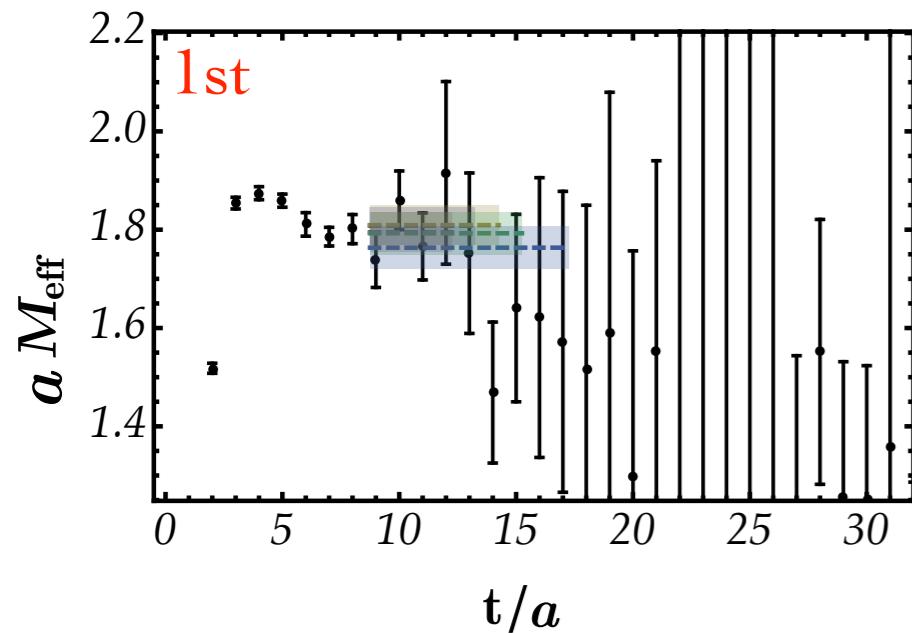
	GS	1st
srcG	1_g5, g5_1	1_g5, g5_1
snkG	1_g5, g5_1	1_g5, g5_1
srcS	10	10
[ti,tf]	10,19	8,13
Mass [GeV]	4.180(57)	4.276(44)

$\Delta M = 0.096 \text{ GeV}$ our
 $\Delta M = 0.333 \text{ GeV}$ RQCD'15

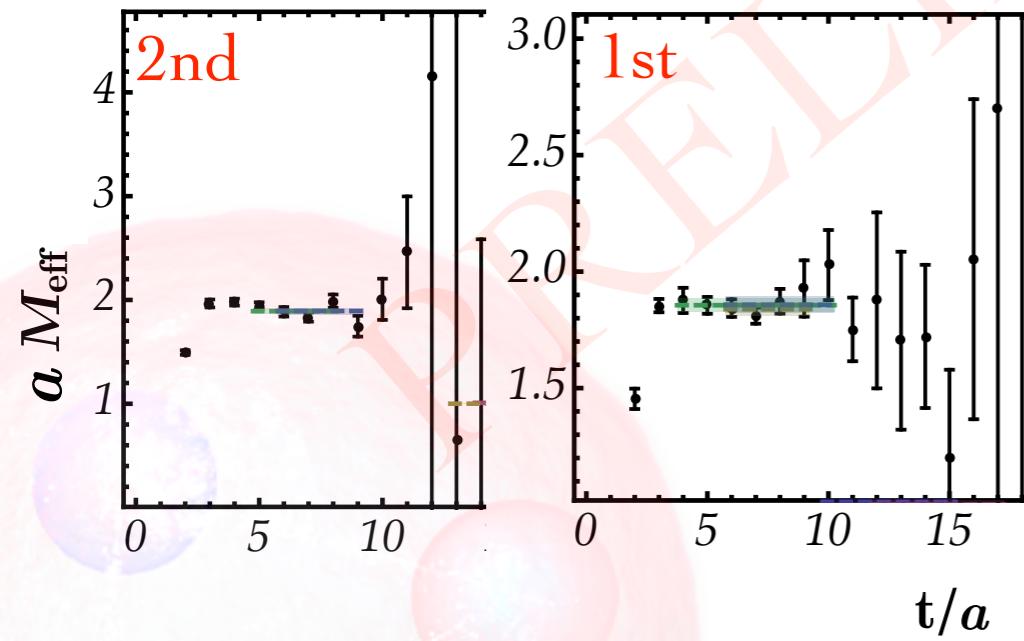
E
[E] CC

#meas ~ 100

$JP = 1/2^+$



$JP = 1/2^-$

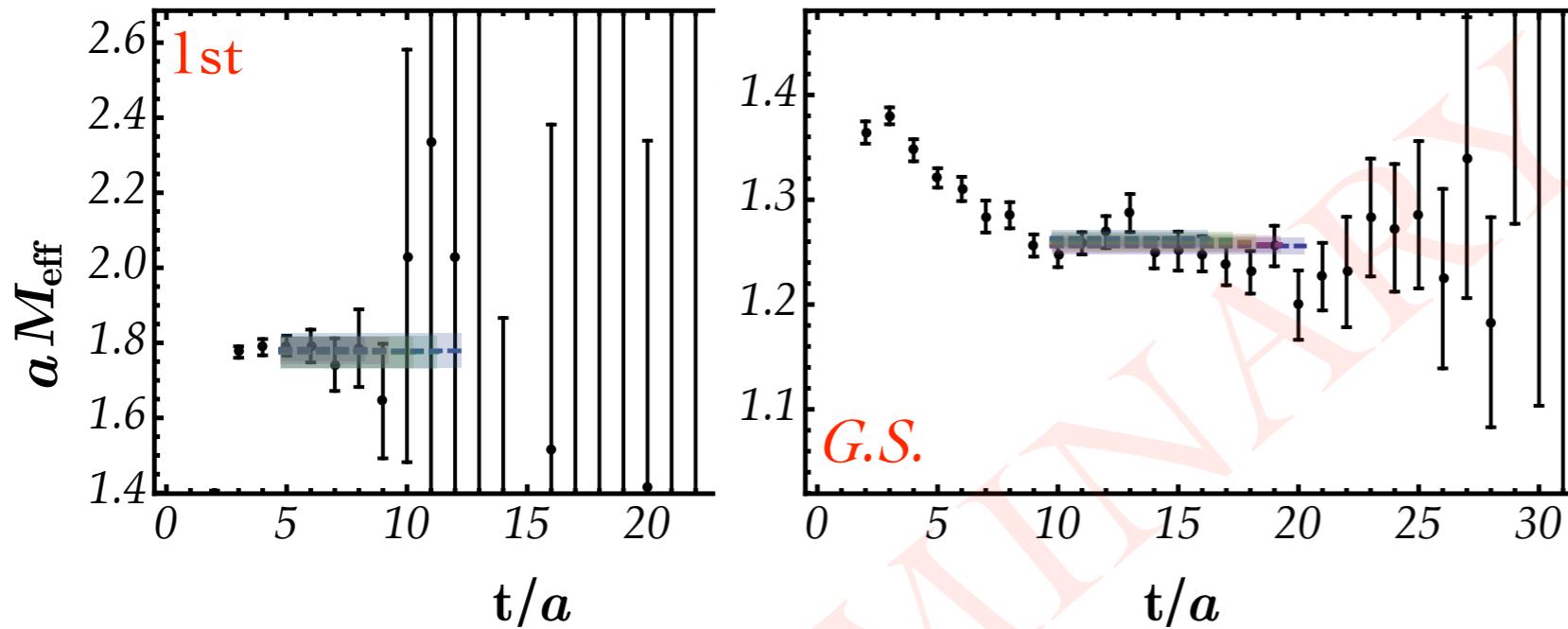


	GS	1st
srcG	1_g5, g ⁴ g5_1	1_g5, g ⁴ g5_1
snkG	1_g5, g ⁴ g5_1	1_g5, g ⁴ g5_1
srcS	150	150
[ti,tf]	6,10	9,13
Mass [GeV]	3.592(26)	3.934(83)
M _{exp} = 3.621(1) GeV LHCb		
$\Delta M = 0.342$ GeV our		
$\Delta M = 0.407$ GeV RQCD'15		

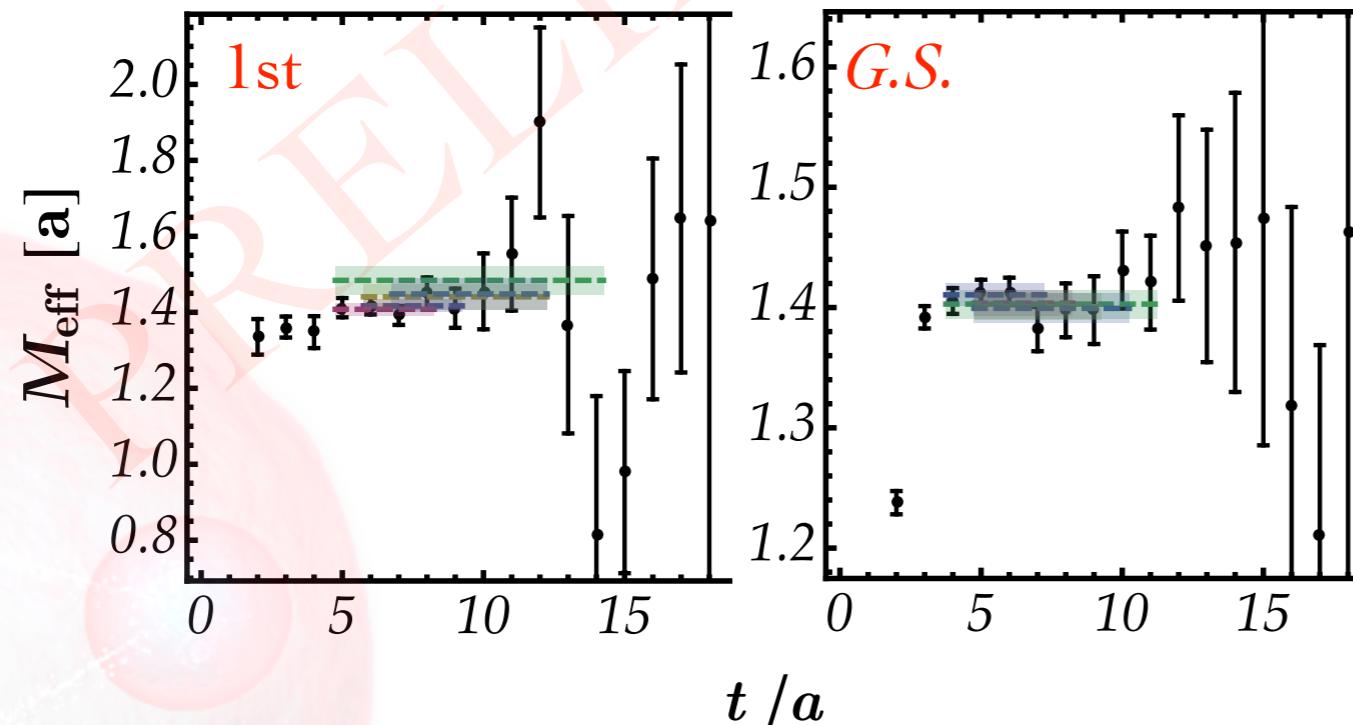
	GS	1st	2nd
srcG	1_g5,g ⁴ g5_1 g5_1	1_g5,g ⁴ g5_1 g5_1	1_g5,g ⁴ g5_1 g5_1
snkG	1_g5,g ⁴ g5_1 g5_1	1_g5,g ⁴ g5_1 g5_1	1_g5,g ⁴ g5_1 g5_1
srcS	150	150	150
[ti,tf]	5,10	4,10	5,8
Mass [GeV]	3.939(63)	4.038(65)	4.117(59)

Ω_C

$JP = 1/2^+$



$JP = 1/2^-$



#meas ~ 145

	GS	1st
srcG	1_g5,g4g5_1 g5_1	1_g5,g4g5_1 g5_1
snkG	1_g5,g4g5_1 g5_1	1_g5,g4g5_1 g5_1
srcS	50	50
[ti,tf]	10,19	5,8
Mass [GeV]	2.735(17)	3.880(70)

$\Delta M = 1.145 \text{ GeV}$ our

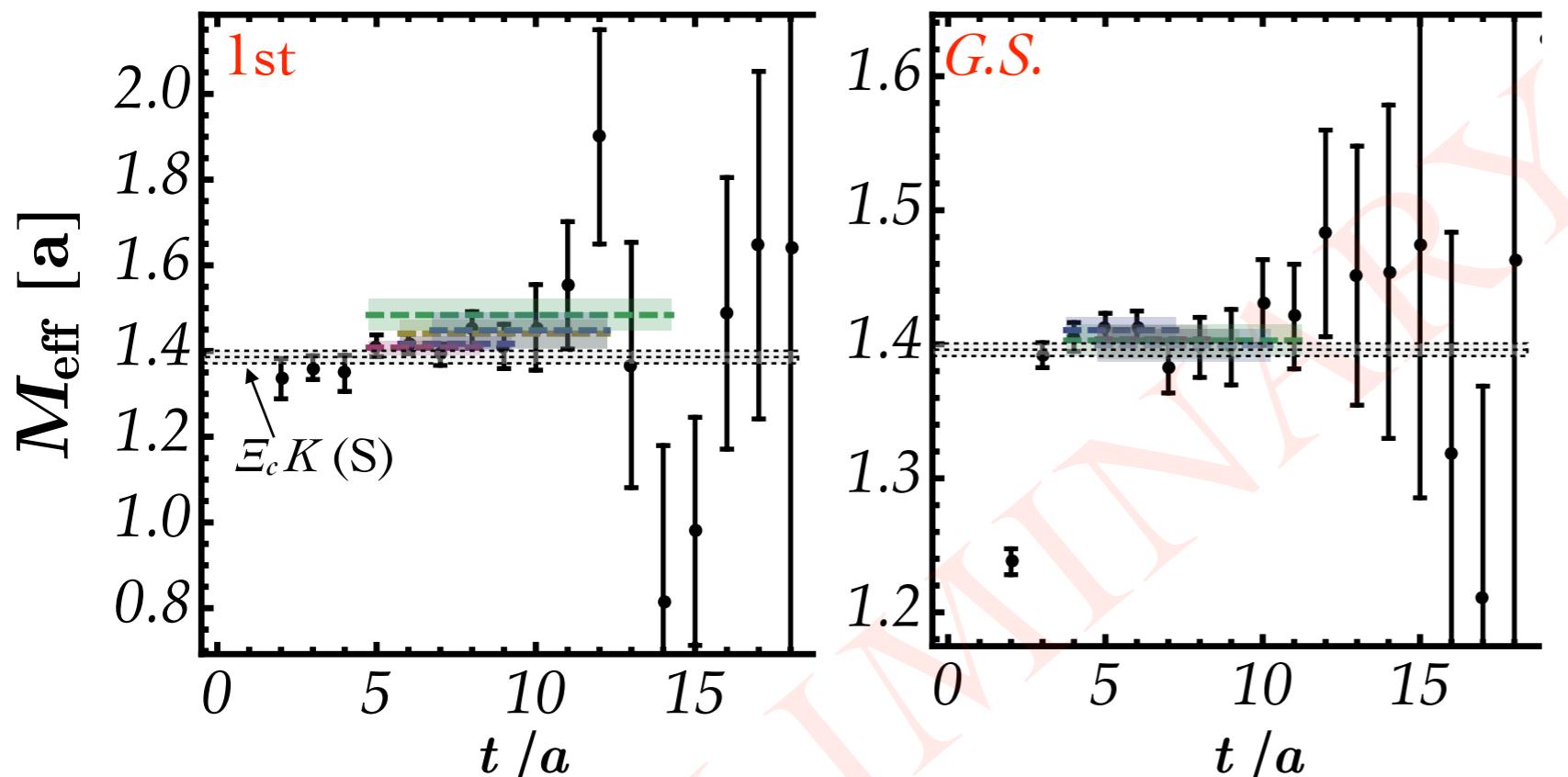
$\Delta M = 0.626 \text{ GeV}$ RQCD'15

	GS	1st
srcG	1_g5,g4g5_1 g5_1	1_g5,g4g5_1 g5_1
snkG	1_g5,g4g5_1 g5_1	1_g5,g4g5_1 g5_1
srcS	150	150
[ti,tf]	4,11	avg([5,8],[6,9])
Mass [GeV]	3.053(26)	3.074(35)(11)

Ω_C

#meas ~ 145

$J^P = 1/2^-$



	GS	1st
srcG	1_g5,g4g5_1 g5_1	1_g5,g4g5_1 g5_1
snkG	1_g5,g4g5_1 g5_1	1_g5,g4g5_1 g5_1
srcS	150	150
[ti,tf]	4,11	avg([5,8],[6,9])
Mass [GeV]	3.053(26)	3.074(35)(11)

$$\Xi_c K(S) = 3.001(28) \text{ GeV}$$

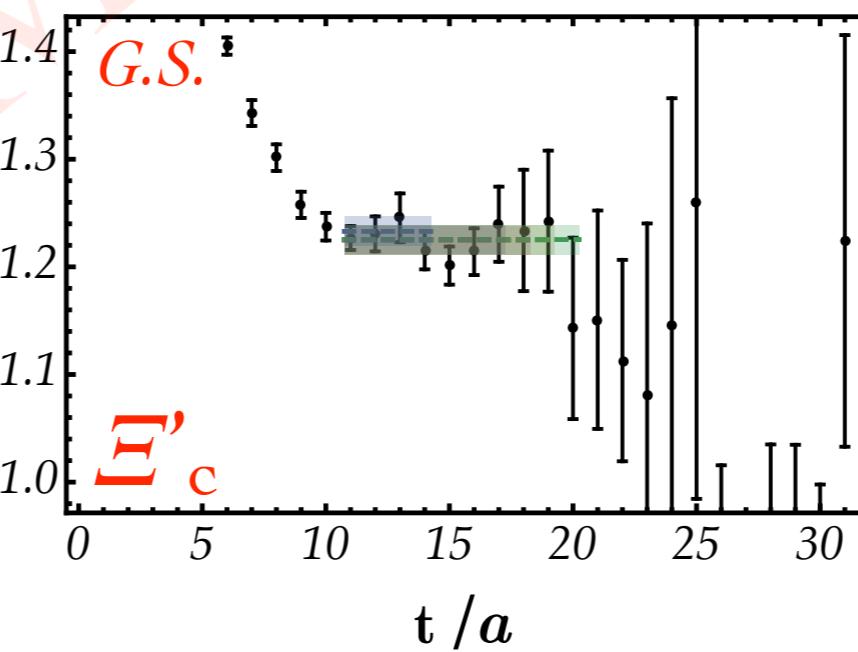
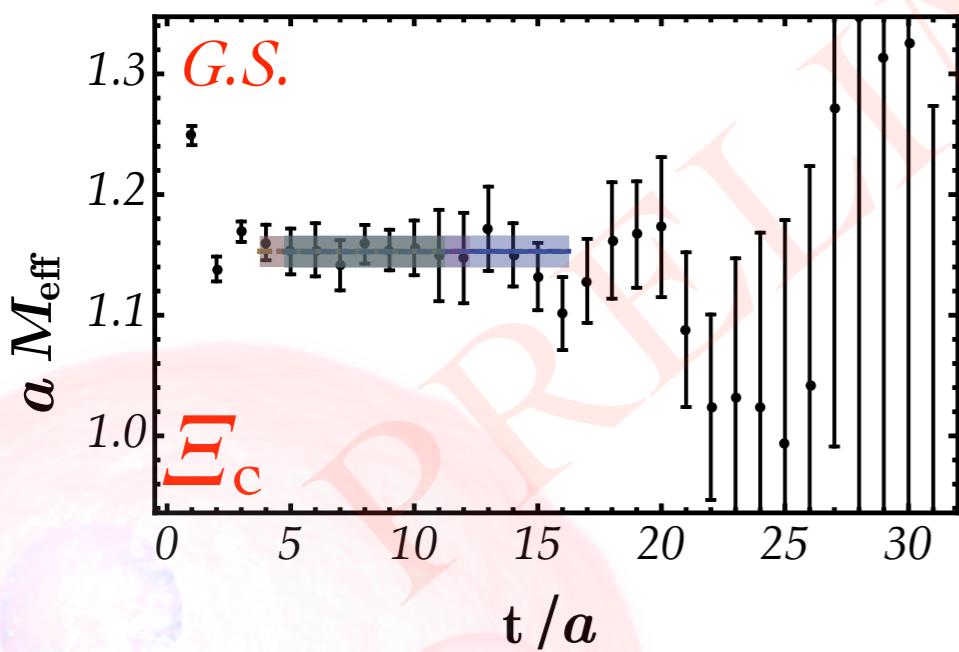
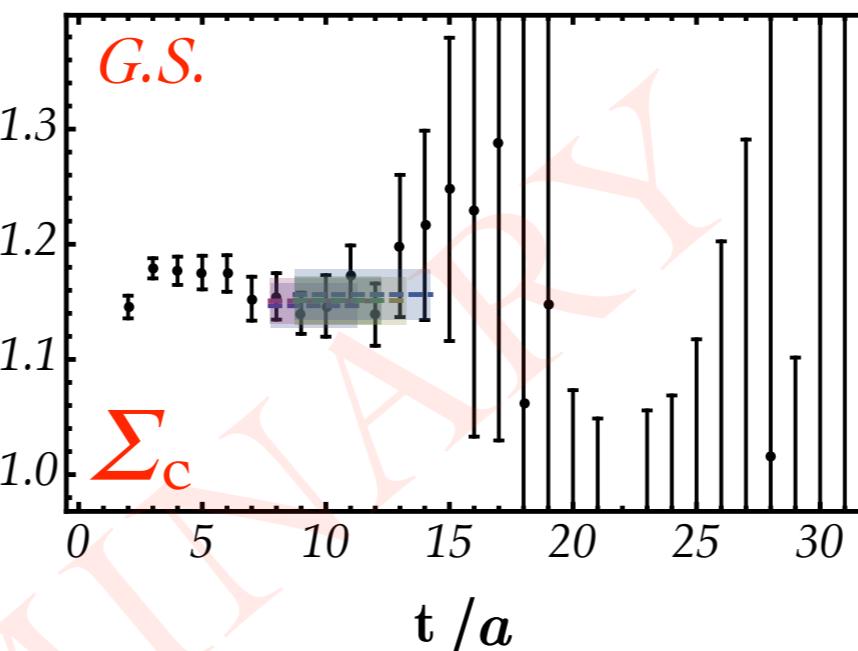
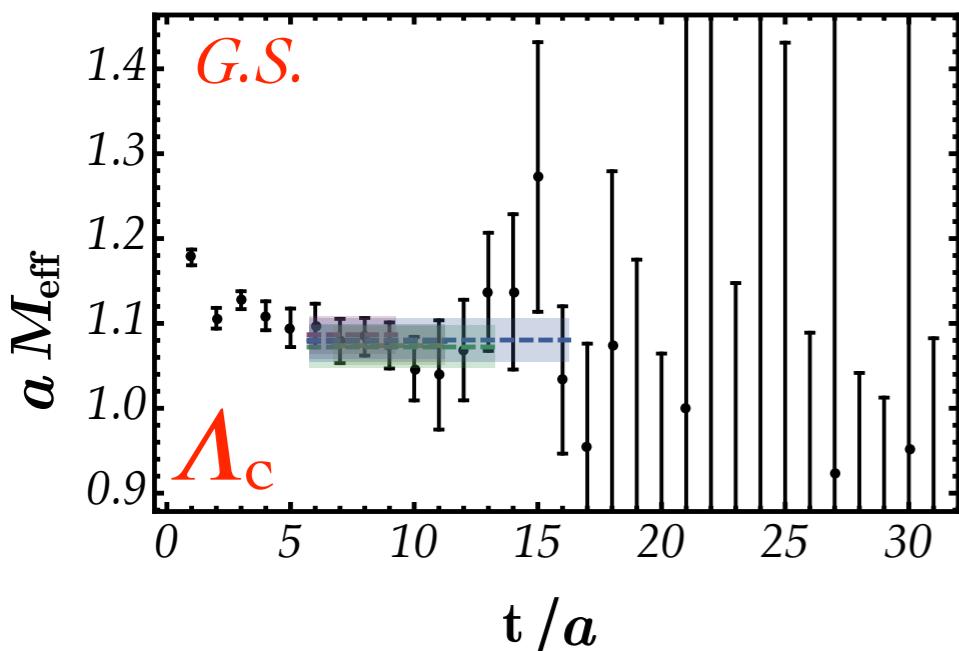


No speculations!



Needs more statistics and a closer, careful inspection

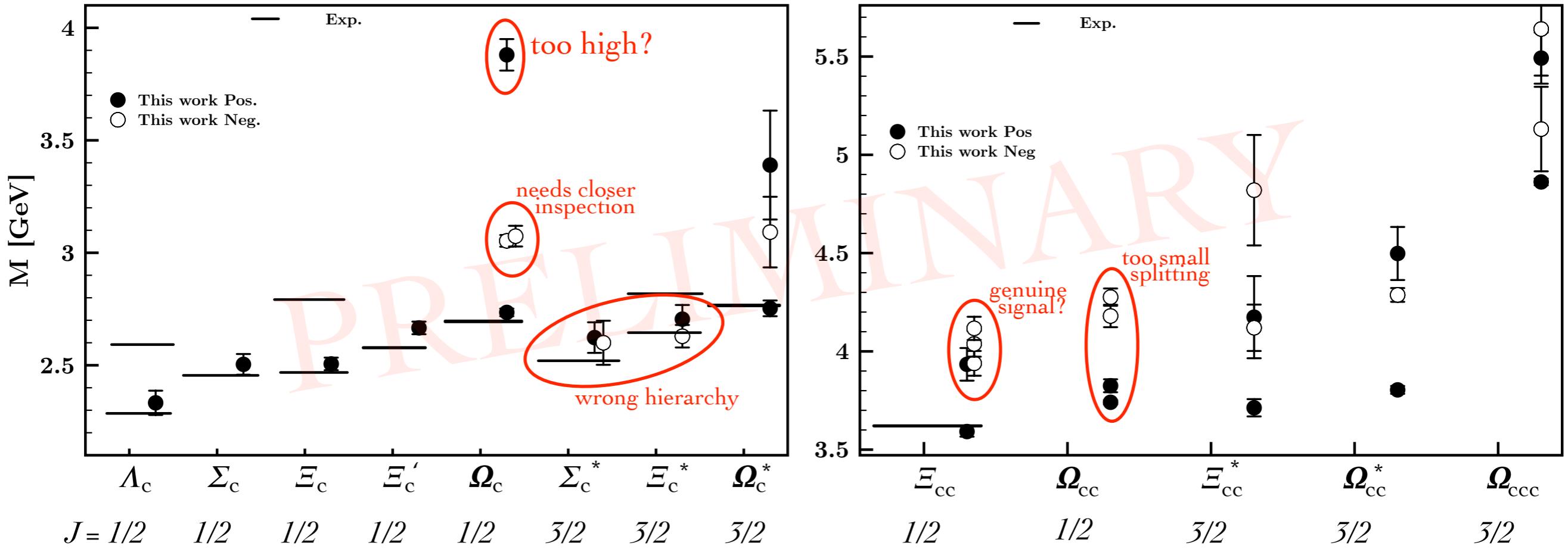
$\Lambda_c, \Sigma_c, \Xi_c \& \Xi'_c$

#meas ~ 100 $J^P = 1/2^+$ 

	GS - Λ_c	GS - Σ_c
srcG	$g5_1$	$g5_1$
snkG	$g5_1$	$g5_1$
srcS	150	150
[ti,tf]	6,13	8,12
Mass [GeV]	$2.333(54)$	$2.504(46)$

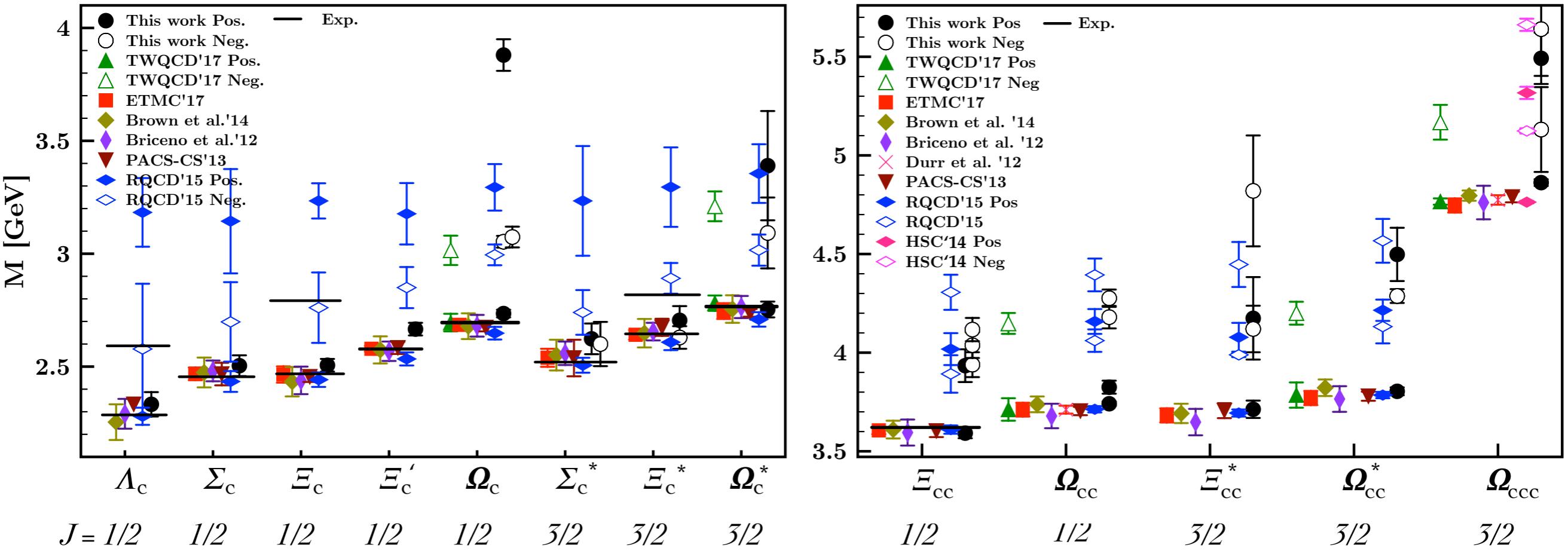
	GS - Ξ_c	GS - Ξ'_c
srcG	$g5_1$	$g5_1$
snkG	$g5_1$	$g5_1$
srcS	150	10
[ti,tf]	5,11	11,17
Mass [GeV]	$2.506(28)$	$2.666(28)$

Summary & Outlook



- ➊ Acquiring statistics
- ➋ Ground states seems fine
- ➌ Some neg. parity and excited signals does not make sense yet

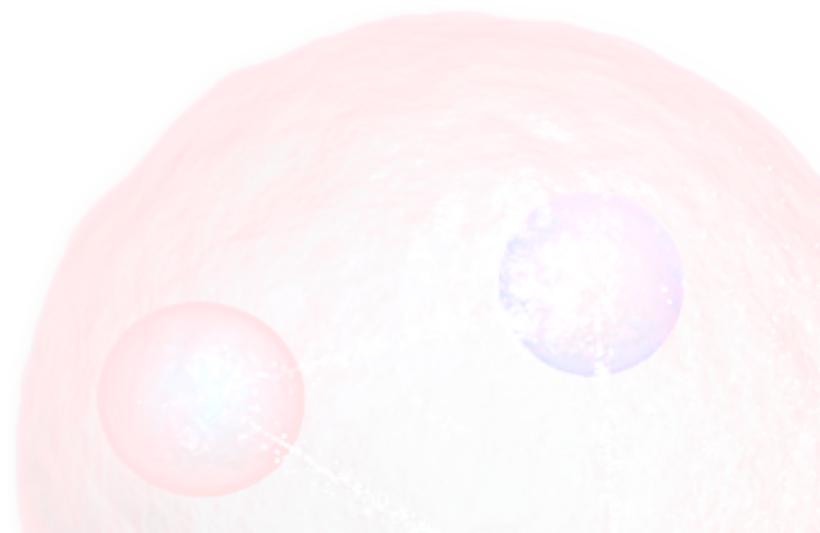
Summary & Outlook



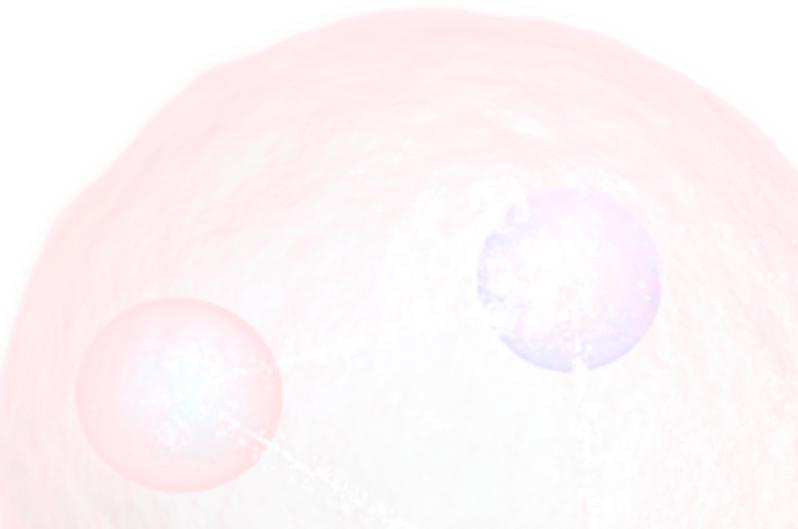
- Work in progress! Expect changes additions to these results

- Ground states are more or less consistent
- Negative parity and excited states need more work
- Room for improvement: Careful plateau id, two-expo. fits for stabilization, check for thresholds

Thank you!



Extras



More on goals

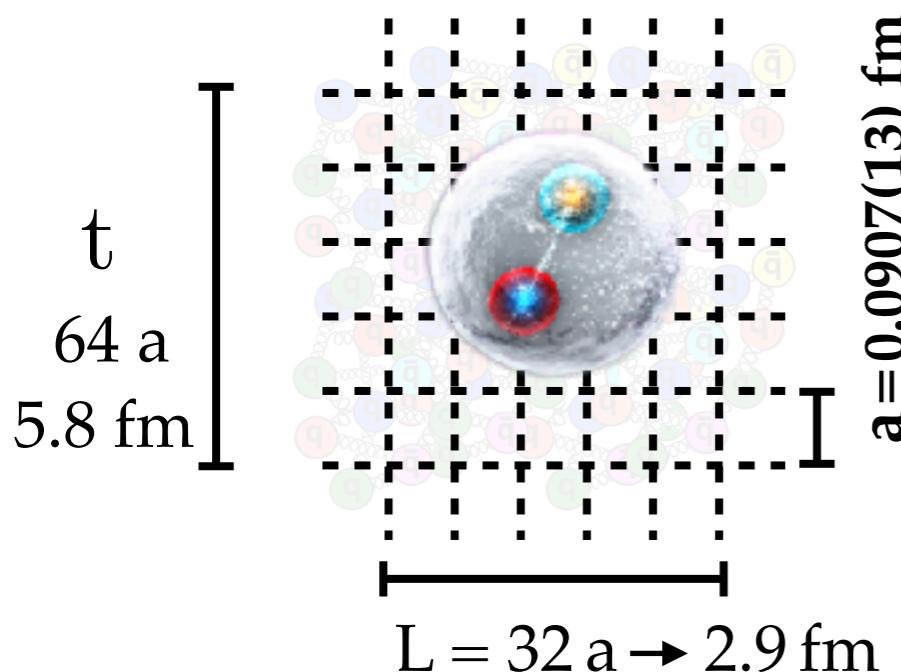
- ➊ Ultimate goal is to go for a pentaquark system
- ➋ Make sure we can produce a charm system properly
- ➌ Best consistency check is a spectrum calculation
 - ➍ Many results to compare with
 - ➎ (+) $m_\pi \sim 156$ MeV configurations
 - ➏ Eliminate systematic error from chiral extrapolation
 - ➏ (+) Use relativistic heavy quark action (Tsukuba action)
 - ➐ Eliminate systematic error from quark action
 - ➑ Our previous works done with Clover action for charm also
 - ➒ (+) Extract at least first excited states of positive and negative parity, spin-1/2 and spin-3/2 charmed baryons
 - ➓ (-) No finite size effects or continuum extrapolation

Tsukuba Action

PACS-CS, $32^3 \times 64$, 2+1 flavor (u/d+s) [Phys. Rev. D79 \(034503\)](#)

$\kappa_{ud} = 0.13781$ $\kappa_s = 0.13640$

$$m_\pi \sim 156 \text{ MeV}$$



- Tsukuba RHQA for charm quark
- Parameters adopted from [Namekawa et.al. PRD84 074505](#)
- κ_Q re-tuned w.r.t. dispersion relation of 1S spin-avg. charmonium mass (next slide)

$$\begin{aligned} D_{x,y} = & \delta_{xy} - \kappa_Q \\ & \times \sum_i [(r_s - \nu \gamma_i) U_{x,i} \delta_{x+\hat{i},y} + (r_s + \nu \gamma_i) U_{x,i}^\dagger \delta_{x,y+\hat{i}}] \\ & - \kappa_Q [(r_t - \gamma_4) U_{x,4} \delta_{x+\hat{4},y} + (r_t + \gamma_4) U_{x,4}^\dagger \delta_{x,y+\hat{4}}] \\ & - \kappa_Q \left[c_B \sum_{i,j} F_{ij}(x) \sigma_{ij} + c_E \sum_i F_{i4}(x) \sigma_{i4} \right] \delta_{xy}, \end{aligned}$$

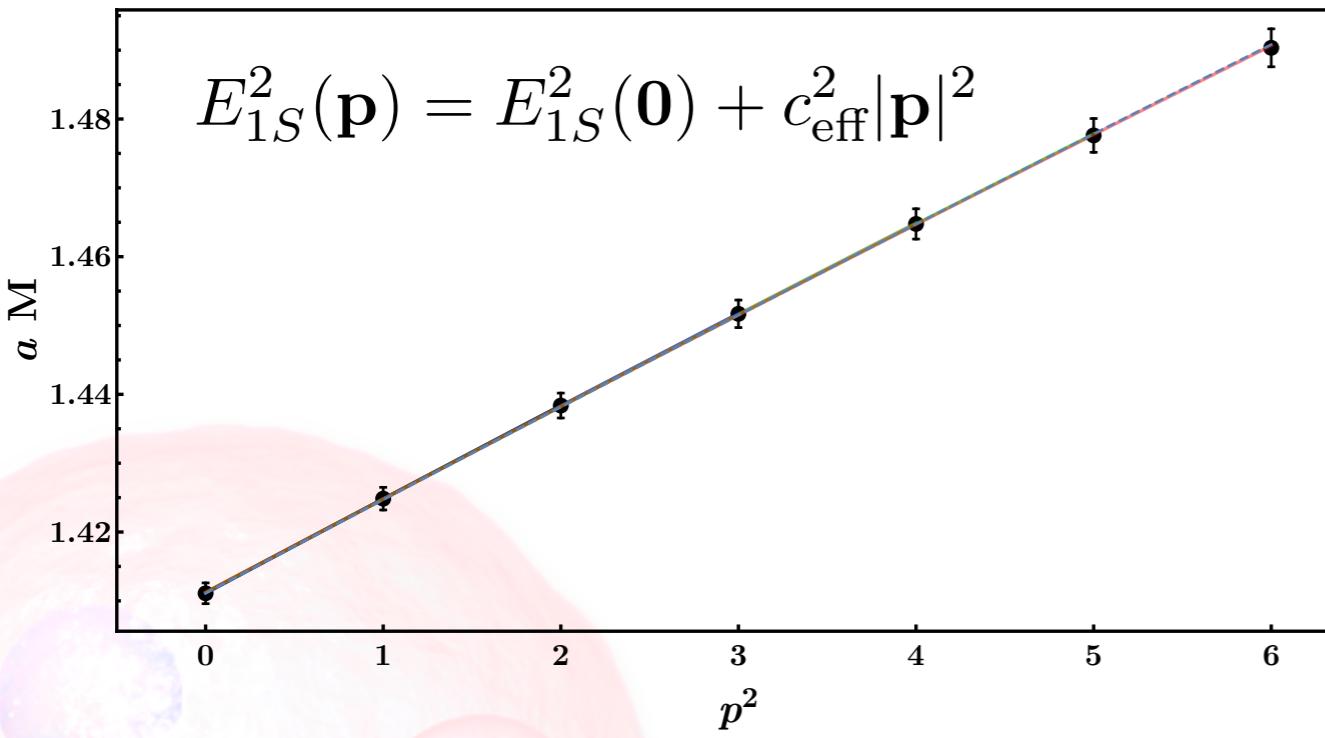
- | | |
|---------------------------|---------------------|
| • $\kappa_Q = 0.10954007$ | • $\nu = 1.1450511$ |
| • $r_s = 1.1881607$ | • $c_B = 1.9849139$ |
| • $r_t = 1.0$ | • $c_E = 1.7819512$ |

- $r_s=1, \nu=1, c_B=c_E \rightarrow O(a)$ -improved Wilson action

Dispersion relation

1S spin-avg: $(\eta_c + 3 J/\psi)/4$:
 $M_{1S} = 3.06853 \text{ GeV}$ (experimental)

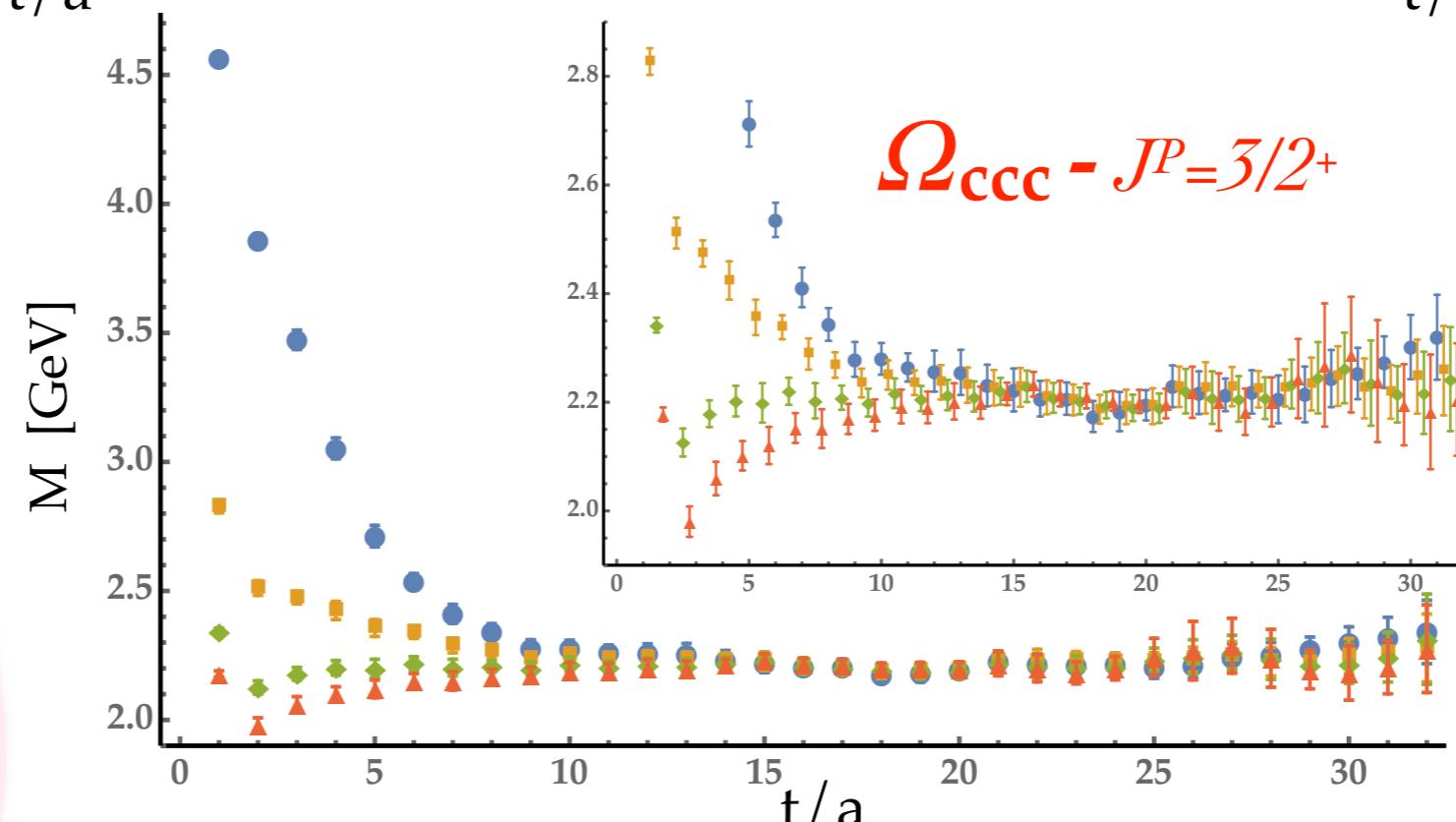
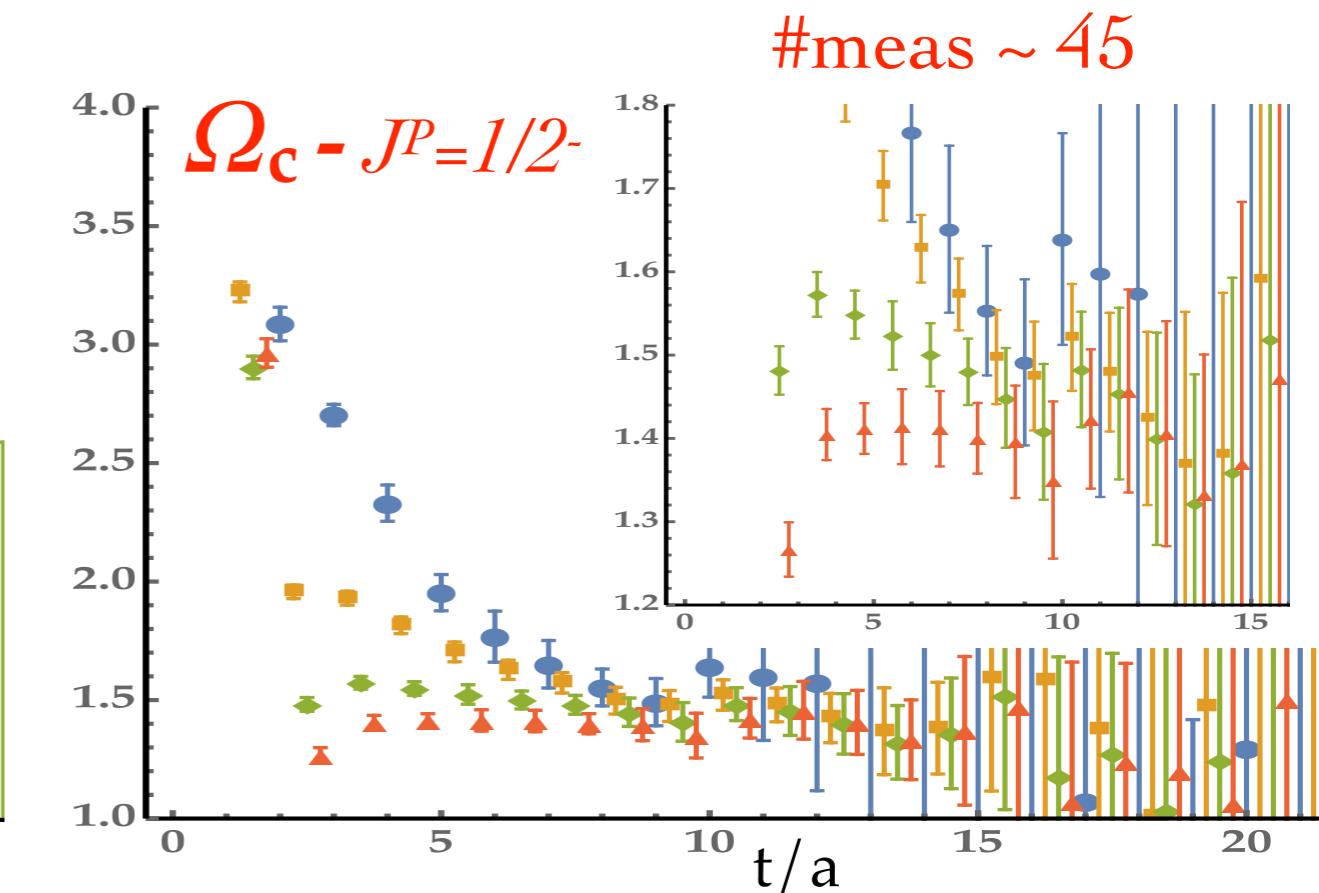
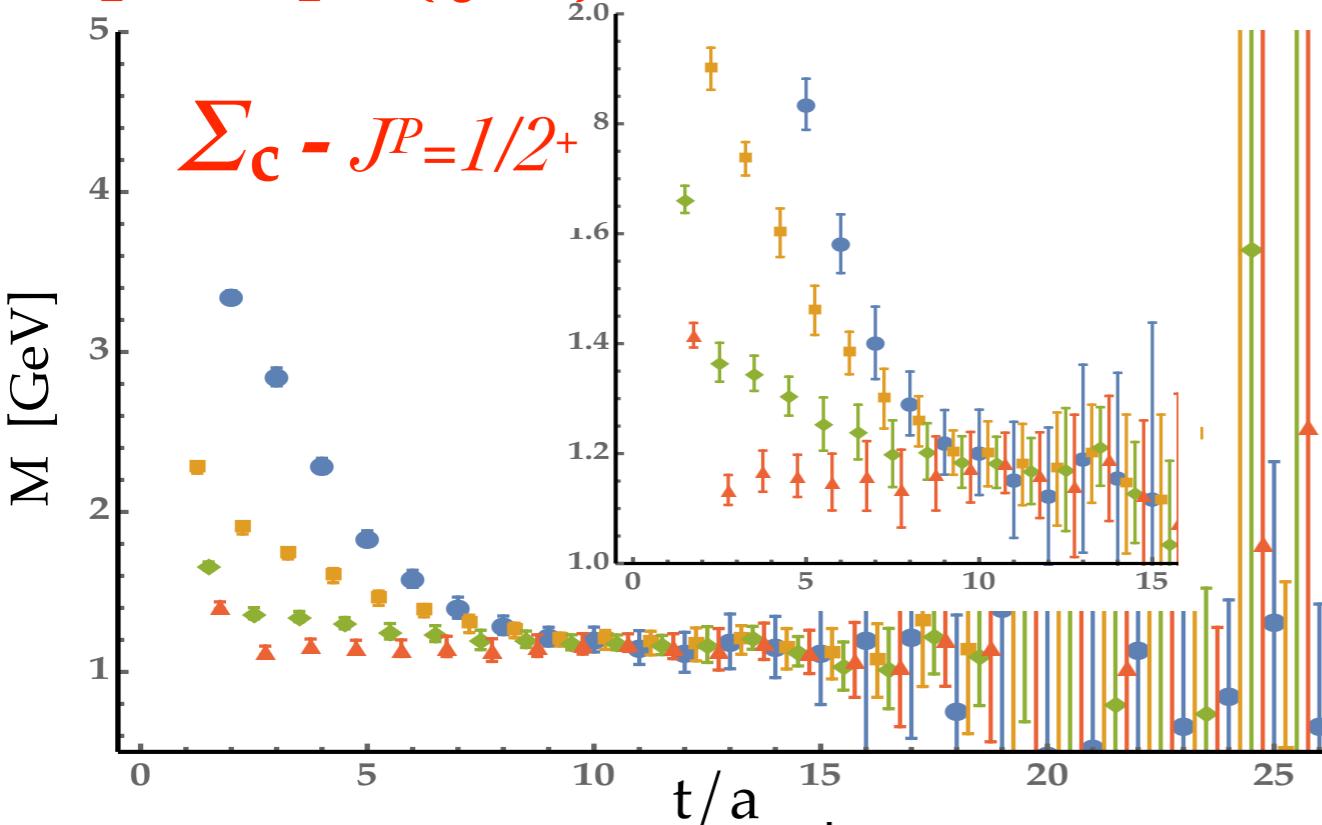
$ \mathbf{p} ^2$	$E_{1S}(\mathbf{0})$ [a]	$E_{1S}(\mathbf{0})$ [GeV]	c_{eff}^2
2	1.41111 ± 0.00150591	3.07058 ± 0.00327686	1.00818 ± 0.0159342
3	1.41113 ± 0.00150235	3.07063 ± 0.00326911	1.00538 ± 0.0169947
4	1.41117 ± 0.00149903	3.07071 ± 0.00326189	1.00186 ± 0.0175885
5	1.41122 ± 0.00149308	3.07082 ± 0.00324894	0.998545 ± 0.0185763
6	1.41127 ± 0.00148551	3.07092 ± 0.00323247	0.995832 ± 0.0197037



	M_{latt}	PDG	
η_c	2.984(2)	2.983	GeV
J/ψ	3.099(4)	3.096	GeV
1S	3.071(3)	3.068	GeV
ΔE_{hyp}	115(4)	113	MeV

Signal vs. smearing

$[\Gamma_1, \Gamma_2] = \{\gamma_5, 1\}$, SH-PT Ground state

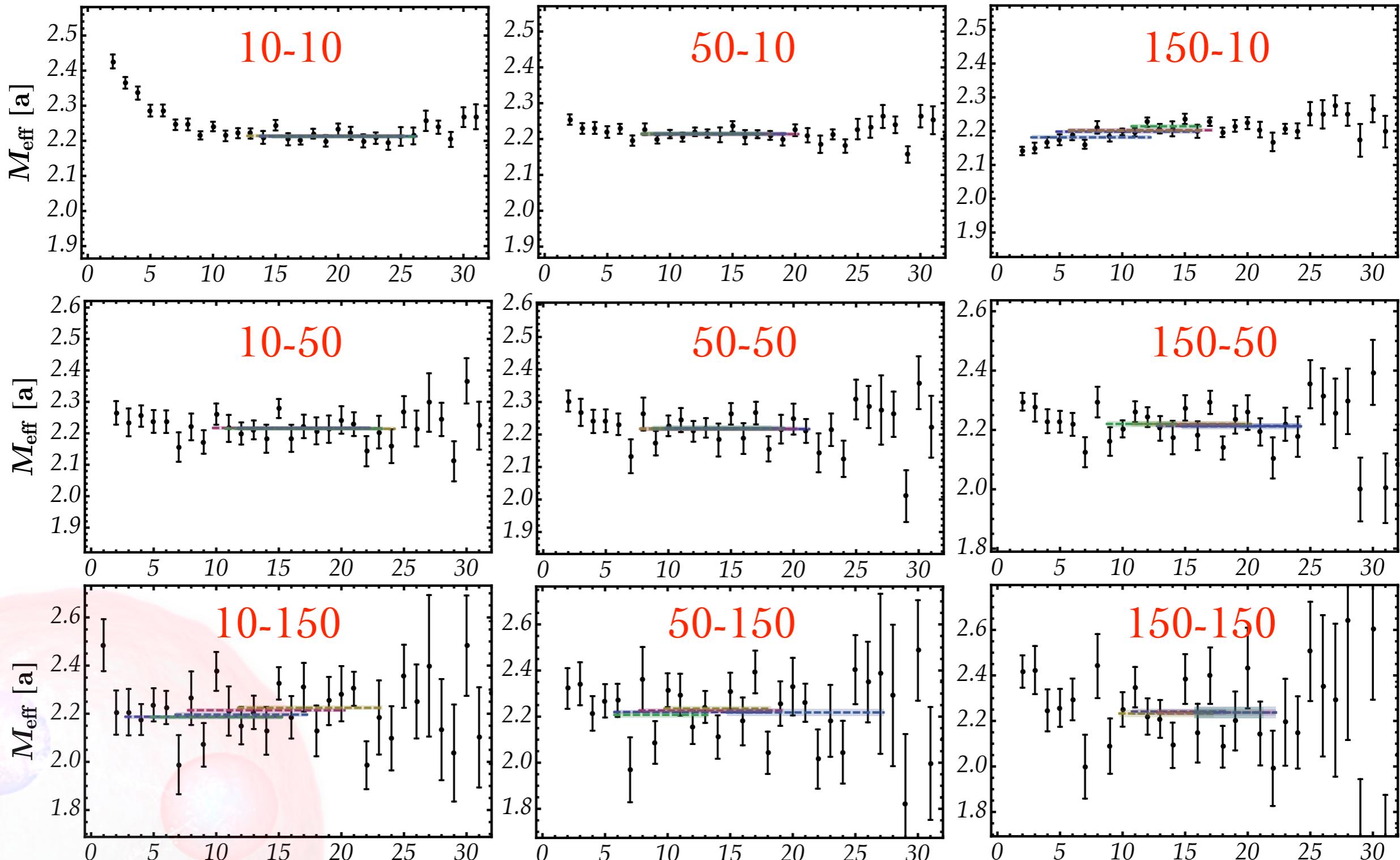


- pt_pt
- sh10_pt
- ◆ sh50_pt
- ▲ sh150_pt

Signal vs. smearing

$\Omega_{\text{ccc}} - J^P = 3/2^+$ SH-SH Ground state

#meas ~ 145



Variational Method

Consistency check

$\Omega_c \ J^P = 1/2^-$

srcG_{1_g5, g5_1}_snkG_{1_g5, g5_1} -- srcS_50_snkS_0

