A LATTICE STUDY OF MINIMAL COMPOSITE DARK MATTER

- motivating the model
- Lagrangian and symmetries
- anticipating the mass spectrum
- lattice methods
- lattice results
- implications for phenomenology

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Baryonic matter gets about 99% of its mass from QCD.
The proton is stable due to an accidental $U(1)_B$ in the Standard Model.

Dark matter might also arise from a non-abelian gauge theory.
The lightest baryon could be stable due to an accidental $U(1)_B$.

What is the minimal such model?

- smallest non-abelian gauge theory: $SU(2)$
- fewest fermions: 1 dark quark
- smallest fermion representation: the fundamental
- no renormalizable couplings to the Standard Model.

Is the dark baryon stable enough to be dark matter?

In general, SU(2) permits decay via $\mathcal{L} = \frac{1}{\Lambda}\bar{q}q|H|^2$.
As we'll see, the 1-flavor case is special, having no such problem.

The Lagrangian of this model,

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \mathcal{L}_{\mathrm{fermion}} + \mathcal{L}_{\mathrm{higher\ dim}}\,,$$

has a global $U(2)$ symmetry because the gauge theory is pseudo-real.

$$\begin{aligned} \mathcal{L}_{\mathrm{fermion}} &= \bar{q}(i\not{D} - m)q \\ &= \bar{\mathcal{Q}} i\not{D}\mathcal{Q} - \frac{m}{2}\left(\mathcal{Q}^T i\sigma^2 C E \mathcal{Q} + \bar{\mathcal{Q}} i\sigma^2 C E \bar{\mathcal{Q}}^T\right) \end{aligned}$$

where C is the charge conjugation matrix, σ^2 is a color Pauli, and

$$\mathcal{Q} = \begin{pmatrix} q_L \\ -i\sigma^2 C \bar{q}_R^T \end{pmatrix}, \qquad E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The global $U(2) = SU(2)_B \times U(1)_A$ has an anomalous axial $U(1)_A$.

The baryonic isospin $SU(2)_B$ remains unbroken.

Therefore there will be no Goldstone bosons.

Operators for scalar, pseudoscalar, vector and axial vector mesons are

$$\begin{aligned}
\bar{q}q &= \tfrac{1}{2}\left(\mathcal{Q}^T i\sigma^2 CE\mathcal{Q} + \bar{\mathcal{Q}} i\sigma^2 CE\bar{\mathcal{Q}}^T\right) \\
\bar{q}\gamma_5 q &= -\tfrac{1}{2}\left(\mathcal{Q}^T i\sigma^2 CE\mathcal{Q} - \bar{\mathcal{Q}} i\sigma^2 CE\bar{\mathcal{Q}}^T\right) \\
\bar{q}\gamma^\mu q &= \bar{\mathcal{Q}}\gamma^\mu\tau^3\mathcal{Q} \qquad \Rightarrow \text{part of an isotriplet} \\
\bar{q}\gamma^\mu\gamma^5 q &= \bar{\mathcal{Q}}\gamma^\mu\mathcal{Q}
\end{aligned}$$

where τ^3 is an isospin Pauli.

The dark matter candidate is the baryon, coupling to $\bar{\mathcal{Q}}\gamma^\mu\tau^\pm\mathcal{Q}$.
It has no dimension 5 decay, so it is cosmologically allowed.*

More than 1 flavour would have given *spin-zero* baryons that can decay at dim 5, eg. $\mathcal{L} = \frac{1}{\Lambda}\bar{q}q|H|^2$, which is a cosmological problem.*

*Appelquist et al., PRD92 (2015) 075030

We use the Wilson action with $\beta = \frac{4}{g^2} = 2.2$

and $\kappa = (8 + 2m)^{-1}$ = several values.

The main study is $12^3 \times 32$ with $>$1000 configurations per ensemble.

We also use $12^3 \times 48$ for some topics.

We use the RHMC algorithm from the HiRep code.

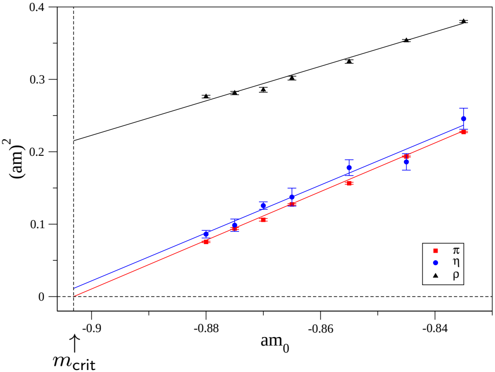
Disconnected quark loops use an unbiased stochastic estimator.

`Del Debbio, Patella, Pica, PRD81 (2010) 094503`

We use Z_2 stochastic wall sources (Z2SEMWall).

`Boyle et al., JHEP 08 (2008) 086`

THE MASSLESS QUARK LIMIT (method #1)



π is not a physical particle but it identifies the massless limit by $m_\pi^2 = 0$.
We can define a shifted bare quark mass $m_q \equiv m_0 - m_{\text{crit}}$.

Topological susceptibility, χ, vanishes as $m_q \to 0$.
χ is a gauge field quantity in the theory's integration measure.
A Euclidean lattice definition is

$$\chi = \frac{\langle Q^2 \rangle}{L^3 T}, \quad Q = \sum_x q(x), \quad q(x) = -\frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left[F_{\mu\nu}(x) F_{\rho\sigma}(x) \right]$$

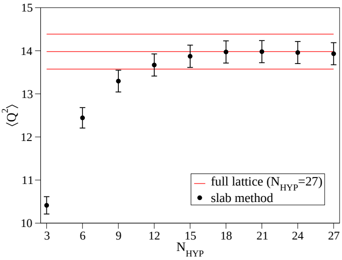
Statistical noise is reduced by the "slab method".

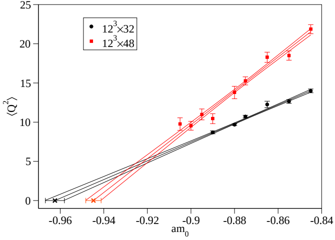
Bietenholz,deForcrand,Gerber,JHEP12(2015)070
JLQCD(Aoki et al.)PTEP2018(2018)043B07

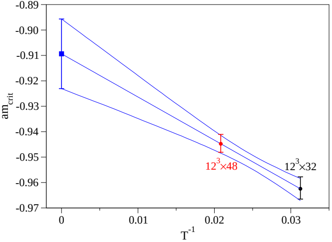
Temporal slabs: $\frac{T}{8}$ <width< $\frac{T}{2}$.
Spatial slabs: $\frac{L}{3}$ <width< $\frac{2L}{3}$.

We also use HYP smearing.

Hasenfratz,Knechtli,PRD64(2001)034504





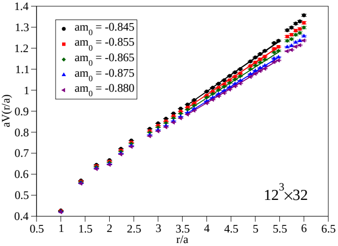


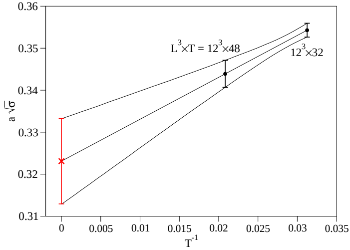
↑ ↑ m_{crit} for each lattice

↑ extrapolated m_{crit}

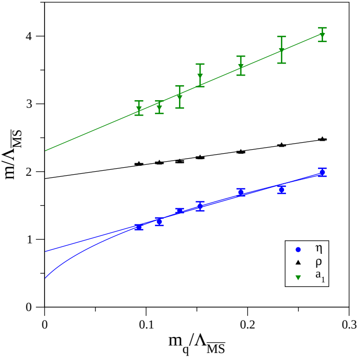
This agrees with the result from method #1.

We calculate the static potential, extract the string tension, then obtain the scale: $\Lambda_{\overline{\mathrm{MS}}} = 0.7712\sqrt{\sigma}$. `Bali Phys.Rep.343(2001)1`





Conclusion: $a\sqrt{\sigma} = 0.323(10) \quad \Rightarrow \quad a\Lambda_{\overline{\mathrm{MS}}} = 0.249(8)$.



Masses are obtained from standard correlator fits.

ρ is the dark matter candidate.

It has no disconnected diagrams.

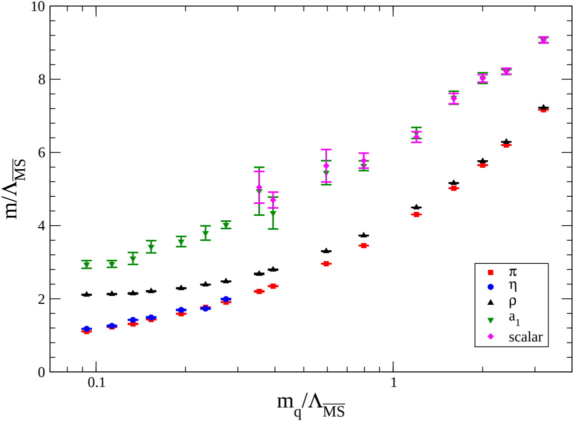
Lattice data for ρ are precise.

For a light quark, $m_\rho \sim 2\Lambda_{\overline{\mathsf{MS}}}$.

η fits are $m_\eta = c_0 + c_1 m_q$ and

$$m_\eta^2 = c_0 + c_1 m_q.$$

The expected degeneracies emerge at large m_q.



HADRON DECAY CONSTANTS

f_η is defined by

$$Z_A \langle 0|A_t(0)|P\rangle = f_\eta m_\eta$$

Fitting lattice data to

$$C_{A_t P}(t) = AB\left(e^{-m_\eta t} - e^{-m_\eta (T-t)}\right)$$
$$C_{PP}(t) = B^2\left(e^{-m_\eta t} + e^{-m_\eta (T-t)}\right)$$

gives the decay constant

$$f_\eta = Z_A A (2\kappa)^2 \sqrt{\frac{2}{m_\eta L^3}}$$

f_ρ is defined by

$$Z_V \langle 0|V_i(0)|V\rangle = f_\rho m_\rho \epsilon_i$$

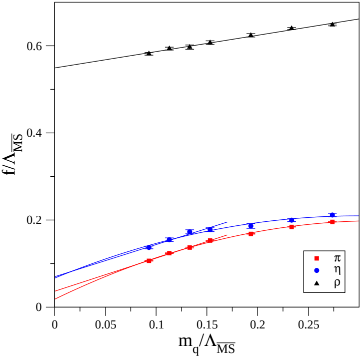
Fitting lattice data to

$$C_{\sigma_t V}(t) = CD\left(e^{-m_\rho t} - e^{-m_\rho (T-t)}\right)$$
$$C_{VV}(t) = D^2\left(e^{-m_\rho t} + e^{-m_\rho (T-t)}\right)$$

gives the decay constant

$$f_\rho = Z_V D (2\kappa)^2 \sqrt{\frac{2}{m_\rho L^3}}$$

One-loop perturbative expressions for Z_V and Z_A are taken from `Del Debbio et al.JHEP06(2008)007.`



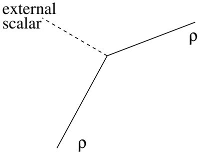
The data for f_ρ are linear.

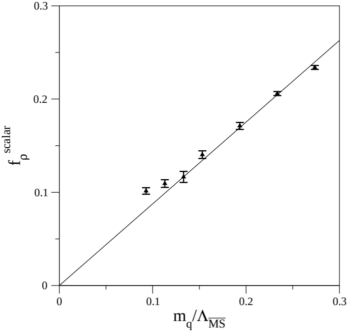
Here, 2 fit options are shown:

$f = c_0 + c_1 m_0$ for small m_0,

$f = c_0 + c_1 m_0 + c_2 m_0^2$.

The dark matter candidate and Higgs may interact by a scalar coupling.





The Feynman-Hellman theorem leads to a dimensionless ratio:

$$f_\rho^{\text{scalar}} = \frac{m_q}{m_\rho}\frac{\partial m_\rho}{\partial m_q}$$

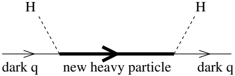
See, for example,

`Junnarkar,Walker-Loud,PRD87(2013)114510`

`Detmold,McCullough,Pochinsky,PRD90(2014)114506`

- The dark hadrons that are stable under dark SU(2) are
  η = lightest meson
  ρ = lightest baryon

- Dark hadrons have no renormalizable couplings to the Higgs.
  Higgs $\to \eta\eta$ might occur if $2m_\eta < m_H$. (Also Higgs $\to \rho\rho$.)
  However, the coupling to the Higgs should be indirect.
  Example:



- Dark hadron decay rates:
  η can decay at dim 5, for example $\mathcal{L} = \frac{1}{\Lambda}\bar{q}q|H|^2$.
  This allows $\tau_\eta \lesssim$ 1 second, evading BBN constraints.
  ρ can decay at dim 6, for example $\mathcal{L} = \frac{1}{\Lambda^2}\bar{q}\gamma_\mu q H \nabla^\mu H$.
  This allows $\tau_\rho \gtrsim 10^{24}$ years, making ρ a dark matter candidate.

This model has another nice feature:
The dark matter relic density is set naturally, through freeze-out.

At early times (high temperatures): $\rho\bar{\rho} \rightleftharpoons \eta\eta$

At late times (low temperatures): $\rho\bar{\rho} \to \eta\eta$

Recall $\tau_\eta \lesssim 1$ second.

This freeze-out mechanism

- works because $m_\rho > m_\eta$, as the lattice results show.
- is special to the 1-flavour theory because ρ is the dark matter.

SUMMARY AND OUTLOOK

Is dark matter the lightest hadron in a non-abelian gauge theory?

Summary:

- The minimal model is SU(2) with 1 fundamental fermion.
- With only 1 fermion, the dark matter particle is spin 1, not spin 0.
- Lattice calculations give hadron masses, decay constants, scalar couplings, and the scale $\Lambda_{\overline{\mathrm{MS}}}$.
- This minimal model can satisfy phenomenological constraints.
- Dark matter freeze-out is natural.

Possible future work:

- extended lattice study, with multiple lattice spacings and volumes.
- study of dark hadron scattering and dark nuclear physics.

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