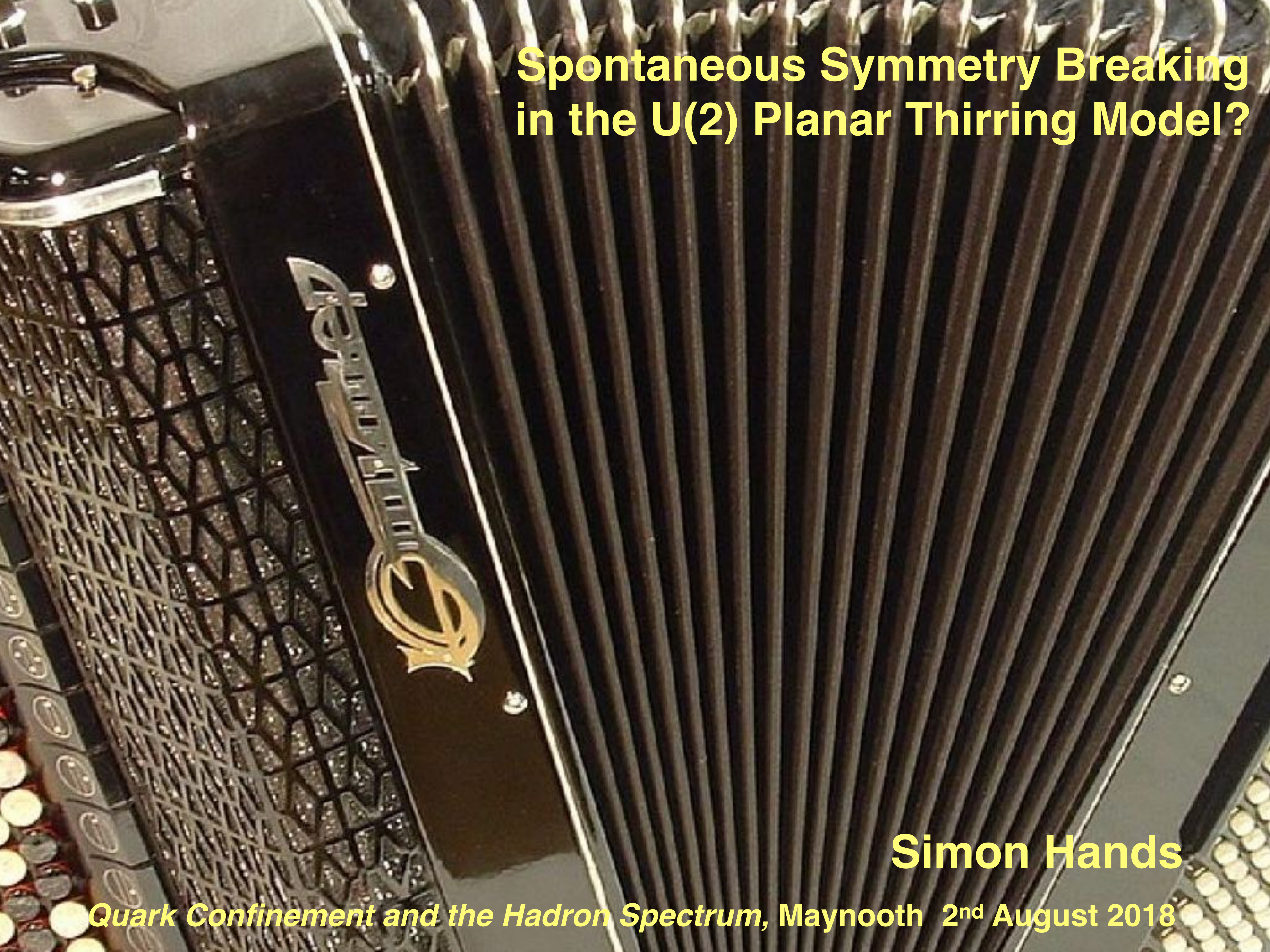
Spontaneous Symmetry Breaking in the U(2) Planar Thirring Model?



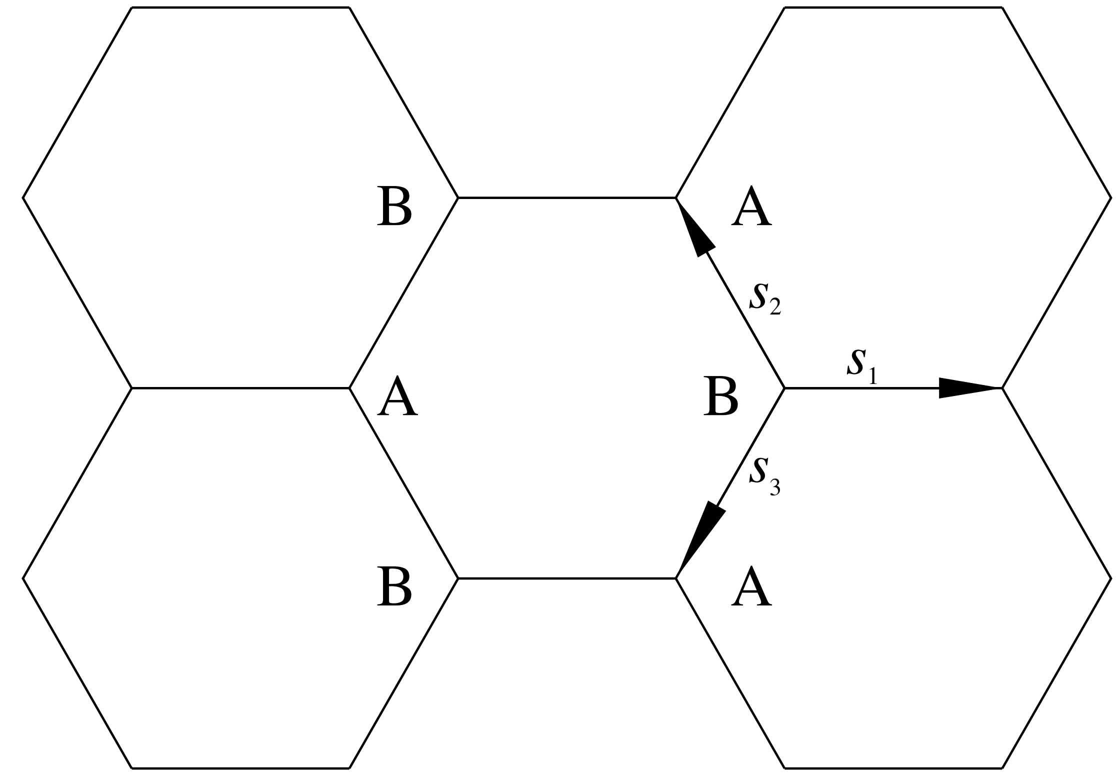
Simon Hands

Quark Confinement and the Hadron Spectrum, Maynooth 2nd August 2018

In this talk I will

- discuss quantum field theories of relativistic fermions in 2+1d focussing on the U(2N)-invariant Thirring model
- review critically old simulation results for QCPs obtained with staggered lattice fermions
- show that domain wall fermions capture the relevant global symmetries more accurately
- present simulation results showing that DWF tell a very different story to staggered

Relativistic Fermions in 2+1d



Several applications
in condensed matter physics

- Nodal fermions in *d*-wave superconductors
- Spin liquids in Heisenberg AFM
- surface states of topological insulators
- ….and graphene

Free reducible fermions in 3 spacetime dimensions

$$\mathcal{S} = \int d^3x \; \bar{\Psi}(\gamma_\mu \partial_\mu)\Psi \; + \; m\bar{\Psi}\Psi$$

$$\mu = 0, 1, 2$$
$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$$
$$\mathrm{tr}(\gamma_\mu \gamma_\mu) = 4$$

For $m=0$ S is invariant under global U(2N) symmetry generated by

(i) $\Psi \mapsto e^{i\alpha}\Psi\,; \quad \bar{\Psi} \mapsto \bar{\Psi}e^{-i\alpha},$ (ii) $\Psi \mapsto e^{i\alpha\gamma_5}\Psi\,; \quad \bar{\Psi} \mapsto \bar{\Psi}e^{i\alpha\gamma_5}$

(iii) $\Psi \mapsto e^{\alpha\gamma_3\gamma_5}\Psi\,; \quad \bar{\Psi} \mapsto \bar{\Psi}e^{-\alpha\gamma_3\gamma_5}$ (iv) $\Psi \mapsto e^{i\alpha\gamma_3}\Psi\,; \quad \bar{\Psi} \mapsto \bar{\Psi}e^{i\alpha\gamma_3}$

For $m\neq0$ γ_3 and γ_5 rotations no longer symmetries

$\Rightarrow$ U(2N) → U(N)⊗U(N)

Mass term $m\bar{\Psi}\Psi$ is hermitian & invariant under parity $x_\mu \mapsto -x_\mu$

Two physically equivalent antihermitian
“twisted” or “Kekulé” mass terms: $im_3\bar{\Psi}\gamma_3\Psi; \quad im_5\bar{\Psi}\gamma_5\Psi$

The “Haldane” mass $m_{35}\bar{\Psi}\gamma_3\gamma_5\Psi$ is not parity-invariant

The Thirring Model in 2+1d

four-fermi form $$\mathcal{L} = \bar\psi_i(\partial\!\!\!/ + m)\psi_i + \frac{g^2}{2N_f}(\bar\psi_i\gamma_\mu\psi_i)^2$$

bosonised form $$\mathcal{L} = \bar\psi_i(\partial\!\!\!/ + i\frac{g}{\sqrt{N_f}}A_\mu\gamma_\mu + m)\psi_i + \frac{1}{2}A_\mu A_\mu$$

- Interacting QFT
- expansion in g^2 non-renormalisable
- Hidden Local Symmetry $\psi \mapsto e^{i\alpha}\psi;\ \ A_\mu \mapsto A_\mu + \partial_\mu\alpha;\ \ \varphi \mapsto \varphi + \alpha$

  if Stückelberg scalar field φ introduced
- expansion in $1/N_f$ exactly renormalisable for $2<d<4$

  $\langle A_\mu A_\nu\rangle \propto \delta_{\mu\nu}/k^{d-2}$ in "Feynman gauge" SJH PRD**51** (1995) 5816
- dynamical chiral symmetry breaking for $g^2 > g_c^2$; $N_f < N_{fc}$?
- Quantum Critical Point at $g_c^2(N<N_{fc})$?

Determination of N_{fc} is a non-perturbative problem in QFT

eg. $N_{fc}=4.32$ strong coupling Schwinger-Dyson (ladder approximation)

Itoh, Kim, Sugiura & Yamawaki Prog. Theor. Phys. **93** (1995) 417

Numerical Lattice Approach

Del Debbio, SJH, Mehegan
NP**B502** (1997) 269; **B552** (1999) 339

Early work used staggered fermions

$$S_{latt} = \frac{1}{2}\sum_{x\mu i} \bar{\chi}_x^i \eta_{\mu x}(1 + iA_{\mu x})\chi_{x+\hat{\mu}}^i - \bar{\chi}_x^i \eta_{\mu x}(1 - iA_{\mu x-\hat{\mu}})\chi_{x-\hat{\mu}}^i$$

$$+ \; m\sum_{xi} \bar{\chi}_x^i \chi_x^i + \frac{N}{4g^2}\sum_{x\mu} A_{\mu x}^2$$

auxiliary boson couples linearly

resembles abelian gauge theory, but link field is **NOT** unit modulus!

$A_{\mu x}$ auxiliary vector field defined on **link** between x and $x+\mu$

$$\eta_{\mu x} \equiv (-1)^{x_0 + \cdots + x_{\mu-1}} \Rightarrow \prod_{\square} \eta\eta\eta\eta = -1$$

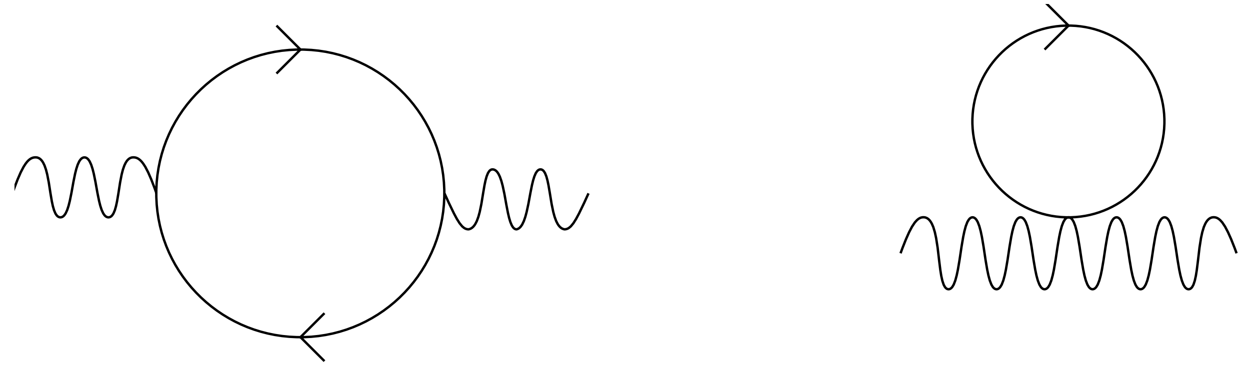
π-flux

Chiral symmetry: U(N)⊗U(N) → U(N) (if $m, \Sigma \neq 0$)

In **weak** coupling continuum limit
$U(2N_f)$ symmetry is recovered, with $N_f = 2N$

Strong coupling limit $g^2 \rightarrow \infty$

The lattice regularisation does *not* respect current conservation



Both diagrams needed to ensure transversity,
(ie. WT identity $\sum_\mu \left[\Pi_{\mu\nu}(x) - \Pi_{\mu\nu}(x-\hat{\mu})\right] = 0$) in lattice QED

$\Rightarrow$ $1/N_f$ expansion yields **additive** renormalisation of g^{-2}

$$g_R^2 = \frac{g^2}{1 - g^2/g_{\lim}^2}$$

$\Rightarrow$ lattice strong coupling limit as $g^2 \rightarrow g_{\lim}^2(N_f)$

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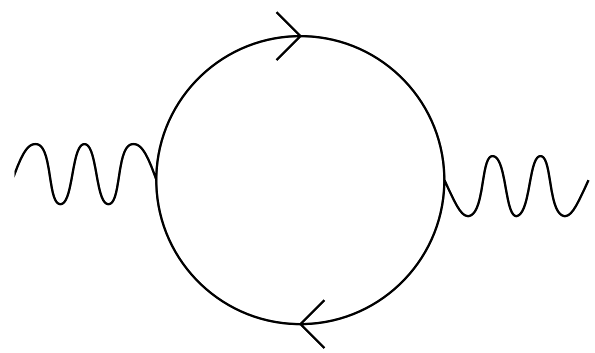

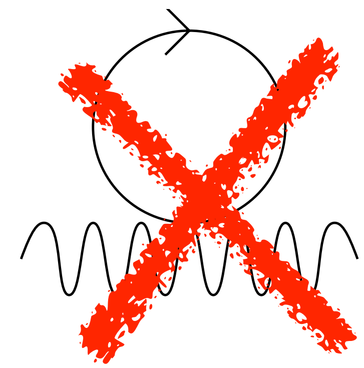

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Only the left hand diagram is present for the lattice Thirring model with linear coupling to auxiliary

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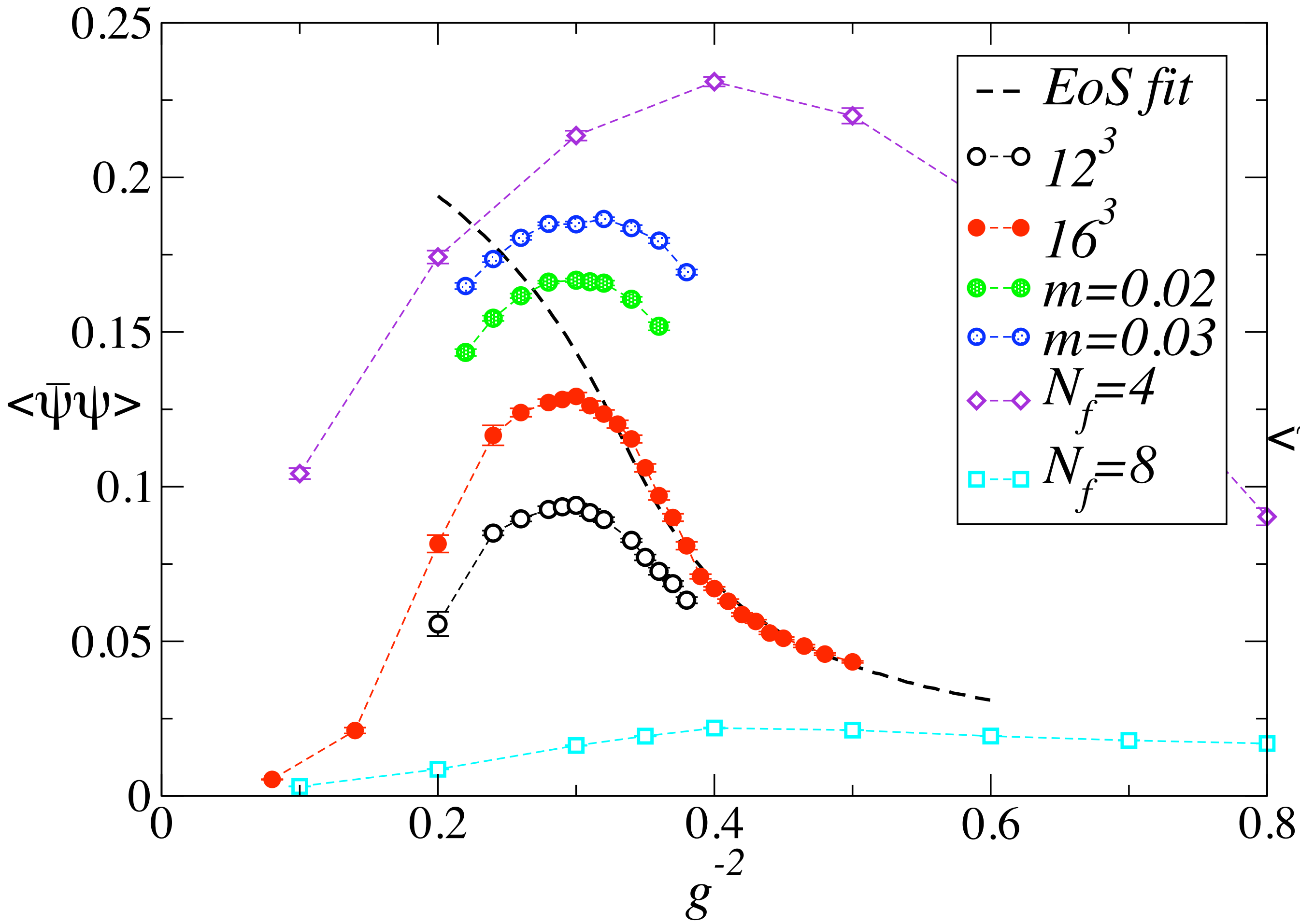
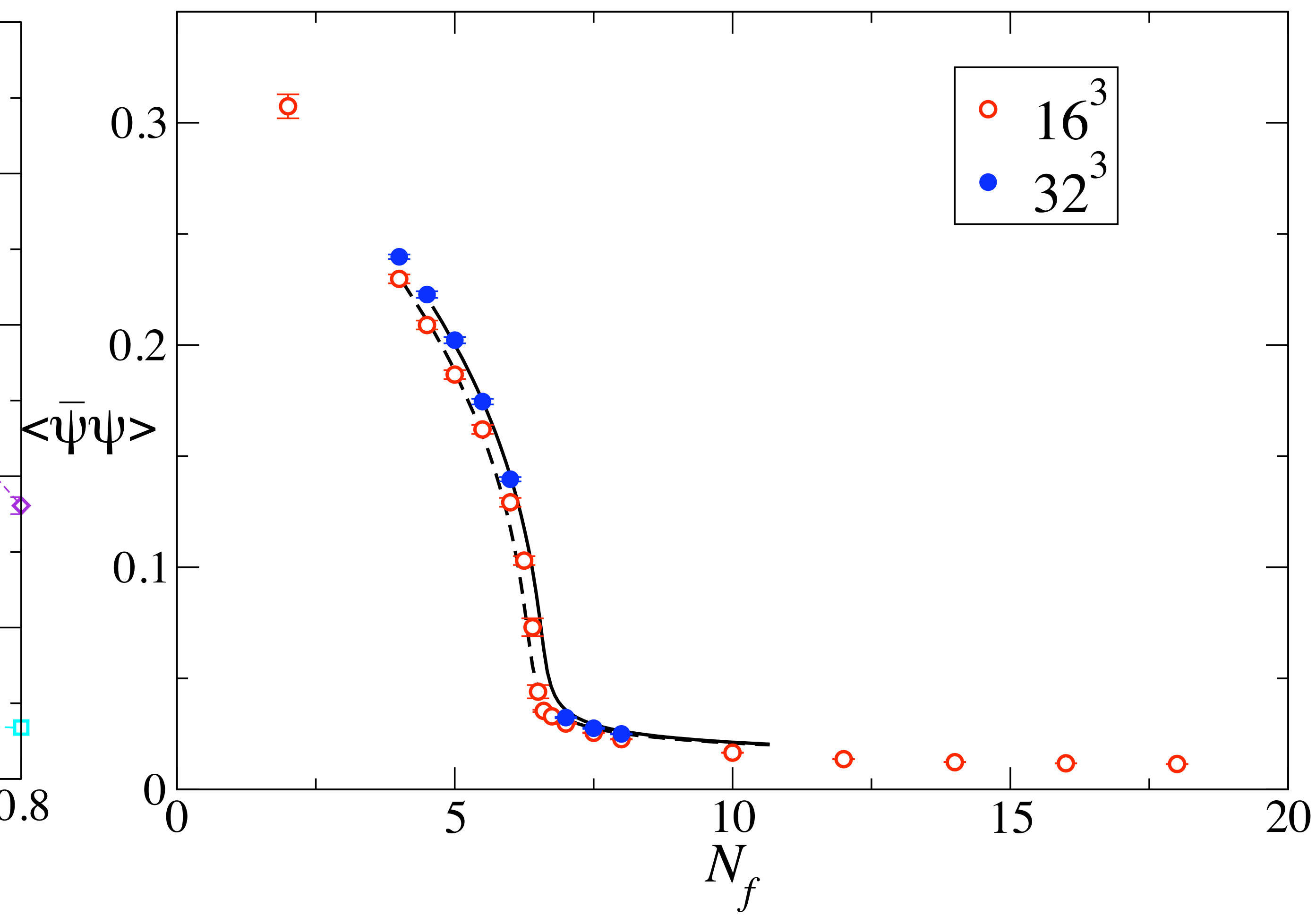
$\Rightarrow$ lattice strong coupling limit as $g^2 \rightarrow g_{\lim}^2(N_f)$

Results in effective strong-coupling limit

Christofi, SJH, Strouthos, PR**D75** (2007) 101701





N_{fc}=6.6(1), $\delta(N_{fc})$=6.90(3)

Chiral symmetry unbroken for all g^2 for $N_f > N_{fc}$

Cf. SDE: N_{fc}=4.32, $\delta(N_{fc})$=1

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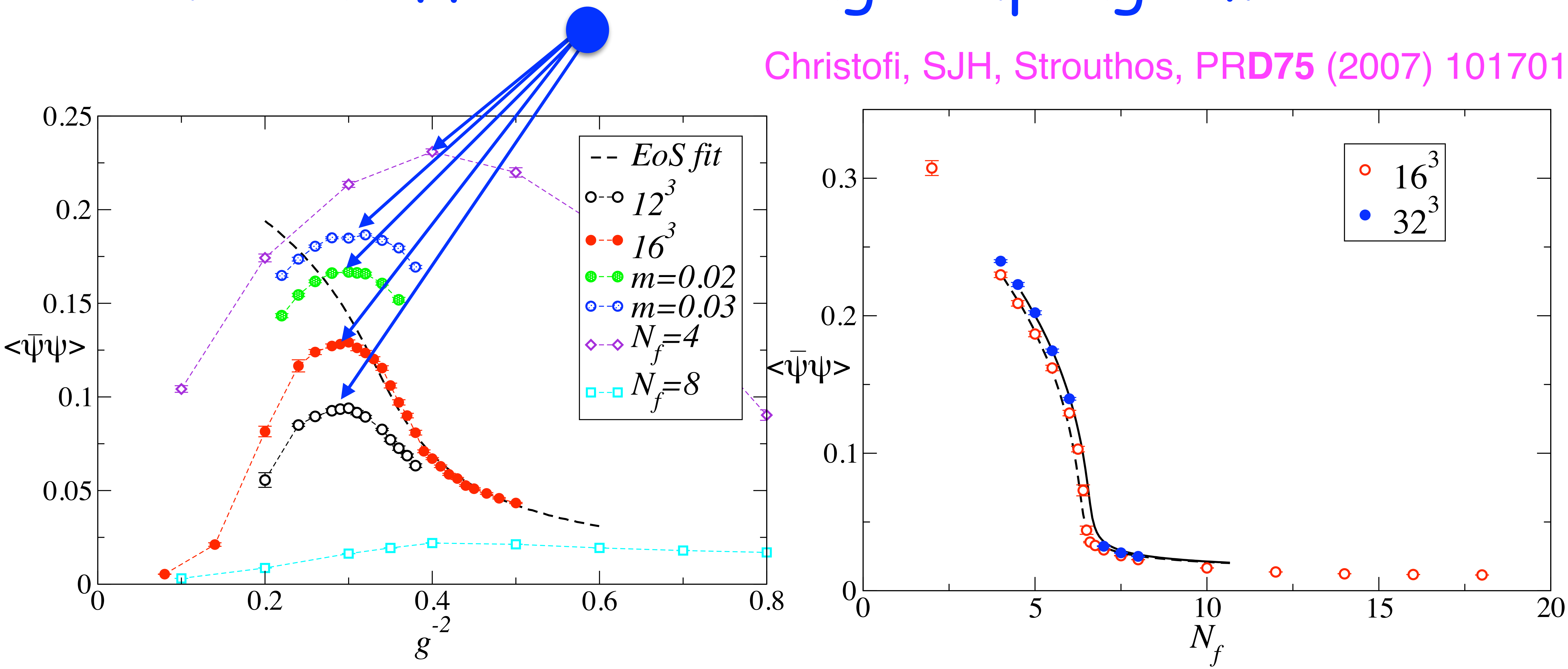

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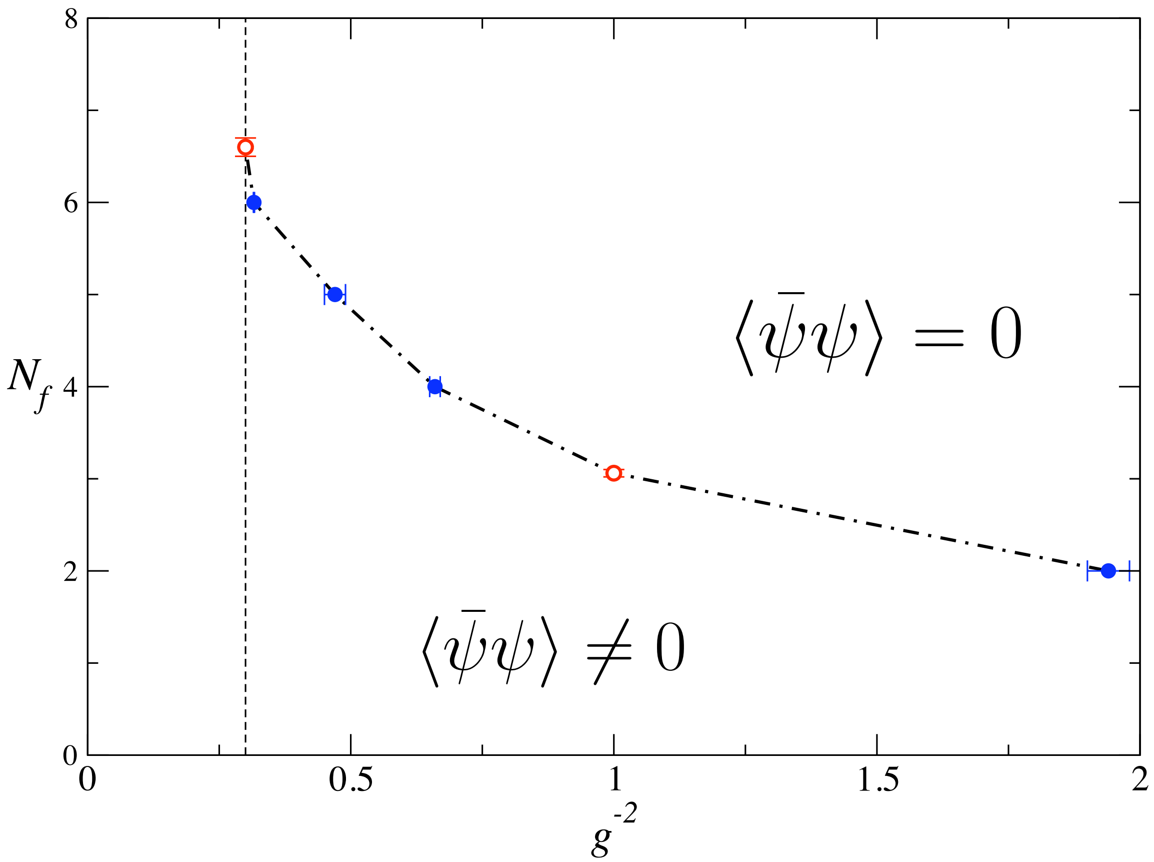
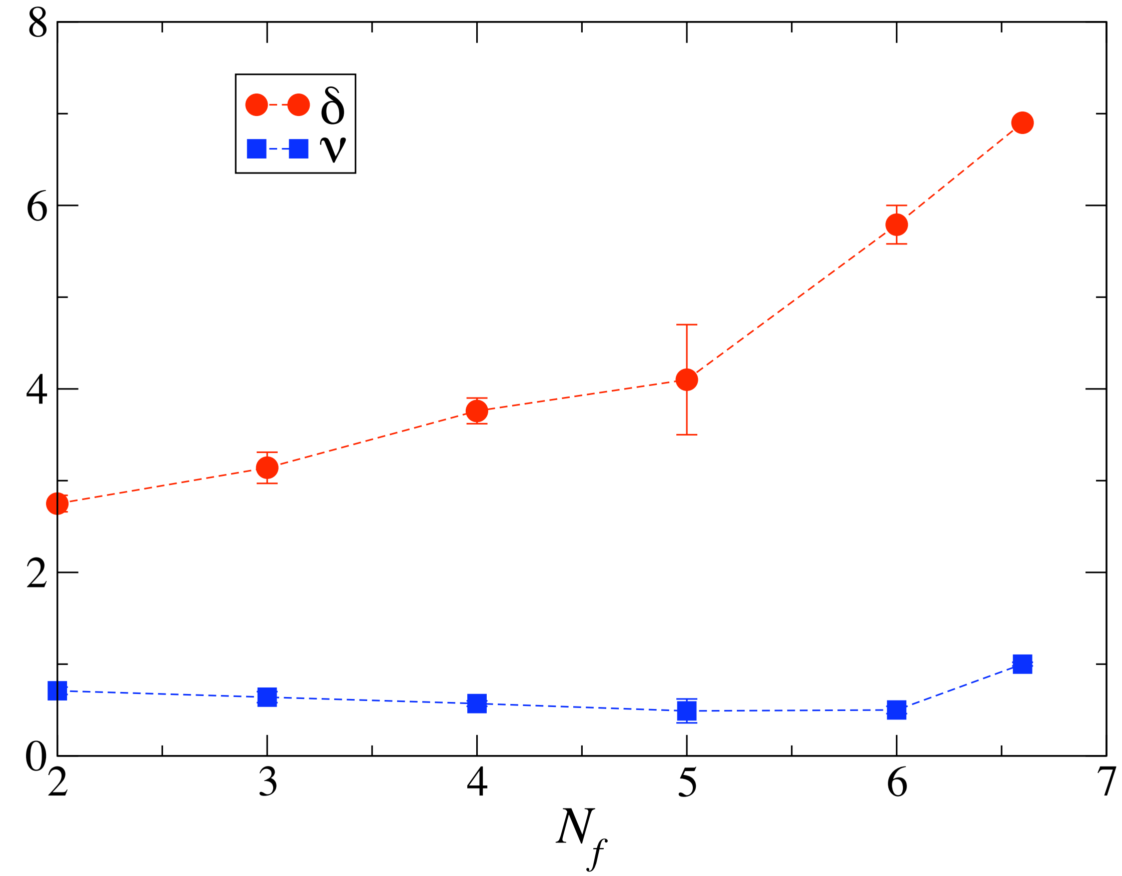
Staggered Thirring Summary

SJH, Lucini, PL**B461** (1999) 263 Christofi, SJH, Strouthos, PR**D75** (2007) 101701





- Chiral symmetry broken for small N_f, large g^2
- Each point (for N_f integer) defines a UV fixed point of RG
- Distinct critical exponents ⇔ distinct interacting QFT
- δ increases with N_f, $\delta(N_{fc})\approx 7$
- Non-covariant form used as EFT for graphene $\Longrightarrow N_{fc} \approx 5$

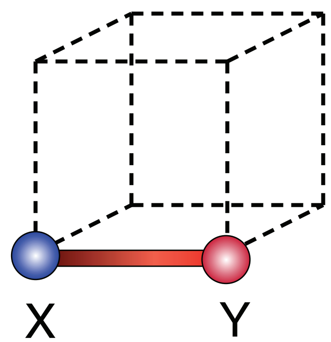
SJH, Strouthos, PR**B78** (2008)165423; Armour, SJH, Strouthos, PR**B81** (2010)125105

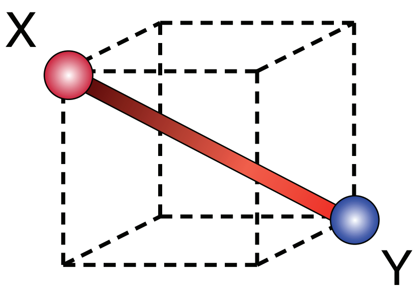
Fermion Bag Algorithm with minimal $N_f = 2$

Chandrasekharan & Li, PRL **108** (2012) 140404; PR**D88** (2013) 021701

Thirring Model: ν=0.85(1), η=0.65(1), η_ψ=0.37(1) ($N_f < N_{fc} \approx 7$)

U(1) GN Model: ν=0.849(8), η=0.633(8), η_ψ=0.373(3) ($N_f \to \infty$: $\nu=\eta=1$)





Interactions between staggered fields $\chi, \bar{\chi}$ spread over elementary cubes.
Only difference between Thirring & GN is body-diagonal term

Staggered fermions not reproducing expected distinction between models a near strongly-coupled fixed point…

see also SLAC fermion approach

Schmidt, Wellegehausen & Wipf, PoS LATTICE2015 (2016) 050
PRD96 (2017) 094504

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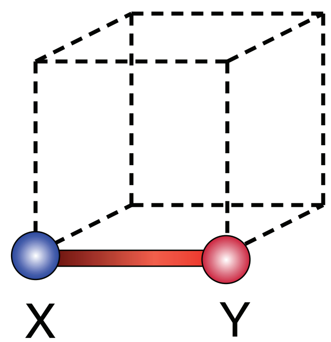

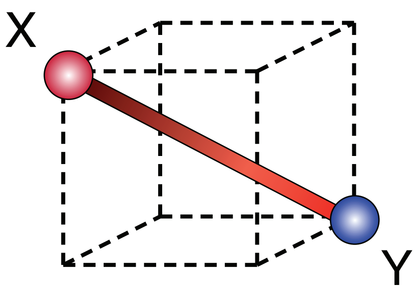

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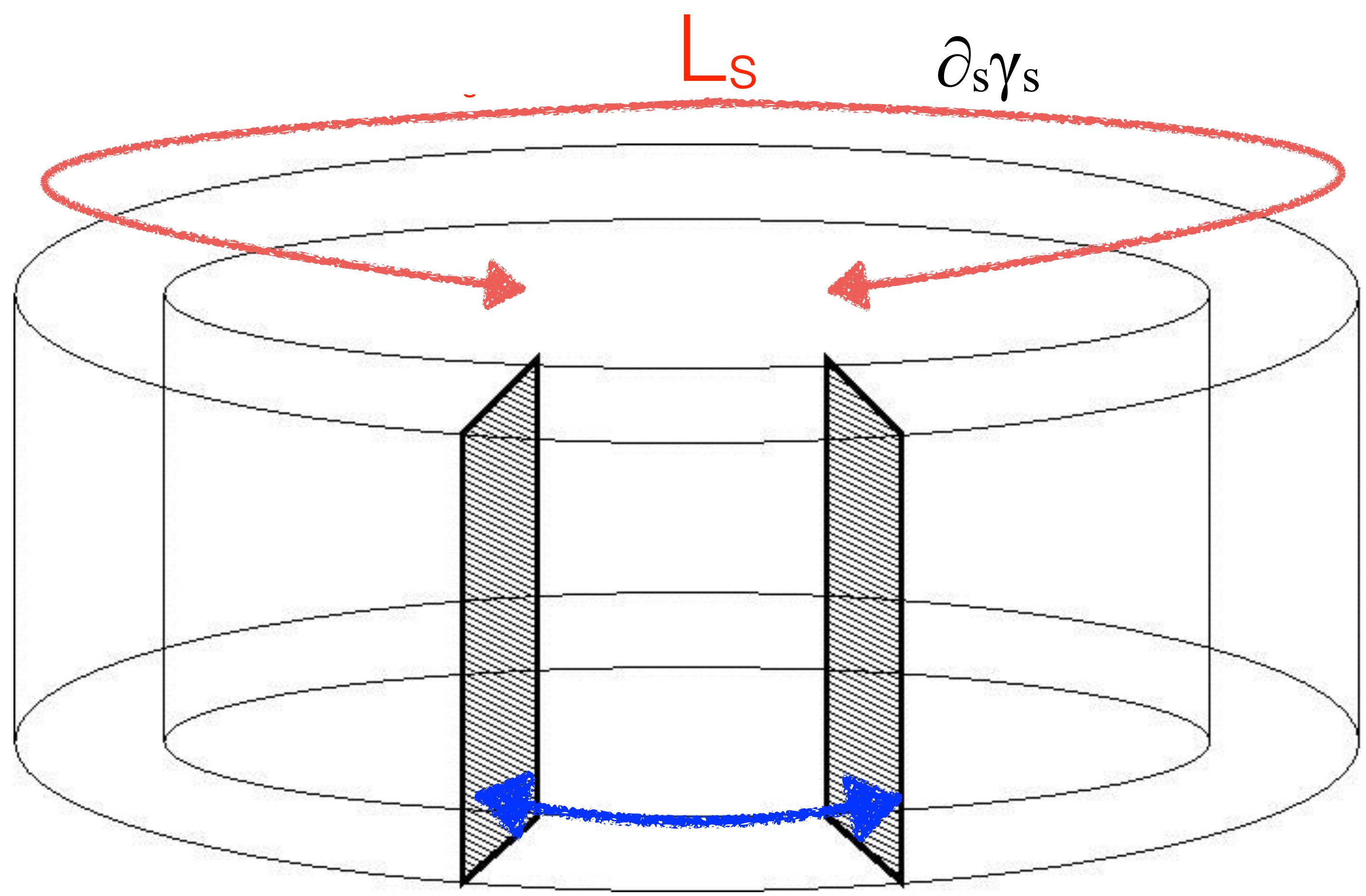
Staggered fermions not reproducing expected distinction between models a near strongly-coupled fixed point…

… so we need better lattice fermions

see also SLAC fermion approach

Schmidt, Wellegehausen & Wipf, PoS LATTICE2015 (2016) 050
PRD96 (2017) 094504

Domain Wall Fermions



Fermions propagate freely along a fictitious third direction of extent L_s with open boundaries

coupling between the walls proportional to explicit massgap m

Basic idea as $L_s \longrightarrow \infty$:

- zero-modes of D_{DWF} localised on walls are $\pm$ eigenmodes of γ_s
- Modes propagating in bulk can be decoupled (with cunning)

"Physical" fields in 2+1d target space

$$\psi(x) = P_- \Psi(x,1) + P_+ \Psi(x, L_s);$$
$$\bar{\psi}(x) = \bar{\Psi}(x, L_s) P_- + \bar{\Psi}(x,1) P_+;$$

with $P_\pm = \frac{1}{2}(1 \pm \gamma_s)$

Bottom Up View…

in DWF approach we simulate
2+1+1d fermions

Desiderata…

- Modes localised on walls carry U(2N)-invariant physics
- Fermion doublers don't contribute to normalisable modes
- Bulk modes can be made to decouple

Claim…

It appears to work for….

- carefully-chosen domain wall height M
- smooth gauge field background

Are DWF in 2+1+1d U(2N) symmetric?

Issue: wall modes are eigenstates of γ_3 as $L_s \to \infty$,

but: U(2N) symmetry demands equivalence under rotations generated by both γ_3 and γ_5

ie. U(2N) → U(N)⊗U(N) symmetry-breaking mass terms

$$m_h \bar{\psi}\psi \qquad im_3\bar{\psi}\gamma_3\psi \qquad im_5\bar{\psi}\gamma_5\psi$$

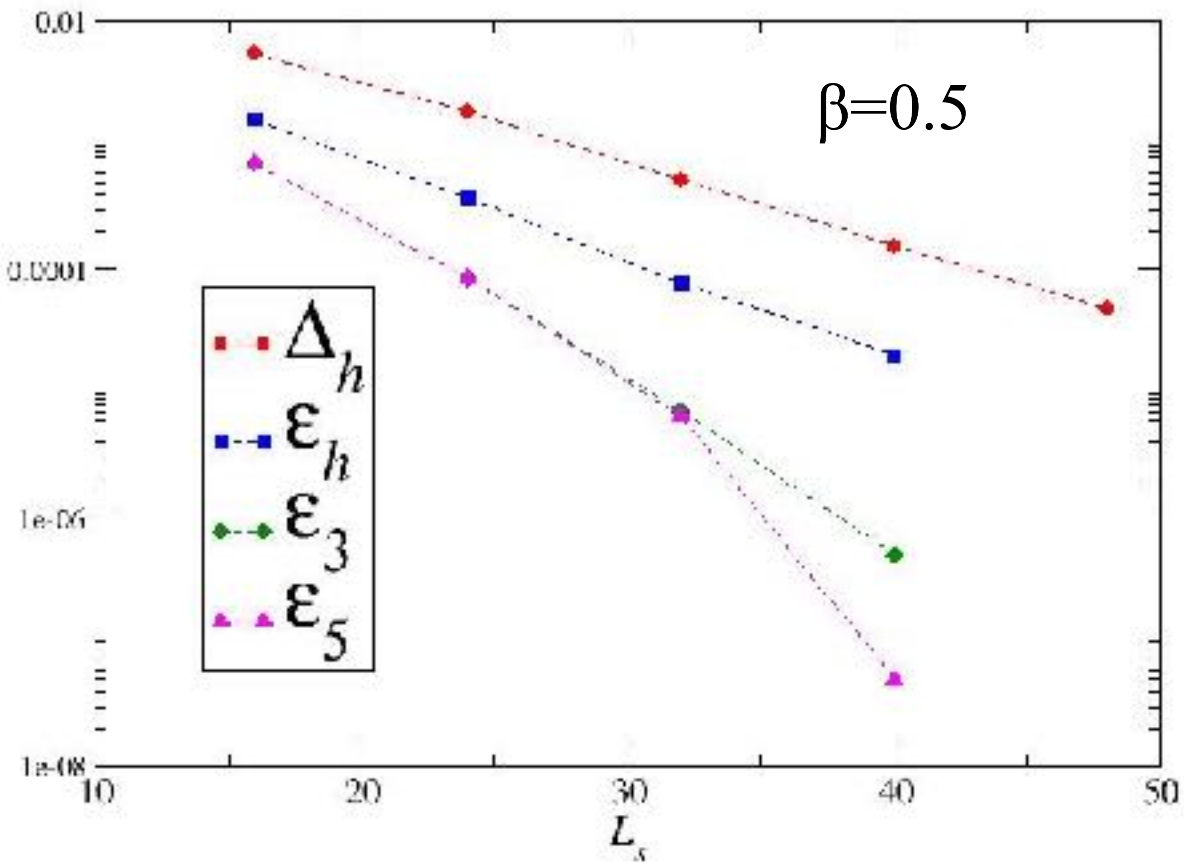
should yield identical physics as $L_s \to \infty$

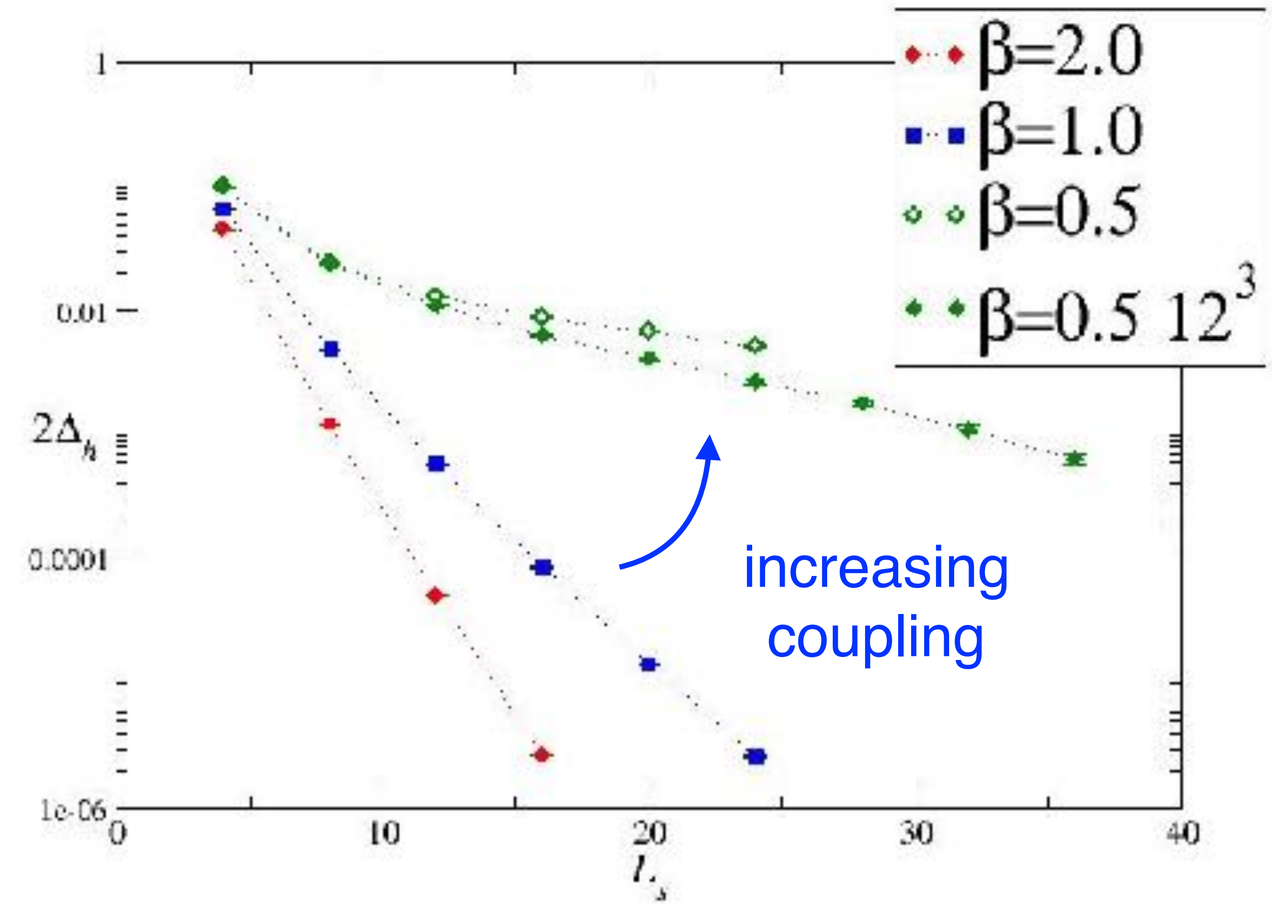
Non-trivial requirement

since m_h, m_3 couple Ψ, $\bar{\Psi}$ on *opposite* walls

while m_5 couples modes on *same* wall

Bilinear Condensates in Quenched QED_3 on $24^3 \times L_s$…





Define main *residual*: $i\langle\bar\Psi(1)\gamma_3\Psi(L_s)\rangle = \frac{i}{2}\langle\bar\psi\gamma_3\psi\rangle_{L_s} + i\Delta_h(L_s)$

real imaginary

$$\frac{1}{2}\langle\bar\psi\psi\rangle_{L_s} = \frac{i}{2}\langle\bar\psi\gamma_3\psi\rangle_{L_S\to\infty} + \Delta_h(L_s) + \epsilon_h(L_s);$$

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$$\frac{i}{2}\langle\bar\psi\gamma_5\psi\rangle_{L_s} = \frac{i}{2}\langle\bar\psi\gamma_3\psi\rangle_{L_S\to\infty} + \epsilon_5(L_s).$$

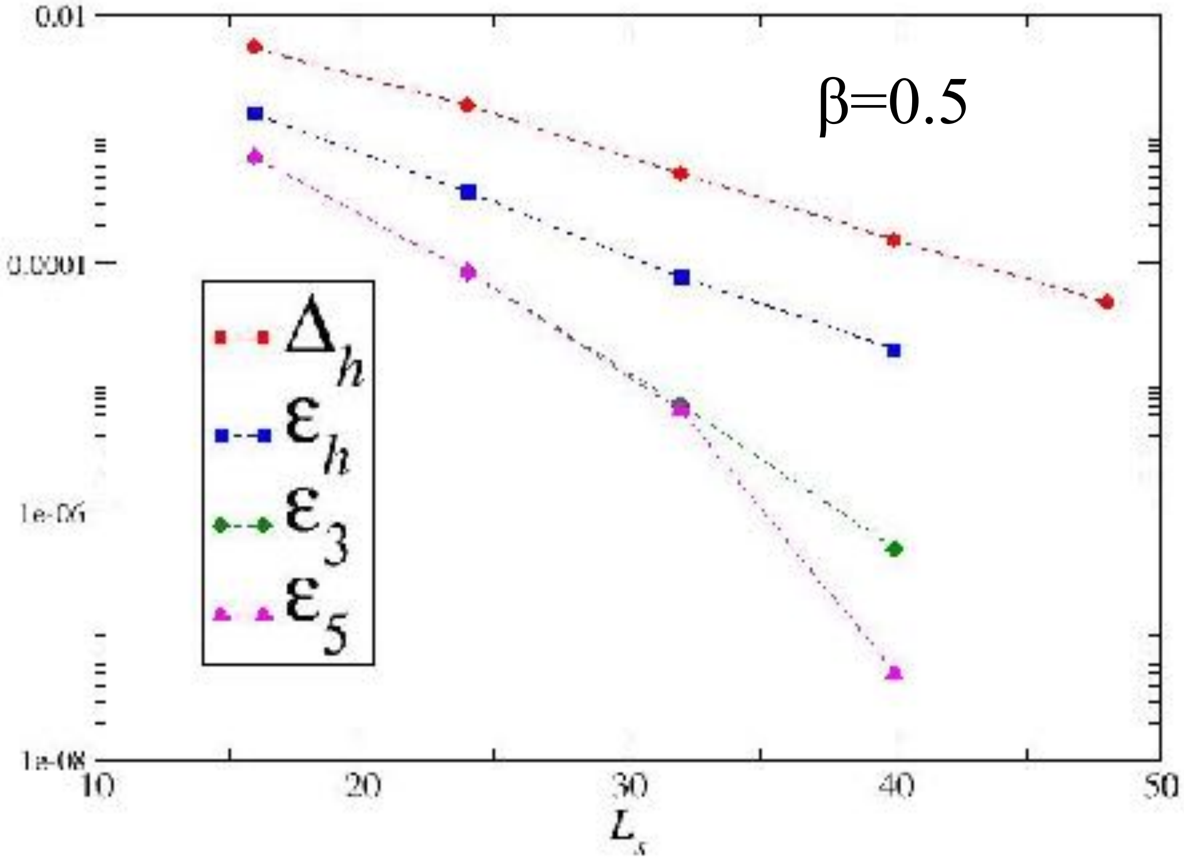
- exponentially suppressed as $L_s \to \infty$
- hierarchy: $\Delta_h > \varepsilon_h > \varepsilon_3 \equiv \varepsilon_5$

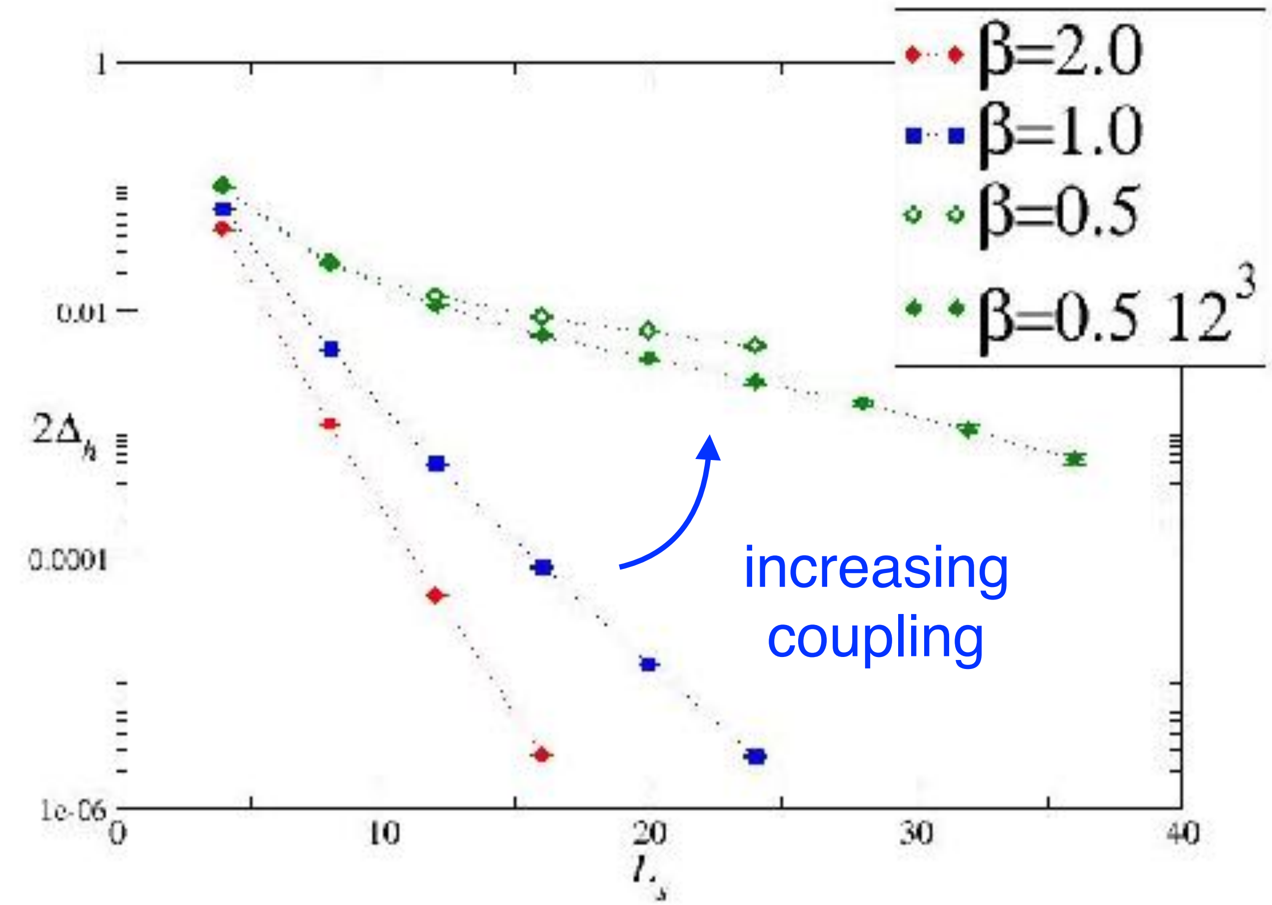
U(2) symmetry restored

$\Leftrightarrow \Delta_h \to 0$

SJH JHEP **09**(2015)047,
PLB **754** (2016) 264

Bilinear Condensates in Quenched QED$_3$ on $24^3 \times L_s$…





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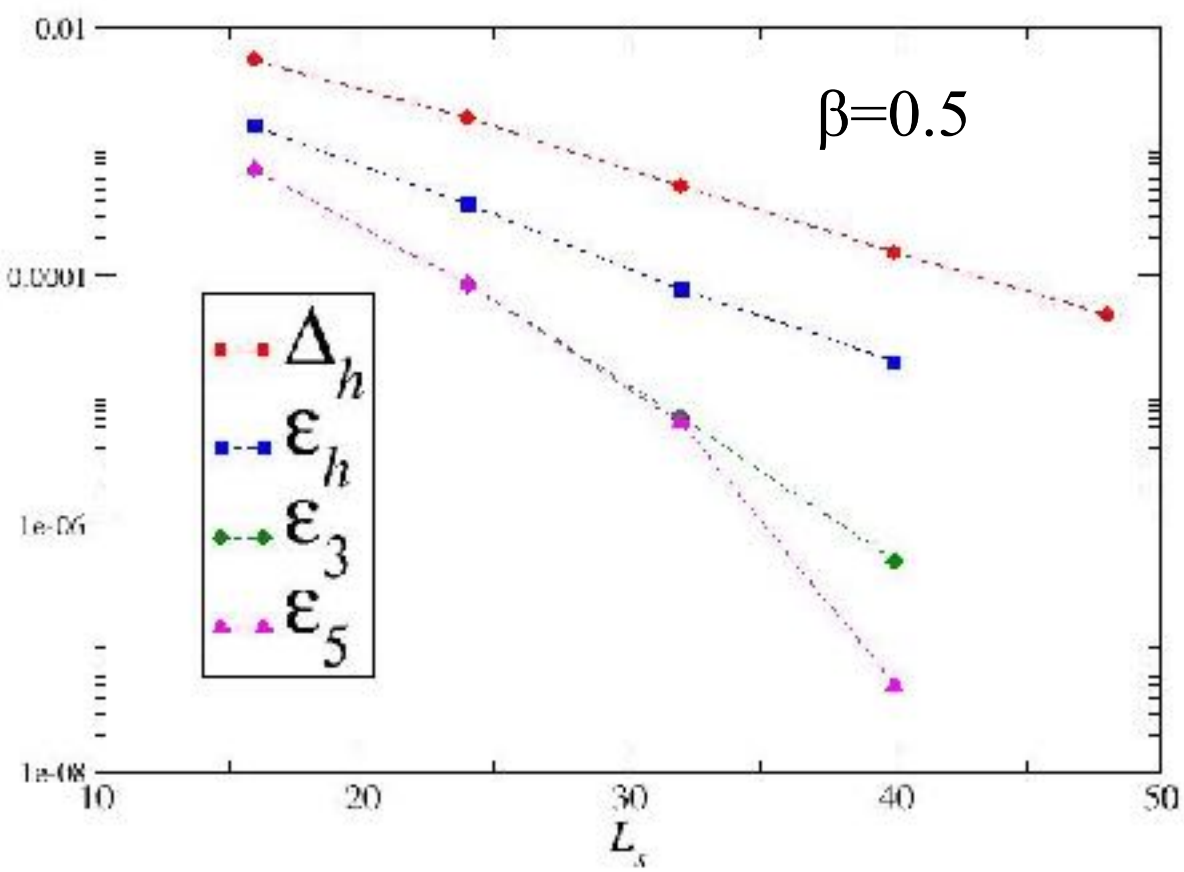

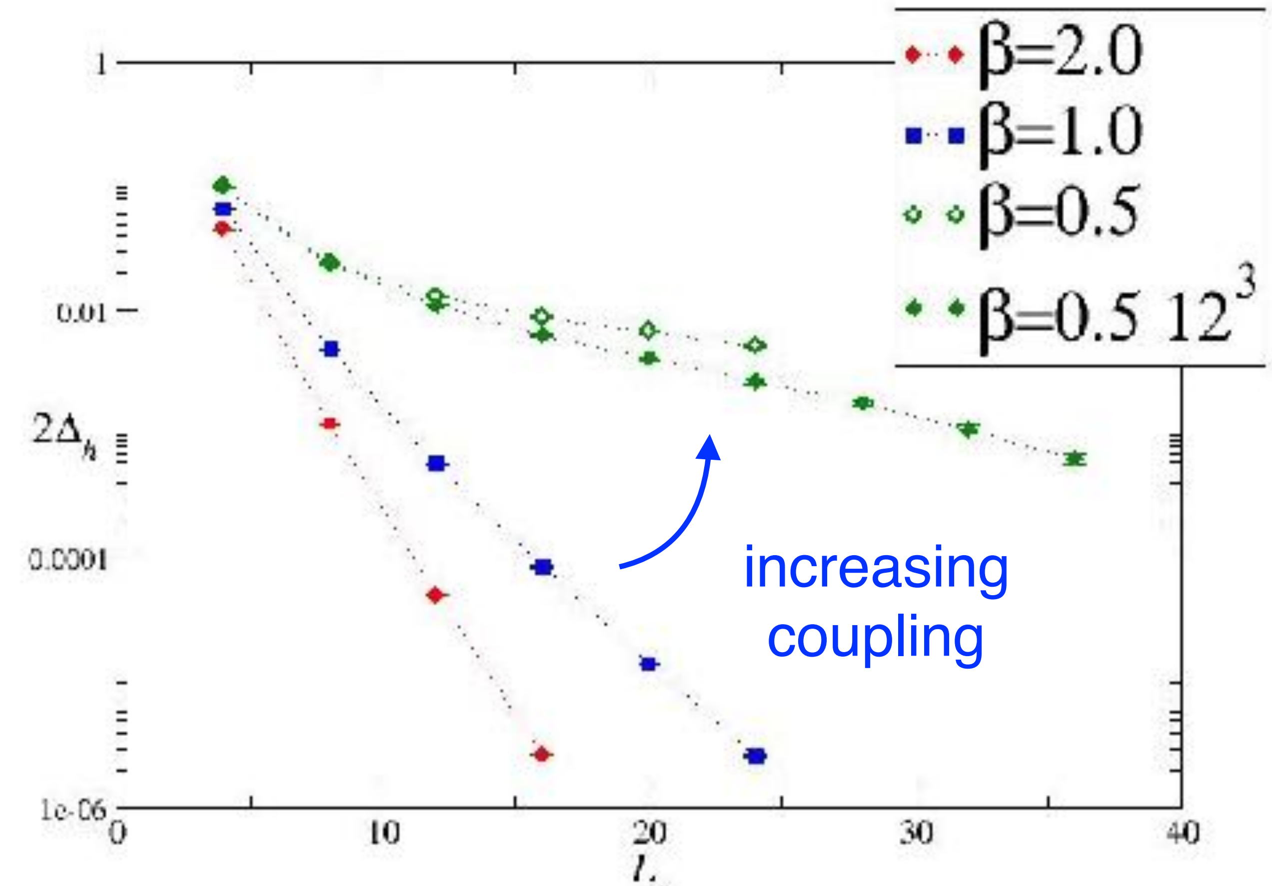

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SJH JHEP **09**(2015)047,
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Top Down View...

The closest approach to continuum symmetries is expressed by **Ginsparg-Wilson** relations

$$\{\gamma_5, D\} = 2D\gamma_5 D$$

RHS is $O(aD)$, so U(2N) recovered in long-wavelength limit if D local

By construction GW is satisfied by the 2+1d *overlap* operator

$$D_{ov} = \frac{1}{2}\left[(1+m_h)+(1-m_h)\frac{A}{\sqrt{A^\dagger A}}\right]$$ with $$\gamma_3 A\gamma_3 = \gamma_5 A\gamma_5 = A^\dagger$$

$A \equiv [2+(D_W-M)]^{-1}[D_W-M]; \quad D_W \text{ local}; \; Ma = O(1)$ D_{ov} not manifestly local

DWF provide a regularisation of overlap with a *local* kernel in 2+1+1d

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Formulational issues for the Thirring Model with DWF

(a) Formulate interaction terms in terms of vector auxiliary $A_\mu(x)$ defined just on walls at $x_3 = 1, L_s$: **"Surface"**

Technical/cost advantage: no Pauli-Villars determinant needed to cancel bulk modes

P. Vranas, I. Tziligakis and J.B. Kogut, Phys. Rev. D 62 (2000) 054507

(b) By analogy with QCD, formulate with $A_\mu(x)$ throughout bulk which are "static" ie. $\partial_3 A_\mu=0$: **"Bulk"**

$$\mathcal{S} = \bar\Psi\mathcal{D}\Psi = \bar\Psi D_W\Psi + \bar\Psi D_3\Psi + m_i S_i$$ with $$\begin{aligned} D_W &= \gamma_\mu D_\mu - (\hat D^2 + M); \\ D_3 &= \gamma_3\partial_3 - \hat\partial_3^2, \end{aligned}$$

Recall link field **not** unit modulus

Bulk formulation

$$[\partial_3, D_\mu] = [\partial_3, \hat D^2] = 0$$

but $[\partial_3, \hat\partial_3^2] \neq 0$ on walls

obstruction to proving $\det\mathcal{D} > 0$ for N=1

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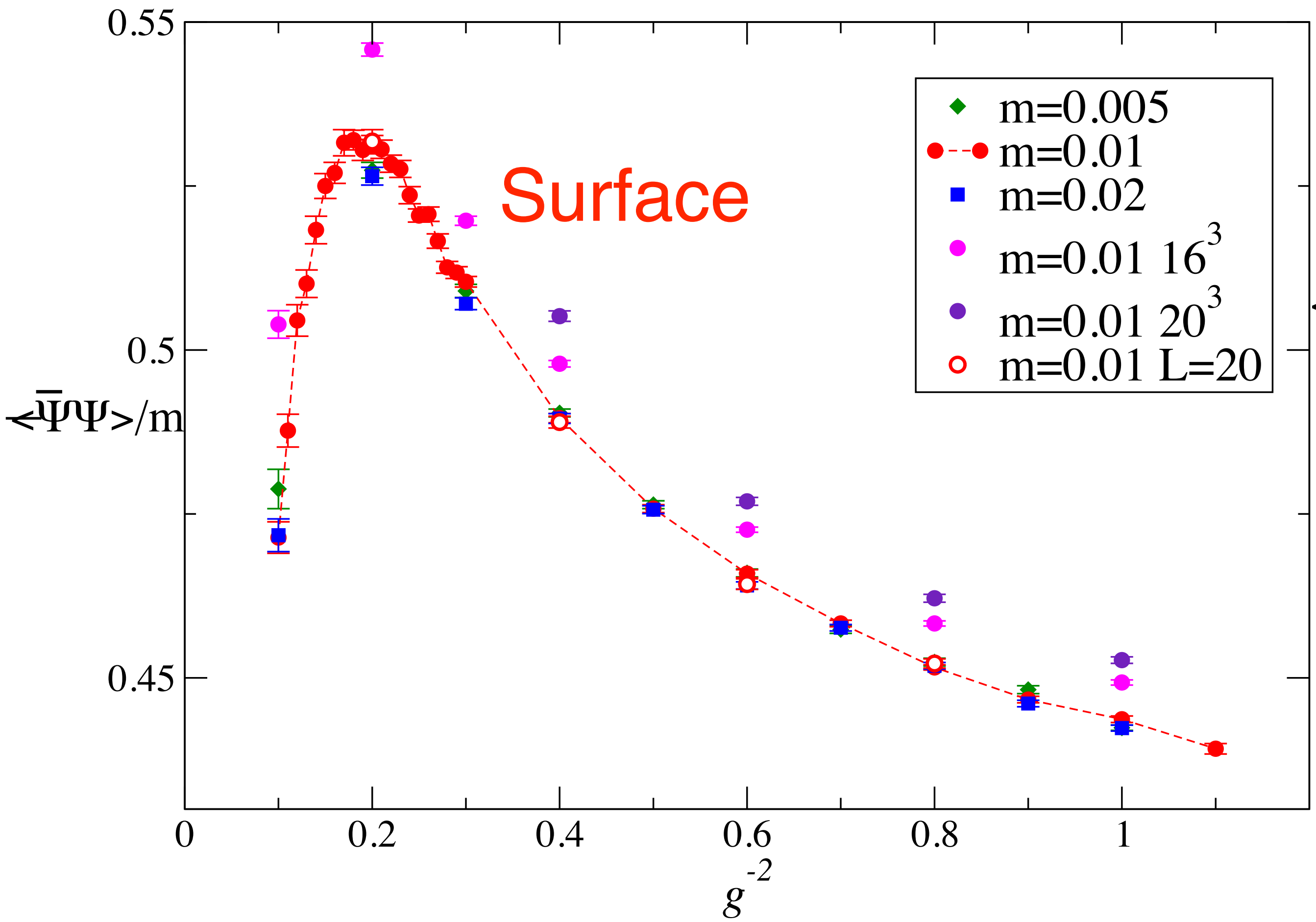
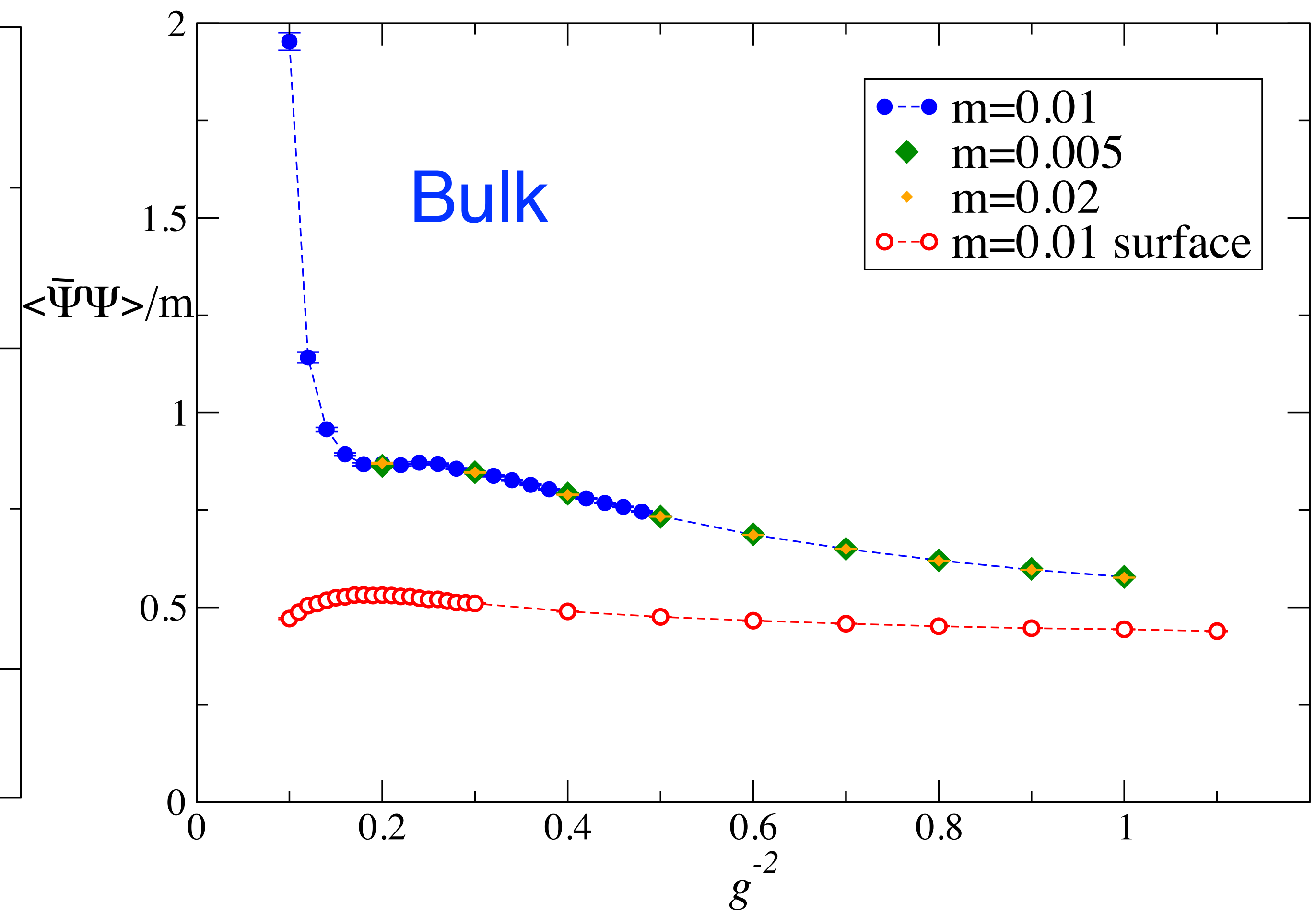
$\Rightarrow$ **need RHMC algorithm for N=1**

HMC Results with N_f=2 on $12^3\times16$

SJH JHEP **11**(2016)015

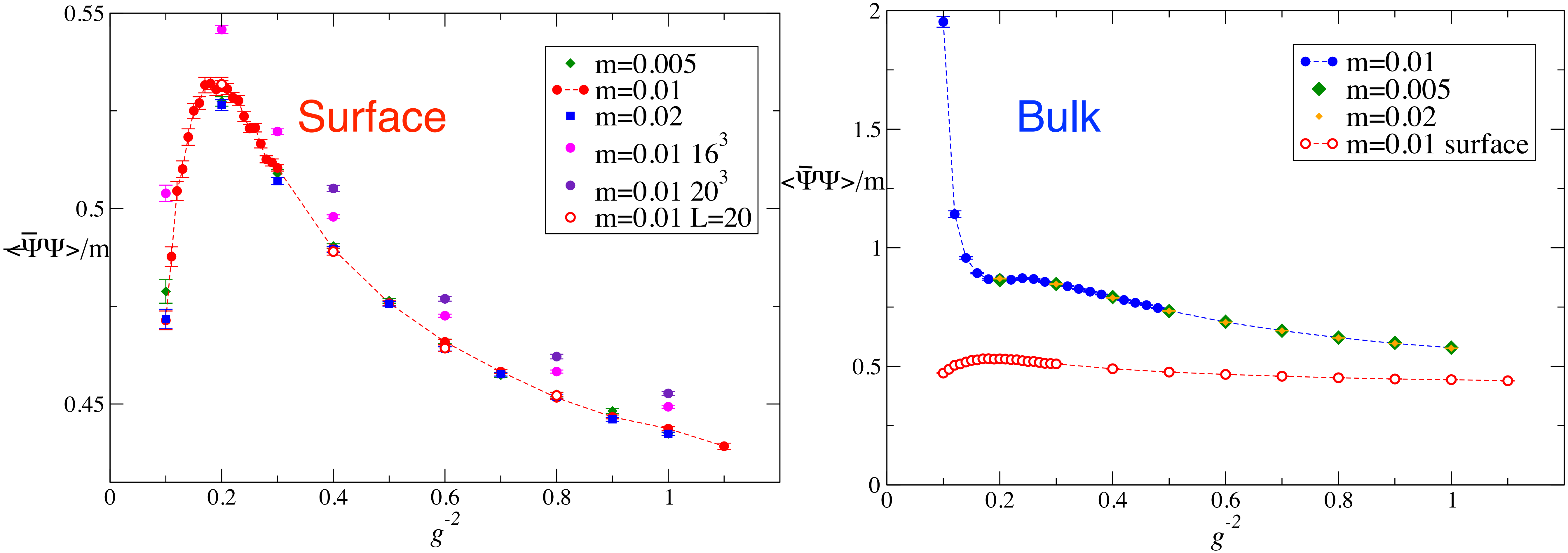




- Breakdown of reflection positivity for strong coupling $ag^{-2}\approx0.2$?
- Strong volume dependence for surface model
- Results at L_s=16 and L_s=20 are consistent
- **No evidence of spontaneous symmetry breaking** anywhere along g^{-2} axis

HMC Results with $N_f=2$ on $12^3\times16$

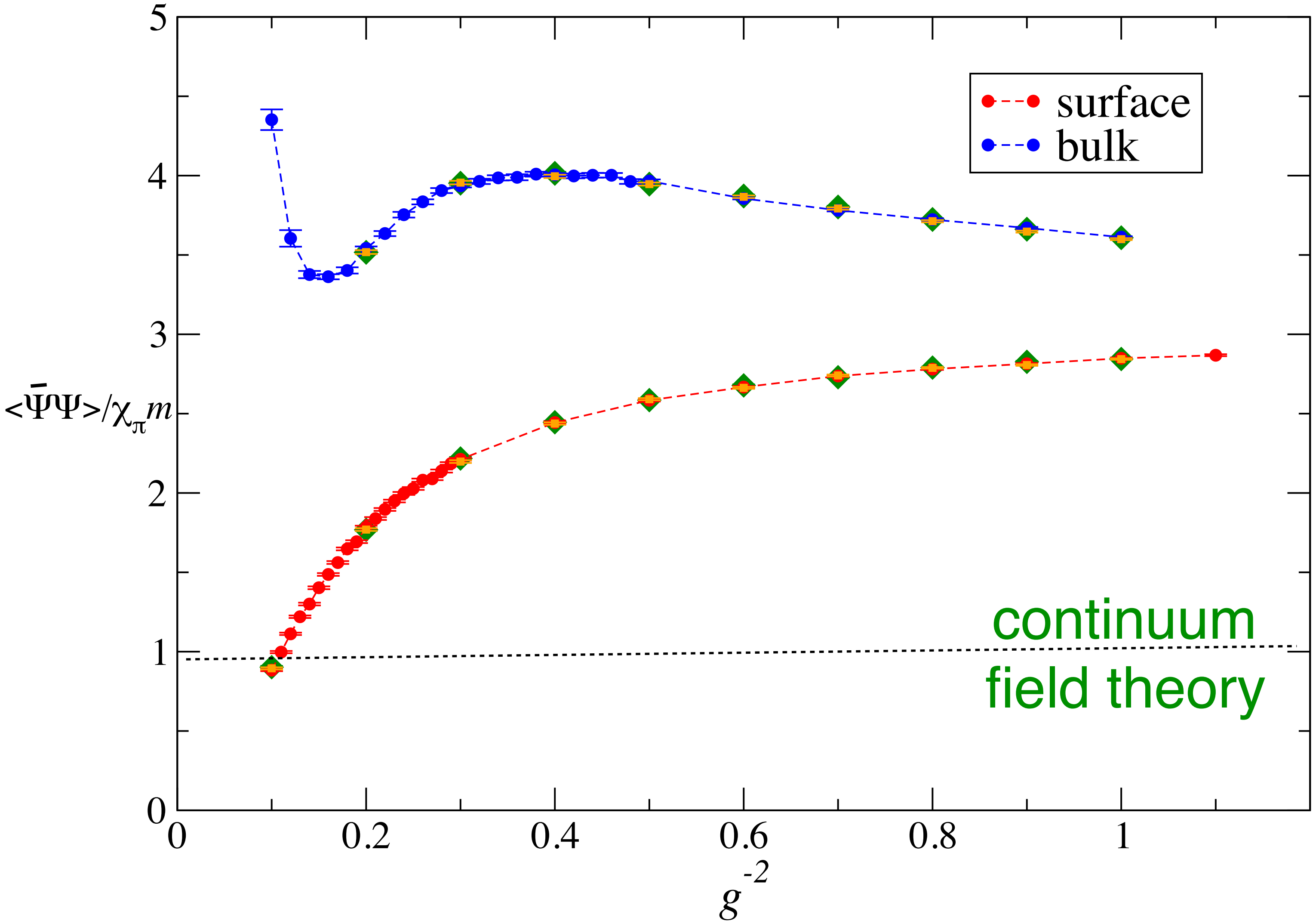
SJH JHEP **11**(2016)015



- Breakdown of reflection positivity for strong coupling $ag^{-2}\approx0.2$?
- Strong volume dependence for surface model
- Results at $L_s=16$ and $L_s=20$ are consistent
- **No evidence of spontaneous symmetry breaking** anywhere along g^{-2} axis

big disparity with
previous staggered results

Axial Ward Identity



Ratio of order parameter to susceptibility is predicted constant by WI

$$\frac{\langle\bar\psi\psi\rangle}{m} = \sum_x \langle\bar\psi\gamma_3\psi(0)\bar\psi\gamma_3\psi(x)\rangle$$

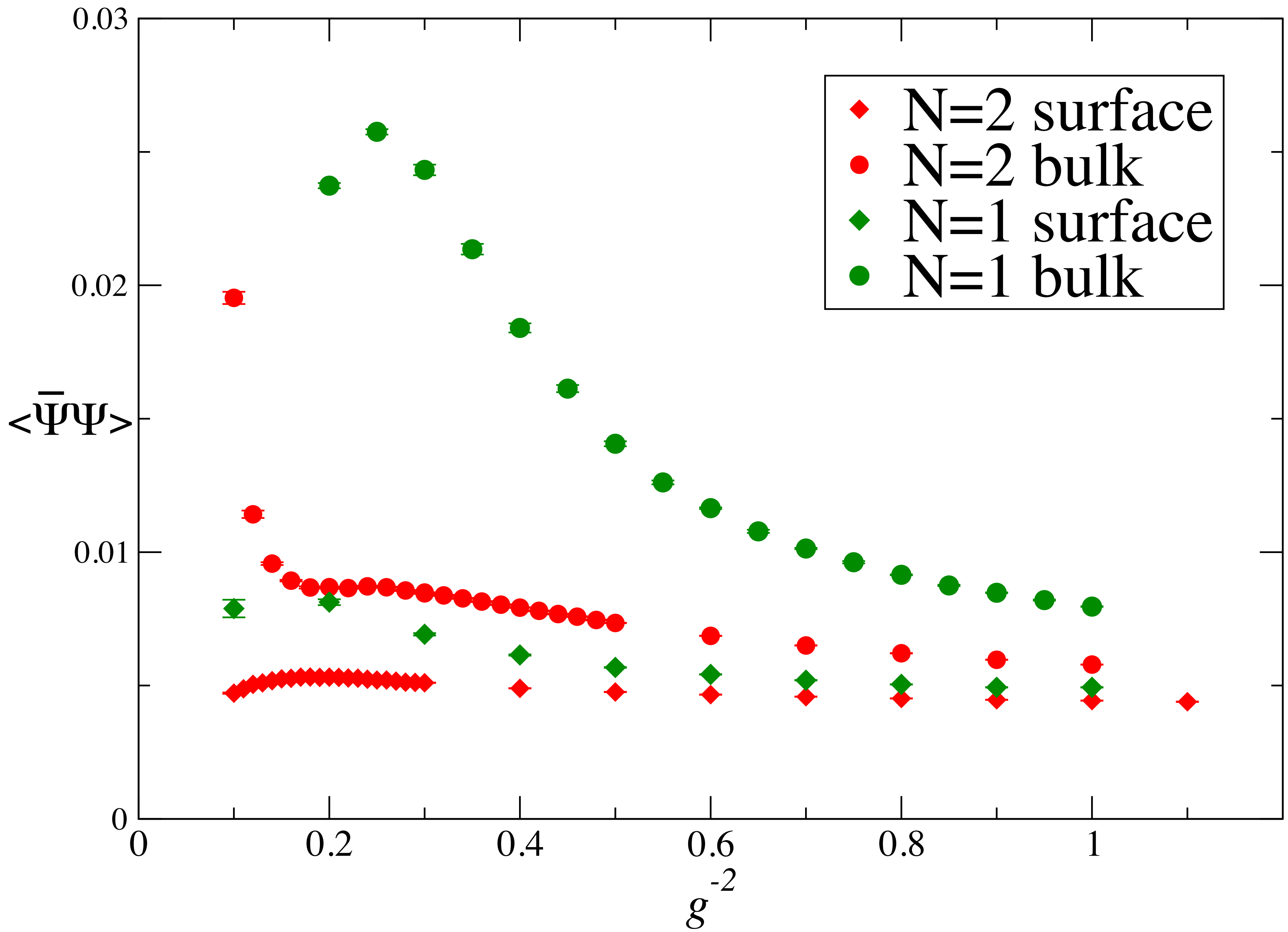
Strong-coupling behaviour suggests
neither Surface nor Bulk model optimal:
work still needed to specify 2+1d states ψ with control over normalisation

Cf. 2+1d Gross-Neveu model, where Ward Identity
is respected, spectroscopy under control…

SJH JHEP **11**(2016)015

RHMC Results for N=1 (12^3x8)

arXiv:1708.07686



N=1 simulations performed with weight $\det(M^\dagger M)^{1/2}$ using **RHMC** algorithm with 25 partial fractions

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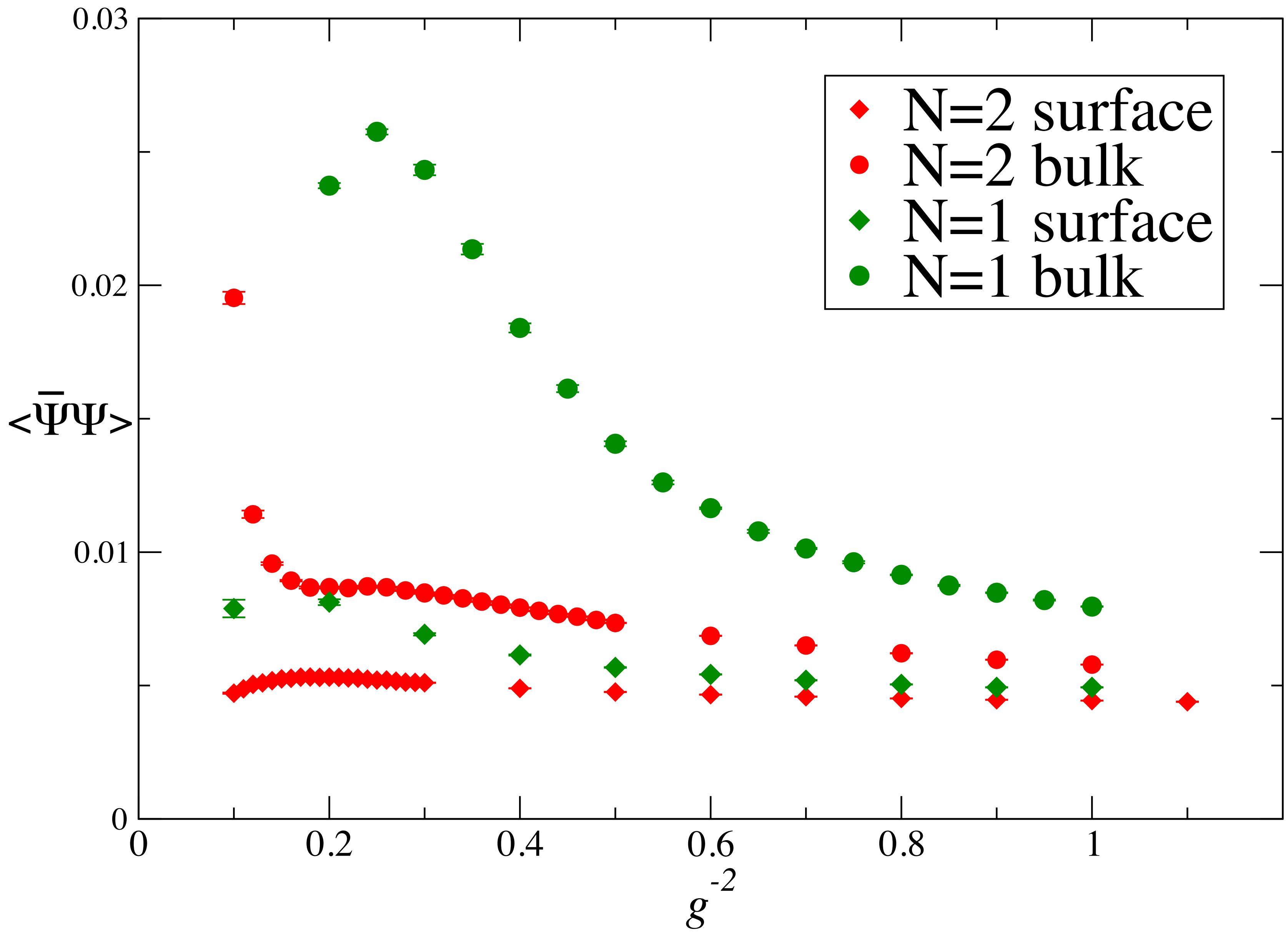

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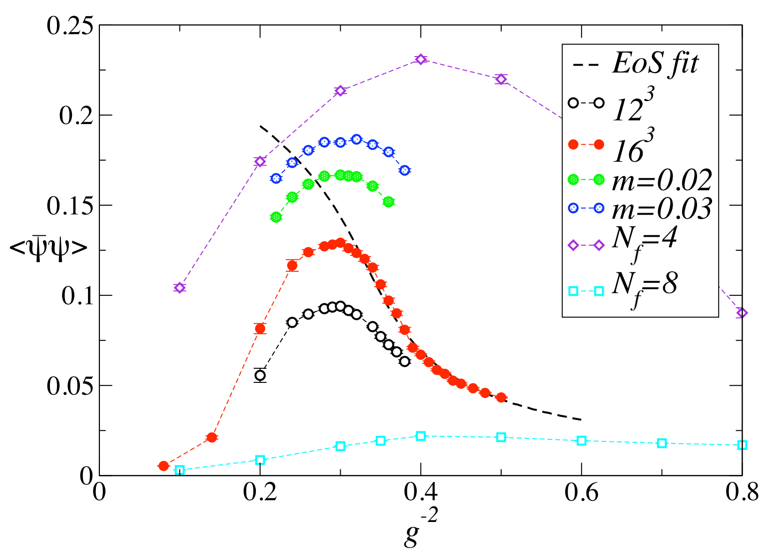

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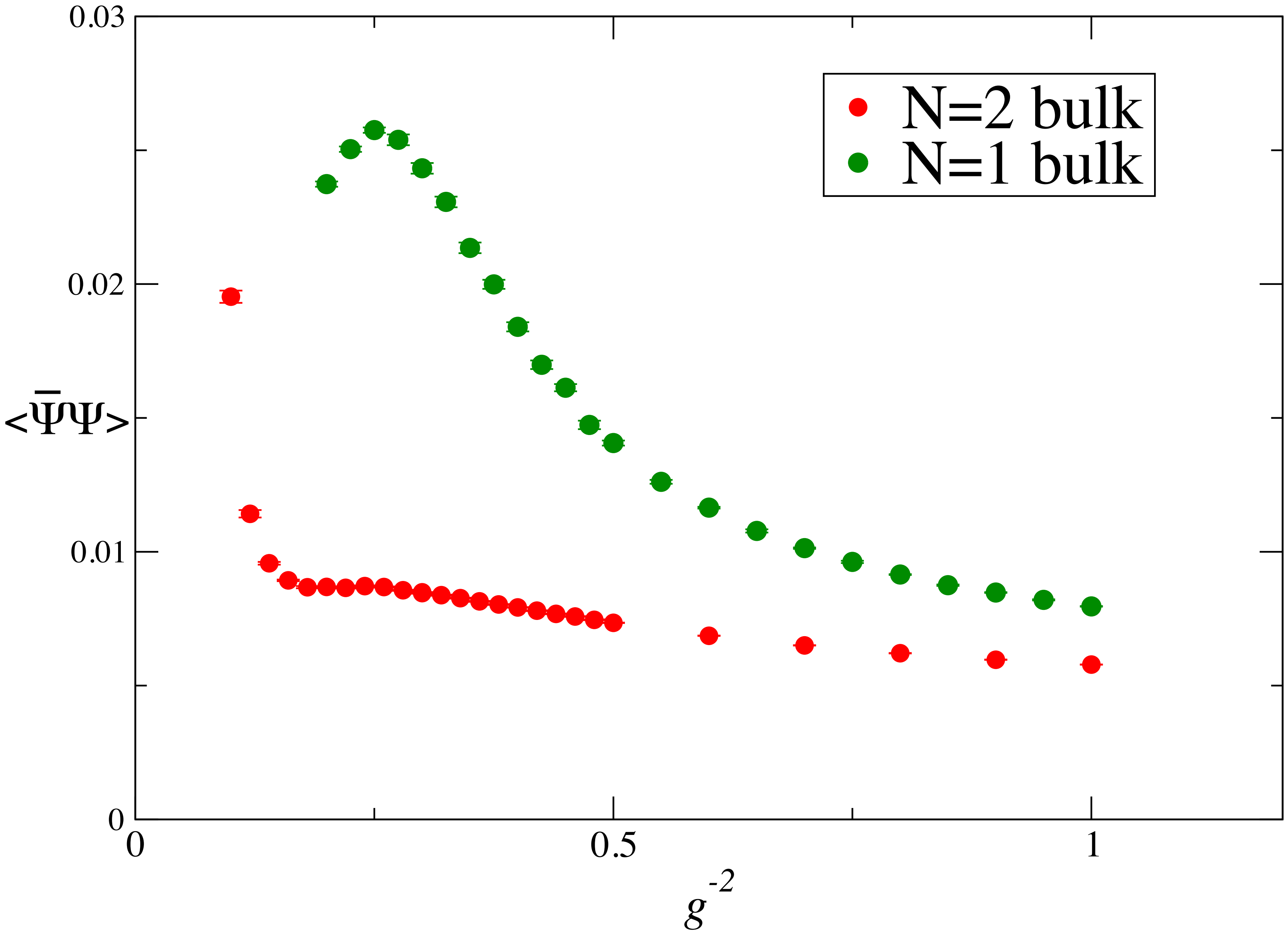

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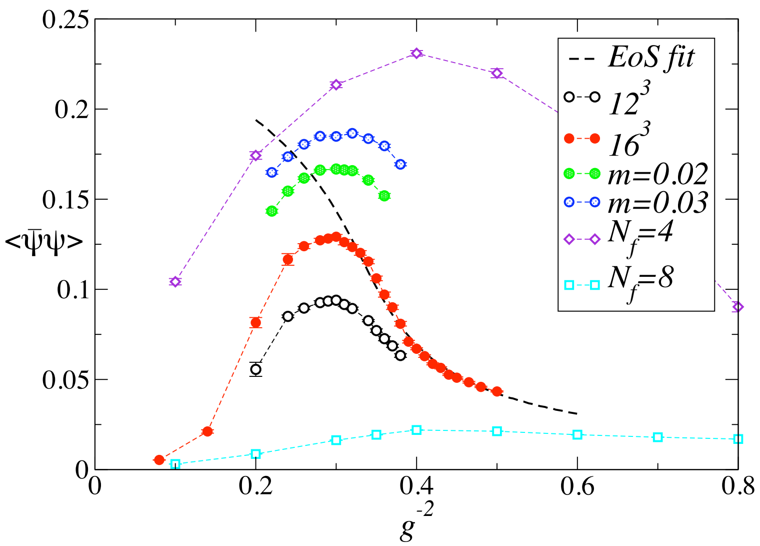

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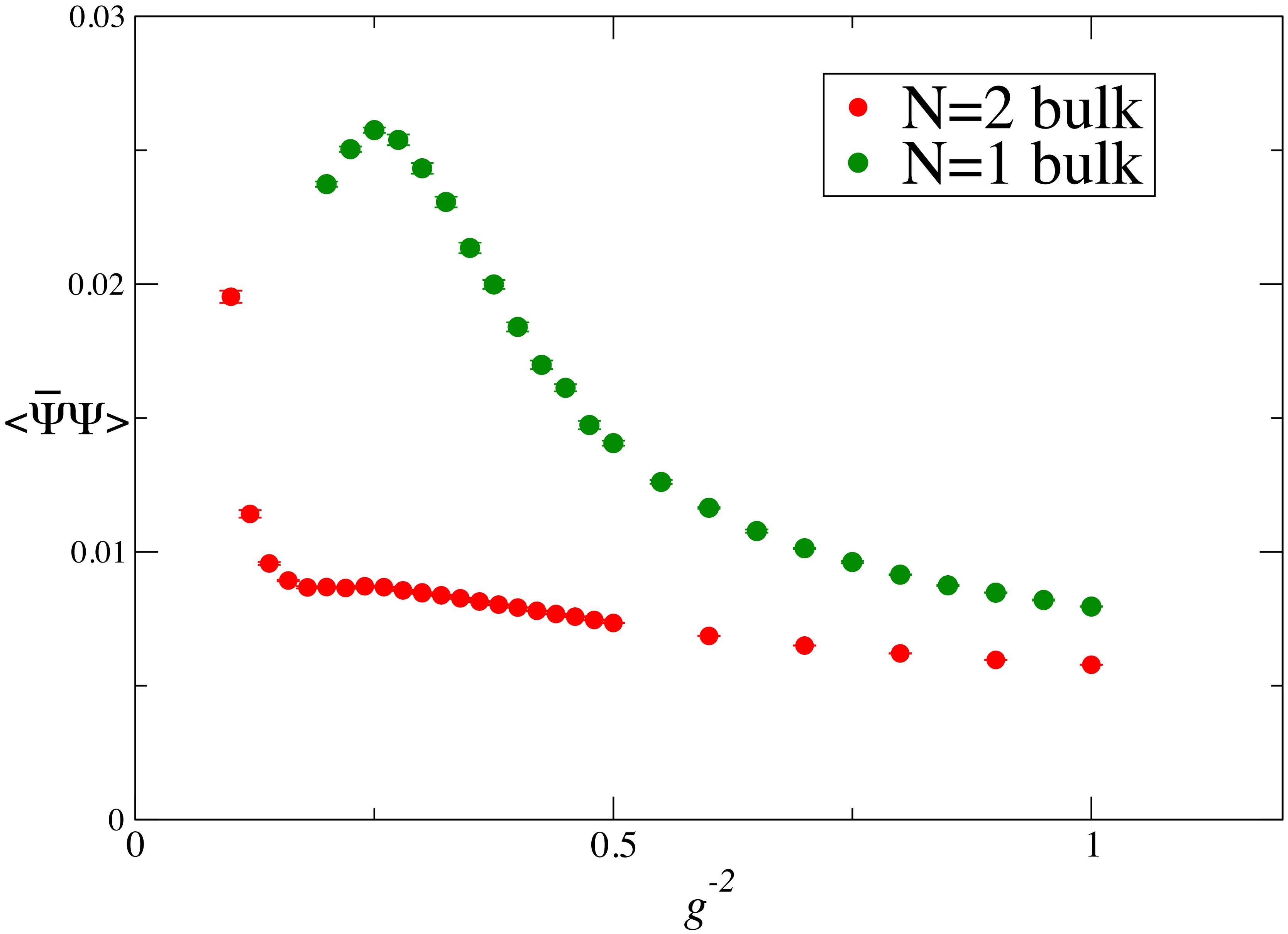

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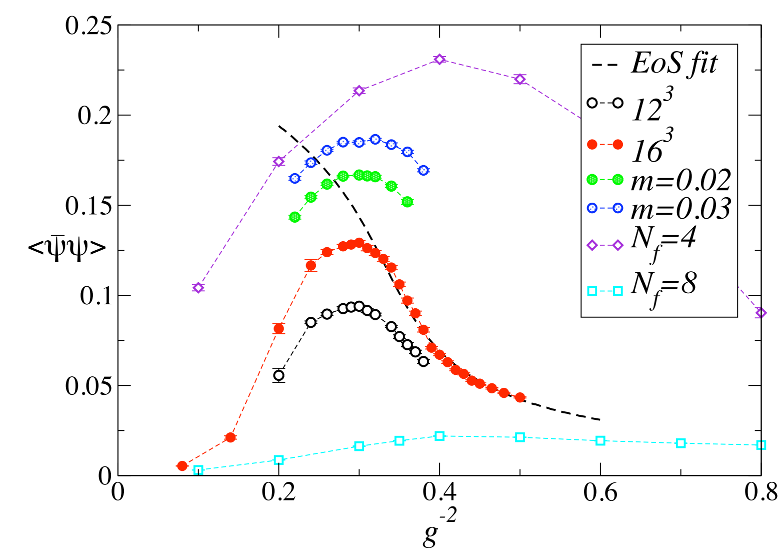
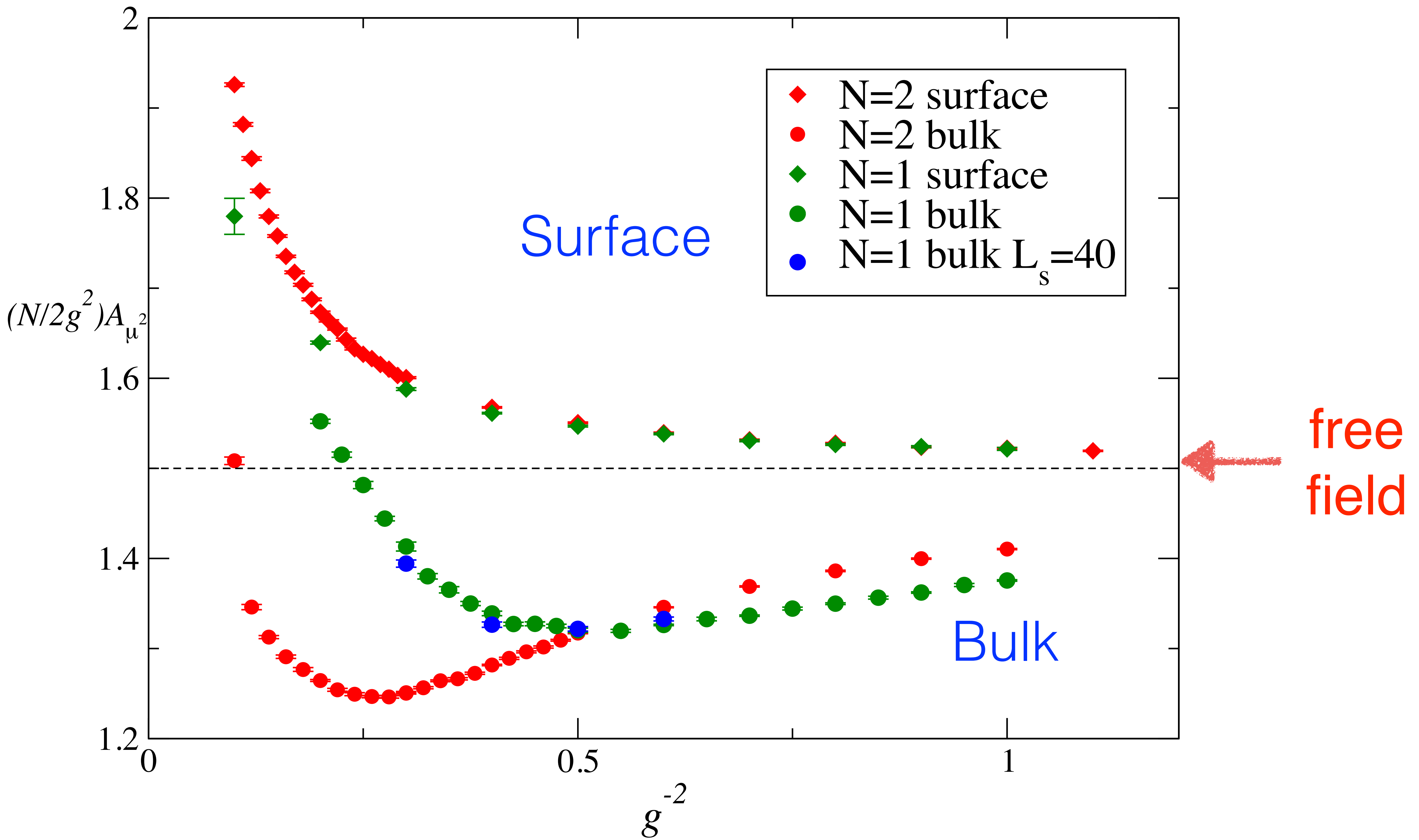


Henceforth focus on **bulk**

Evidence for enhanced pairing for N=1 and $ag^{-2} < 0.5$?

Boson Action

an interesting diagnostic

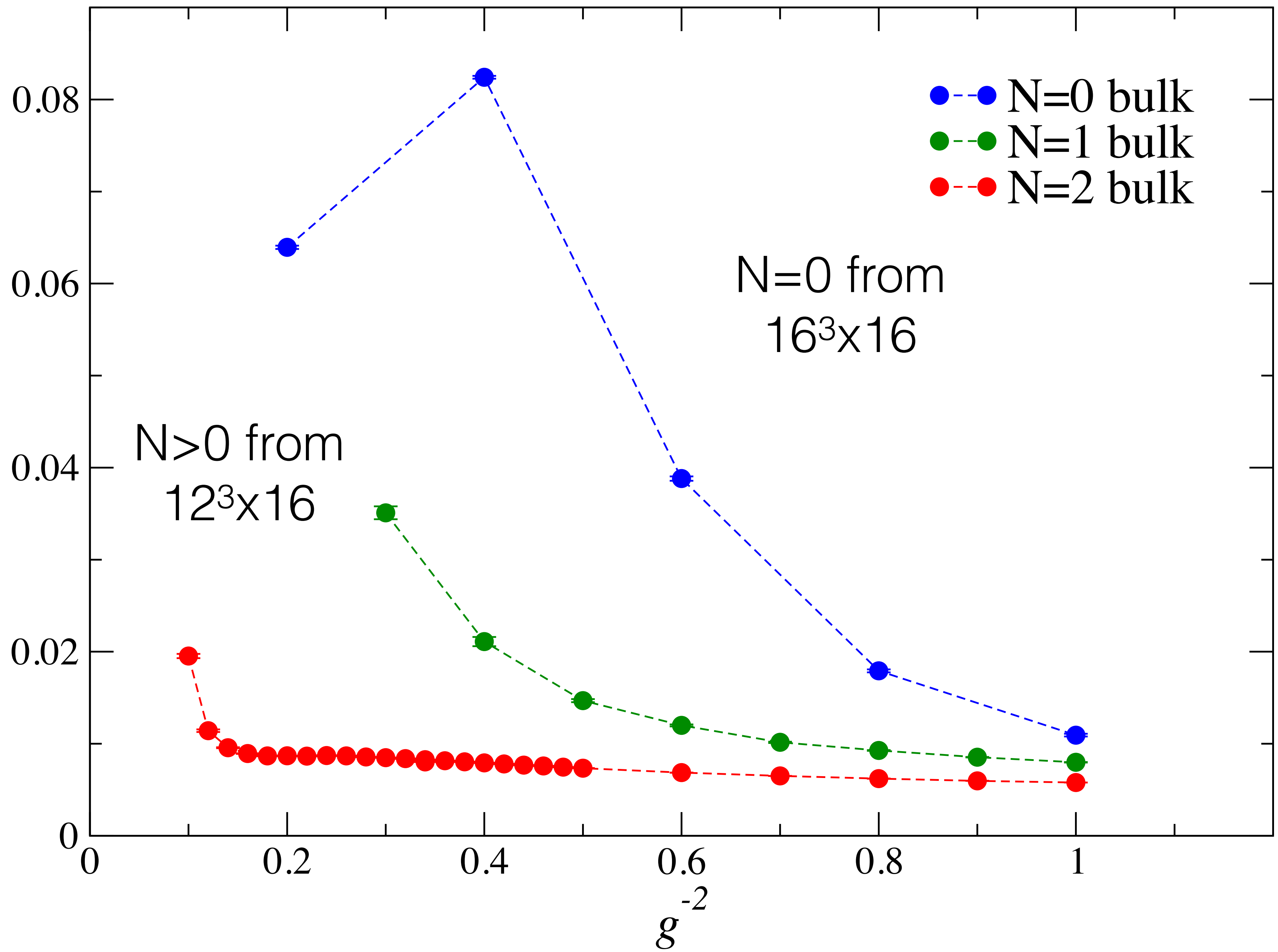


Surface and Bulk models show different behaviour

N=1: change of behaviour for $ag^{-2} < 0.5$?

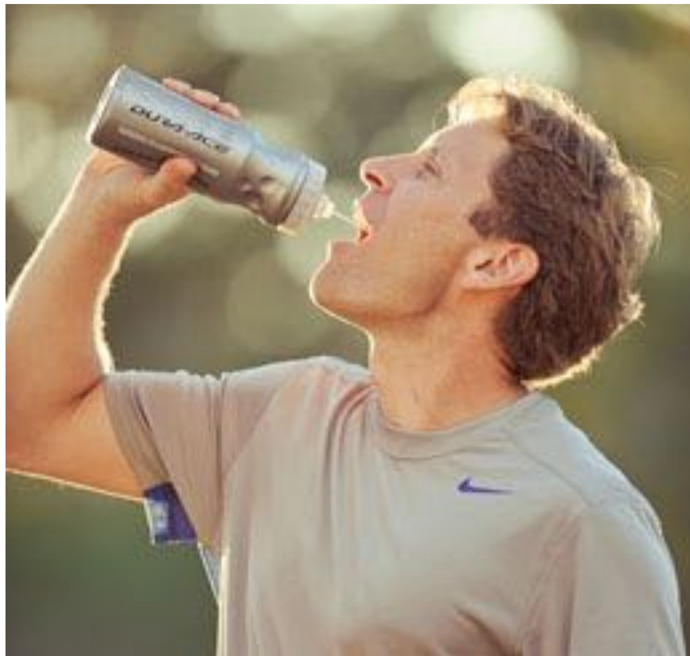
Quenched Interlude

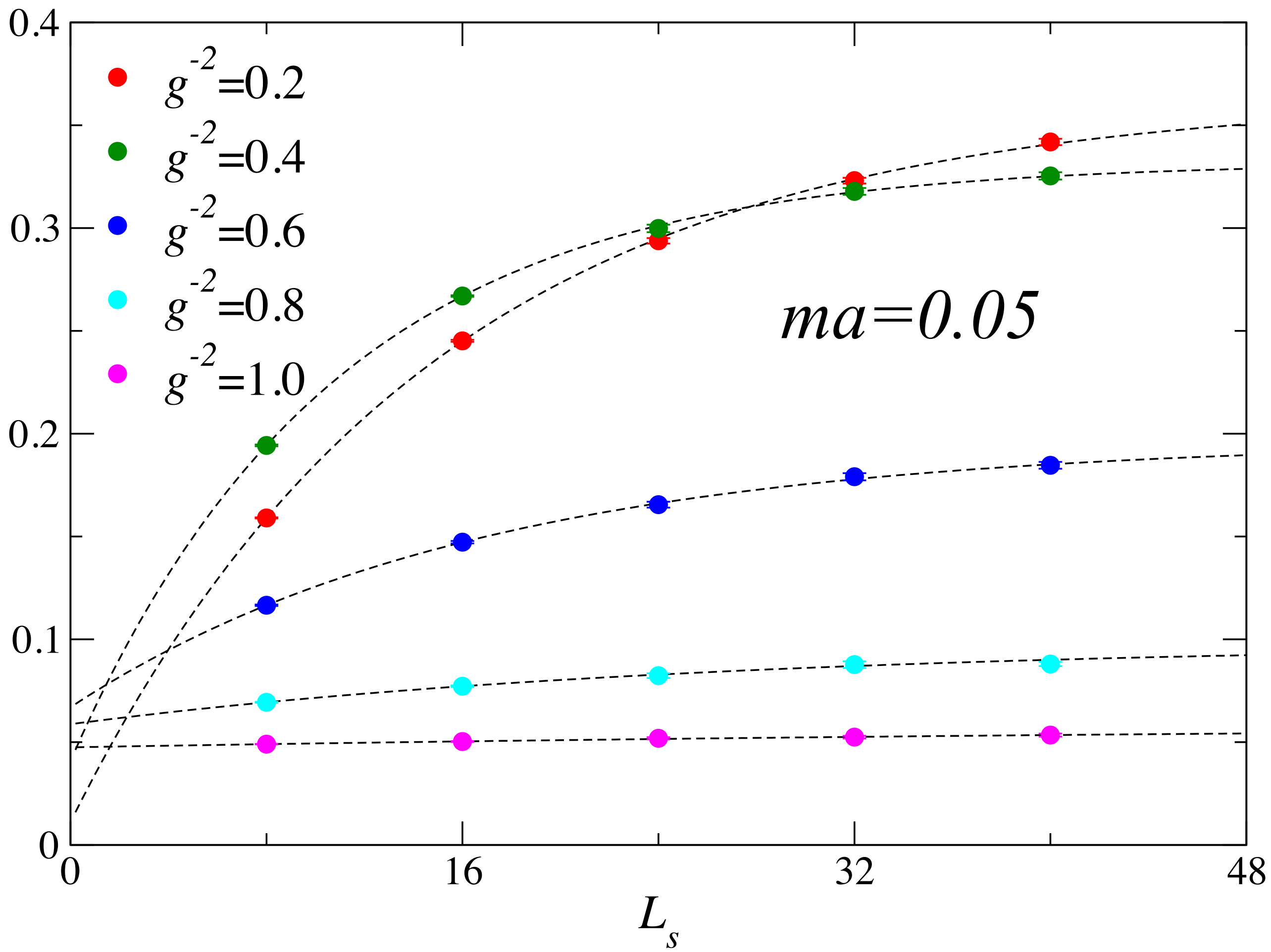
what does U(2N) symmetry-breaking look like with DWF?



comparison of **bulk** models with

N=0,1,2 with L_s=16, ma=0.01

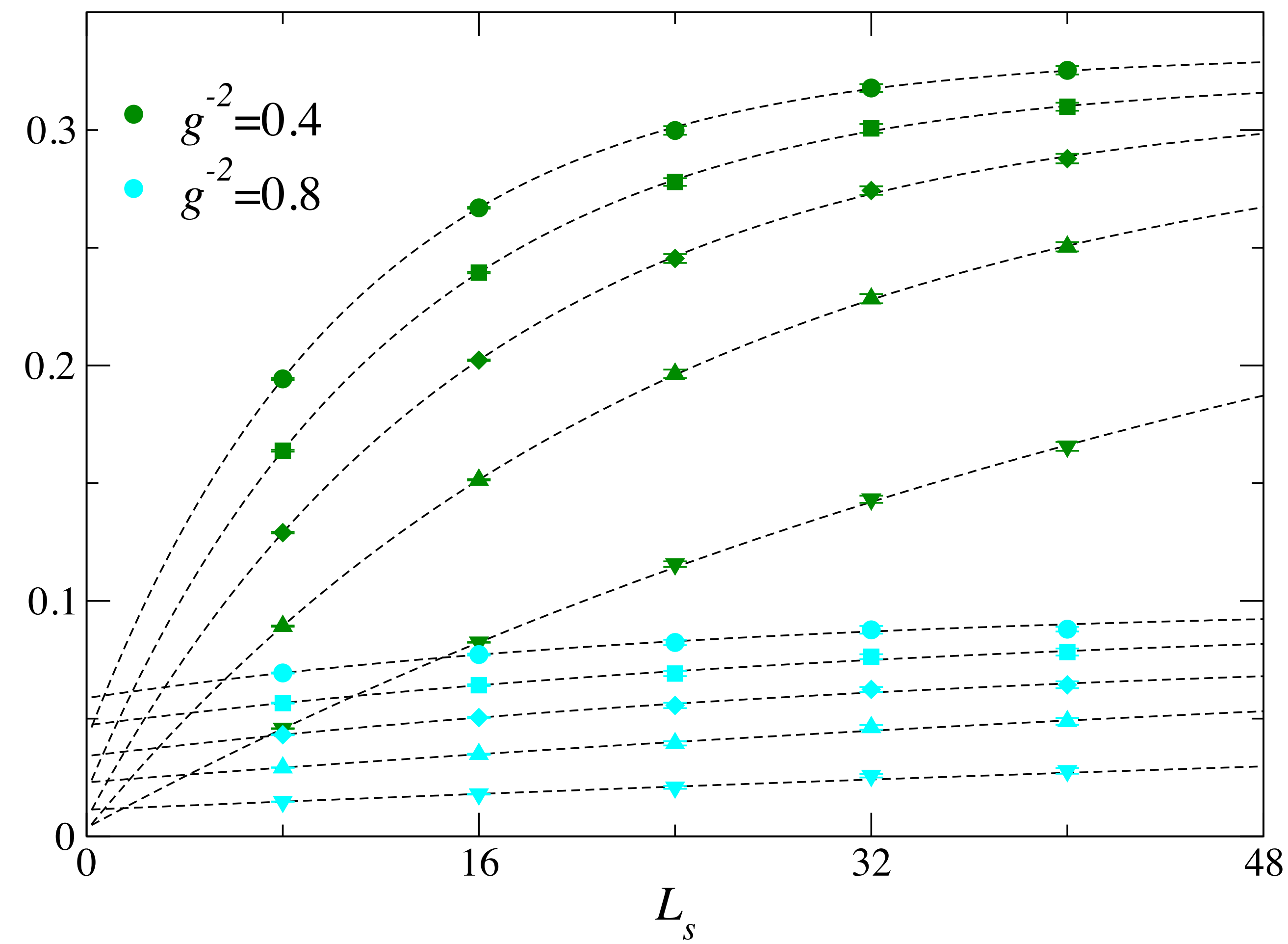




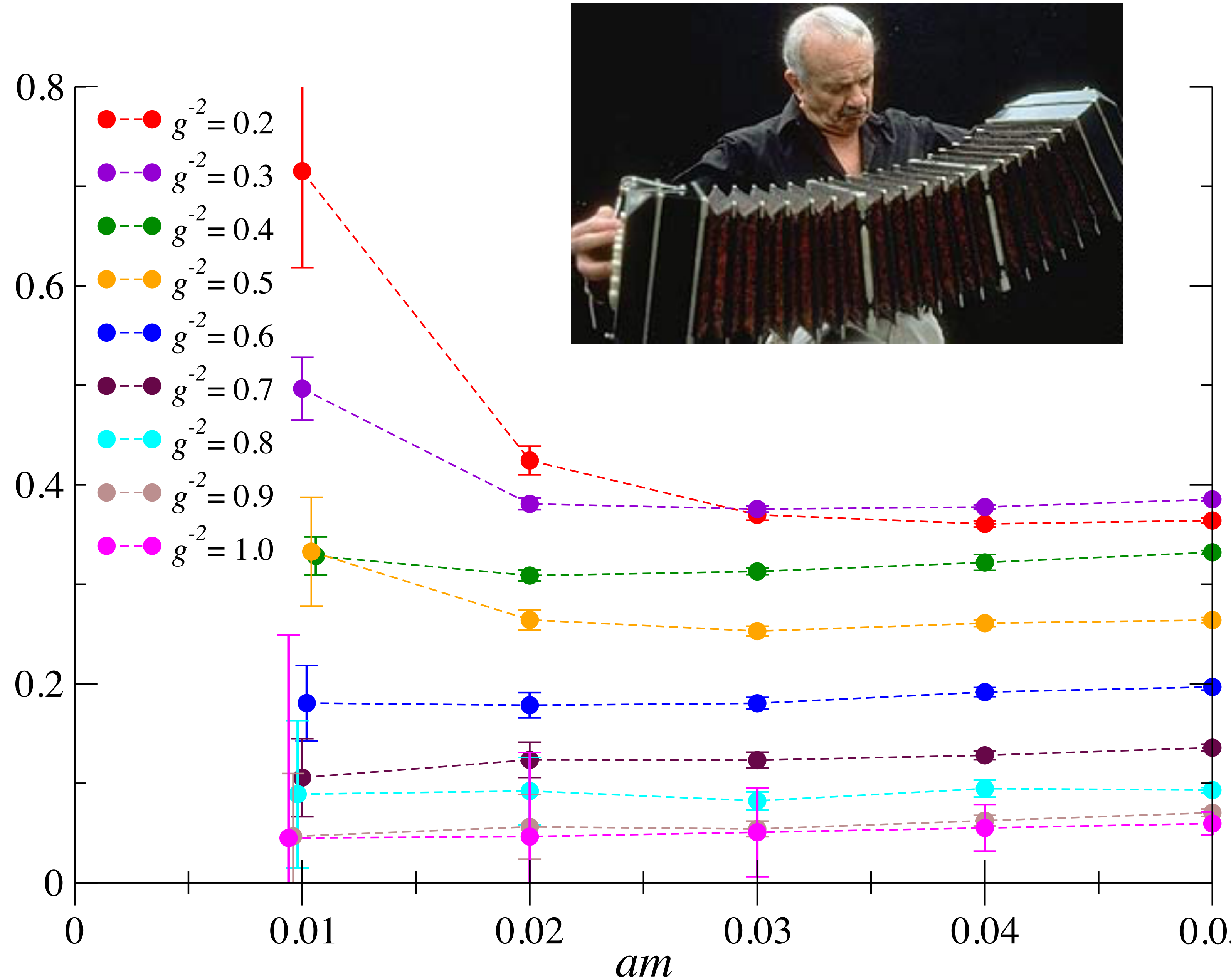
Finite-L_s corrections *much* more significant in quenched simulations

$$\langle\bar\psi\psi\rangle_{L_s} = \langle\bar\psi\psi\rangle_\infty - A(m,g^2)e^{-\Delta(m,g^2)L_s}$$

Amplitude A & decay constant Δ both increase with size of signal



$L_s \to \infty$ for quenched theory



$ag^{-2} \lesssim 0.2$ strong coupling lattice artefacts?

$ag^{-2} \gtrsim 0.8$ $m \to 0$ limit hard to extract, consistent with zero

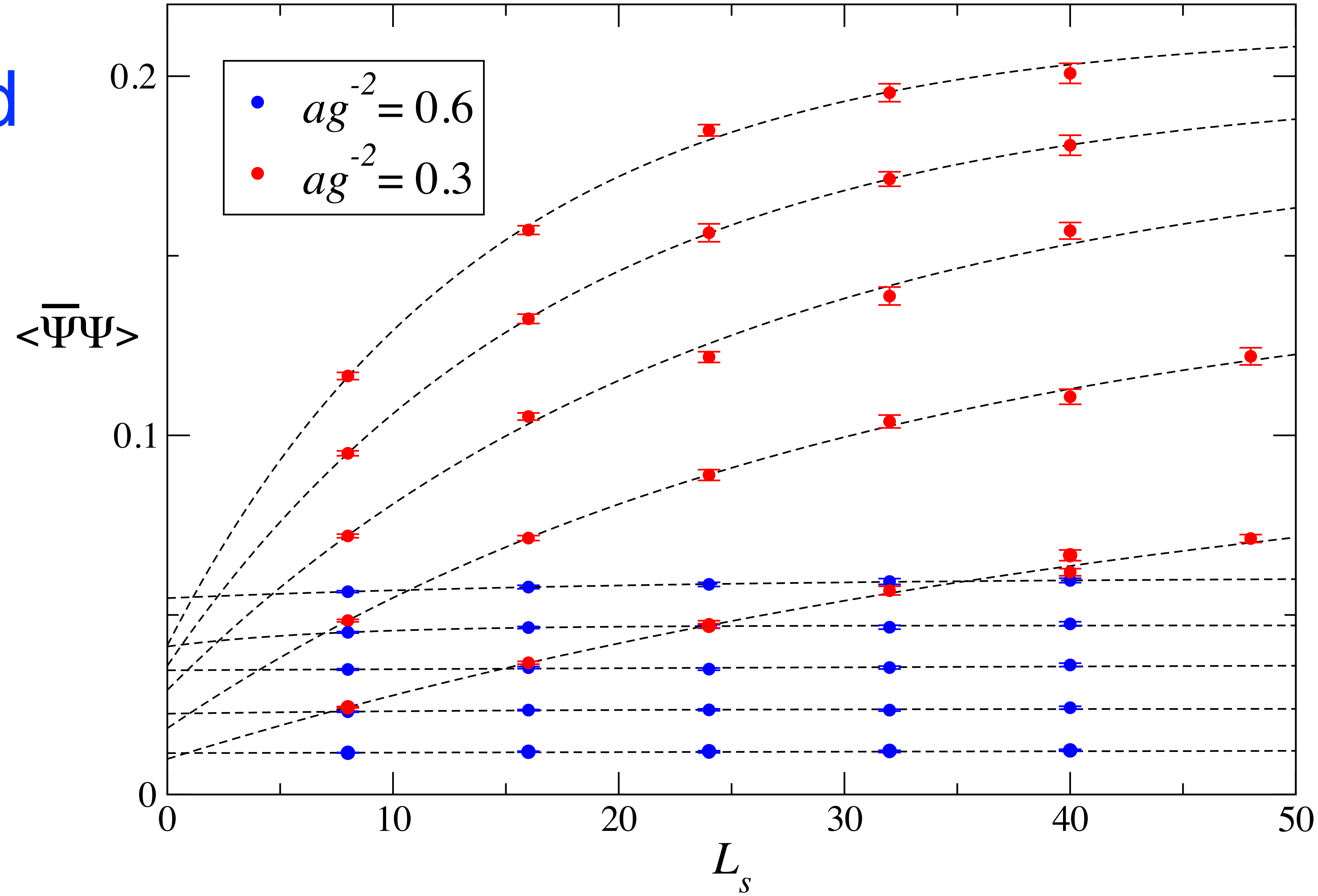
$ag^{-2} \in (0.3, 0.7)$ $m \to 0$ **has non-vanishing intercept consistent with symmetry breaking**

Cf. quenched QED_4 in the old days….

Kocić, SJH, Kogut, Dagotto, NPB **347**(1990)217

$$\Rightarrow N_c > 0?$$

Have now repeated analysis for $N=1, 12^3 \times L_s$



lines are exponential extrapolations $L_s \to \infty$

Again, a big contrast

weak $ag^{-2}=0.6$ vs. strong $ag^{-2}=0.3$

$L_s=48$, $am=0.01$, $ag^{-2}=0.3$:

RHMC Hamiltonian step requires ~9500 QMR iterations

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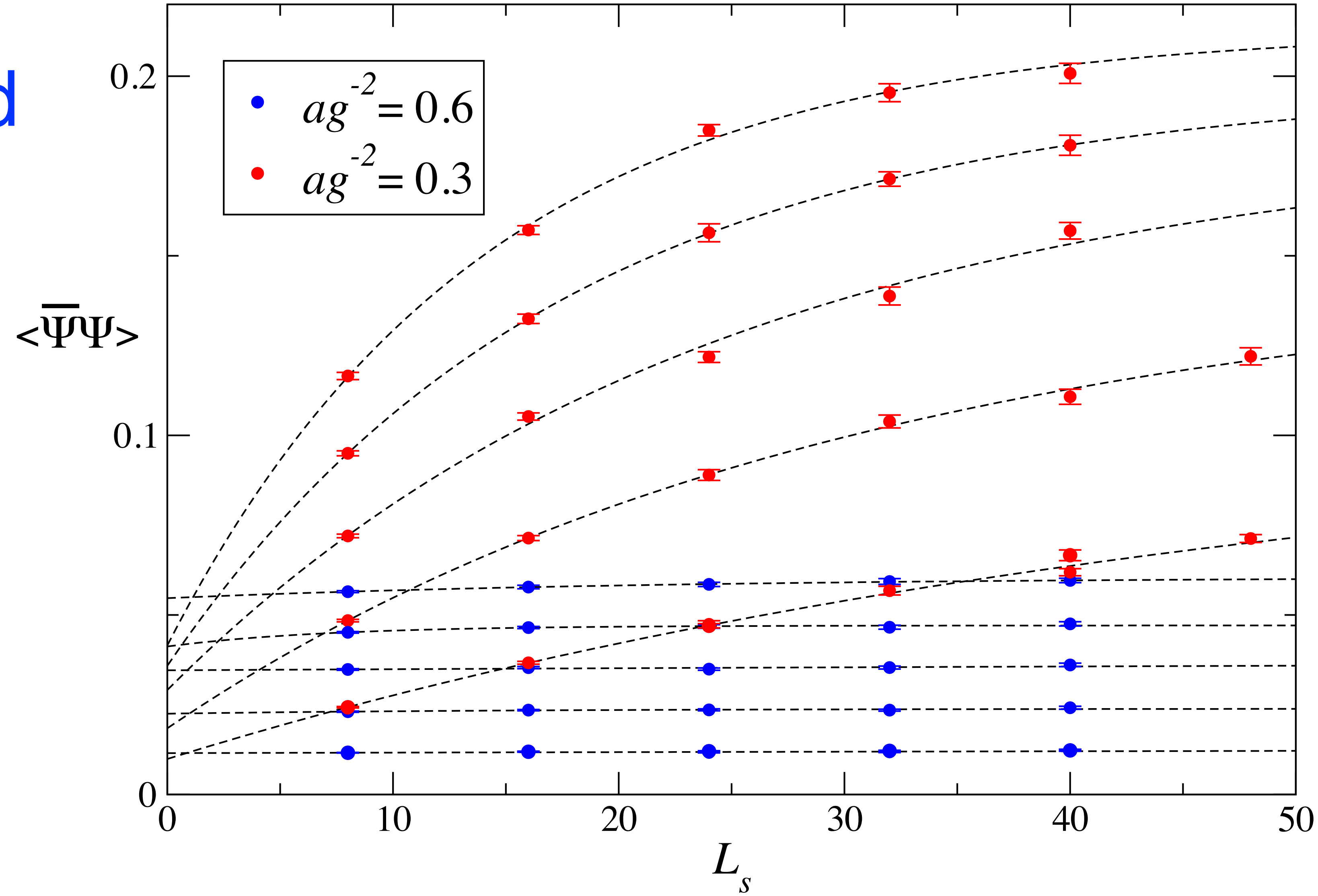

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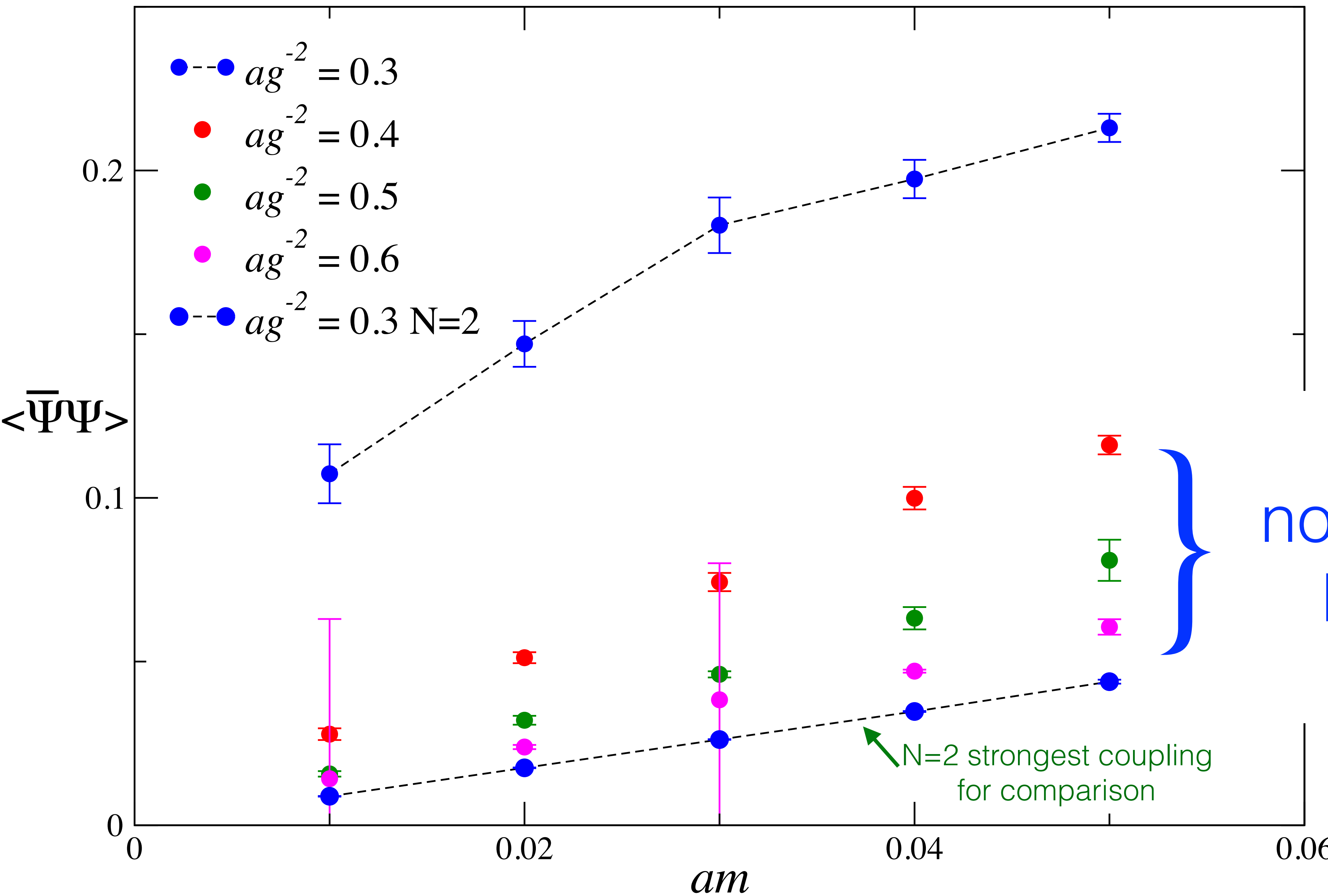
No-one said strong coupling would be easy….

N=1 $L_s \to \infty$

$12^3 \times L_s$, $L_s=8,\ldots,40(48)$; $ag^{-2}=0.6,5,4,3$;
$ma = 0.01,2,3,4,5$ ⇔

O(6 months) on cluster, 4 cores per run



Qualitative difference in $\langle\bar{\Psi}\Psi(m)\rangle$ at the strongest coupling examined $ag^{-2} = 0.3$

no symmetry breaking

$\Rightarrow 1 < N_c < 2$? $0.3 < ag_c^{-2} < 0.4$??

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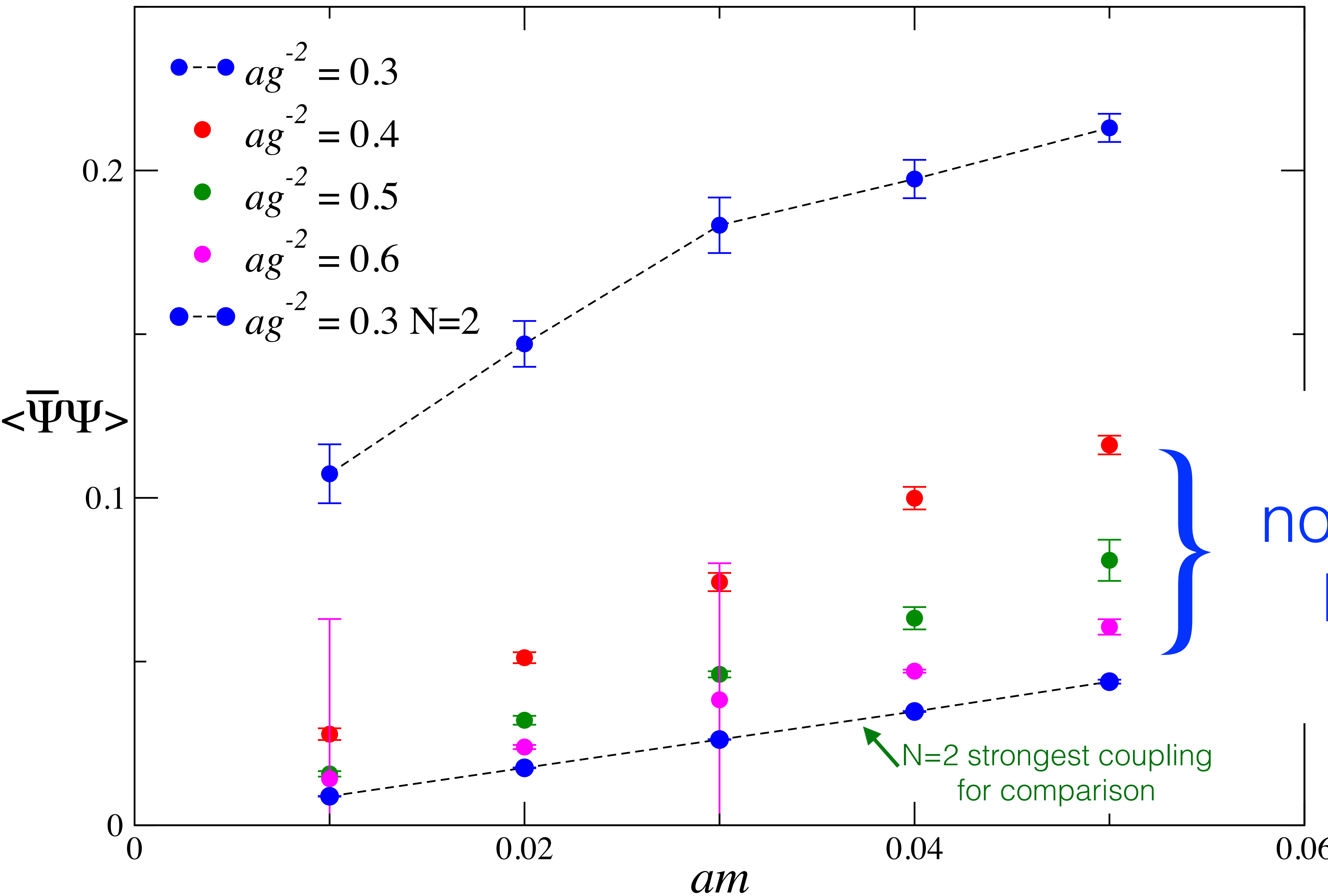

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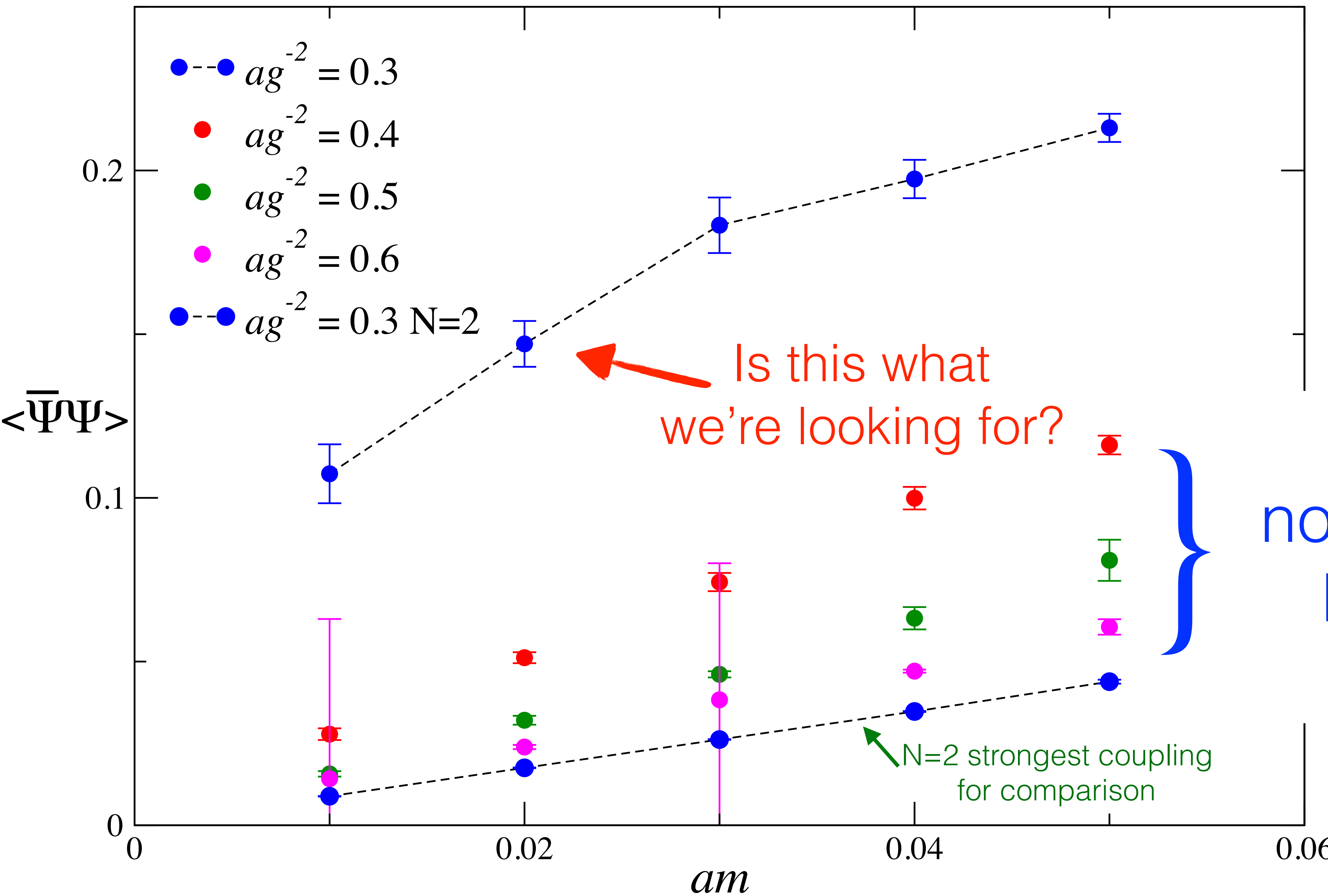

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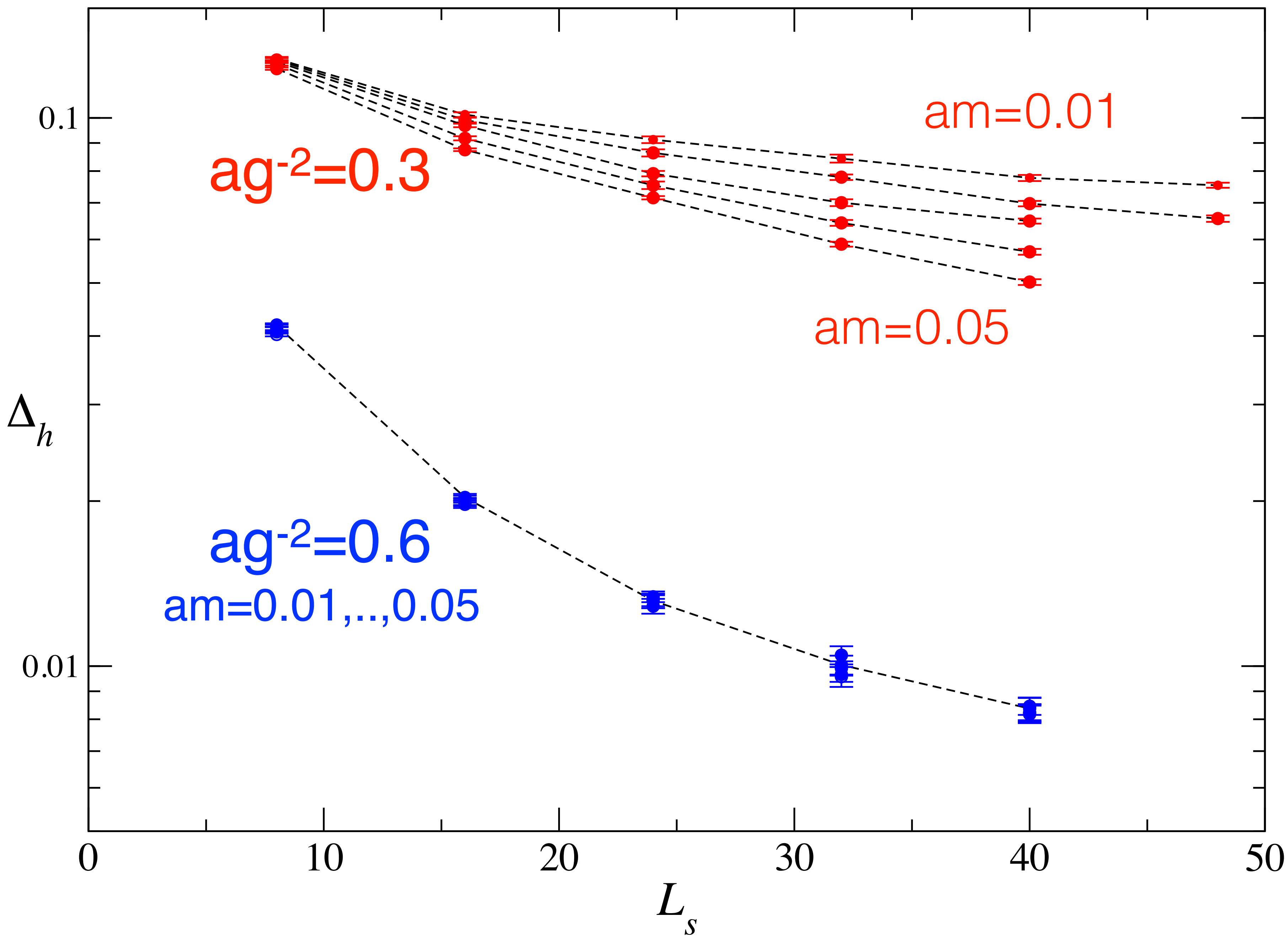
U(2) symmetry restoration as $L_s \to \infty$



Qualitatively different at strong and weak coupling,
and *slow*...

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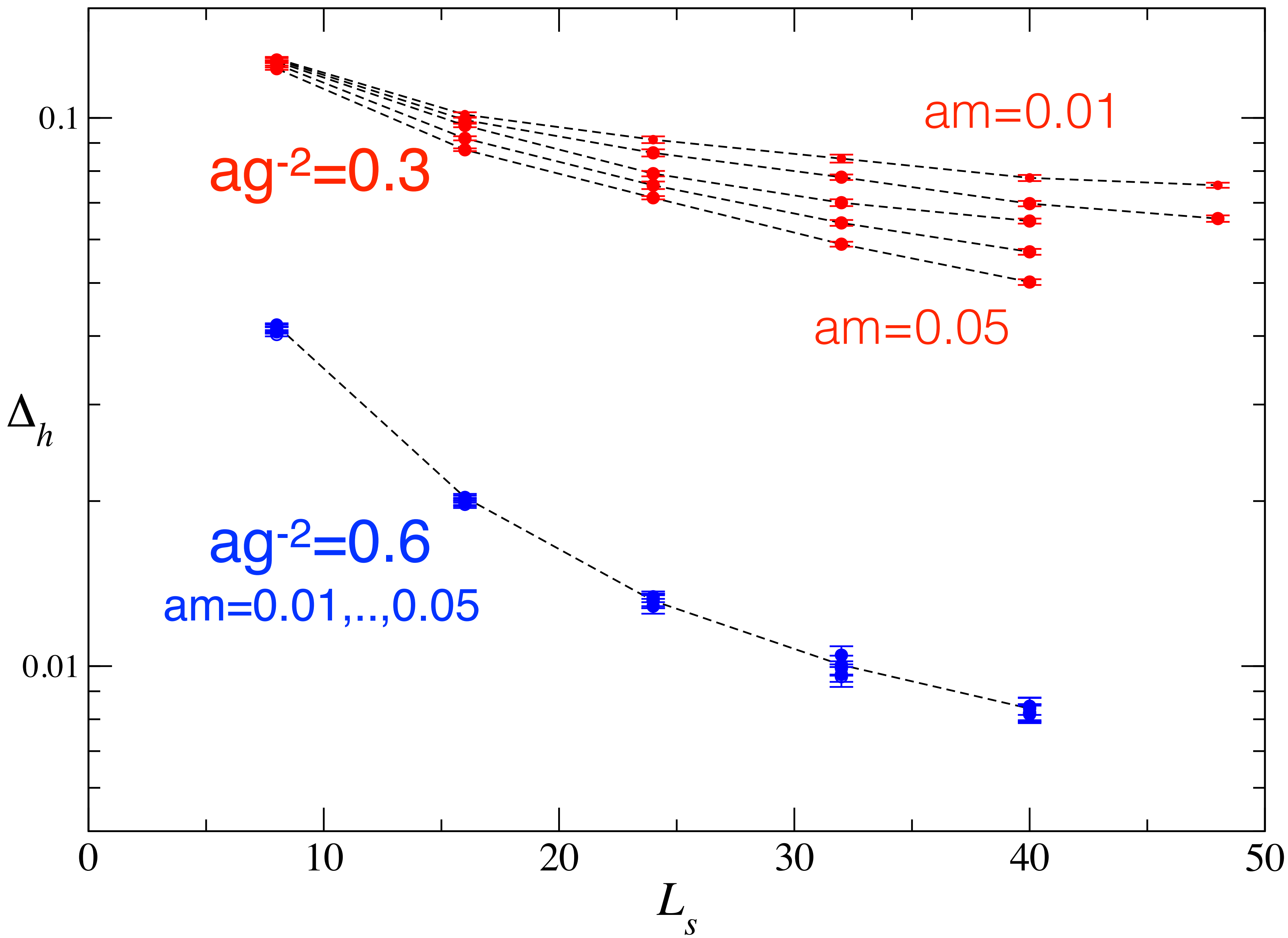

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Summary & Outlook

- No obstruction found to simulating U(2N) fermions
- "twisted mass" $im_3\bar{\psi}\gamma_3\psi$ optimises $L_s\to\infty$
- Robust conclusion: $N_{fc}<2$ for both bulk and surface
- Tentative evidence for SSB for $N=1$ at strong coupling

  Cf. QED$_3$ $N_{fc}<1$ Karthik & Narayanan PR**D93** 045020, **D94** 065026 (2016)
- Staggered Thirring Model shouldn't be forgotten — very non-trivial sensitivity to N
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- Try Haldane mass $m_{35}\neq 0$?
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- Analysis of critical scaling at QCP requires improved code!

Science & Technology Facilities Council

JHEP **1509** (2015) 047
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