Spontaneous Symmetry Breaking in the U(2) Planar Thirring Model?

Simon Hands

Quark Confinement and the Hadron Spectrum, Maynooth 2nd August 2018
In this talk I will

• discuss quantum field theories of relativistic fermions in 2+1d focussing on the U(2N)-invariant Thirring model

• review critically old simulation results for QCPs obtained with staggered lattice fermions

• show that domain wall fermions capture the relevant global symmetries more accurately

• present simulation results showing that DWF tell a very different story to staggered
Relativistic Fermions in 2+1d

Several applications in condensed matter physics

• Nodal fermions in $d$-wave superconductors
• Spin liquids in Heisenberg AFM
• Surface states of topological insulators
• …and graphene
Free reducible fermions in 3 spacetime dimensions

\[ S = \int d^3x \, \bar{\Psi} (\gamma_\mu \partial_\mu) \Psi + m \bar{\Psi} \Psi \]

For \( m=0 \) \( S \) is invariant under global \( U(2N) \) symmetry generated by

- (i) \( \Psi \rightarrow e^{i\alpha} \Psi \); \( \bar{\Psi} \rightarrow \bar{\Psi} e^{-i\alpha} \)
- (ii) \( \Psi \rightarrow e^{i\alpha \gamma_5} \Psi \); \( \bar{\Psi} \rightarrow \bar{\Psi} e^{i\alpha \gamma_5} \)
- (iii) \( \Psi \rightarrow e^{\alpha \gamma_3 \gamma_5} \Psi \); \( \bar{\Psi} \rightarrow \bar{\Psi} e^{-\alpha \gamma_3 \gamma_5} \)
- (iv) \( \Psi \rightarrow e^{i\alpha \gamma_3} \Psi \); \( \bar{\Psi} \rightarrow \bar{\Psi} e^{i\alpha \gamma_3} \)

For \( m\neq 0 \) \( \gamma_3 \) and \( \gamma_5 \) rotations no longer symmetries

\[ \Rightarrow U(2N) \rightarrow U(N) \otimes U(N) \]

Mass term \( m \bar{\Psi} \Psi \) is hermitian & invariant under parity \( x_\mu \rightarrow -x_\mu \)

Two physically equivalent antihermitian “twisted” or “Kekulé” mass terms:

\[ i m_3 \bar{\Psi} \gamma_3 \Psi; \quad i m_5 \bar{\Psi} \gamma_5 \Psi \]

The “Haldane” mass \( m_{35} \bar{\Psi} \gamma_3 \gamma_5 \Psi \) is not parity-invariant
The Thirring Model in 2+1d

\[ \mathcal{L} = \bar{\psi}_i (\partial + m) \psi_i + \frac{g^2}{2N_f} (\bar{\psi}_i \gamma_\mu \psi_i)^2 \]

- four-fermi form
- bosonised form \[ \mathcal{L} = \bar{\psi}_i (\partial + i \frac{g}{\sqrt{N_f}} A_\mu \gamma_\mu + m) \psi_i + \frac{1}{2} A_\mu A_\mu \]

- Interacting QFT
- expansion in \( g^2 \) non-renormalisable
- Hidden Local Symmetry \( \psi \mapsto e^{i\alpha} \psi; \ A_\mu \mapsto A_\mu + \partial_\mu \alpha; \ \varphi \mapsto \varphi + \alpha \)
  if Stückelberg scalar field \( \varphi \) introduced
- expansion in \( 1/N_f \) exactly renormalisable for \( 2<d<4 \)
  \[ \langle A_\mu A_\nu \rangle \propto \delta_{\mu\nu}/k^{d-2} \] in “Feynman gauge” \( \text{SJH PRD51 (1995) 5816} \)
- dynamical chiral symmetry breaking for \( g^2 > g_c^2; N_f < N_{fc} \)?
- Quantum Critical Point at \( g_c^2 (N<N_{fc}) \) ?

Determination of \( N_{fc} \) is a non-perturbative problem in QFT

eg. \( N_{fc}=4.32 \) strong coupling Schwinger-Dyson (ladder approximation) \( \text{Itoh, Kim, Sugiura & Yamawaki Prog. Theor. Phys. 93 (1995) 417} \)
Numerical Lattice Approach

Early work used staggered fermions

\[
S_{latt} = \frac{1}{2} \sum_{x, \mu i} \bar{\chi}_x^i \eta_{\mu x} (1 + i A_{\mu x}) \chi_{x + \hat{\mu}}^i - \bar{\chi}_x^i \eta_{\mu x} (1 - i A_{\mu x - \hat{\mu}}) \chi_{x - \hat{\mu}}^i \\
+ m \sum_{x i} \bar{\chi}_x^i \chi_x^i + \frac{N}{4g^2} \sum_{x \mu} A_{\mu x}^2
\]

auxiliary boson couples linearly

resembles abelian gauge theory, but link field is NOT unit modulus!

\[
A_{\mu x} \text{ auxiliary vector field defined on link between } x \text{ and } x + \mu
\]

\[
\eta_{\mu x} \equiv (-1)^{x_0 + \cdots + x_{\mu - 1}} \Rightarrow \prod_{\square} \eta \eta \eta = -1 \quad \pi\text{-flux}
\]

Chiral symmetry: \( U(N) \otimes U(N) \rightarrow U(N) \) (if \( m, \Sigma \neq 0 \))

In weak coupling continuum limit
\( U(2N_f) \) symmetry is recovered, with \( N_f = 2N \)
Strong coupling limit \( g^2 \rightarrow \infty \)

The lattice regularisation does not respect current conservation

Both diagrams needed to ensure transversity, (ie. WT identity \( \sum_\mu [\Pi_{\mu\nu}(x) - \Pi_{\mu\nu}(x - \hat{\mu})] = 0 \)) in lattice QED

\[ g^2 \rightarrow g_{\text{lim}}^2(N_f) \]
**Strong coupling limit** \( g^2 \rightarrow \infty \)

The lattice regularisation does **not** respect current conservation

Both diagrams needed to ensure transversity, (ie. WT identity \( \sum_{\mu} \left[ \Pi_{\mu\nu}(x) - \Pi_{\mu\nu}(x - \hat{\mu}) \right] = 0 \) ) in lattice QED

Only the left hand diagram is present for the lattice Thirring model with linear coupling to auxiliary

\[ \Rightarrow \frac{1}{N_f} \text{ expansion yields additive} \quad g^{-2} \]

\[ g^2_R = \frac{g^2}{1 - g^2 / g^2_{\text{lim}}} \]

\[ \Rightarrow \text{lattice strong coupling limit as} \quad g^2 \rightarrow g_{\text{lim}}^2(N_f) \]
Results in effective strong-coupling limit

\[ N_{fc} = 6.6(1), \quad \delta(N_{fc}) = 6.90(3) \]

Chiral symmetry unbroken for all \( g^2 \) for \( N_f > N_{fc} \)

Cf. SDE: \( N_{fc} = 4.32, \quad \delta(N_{fc}) = 1 \)

“conformal phase transition”
Results in effective strong-coupling limit

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Cf. SDE: \( N_{fc}=4.32, \quad \delta(N_{fc})=1 \)

“conformal phase transition”
Staggered Thirring Summary

\[ \langle \bar{\psi} \psi \rangle = 0 \]

- Chiral symmetry broken for small \( N_f \), large \( g^2 \)
- Each point (for \( N_f \) integer) defines a UV fixed point of RG
- Distinct critical exponents \( \Leftrightarrow \) distinct interacting QFT
- \( \delta \) increases with \( N_f \), \( \delta(N_{fc}) \approx 7 \)
- Non-covariant form used as EFT for graphene \( \Rightarrow N_{fc} \approx 5 \)

**Fermion Bag Algorithm with minimal** $N_f=2$


**Thirring Model:** $v=0.85(1)$, $\eta=0.65(1)$, $\eta_\psi=0.37(1)$ \hspace{1cm} ($N_f < N_{fc} \approx 7$)

**U(1) GN Model:** $v=0.849(8)$, $\eta=0.633(8)$, $\eta_\psi=0.373(3)$ \hspace{1cm} ($N_f \to \infty$: $v=\eta=1$)

Interactions between staggered fields $\chi$, $\chi$ spread over elementary cubes. Only difference between Thirring & GN is body-diagonal term

Staggered fermions not reproducing expected distinction between models a near strongly-coupled fixed point...

see also SLAC fermion approach

Schmidt, Wellegehausen & Wipf, PoS LATTICE2015 (2016) 050

PRD96 (2017) 094504
Fermion Bag Algorithm with minimal $N_f = 2$


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Staggered fermions not reproducing expected distinction between models a near strongly-coupled fixed point…

… so we need better lattice fermions

see also SLAC fermion approach

Schmidt, Wellegehausen & Wipf, PoS LATTICE2015 (2016) 050
PRD96 (2017) 094504
Fermions propagate freely along a fictitious third direction of extent $L_s$ with open boundaries.

**Basic idea as $L_s \to \infty$:**

- zero-modes of $D_{\text{DWF}}$ localised on walls are $\pm$ eigenmodes of $\gamma_s$
- Modes propagating in bulk can be decoupled (with cunning)

\[
\psi(x) = P_- \Psi(x, 1) + P_+ \Psi(x, L_s);
\]

\[
\bar{\psi}(x) = \bar{\Psi}(x, L_s) P_- + \bar{\Psi}(x, 1) P_+; \quad \text{with } P_\pm = \frac{1}{2}(1 \pm \gamma_s).
\]
Bottom Up View…

in DWF approach we simulate 2+1+1d fermions

Desiderata…

• Modes localised on walls carry U(2N)-invariant physics
• Fermion doublers don’t contribute to normalisable modes
• Bulk modes can be made to decouple

Claim…

It appears to work for….
• carefully-chosen domain wall height M
• smooth gauge field background
Are DWF in 2+1+1d U(2N) symmetric?

**Issue:** wall modes are eigenstates of \( \gamma_3 \) as \( L_s \to \infty \),

but: \( U(2N) \) symmetry demands equivalence under rotations generated by both \( \gamma_3 \) and \( \gamma_5 \)

ie. \( U(2N) \to U(N) \otimes U(N) \) symmetry-breaking mass terms

\[
\begin{align*}
m_h \bar{\psi} \psi & \quad \text{im}_3 \bar{\psi} \gamma_3 \psi \\
\end{align*}
\]

should yield identical physics as \( L_s \to \infty \)

**Non-trivial requirement**

since \( m_h, m_3 \) couple \( \Psi, \bar{\Psi} \) on *opposite* walls

while \( m_5 \) couples modes on *same* wall
Define main residual: \[ i\langle \bar{\Psi}(1)\gamma_3\Psi(L_s)\rangle = \frac{i}{2}\langle \bar{\psi}\gamma_3\psi\rangle_{L_s\to\infty} + i\Delta_h(L_s) + \epsilon_h(L_s); \]

\[ \frac{1}{2}\langle \bar{\psi}\psi\rangle_{L_s} = \frac{i}{2}\langle \bar{\psi}\gamma_3\psi\rangle_{L_s\to\infty} + \Delta_h(L_s) + \epsilon_3(L_s); \]

\[ \frac{i}{2}\langle \bar{\psi}\gamma_5\psi\rangle_{L_s} = \frac{i}{2}\langle \bar{\psi}\gamma_3\psi\rangle_{L_s\to\infty} + \epsilon_5(L_s). \]

- exponentially suppressed as \( L_s \to \infty \)
- hierarchy: \( \Delta_h > \epsilon_h > \epsilon_3 \equiv \epsilon_5 \)

U(2) symmetry restored

\[ \iff \Delta_h \to 0 \]

Bilinear Condensates in Quenched QED$_3$ on $24^3 \times L_s$...

**Define main residual:**

\[
\begin{align*}
\frac{1}{2} \langle \bar{\psi} \psi \rangle_{L_s} &= \frac{i}{2} \langle \bar{\psi} \gamma_3 \psi \rangle_{L_s \to \infty} + \Delta_h(L_s) + \epsilon_h(L_s); \\
\frac{i}{2} \langle \bar{\psi} \gamma_3 \psi \rangle_{L_s} &= \frac{i}{2} \langle \bar{\psi} \gamma_3 \psi \rangle_{L_s \to \infty} + \epsilon_3(L_s); \\
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\end{align*}
\]

- exponentially suppressed as $L_s \to \infty$
- hierarchy: $\Delta_h > \epsilon_h > \epsilon_3 \equiv \epsilon_5$

\[\begin{array}{c}
\begin{array}{c}
\beta = 0.5 \\
\beta = 1.0 \\
\beta = 2.0 \\
\beta = 0.5 \ 12^3
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
\text{increasing coupling}
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
\text{U(2) symmetry restored}
\end{array}
\end{array}\]

$\Leftrightarrow \Delta_h \to 0$

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\end{align*} \]

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**U(2) symmetry restored**

\[ \Leftrightarrow \Delta_h \to 0 \]

Top Down View...

The closest approach to continuum symmetries is expressed by Ginsparg-Wilson relations

$$\{\gamma_5, D\} = 2D\gamma_5 D$$

RHS is $O(aD)$, so U(2N) recovered in long-wavelength limit if $D$ local

By construction GW is satisfied by the 2+1d overlap operator

$$D_{ov} = \frac{1}{2} \left[ (1 + m_h) + (1 - m_h) \frac{A}{\sqrt{A^\dagger A}} \right]$$

with $\gamma_3 A\gamma_3 = \gamma_5 A\gamma_5 = A^\dagger$

$A \equiv [2 + (D_W - M)]^{-1}[D_W - M]$; $D_W$ local; $Ma = O(1)$ \[D_{ov} \] not manifestly local

DWF provide a regularisation of overlap with a local kernel in 2+1+1d

$$\frac{\det D_{\text{DWF}}(m_i)}{\det D_{\text{DWF}}(m_h = 1)} = \det D_{Ls}(m_i)$$

$$\lim_{Ls \to \infty} D_{Ls} = D_{ov}$$

SJH PLB 754 (2016) 264
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\[
\begin{align*}
\{ \gamma_5, D \} &= 2D \gamma_5 D \\
\{ \gamma_3, D \} &= 2D \gamma_3 D \\
[\gamma_3 \gamma_5, D] &= 0
\end{align*}
\]

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\[
\lim_{L_s \to \infty} D_{L_s} = D_{ov}
\]

SJH PLB 754 (2016) 264
(a) Formulate interaction terms in terms of vector auxiliary 
\( A_\mu(x) \) defined just on walls at \( x_3 = 1 \), \( L_s \): "Surface"

Technical/cost advantage: no Pauli-Villars determinant needed to cancel bulk modes


(b) By analogy with QCD, formulate with \( A_\mu(x) \) throughout bulk
which are "static" ie. \( \partial_3 A_\mu = 0 \): "Bulk"

\[
S = \bar{\Psi} D \Psi = \bar{\Psi} D_W \Psi + \bar{\Psi} D_3 \Psi + m_i S_i
\]

with

\[
D_W = \gamma_\mu D_\mu - (\hat{D}^2 + M);
\]

\[
D_3 = \gamma_3 \partial_3 - \hat{\partial}_3^2;
\]

Recall link field not unit modulus

Bulk formulation

\[
[\partial_3, D_\mu] = [\partial_3, \hat{D}^2] = 0
\]

but \( [\partial_3, \hat{\partial}_3^2] \neq 0 \) on walls

obstruction to proving \( \det D > 0 \) for \( N=1 \)
Formulational issues for the Thirring Model with DWF

(a) Formulate interaction terms in terms of vector auxiliary $A_\mu(x)$ defined just on walls at $x_3 = 1$, $L_s$: “Surface”

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(b) By analogy with QCD, formulate with $A_\mu(x)$ throughout bulk which are “static” ie. $\partial_3 A_\mu = 0$: “Bulk”

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with $D_W = \gamma_\mu D_\mu - (\hat{D}^2 + M)$; $D_3 = \gamma_3 \partial_3 - \hat{D}_3$

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obstruction to proving $\det D > 0$ for $N=1$

$\Rightarrow$ need RHMC algorithm for $N=1$
• Breakdown of reflection positivity for strong coupling $a g^{-2} \approx 0.2$?
• Strong volume dependence for surface model
• Results at $L_s = 16$ and $L_s = 20$ are consistent
• No evidence of spontaneous symmetry breaking anywhere along $g^{-2}$ axis
HMC Results with $N_f=2$ on $12^3 \times 16$

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- No evidence of spontaneous symmetry breaking anywhere along $g^{-2}$ axis

big disparity with previous staggered results
Axial Ward Identity

Strong-coupling behaviour suggests neither Surface nor Bulk model optimal:
work still needed to specify 2+1d states $\psi$ with control over normalisation

Cf. 2+1d Gross-Neveu model, where Ward Identity is respected, spectroscopy under control…
RHMC Results for $N=1$ (12$^3$x8)

N=1 simulations performed with weight $\det(M^\dagger M)^{1/2}$ using RHMC algorithm with 25 partial fractions.
RHMC Results for $N=1$ $(12^3 \times 8)$

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$arXiv:1708.07686$
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RHMC Results for $N=1$ (12$^3$x8)

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Henceforth focus on **bulk**

Evidence for enhanced pairing for $N=1$ and $ag^{-2} < 0.5$?
Boson Action

Surface and Bulk models show different behaviour

$N=1$: change of behaviour for $ag^{-2} < 0.5$?
Quenched Interlude

what does $U(2N)$ symmetry-breaking look like with DWF?

comparison of bulk models with $N=0,1,2$ with $L_s=16$, $ma=0.01$
Finite-$L_s$ corrections *much* more significant in quenched simulations

\[
\langle \bar{\psi}\psi \rangle_{L_s} = \langle \bar{\psi}\psi \rangle_{\infty} - A(m, g^2) e^{-\Delta(m, g^2)L_s}
\]

Amplitude $A$ & decay constant $\Delta$ both increase with size of signal
\[ L_s \to \infty \]

For quenched theory

\[
\begin{align*}
ag^{-2} & \lesssim 0.2 \\
ag^{-2} & \approx 0.8 \\
ag^{-2} & \in (0.3, 0.7)
\end{align*}
\]

Strong coupling lattice artefacts? Limit hard to extract, consistent with zero. Has non-vanishing intercept consistent with symmetry breaking.

Cf. quenched QED\(_4\) in the old days….

Kocić, SJH, Kogut, Dagotto, NPB 347(1990)217

\[ \Rightarrow N_c > 0? \]
Have now repeated analysis for \(N=1,12^3 \times L_s\)

lines are exponential extrapolations \(L_s \to \infty\)

Again, a big contrast

weak \(ag^{-2} = 0.6\) vs. strong \(ag^{-2} = 0.3\)

\(L_s = 48, \ am = 0.01, \ ag^{-2} = 0.3\): RHMC Hamiltonian step requires \(\sim 9500\) QMR iterations
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No-one said strong coupling would be easy….
\( N = 1 \) \( L_s \rightarrow \infty \)

\[ 12^3 \times L_s, \ L_s = 8, \ldots, 40(48); \ \text{ag}^{-2} = 0.6, 5, 4, 3; \ \text{ma} = 0.01, 2, 3, 4, 5 \ \iff \]

O(6 months) on cluster, 4 cores per run

\[ 0.01 < \text{ag}^{-2} < 0.4 \ ? \ 0.3 < \text{ag}_{c^{-2}} < 0.4 \ ?? \]
$N=1 \quad L_s \to \infty$

$12^3 \times L_s, \quad L_s = 8, \ldots, 40(48); \quad a g^{-2} = 0.6, 5, 4, 3; \quad m a = 0.01, 2, 3, 4, 5 \iff$

O(6 months) on cluster, 4 cores per run 😞

Qualitative difference in $\langle \overline{\Psi} \Psi(m) \rangle$ at the strongest coupling examined

$ag^{-2} = 0.3$

no symmetry breaking

$\Rightarrow 1 < N_c < 2 \ ? \ \ 0.3 < ag_{c}^{-2} < 0.4 \ ???$
$N=1 \quad L_s \to \infty$

$12^3 \times L_s$, $L_s=8,\ldots,40(48)$;  $ag^{-2}=0.6,5,4,3$;  
$ma = 0.01,2,3,4,5$ ↔

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$\Rightarrow 1 < N_c < 2$  ?  $0.3 < ag_{c^{-2}} < 0.4$  ??
U(2) symmetry restoration as $L_s \rightarrow \infty$

Qualitatively different at strong and weak coupling, and \textit{slow}...
U(2) symmetry restoration as $L_s \to \infty$

Qualitatively different at strong and weak coupling, and slow… 😞
Summary & Outlook

- No obstruction found to simulating U(2N) fermions
- "twisted mass" $im_3 \bar{\psi} \gamma_3 \psi$ optimises $L_s \to \infty$
- Robust conclusion: $N_{fc} < 2$ for both bulk and surface
- Tentative evidence for SSB for $N=1$ at strong coupling

Cf. QED$_3$ $N_{fc} < 1$ Karthik & Narayanan PRD93 045020, D94 065026 (2016)

- Staggered Thirring Model shouldn't be forgotten — very non-trivial sensitivity to $N$
- Need to check $V \to \infty$, the effect of varying $M_{\text{wall}}$
- Try Haldane mass $m_{35} \neq 0$?
- Need to examine locality of corresponding $D_{\text{ov}}$
- Analysis of critical scaling at QCP requires improved code!
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