

Infrared Behavior of SU(2) gauge theory

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Confinement XIII, Maynooth

Motivation

- Understanding strongly coupled theories
- Phases of gauge theories
 - Infrared behavior: χSB /Walking/IRFP
 - IRFP properties i.e. anomalous dimensions
- Phenomenological motivation
 - Composite Higgs
 - (Extended) Technicolor

β -function

$$\beta(g) = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}, \quad \left\{ \begin{array}{l} \beta_0 = \frac{11}{3} N_c - \frac{4}{3} T_r N_f \\ \beta_1 = \frac{34}{3} N_c^2 - \left(\frac{20}{3} N_c - 4 C_r\right) T_r N_f \end{array} \right.$$

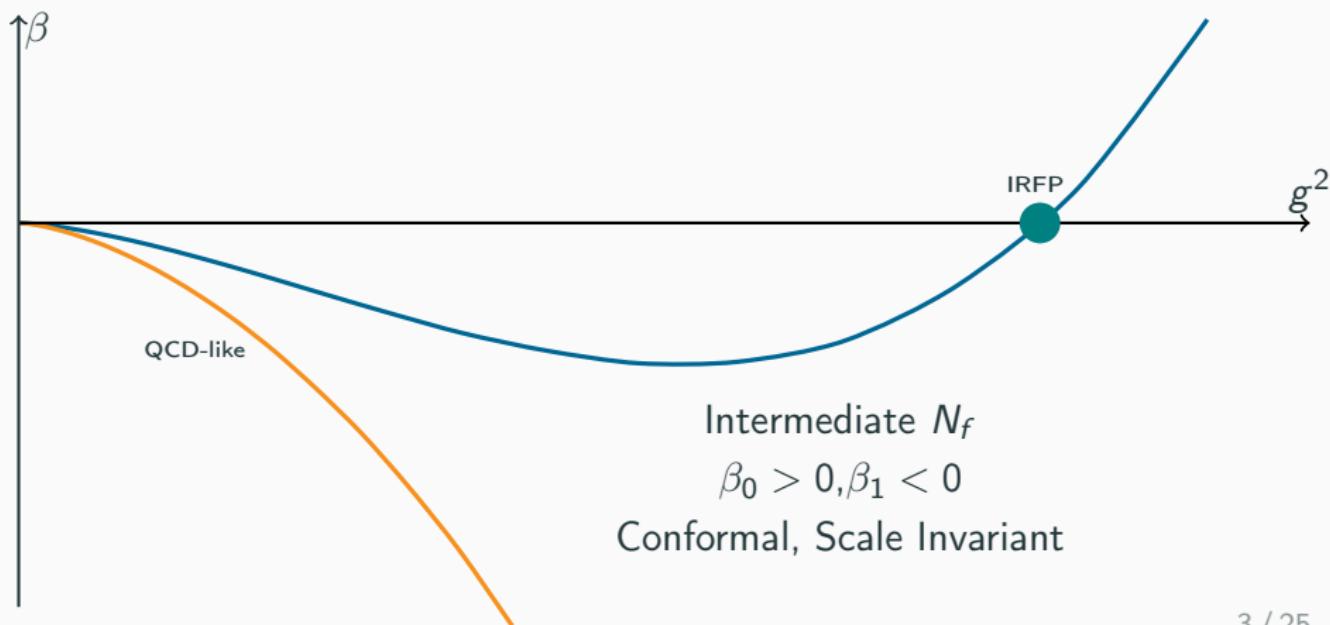
- Behavior depends on **group**, **representation** and **number of fermions**



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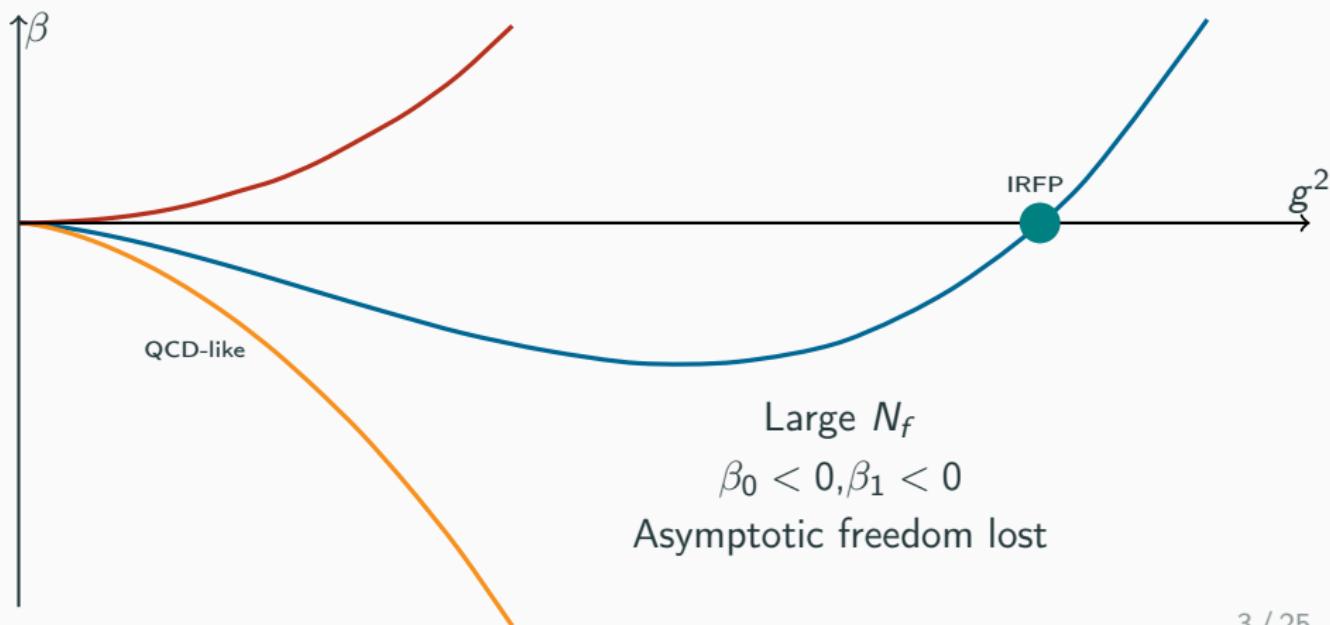
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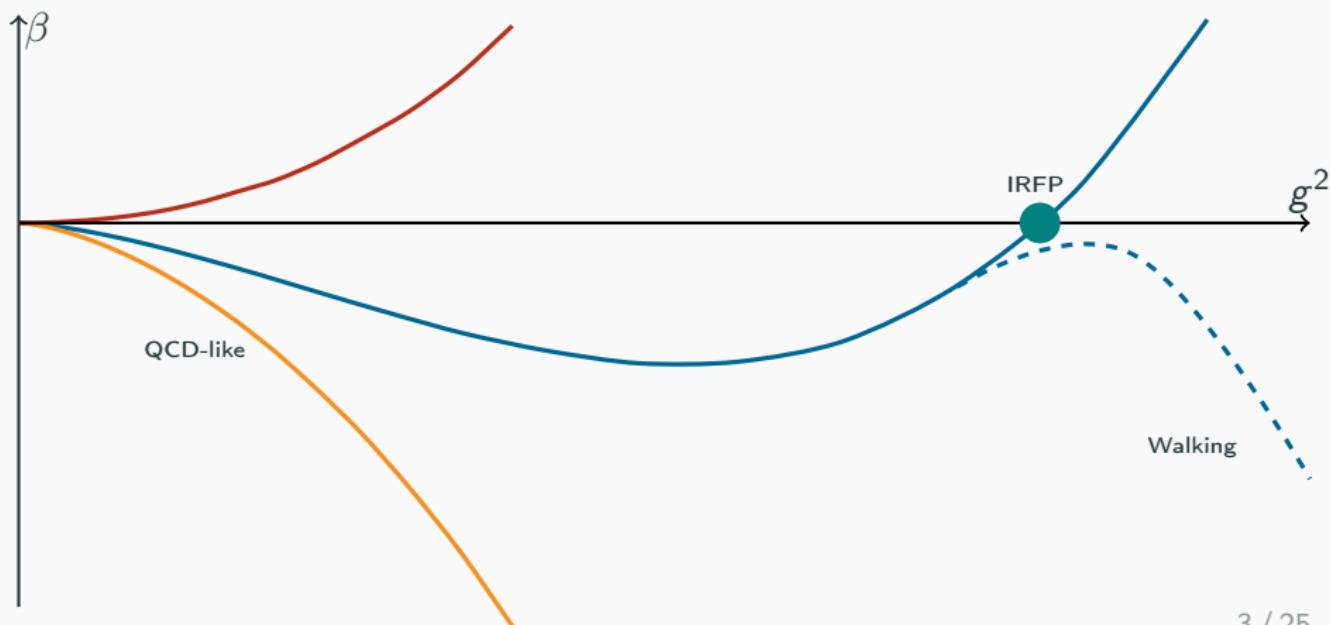
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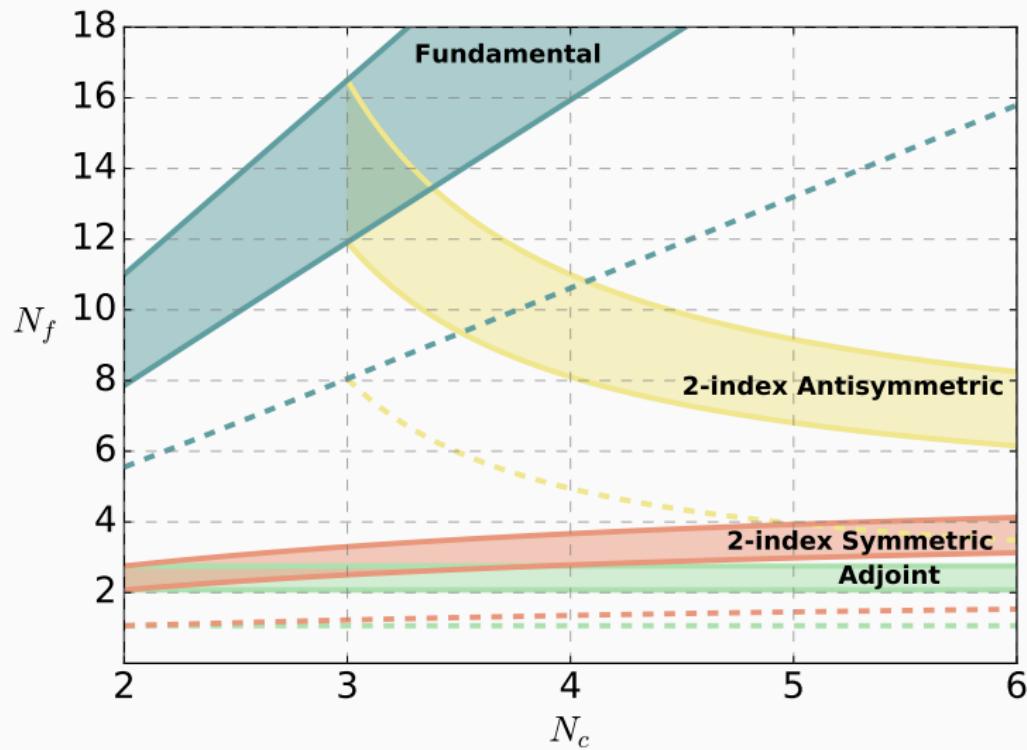
β -function

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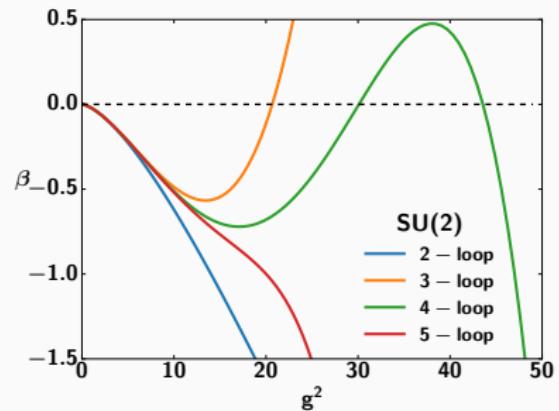
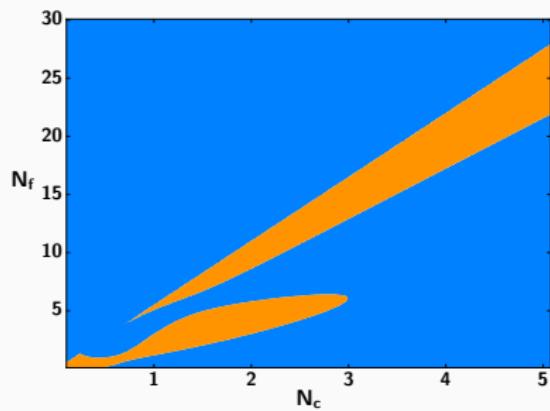
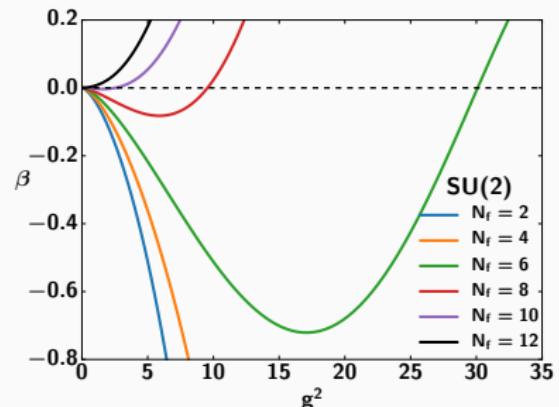
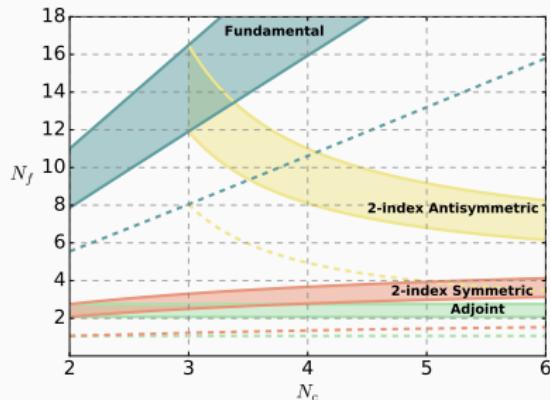
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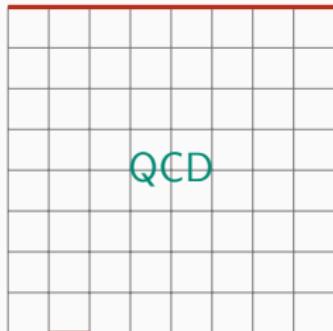
Conformal Window



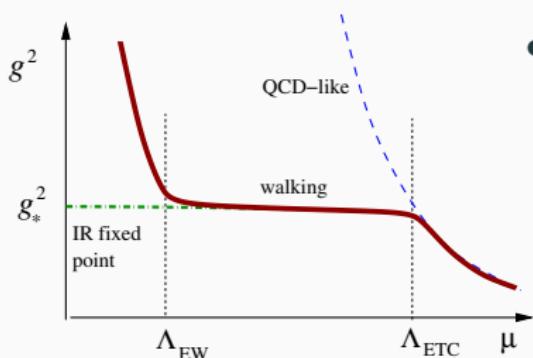
SU(2) Conformal Window



Simulations QCD \leftrightarrow Conformal



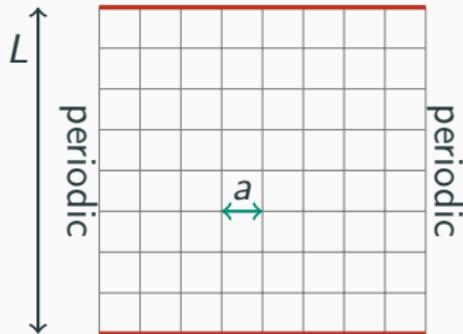
- In QCD
 - Asymptotic freedom
 - Clear cut between strong and weak coupling
 - Coupling depends nicely on lattice scale



- On a walking/conformal theory, coupling large and mostly independent on scale
 - Coupling equally strong almost everywhere
 - Small β required, regardless of L

Lattice details: Boundary conditions

$$T=L, U=1, \psi=0$$



$$T=0, U=1, \psi=0$$

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V)$$

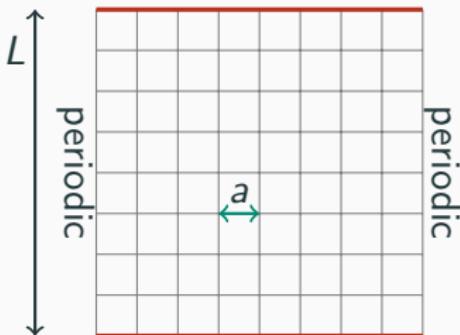
$$S_G = \beta \sum_{\square} \left(1 - \frac{1}{2} \text{Tr} \square(x) \right)$$

$$S_F = a^4 \sum_{N_f} \sum_x \left(\bar{\psi} (iD + m_0) \psi + \frac{ai}{4} c_{sw} \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi \right)$$

- Schrödinger functional boundaries $c_t = 1$
 - Periodic boundary conditions on spatial boundaries
 - Dirichlet boundary conditions on time boundaries
- Easier to tune fermion masses to zero
- Enables measurement of mass anomalous dimension

Lattice details: Action

$$T=L, U=1, \psi=0$$



$$T=0, U=1, \psi=0$$

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V)$$

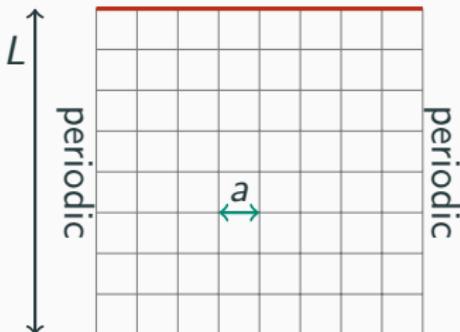
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- Use HEX-smearing
 - Moves bulk phase and allows reaching zero mass at higher couplings
- Mix smeared (V) and unsmeared (U) gauge fields with c_g
- Clover improved Wilson action $c_{sw} = 1$

Lattice details: General

$$T=L, U=1, \psi=0$$



$$T=0, U=1, \psi=0$$

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V)$$

$$S_G = \beta \sum_{\square} \left(1 - \frac{1}{2} \text{Tr} \square(x) \right)$$

$$S_F = a^4 \sum_{N_f} \sum_x (\bar{\psi} (iD + m_0) \psi + \frac{ai}{4} c_{sw} \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi)$$

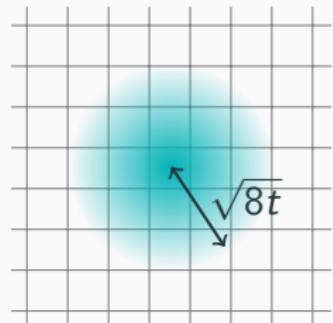
- $N_f = 6$: $L = 8^4 - 30^4$ and $\beta = 8 - 0.5$
- $N_f = 8$: $L = 6^4 - 32^4$ and $\beta = 8 - 0.4$
- Tune the fermion masses to zero with axial ward identity ($L=24$)

Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu},$$

$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}].$$

$B_{0,\mu} = A_\mu$ ← the original gauge field

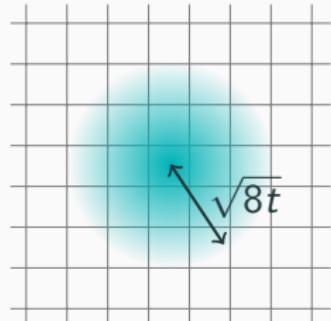


- Only gauge fields
- Evolve with fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- We use Lüscher-Weisz action for S_{YM}

Gradient flow

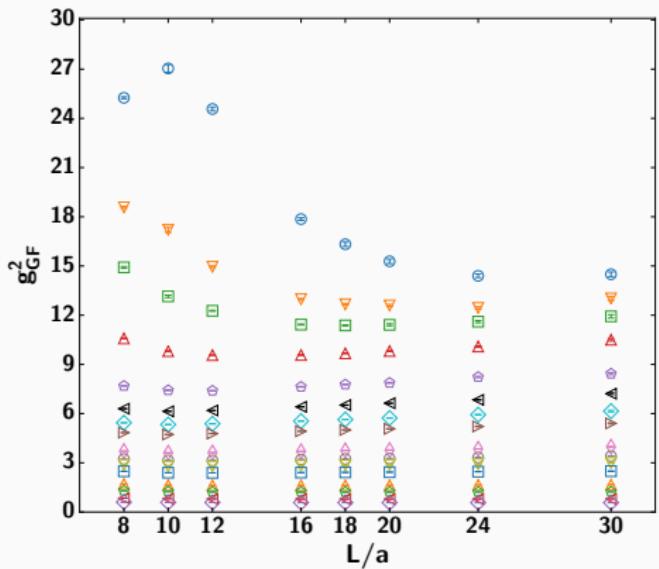
$$\begin{aligned}\langle E(t) \rangle &= \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle \\ &= \frac{3(N^2 - 1)g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4),\end{aligned}$$

$$g_{\text{GF}}^2(\mu) = \mathcal{N}^{-1} t^2 \langle E(t) \rangle |_{x_0=L/2, t=1/8\mu^2},$$

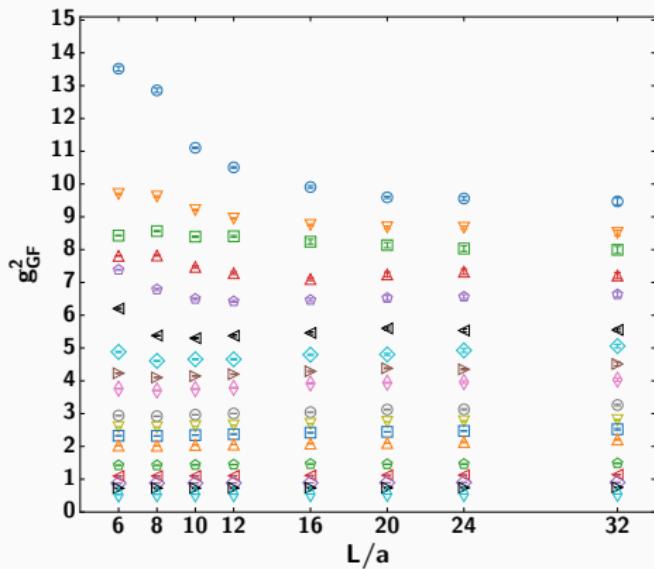


- Evolve the flow equation to time t
- Coupling at scale $\mu^{-1} = \sqrt{8t} = c_t L$
- We use Clover for the discretization of E
- We use $c_t = 0.3$ for $N_f = 6$, and $c_t = 0.4$ for $N_f = 8$
- c_t defines the scheme
- Boundary conditions break time translation, only use $x_0 = L/2$

Raw couplings



$$N_f = 6$$



$$N_f = 8$$

- Strong finite size effects on small lattices
→ Only use lattices of size 10 or bigger

Step scaling function

- Step scaling function in the lattice and continuum:

$$\Sigma(s, u, a/L) = g_{GF}^2(g_0, s \frac{L}{a}) \Big|_{g_{GF}^2(g_0, \frac{L}{a})=u}, \quad \sigma(u, s) = \lim_{a/L \rightarrow 0} \Sigma(u, s, a/L)$$

- Use interpolated couplings for consistent u and do the limit as:

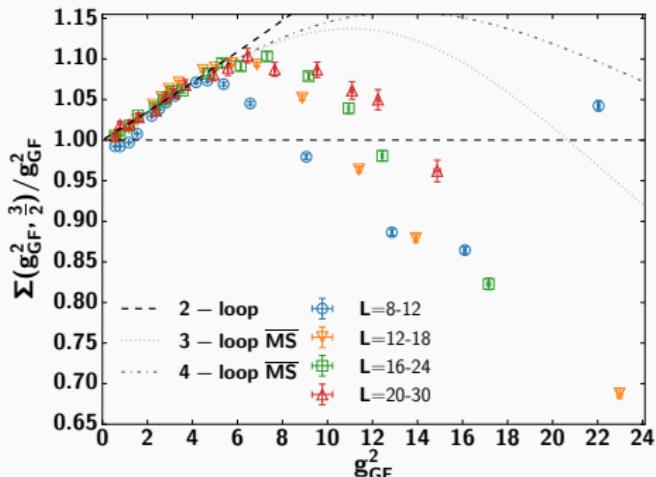
$$\Sigma(u, s, a/L) = \sigma(u, s) + c(u) \left(\frac{L}{a} \right)^{-2}$$

- At fixed point $\sigma(u)/u = 1$
- Related to beta function:

$$-2 \ln(s) = \int_{\sqrt{u}}^{\sqrt{\sigma(u,s)}} \frac{dx}{\beta(x)}, \quad \beta(g) = \frac{g}{2 \ln(s)} \left(1 - \frac{\sigma(g^2, s)}{g^2} \right)$$

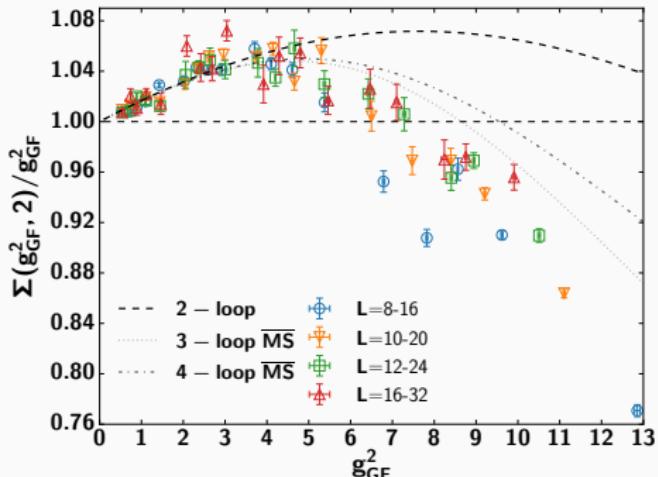
- For $N_f = 8$ we use $s = 2$, Pairs: 8 – 16, 10 – 20, 12 – 24, 16 – 32
- For $N_f = 6$ we use $s = 1.5$, Pairs: 8 – 12, 12 – 18, 16 – 24, 20 – 30

Raw step scaling function



$$N_f = 6$$

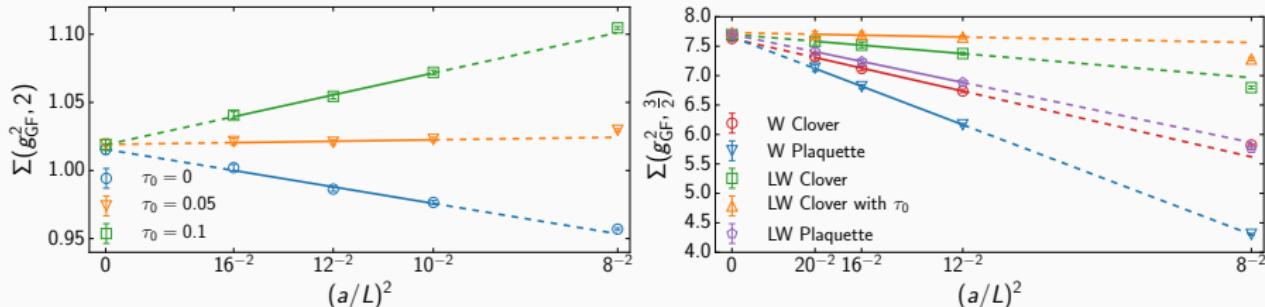
$$s = 3/2, c = 0.3$$



$$N_f = 8$$

$$s = 2, c = 0.4$$

Discretizations and τ -correction

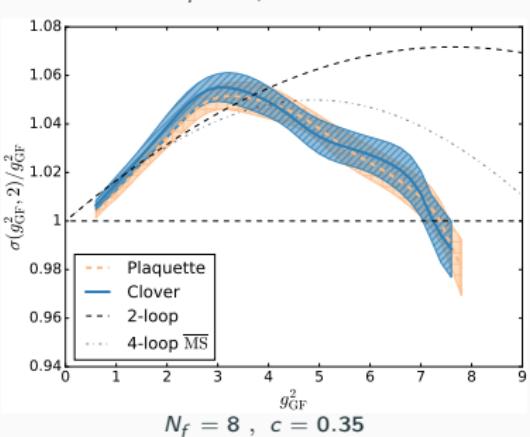
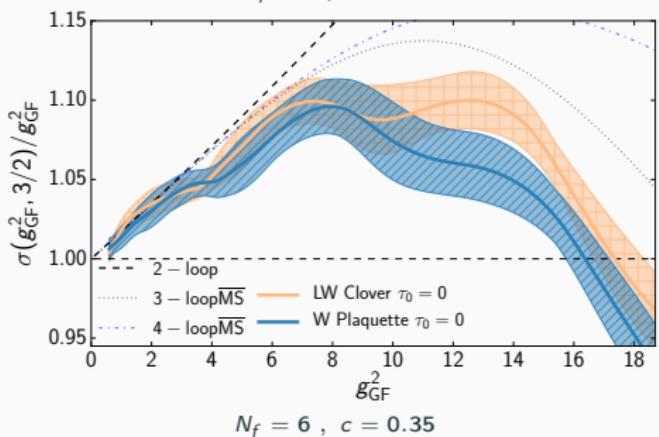
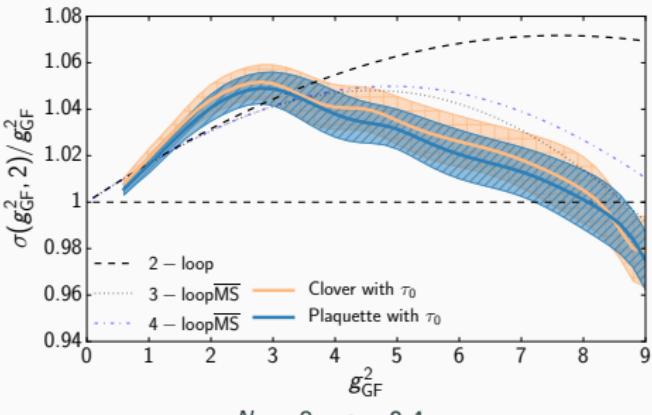
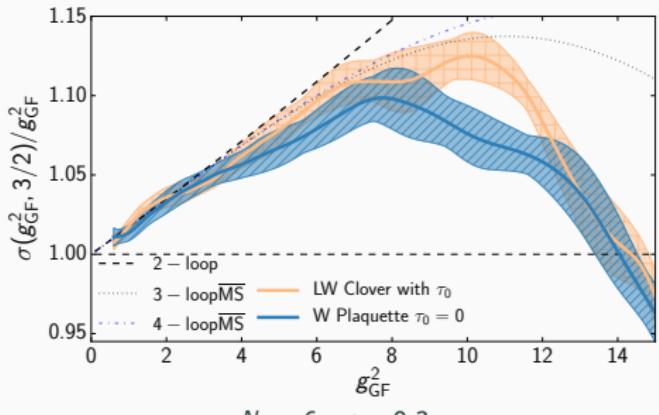


- Different actions of simulation, flow and energy density give different discretization effects
- Reduce effects by τ -shift:

$$g_{GF}^2 = \mathcal{N}^{-1} t^2 \langle E(t + \tau_0 a^2) \rangle = \mathcal{N}^{-1} t^2 \langle E(t) \rangle + \mathcal{N}^{-1} t^2 \langle \frac{\partial E(t)}{\partial t} \rangle \tau_0 a^2$$

- $N_f = 8$: $\tau = 0.06 \log(1 + g_{GF}^2)$, $N_f = 6$: $\tau = 0.025 \log(1 + 2 * g_{GF}^2)$
- Chosen discretization: LW evolved flow with Clover measurement

Continuum Step scaling function

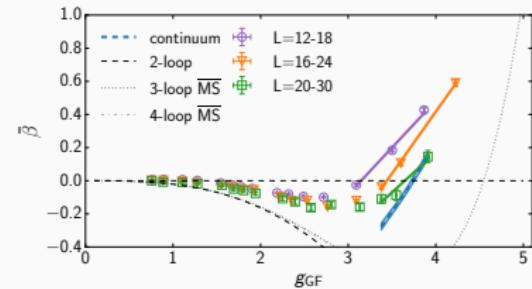


Slope of β -function: γ_g^*

- Scheme independent observable at fixed point
- β -function related to step scaling function as:

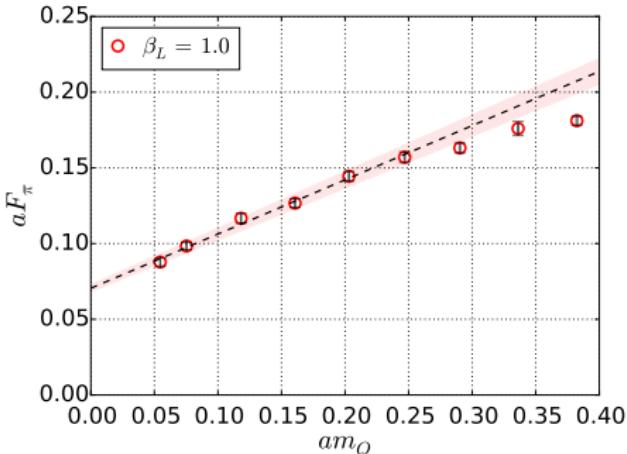
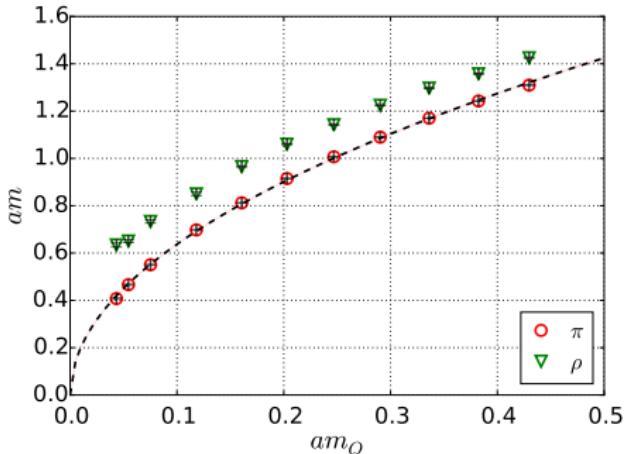
$$\beta(g) = \frac{g}{2 \ln(s)} \left(1 - \frac{\sigma(g^2, s)}{g^2} \right)$$

- We can fit a line to points around IRFP



	$c_t = 0.3$	$c_t = 0.35$	R&S
$N_f = 6$	$0.66(4)^{+0.25}_{-0.13}$	$0.67(11)^{+0.21}_{-0.11}$	0.6515
$N_f = 8$	$0.19(8)^{+0.21}_{-0.09}$	$0.2(1)$	0.25

Mass scaling



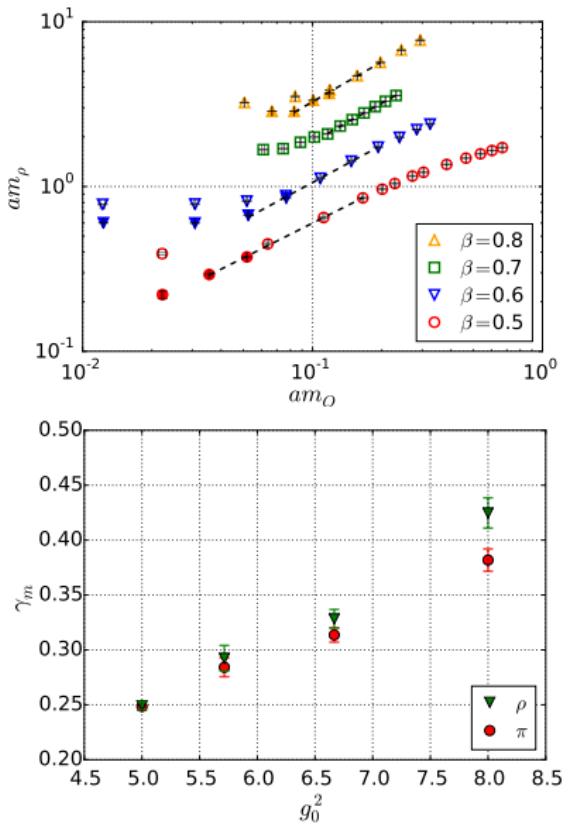
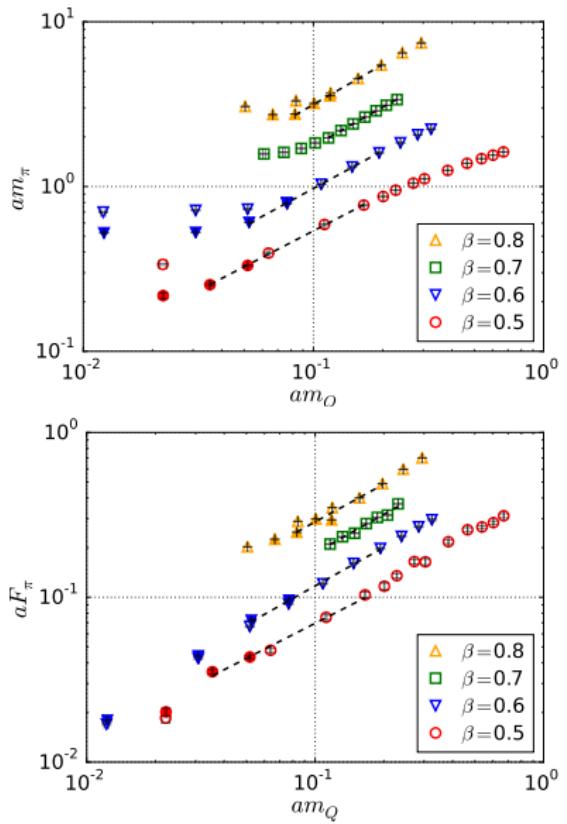
- Above: $N_f = 2$, Chiral symmetry breaking, $M_\pi \sim m_Q^{1/2}$
- Mass anomalous dimension defined by:

$$\mu \frac{dm(\mu)}{d\mu} = -\gamma(g^2)m(\mu)$$

- In conformal theories masses should follow a power law:

$$m_q^{1/(1+\gamma_m(g))}$$

$N_f = 6$ mass spectrum



Mass anomalous dimension: Step scaling method

- Measure the pseudoscalar density renormalization constant:

$$Z_P(g_0, \frac{L}{a}) = \frac{\sqrt{Nf_1}}{f_p(\frac{1}{2} \frac{L}{a})}$$

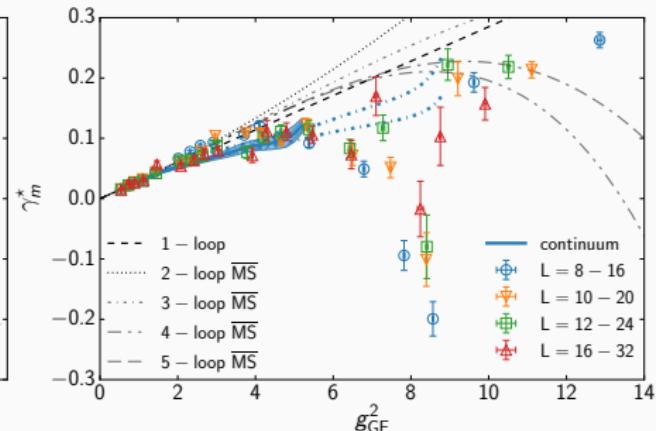
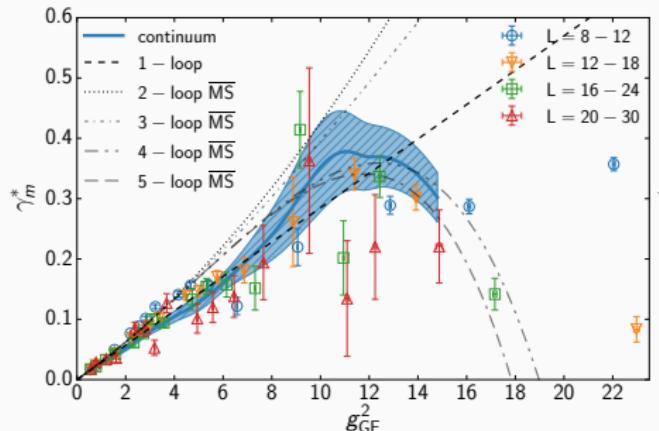
- Step scaling procedure similar to coupling

$$\Sigma_P(u, \frac{a}{L}) = \left. \frac{Z_P(g_0, \frac{sL}{a})}{Z_P(g_0, \frac{L}{a})} \right|_{u=g_{GF}^2}, \quad \begin{aligned} \Sigma_P(u) &= \sigma_P(u, s) + c(u)(\frac{L}{a})^{-2} \\ \sigma_P(g^2) &= \lim_{a \rightarrow 0} \Sigma_P(g^2, \frac{a}{L}) \end{aligned}$$

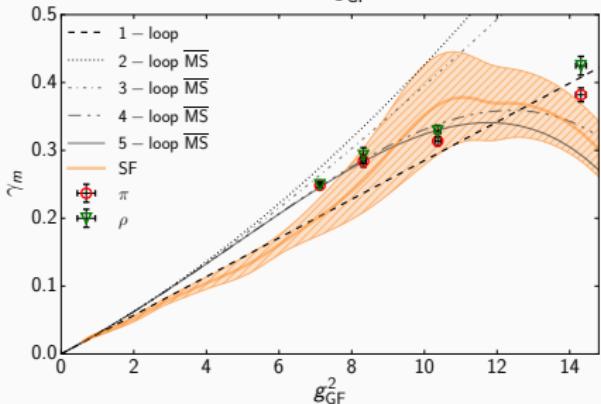
- Related to mass anomalous dimension γ by:

$$\gamma^* = -\frac{\log \sigma_P(g^2)}{\log s}$$

Mass anomalous dimension: Step scaling method



- Gives results comparable to perturbation theory
- Method breaks at large coupling



Mass anomalous dimension: Spectral method

- The mode number of Dirac operator defined from eigenvalue density:

$$\nu(\Lambda) \equiv 2 \int_0^{\sqrt{\Lambda^2 - m^2}} \rho(\lambda) d\lambda$$

- For massless theory at IRFP should follow the scaling:

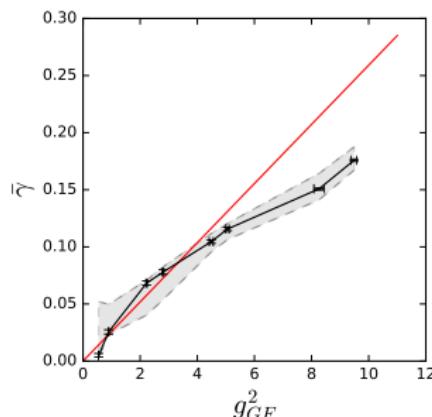
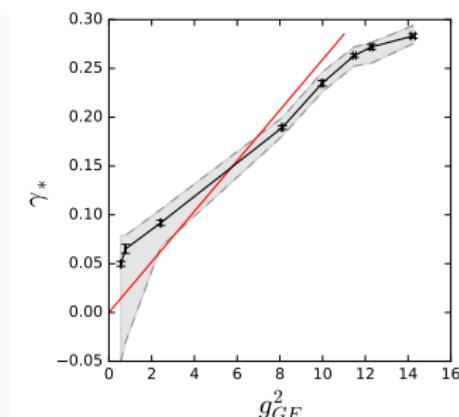
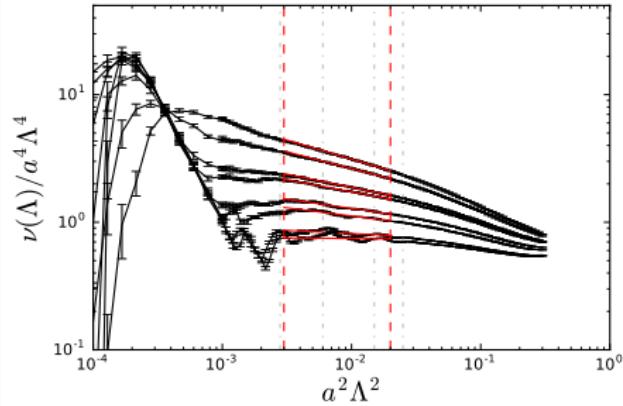
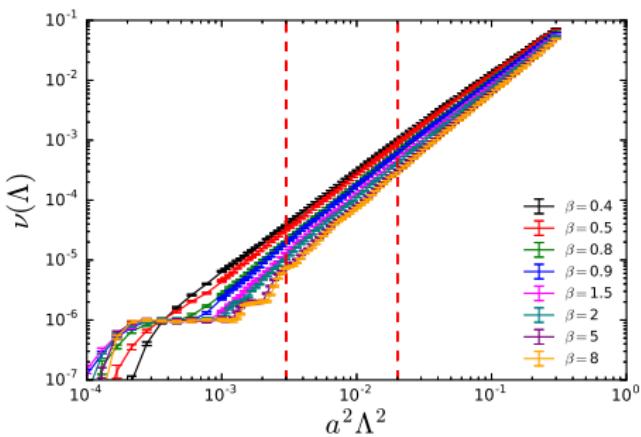
$$\nu(\Lambda) \simeq C \Lambda^{4/(1+\gamma_*)}$$

- Measure mode number from the lattice configurations:

$$\nu(\Lambda) = \lim_{V \rightarrow \infty} \frac{1}{V} \langle \text{tr } \mathbb{P}(\Lambda) \rangle,$$

- \mathbb{P} projects full eigenspace of $M = m^2 - \not{D}^2$ to the eigenspace of eigenvalues smaller than Λ^2 ; Stochastically do the trace

Mass anomalous dimension: Spectral method



Conclusions

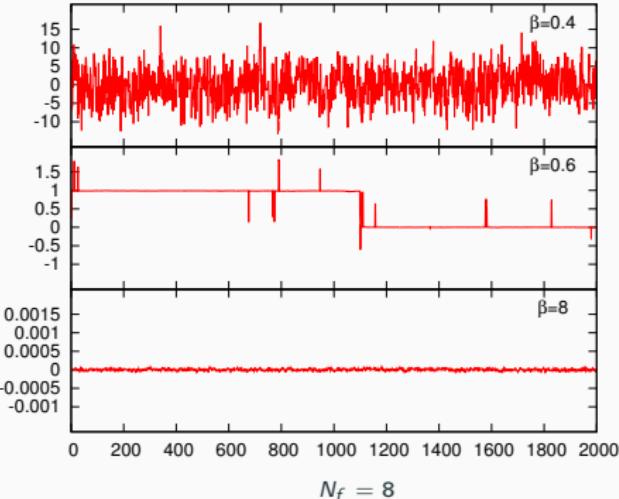
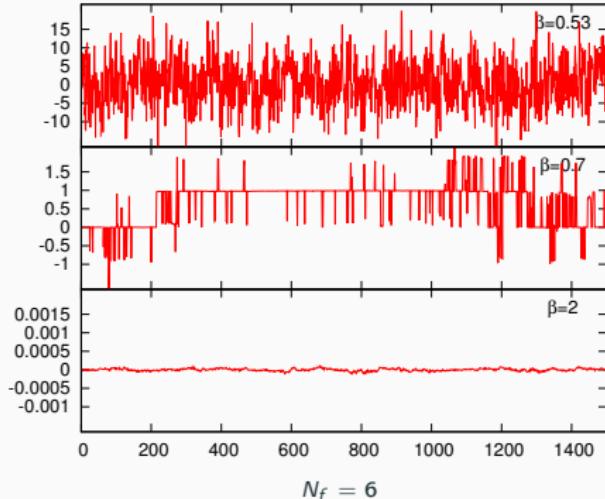
- We can accurately reach strong coupling with GF
- Clear indication for fixed point
 - $N_f = 6$: $g_*^2 = 14.5(3)^{+0.41}_{-1.38}$
 - $N_f = 8$: $g_*^2 = 8.24(59)^{+0.97}_{-1.64}$

⇒ Conformal window edge between $N_f = 4 - 6$
- Mass anomalous dimension:
 - Mass step scaling works for small couplings
 - Spectral method works for large couplings
 - γ^* has relatively small value
 - $N_f = 6$: $\gamma_m^* = 0.283(2)^{+0.01}_{-0.01}$
 - $N_f = 8$: $\gamma_m^* = 0.15(2)$
- The slope of the beta function close to the Ryttov&Shrock result

Thank you

Backup slides

Topology

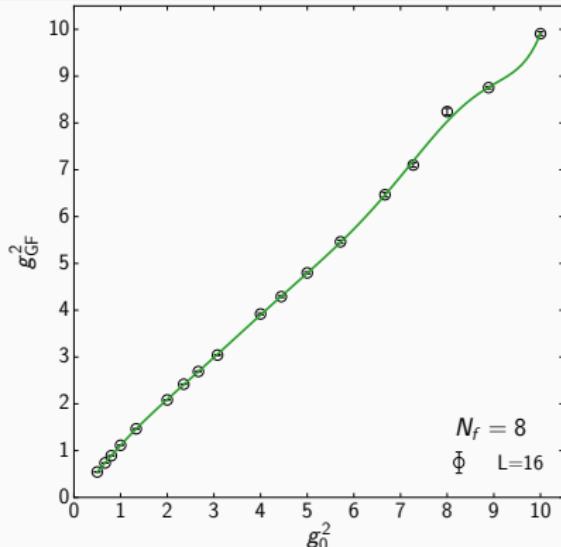
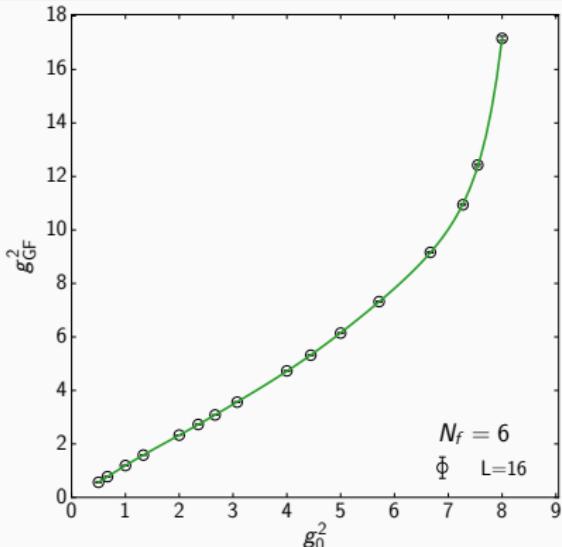


- Gradient flow allows measurement of topological charge

$$Q = \frac{1}{32\pi^2} \sum_x \epsilon_{\mu\nu\alpha\beta} G_{\mu\nu}^a(x; t) G_{\alpha\beta}^a(x; t)$$

- Frozen at small coupling, unfrozen at large couplings
- On intermediate couplings frozen to nonzero

Interpolation

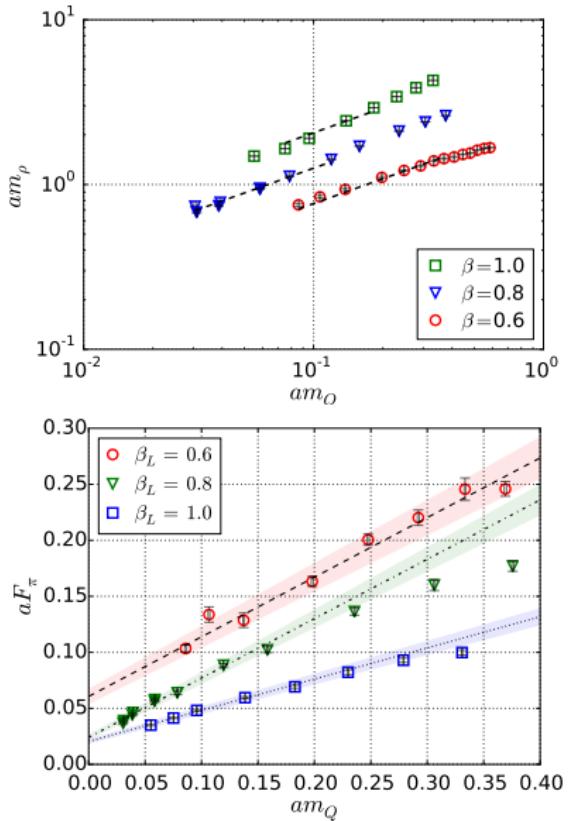
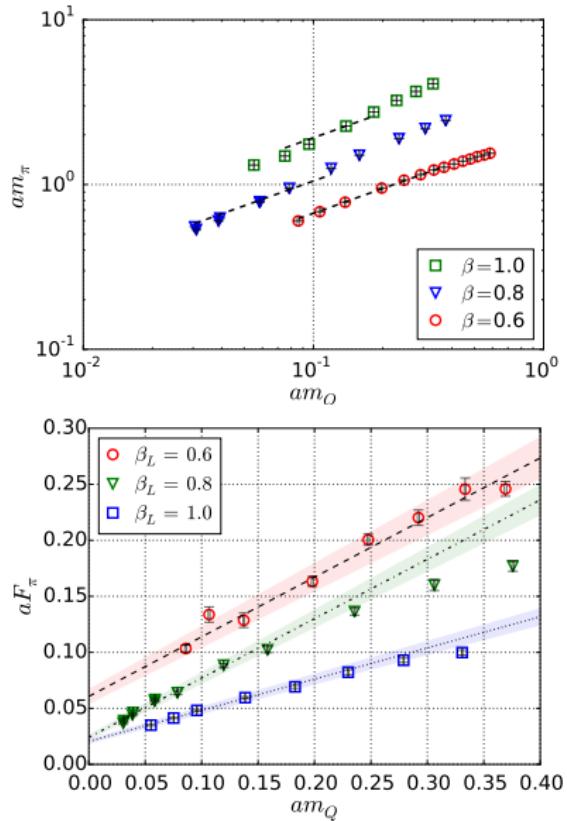


$$g_{GF}^2 = g_0^2 \left(1 + \sum_{i=1}^n a_i g_0^{2i}\right),$$

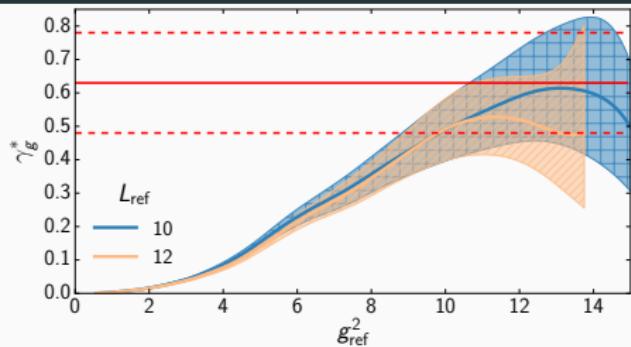
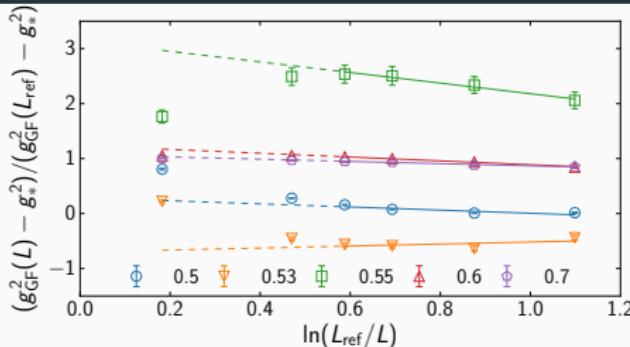
$$g_{GF}^2 = g_0^2 \frac{1 + \sum_{i=1}^n a_i g_0^{2i}}{1 + \sum_{j=1}^m b_j g_0^{2j}}$$

- For $N_f = 6$ $s = 1.5$, Interpolate using polynomial function $n = 9$
- For $N_f = 8$ $s = 2$, Interpolate using rational function $n = 7$ $m = 1$
- Estimate systematic errors by changing n, m by 1

$N_f = 4$ mass spectrum



Slope of β -function: γ_g^*



- Previous slide relies on reliability of continuum limit at IRFP
- Alternative method:

$$\beta(g_{\text{GF}}^2) = -\mu \frac{dg_{\text{GF}}^2}{d\mu} = \gamma_g^*(g_{\text{GF}}^2 - g_*^2)$$

- Integrate from L_{ref} to L

$$g_{\text{GF}}^2(\beta, L) - g_*^2 = [g_{\text{GF}}^2(\beta, L_{\text{ref}}) - g_*^2] \left(\frac{L_{\text{ref}}}{L} \right)^{\gamma_g^*}.$$