Sextet sigma particle or dilaton?
2018

Lattice Higgs Collaboration (L$_{at}$HC)

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XIIIth Quark Confinement and the Hadron Spectrum

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What is the composite scalar (a.k.a. Higgs) paradigm?

the Higgs doublet field

elementary scalar?

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_2 + i\,\pi_1 \\ \sigma - i\,\pi_3 \end{pmatrix} \qquad \frac{1}{\sqrt{2}}(\sigma + i\,\vec{\tau}\cdot\vec{\pi}) \equiv M$$

$$D_\mu M = \partial_\mu M - i\,g\,W_\mu M + i\,g' M\,B_\mu\,, \qquad \text{with} \qquad W_\mu = W_\mu^a \frac{\tau^a}{2}\,, \quad B_\mu = B_\mu \frac{\tau^3}{2}$$

The Higgs Lagrangian is

spontaneous symmetry breaking

Higgs mechanism

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left[D_\mu M^\dagger D^\mu M\right] - \frac{m_M^2}{2}\mathrm{Tr}\left[M^\dagger M\right] - \frac{\lambda}{4}\mathrm{Tr}\left[M^\dagger M\right]^2$$

near-conformal strongly coupled gauge theory

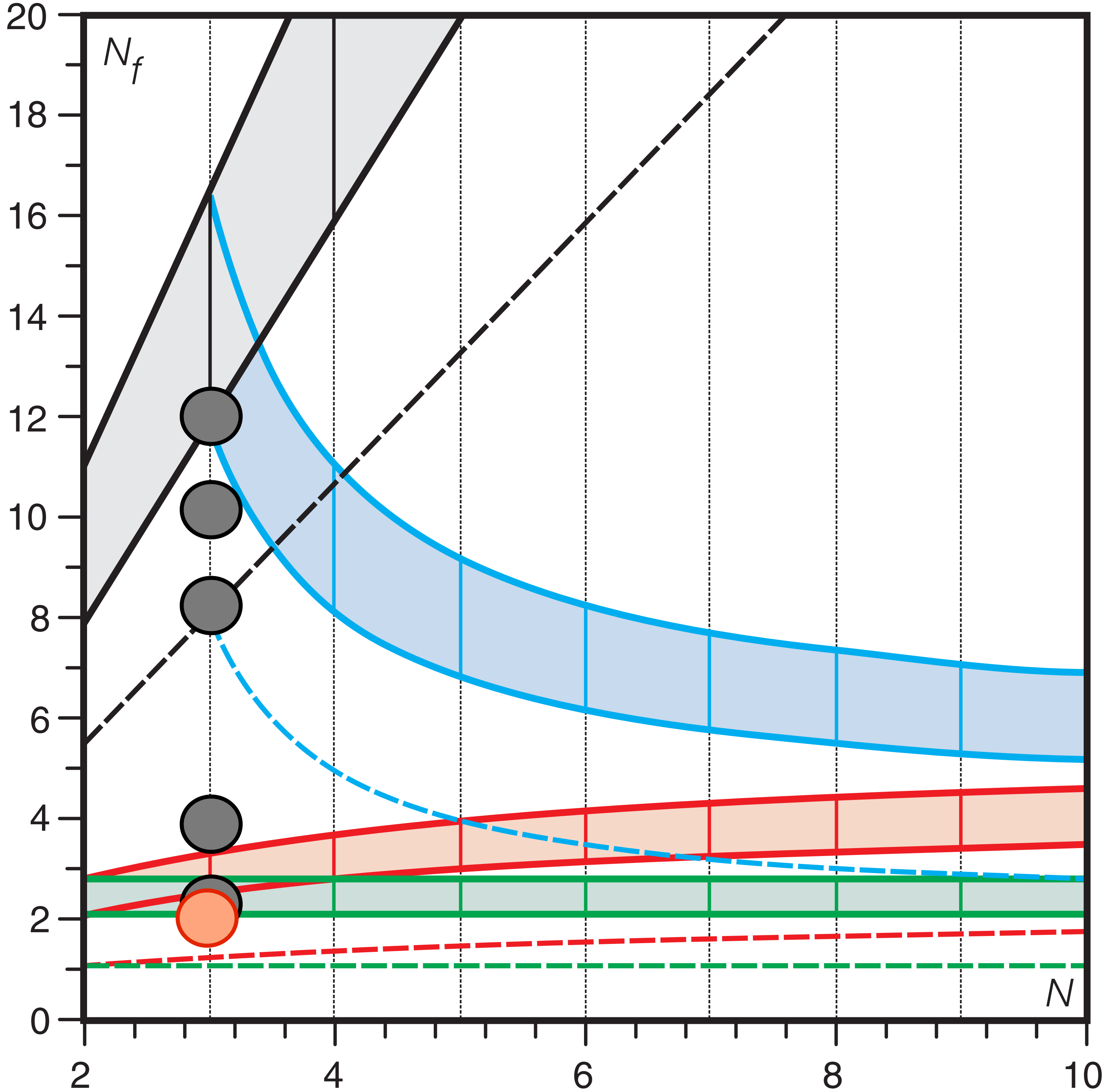
fermions (Q) in gauge group reps in flavor/color space:

$$\mathcal{L}_{Higgs} \rightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{Q}\gamma_\mu D^\mu Q + \ldots$$

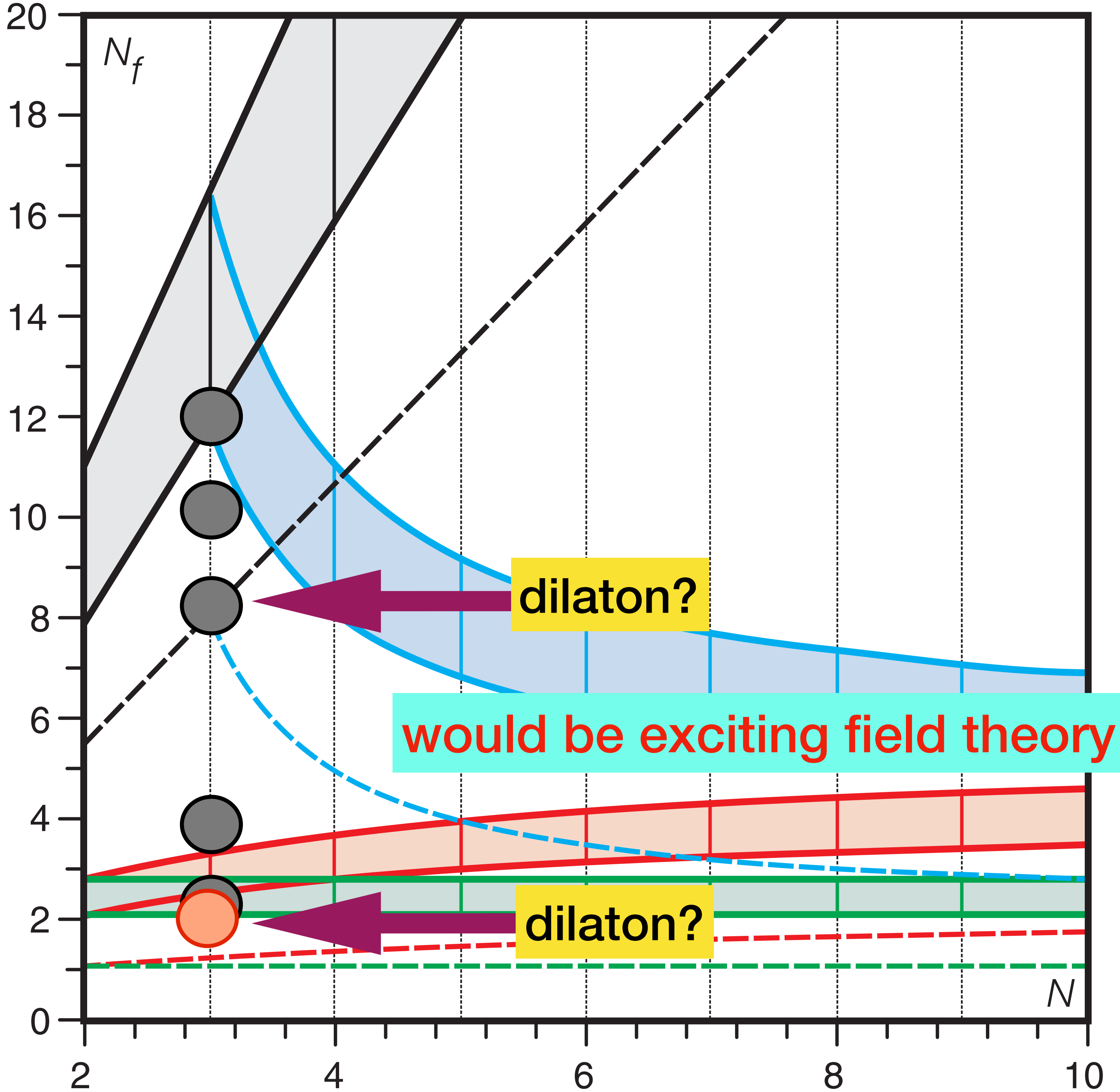
light scalar separated from
multi-TeV resonance spectrum

unlike QCD
requires BSM field theory tools for LHC apps
dilaton with broken scale symmetry and chiSB?

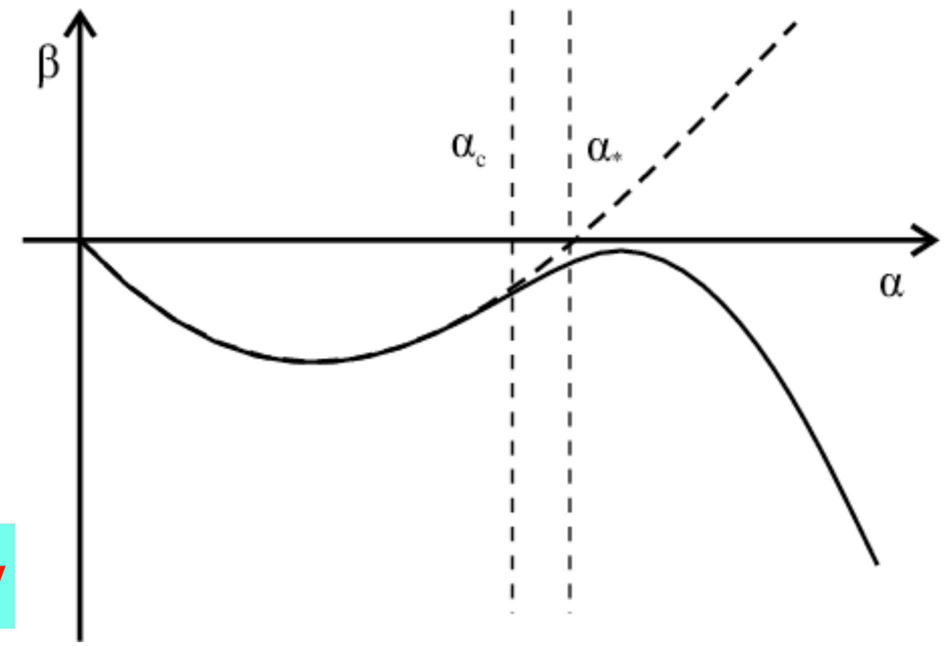
SCGT Theory Space



SCGT Theory Space



target of lattice BSM?



near-conformal

$(\bar{\psi}\psi)^{\Delta}$ deforming the would be IRFP

fermion mass

m=0

chiral symmetry breaking

$f_d > f_\pi$

scale symmetry breaking

$$L = \frac{1}{2}\partial_\mu \chi \partial_\mu \chi - V_d(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right] - \frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma + \Sigma^\dagger\right)$$

based on: DOI:10.1103/PhysRevD.94.091501, arXiv:1712.08594, arXiv:1710.09262, arXiv:1711.04833,arXiv:1711.05299, PLB B779 (2018) 230-236

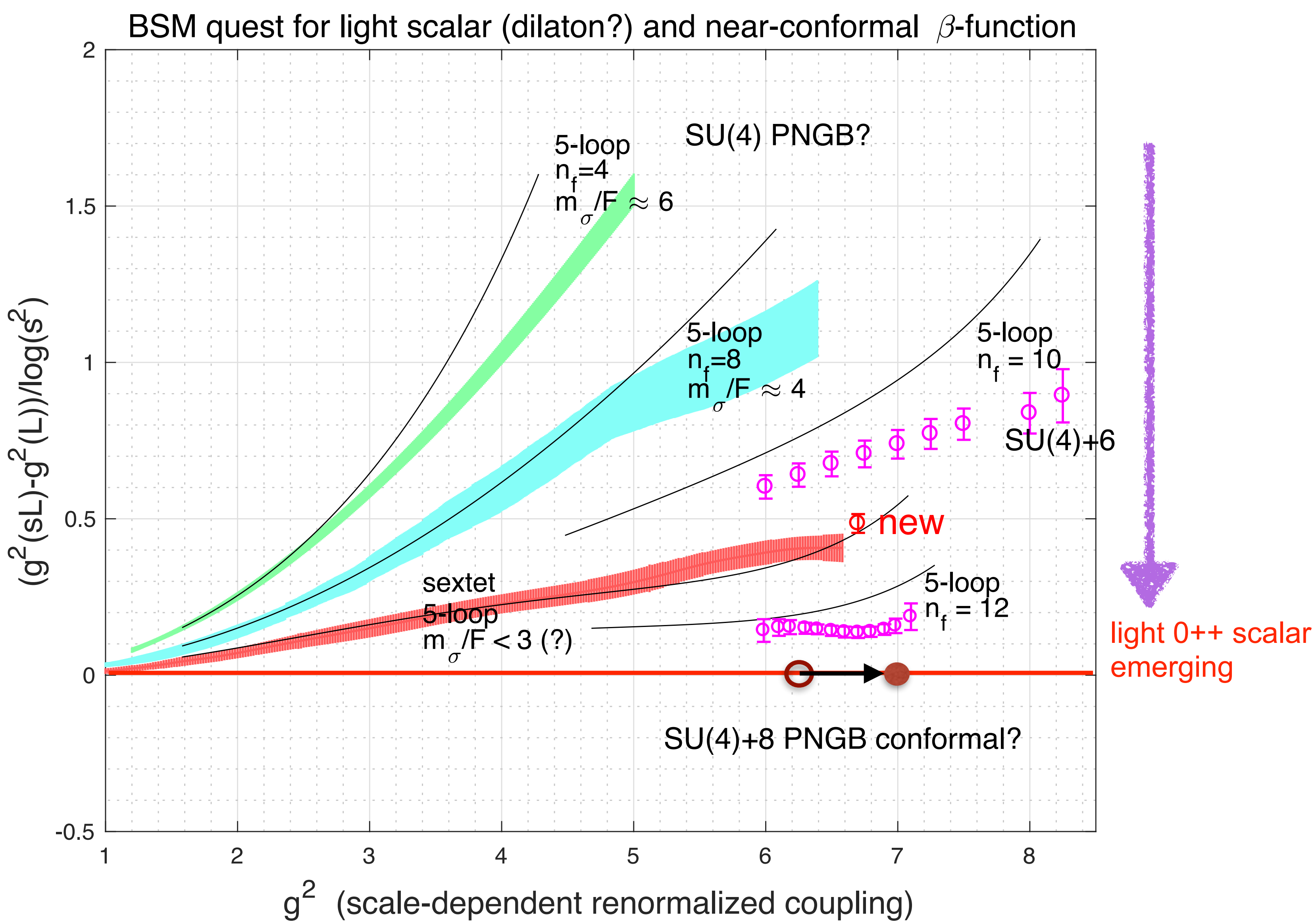
- walking β-functions in χSB phase? which models are dilaton candidates?
- linear σ-model with low mass $m_\pi \gtrsim m_\sigma$ requires extensions ➙ dilaton?
- dilaton signatures in the p-regime of the sextet model 2017 BU workshop: while we are struggling with the sextet analysis, Appelquist et al.: it works for nf=8 anyway. Hmmm ….
- dilaton signatures in the $\mathcal{E}$-regime ?
- simulating the effective potential of the composite scalar

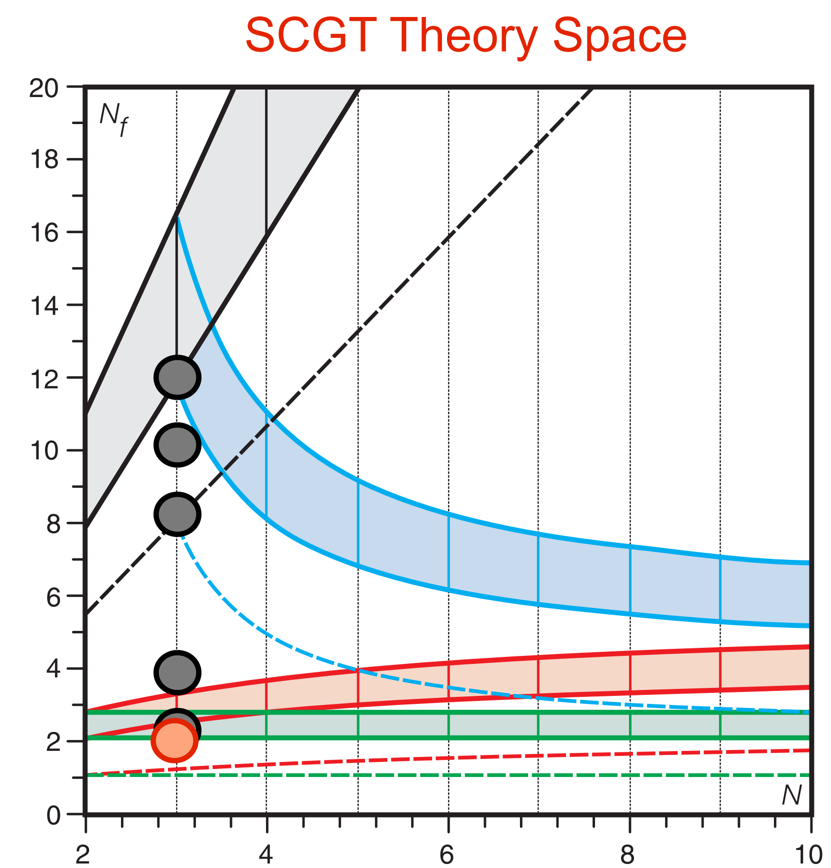
if I get carried away:

(from *Julius Caesar*, spoken by Marc Antony)

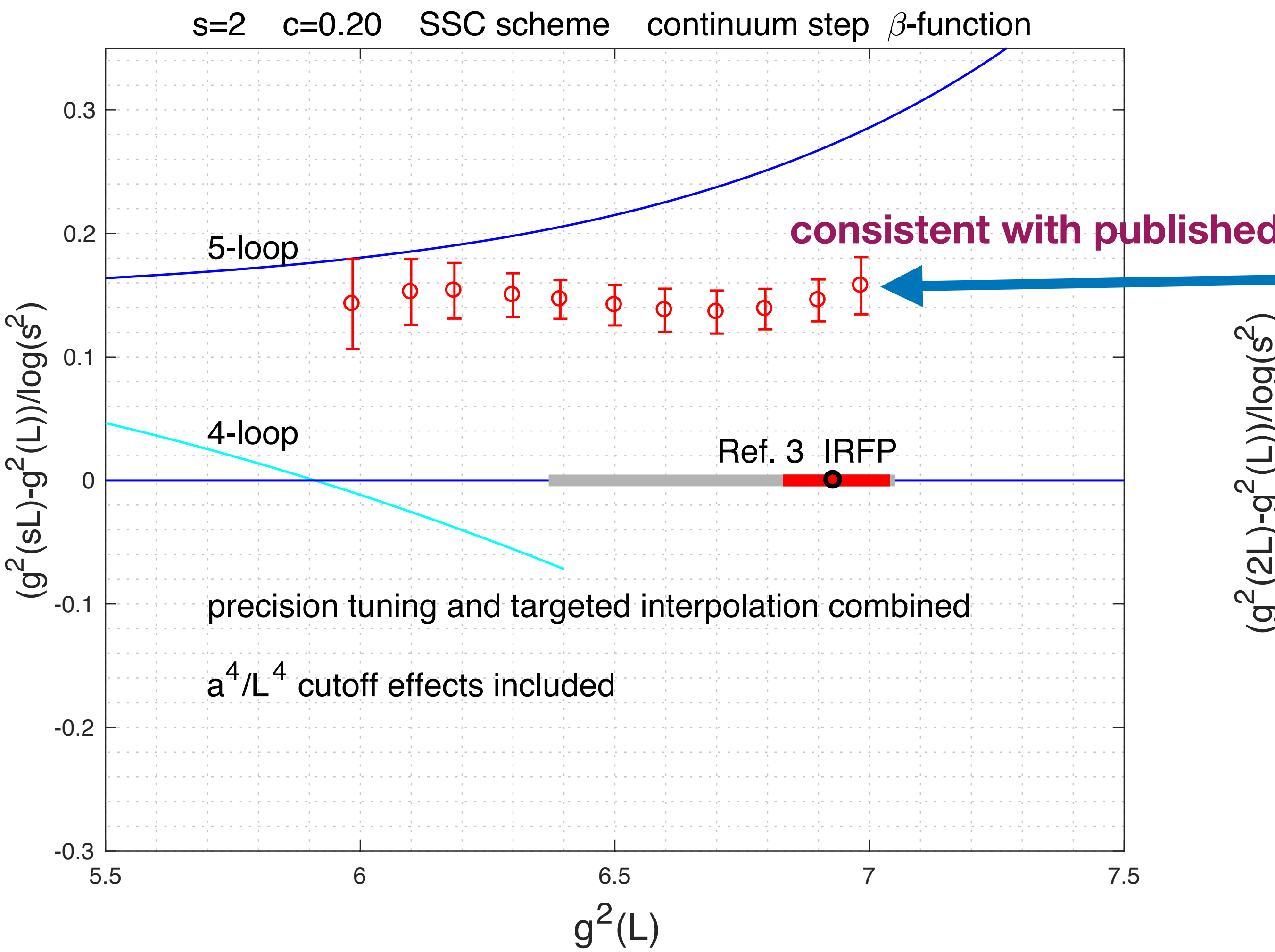
I come to bury Caesar, not to praise him.

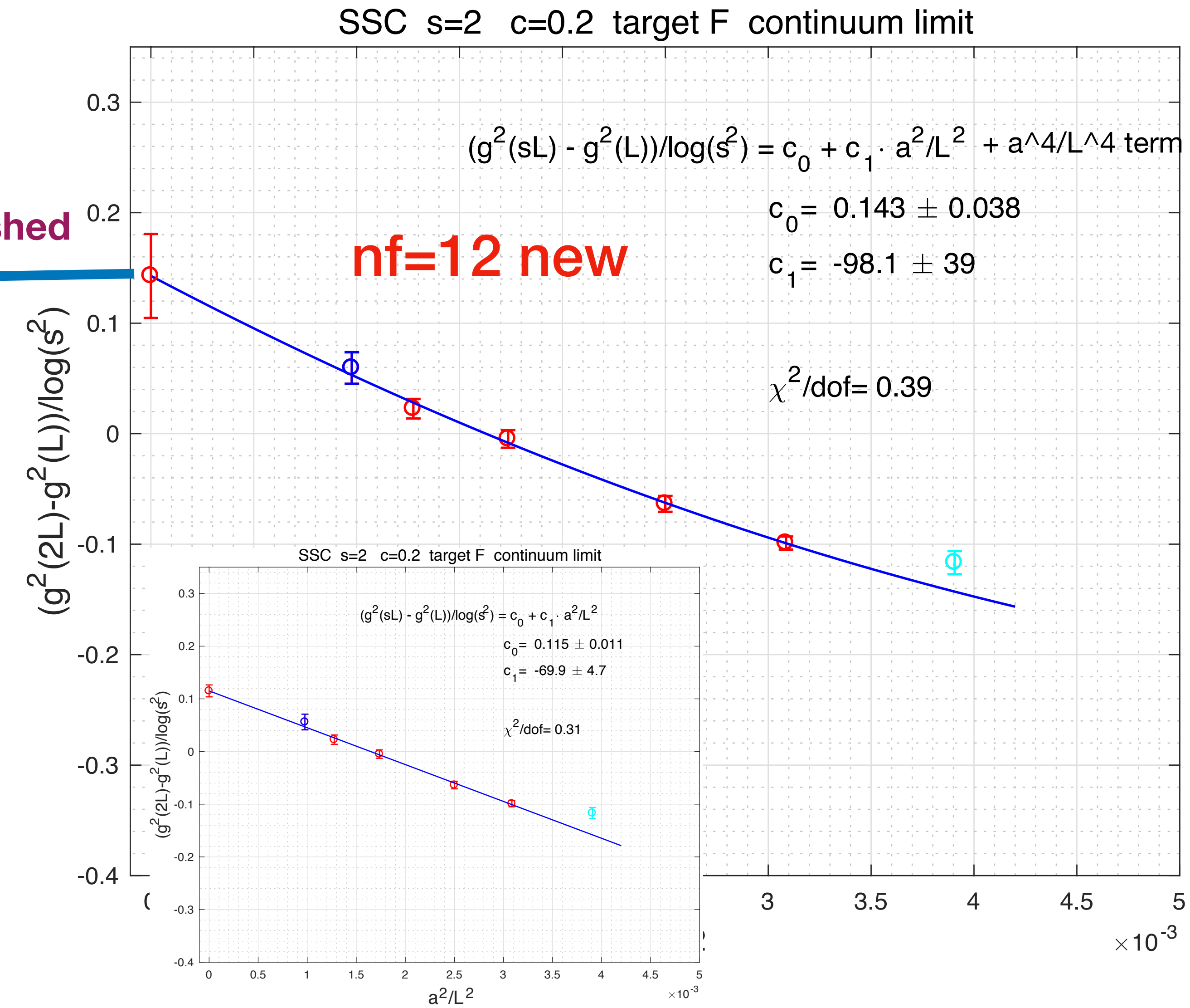
testing scale-dependent BSM gauge couplings and β-functions:

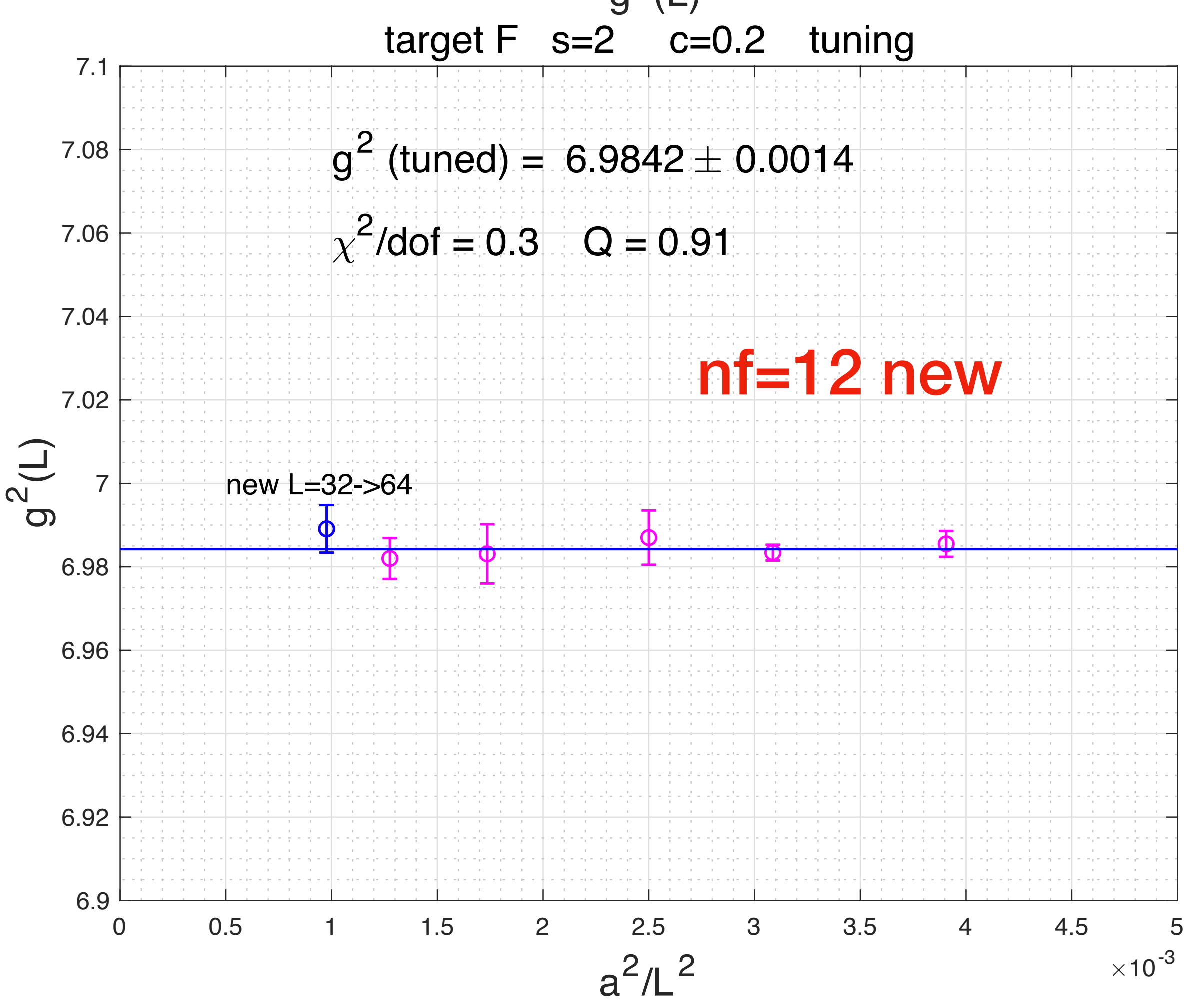




- with established χSB, sextet model closest to CW in explored range of β-function
- nf=10 is not conformal in explored β-function range of our analysis
- nf=12 is not conformal in explored β-function range of our analysis new check
- nf=13 is conformal
- sextet SU(2) flavor group simplest with light 0++ scalar — dilaton analysis







LatHC PLB B779 (2018) 230-236 arXiv:1710.09262
confirmed with new updated results:

L=32 -> L=64 step at three tuned g^2 targets
adds further evidence against nf=12 IRFP

staggered "non-universality argument" based on 3d spin models is misguided

New Boulder-BU poster with DW fermions so far is not in contradiction with our Nf=12 analysis

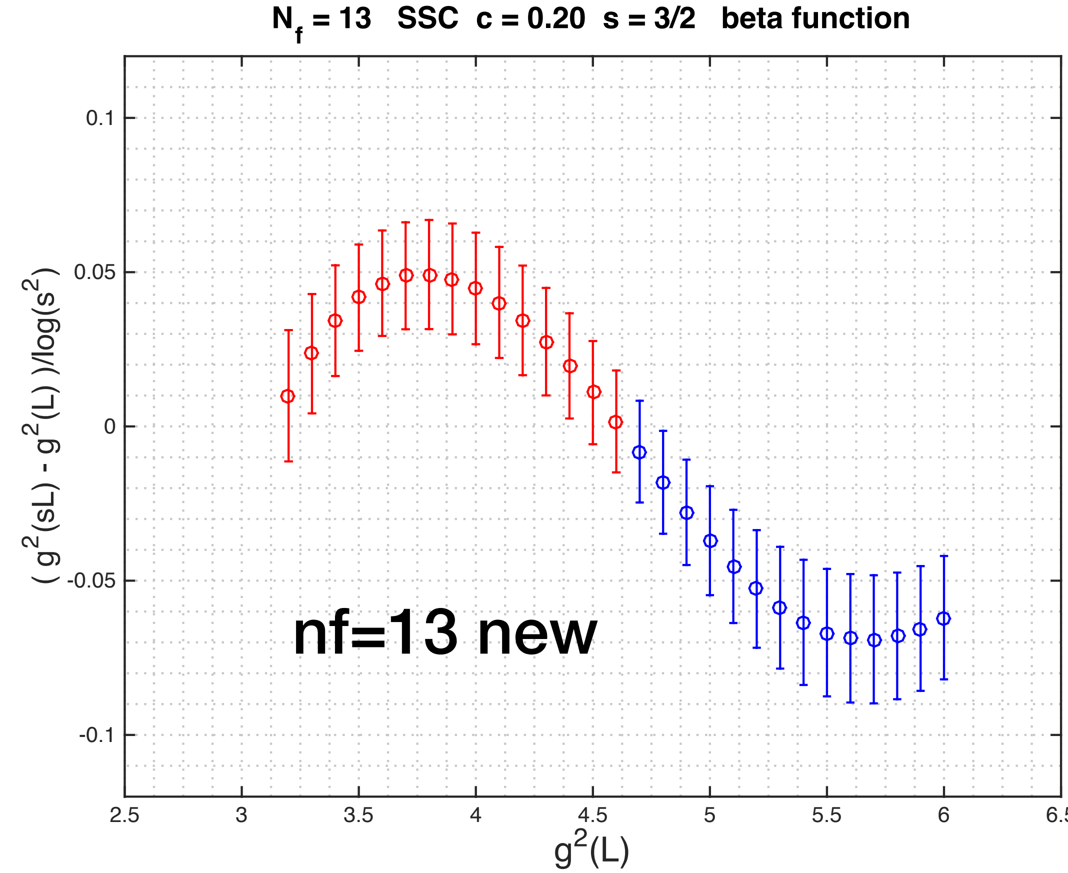
quadratic continuum extrapolation across range of renormalized couplings

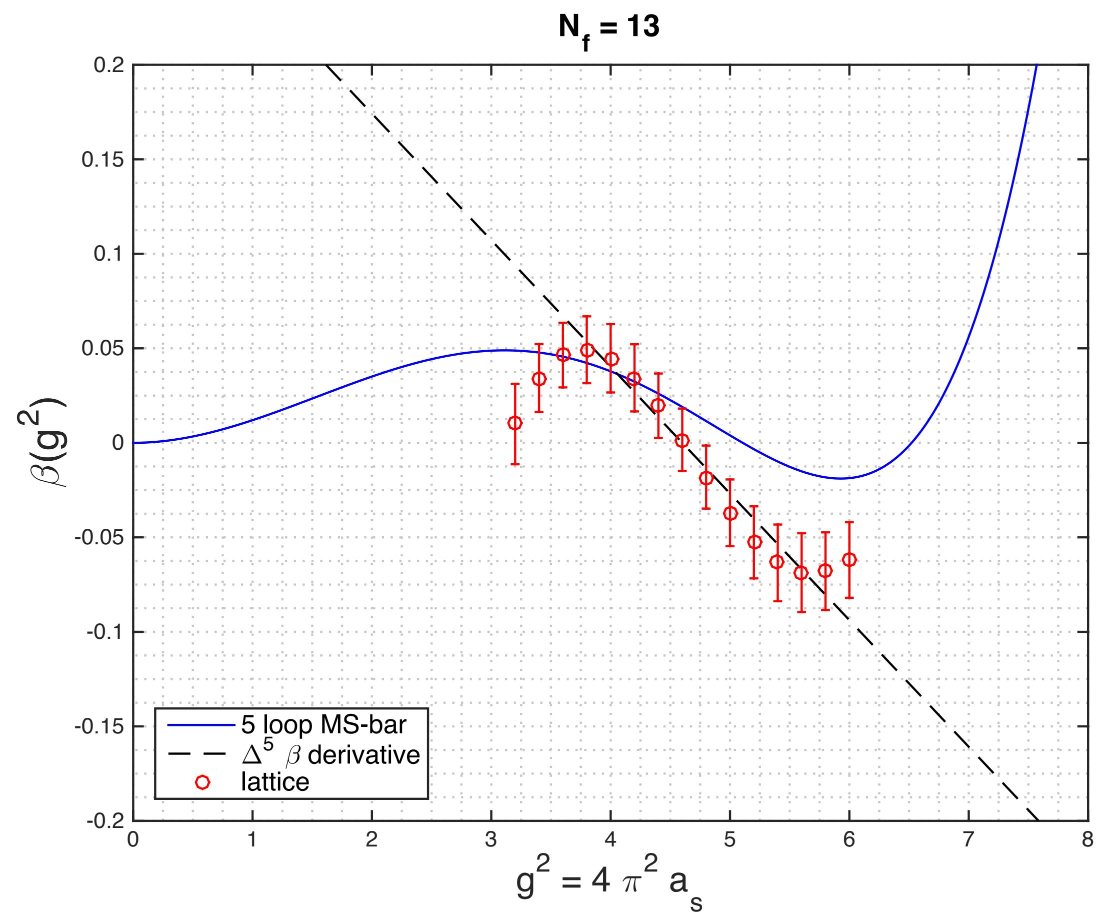
physical property: slope of beta function at IRFP

$$\beta' = d\beta/dg^2|_{g_*^2}$$

nf=13 new

MS-bar loop order	β'	Ryttov-Shrock delta order	β'
2	0.087	2	0.051
3	0.078	3	0.072
4	0.075	4	0.069
5	0.037	5	0.067





perturbative results quite close to lattice data

5-loop MS-bar beta function less steep than lower orders

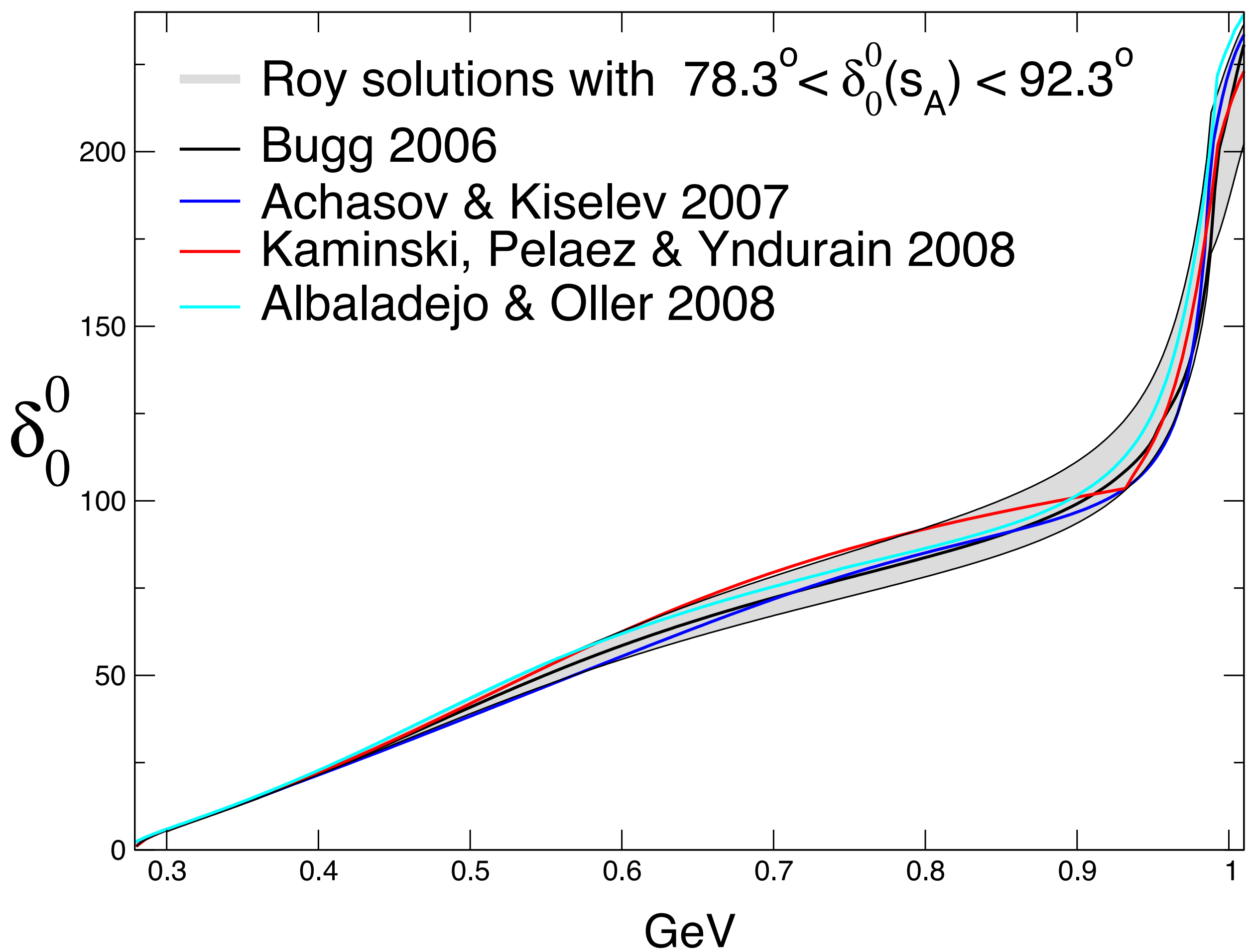
The light 0++ scalar

QCD (aka old TC) 80', 90'

the failure of old Higgs-less technicolor:
0^{++} scalar in QCD (bad Higgs impostor)

$\sqrt{s_\sigma}$ = (400 - 1200) - i (250 - 500) MeV estimate in Particle Data Book

π-π phase shift in 0^{++} "Higgs" channel



broad M_σ ~ 1.5 TeV in old technicolor, based on scaled up QCD, hence the tag "Higgs-less"

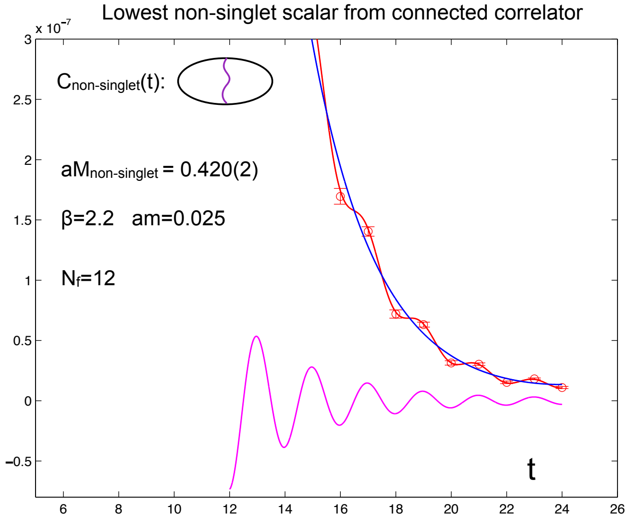
beautiful lattice study of 0++ from JLAB group

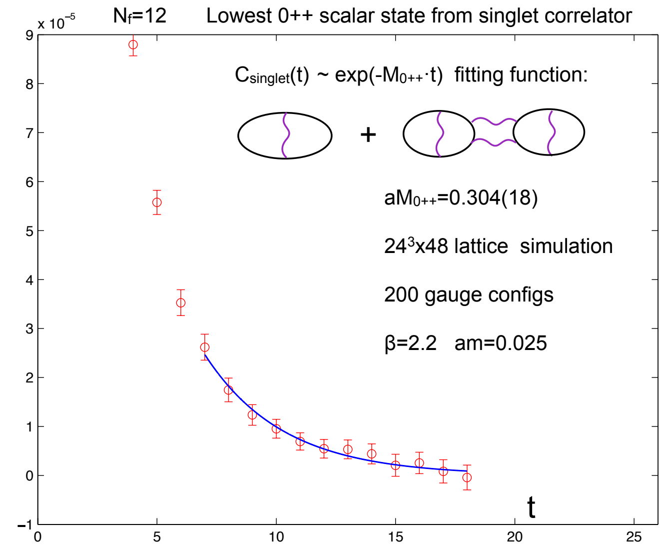
This is expected to be different in near-conformal strongly coupled gauge theories

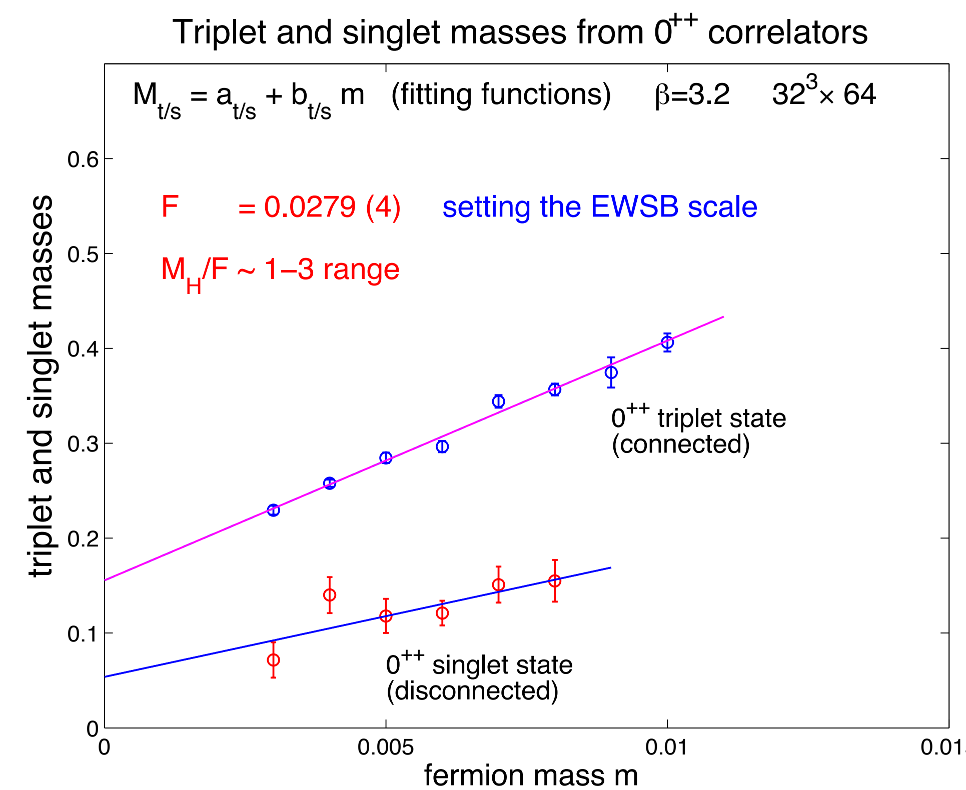
$$\sqrt{s_\sigma} = 441\,^{+16}_{-8} - i\,272\,^{+9}_{-12.5}\ \text{MeV}$$

Leutwyler:
dispersion theory combined with ChiPT

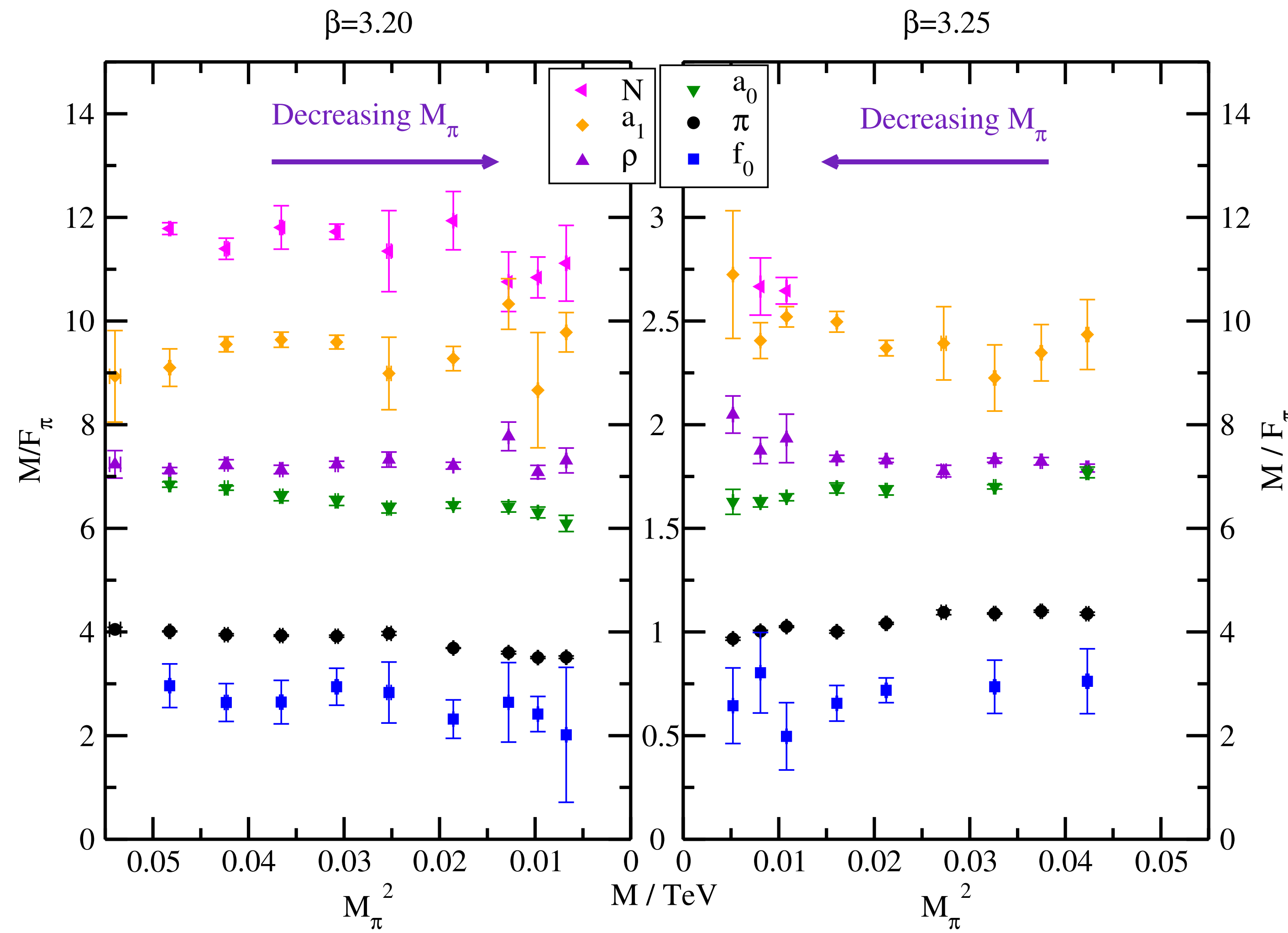
light 0++ scalar and spectrum   sextet model   LatHC







LatHC light sextet 0++ from disconnected correlator



0++ is tracking the Goldstone pion $m_\pi^2 \geq m_\sigma^2$

Light scalar sigma particle or dilaton ?

linear σ-model in simulations in low mass range with $m_\pi \gtrsim m_\sigma$ requires extension

$SU(2)\otimes SU(2) \sim O(4)$ for sextet model

$$L = \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}\mu^2(\sigma^2+\vec{\pi}^2) + \frac{1}{4}g(\sigma^2+\vec{\pi}^2)^2 - \varepsilon\sigma$$

$m_\sigma^2 \geq 3m_\pi^2$ tree level relation

$$L = \frac{F^2}{4} tr(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - \frac{F^2 M^2}{4} tr(\Sigma + \Sigma^\dagger)$$

pion field $\Sigma = e^{i\pi_a\tau_a/F}$ with τ_a Pauli matrices,

tree level pion mass $M^2 = 2Bm$

$$M_\pi^2 = M^2\left\{1 - \frac{1}{2}\frac{M^2}{16\pi^2 F^2}\bar{l}_3 + O(M^4)\right\} \qquad \bar{l}_3 = \frac{16\pi^2}{g_R} - \ln\frac{M_\pi^2}{M_R^2} - \frac{14}{3}$$

$$F_\pi = F\left\{1 + \frac{M^2}{16\pi^2 F^2}\bar{l}_4 + O(M^4)\right\} \qquad \bar{l}_4 = \frac{8\pi^2}{g_R} - \ln\frac{M_\pi^2}{M_R^2} - \frac{1}{3}$$

triviality analysis

(and loop expansion)

circa 1987-1988

$m_\sigma^2 \geq 2m_\pi^2$ 1-loop relation

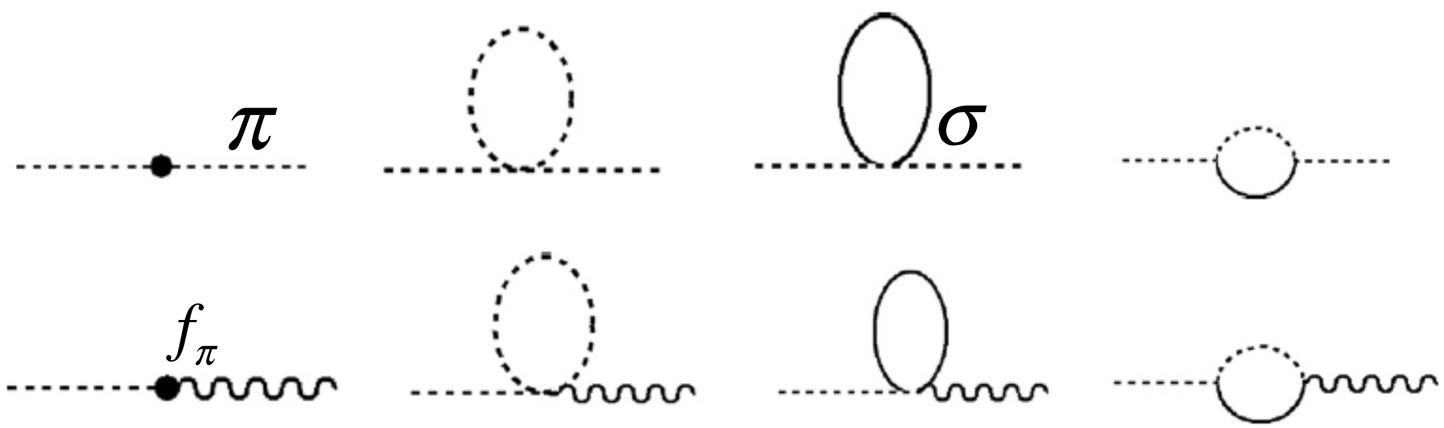
$$\frac{1}{2}\frac{M_\pi^2}{16\pi^2F^2}\bar{l}_3 < 0.5 \Rightarrow \frac{M_\sigma}{M} > \sqrt{2} \text{ with the condition } \frac{M_\sigma}{F} = 2$$

similar condition from $\bar{l}_4 = \frac{8\pi^2}{g_R} - \ln\frac{M_\pi^2}{M_R^2} - \frac{1}{3}$

generalize linear O(4) σ-model in low mass $m_\pi \geq m_\sigma$ range

→ nonlinear σ-model, perhaps dilaton?

extended EFT of σ-π entanglement in the BSM Higgs sector:



Soto et al. targeting QCD

Nuclear Physics B 866 (2013) 270–292

Sanino et al. added new terms for BSM

$$L = \frac{1}{2}\partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right)\cdot\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \ldots\right)$$

$\Sigma = e^{i\pi^a\tau^a/v}$ with τ^a Pauli matrices

$$V(h) = \frac{1}{2}m_h^2 \cdot h^2 + d_3\left(\frac{m_h^2}{2v}\right)\cdot h^3 + d_4\left(\frac{m_h^2}{8v^2}\right)\cdot h^4 + \ldots$$

light 0++ scalar:

σ-particle or dilaton?

dilaton, or non-linear σ-model parameters

σ-model limit (SM): $a = b = d_3 = d_4 = 1$ (or more relaxed in χSB framework)

dilaton model limit: $a = b^2$, $b_3 = 0$ scale symmetry breaking set by f_d (in far IR χSB can be triggered)

M_π, F_π, M_σ are calculated to 1-loop: extended SU(2) flavor chiral dynamics

We have been analyzing the small pion mass region in the M_π = 0.07- 0.015 range

of the p-regime, also targeting the ε-regime

linear sigma model limit in of χPT p-regime simulations requires very small pion masses

$m_\pi \ll m_\sigma$ not reached in p-regime simulations

Low energy effective theory of the $\sigma(x)$ dilaton field and the $\pi^a(x)$ Goldstone bosons separated from the higher resonance states with SU(2) flavor in sextet model:

$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi-V_{\rm d}(\chi)+\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right]-\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

after long early history, recent:
Golterman and Shamir
Appelquist et al., LatHC
Matsuzaki and Yamawaki
LatKMI …

$y=3-\gamma$ where γ is the mass anomalous dimension

$\chi(x)=f_d\, e^{\sigma(x)/f_d}$ describes the dilaton field $\sigma(x)$

pion field $\Sigma=\mathrm{e}^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices, tree level pion mass $m_\pi^2=2Bm$

we adapt Appelquist et al. notation for comparison

$$V_{\rm d1}=\frac{m_d^2}{2f_d^2}\left(\frac{\chi^2}{2}-\frac{f_d^2}{2}\right)^2$$ relevant deformation of IRFP theory

$$V_{\rm d2}=\frac{m_d^2}{16f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d}-1\right)$$ nearly marginal deformation

Golterman and Shamir
Appelquist et al.

two different dilaton potentials illustrate scope of the analysis

Appelquist et al. test Nf=8 fundamental rep and fit obsolete sextet data with paper and pencil!

test V ~ χ^p for large χ

dictionary for the effective dilaton theory coupled to Goldstone pions:

f_π	Goldstone decay constant	chiral limit	$\Rightarrow$	F_π	with fermion mass deformations m
$m_\pi=2mB$	Goldstone pions			M_π	
f_d	dilaton decay constant			F_d	
m_d	dilaton mass			M_d	

How do we test dilaton theory? General scaling laws:

Golterman and Shamir

Appelquist et al. nf=8 tests

F_d minimum of dilaton potential after fermion mass m is turned on:

for V_{d1} potential: $\left(\frac{F_d^2}{f_d^2}\right)^{2-y/2}\left(1-\frac{F_d^2}{f_d^2}\right)=\frac{2yF^2}{f_d^2}\left(\frac{m_\pi^2}{m_d^2}\right)$

for V_{d2} potential: $\left(\frac{F_d^2}{f_d^2}\right)^{2-y/2}\ln\left(\frac{F_d^2}{f_d^2}\right)=\frac{2yF^2}{f_d^2}\left(\frac{m_\pi^2}{m_d^2}\right)$

$$\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2\cdot V_\pi(\chi)\cdot tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right]$$

$$\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y\cdot V_\Sigma(\chi)\cdot tr\left(\Sigma+\Sigma^\dagger\right)$$

spoiler alert: $V_\pi(\chi)$ and $V_\Sigma(\chi)$ set to one as higher order only in tripple 1/N expansion (Golterman/Shamir) otherwise not determined

F_π^2 and M_π^2 at finite fermion mass m:

$$\left.\begin{array}{l}\dfrac{F_\pi^2}{f_\pi^2}=\dfrac{F_d^2}{f_d^2}\\[2ex]\dfrac{M_\pi^2}{m_\pi^2}=\left(\dfrac{F_d^2}{f_d^2}\right)^{y/2-1}\end{array}\right\}\Rightarrow\quad M_\pi^2\left(F_\pi^2\right)^{1-y/2}=\underbrace{2B_\pi\left(f_\pi^2\right)^{1-y/2}}_{\substack{C\ \text{fitted}\\ y\ \text{fitted}}}\cdot m$$

scaling test I: non-chiPT M_π^2 and F_π^2

independent of dilaton potential !

now we test this in the sextet model

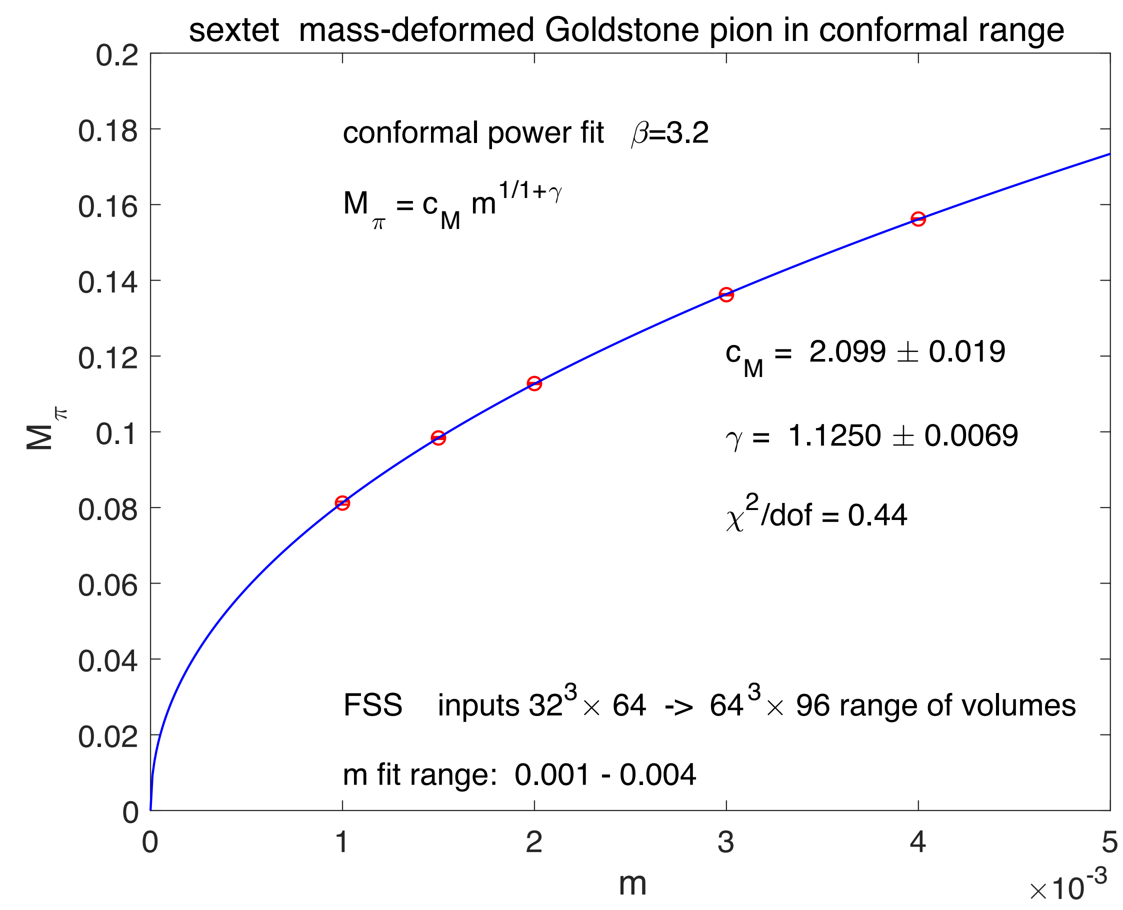
spoiler alert:

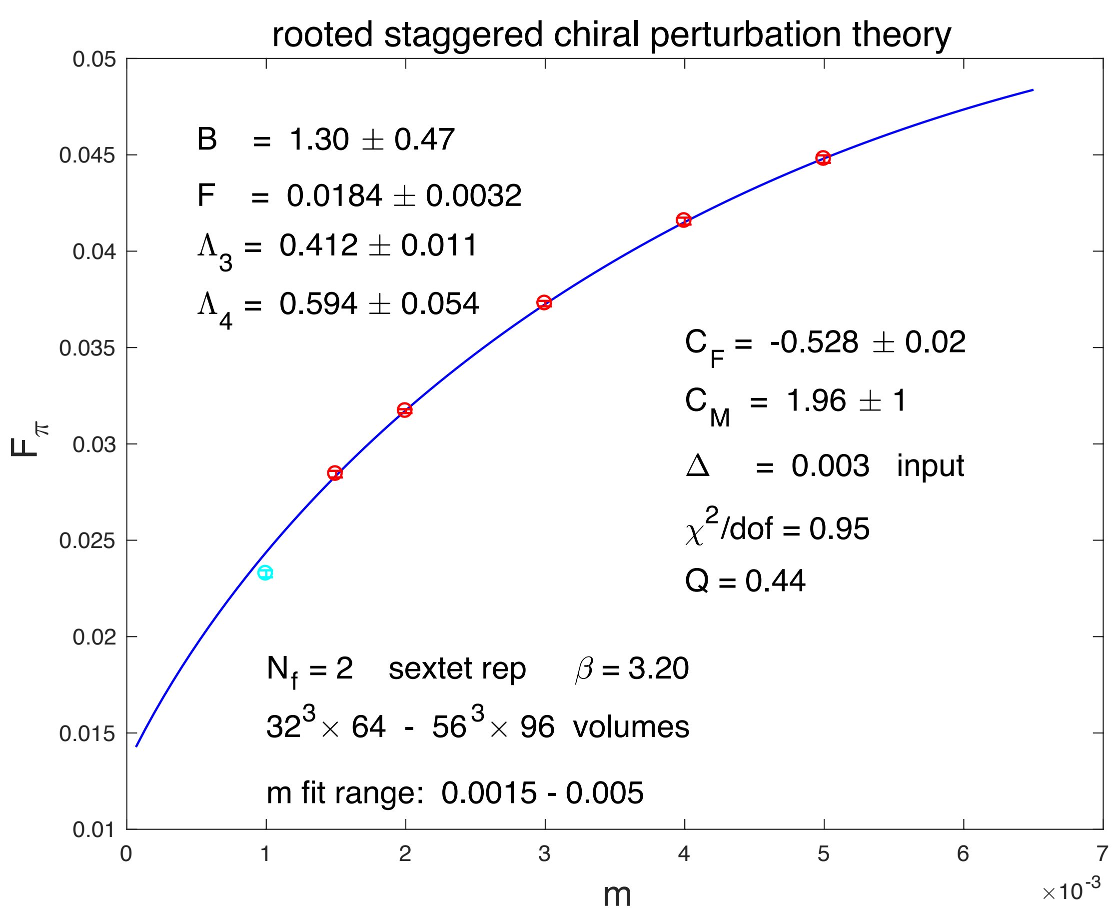
Fitted range conformal mass deformation?

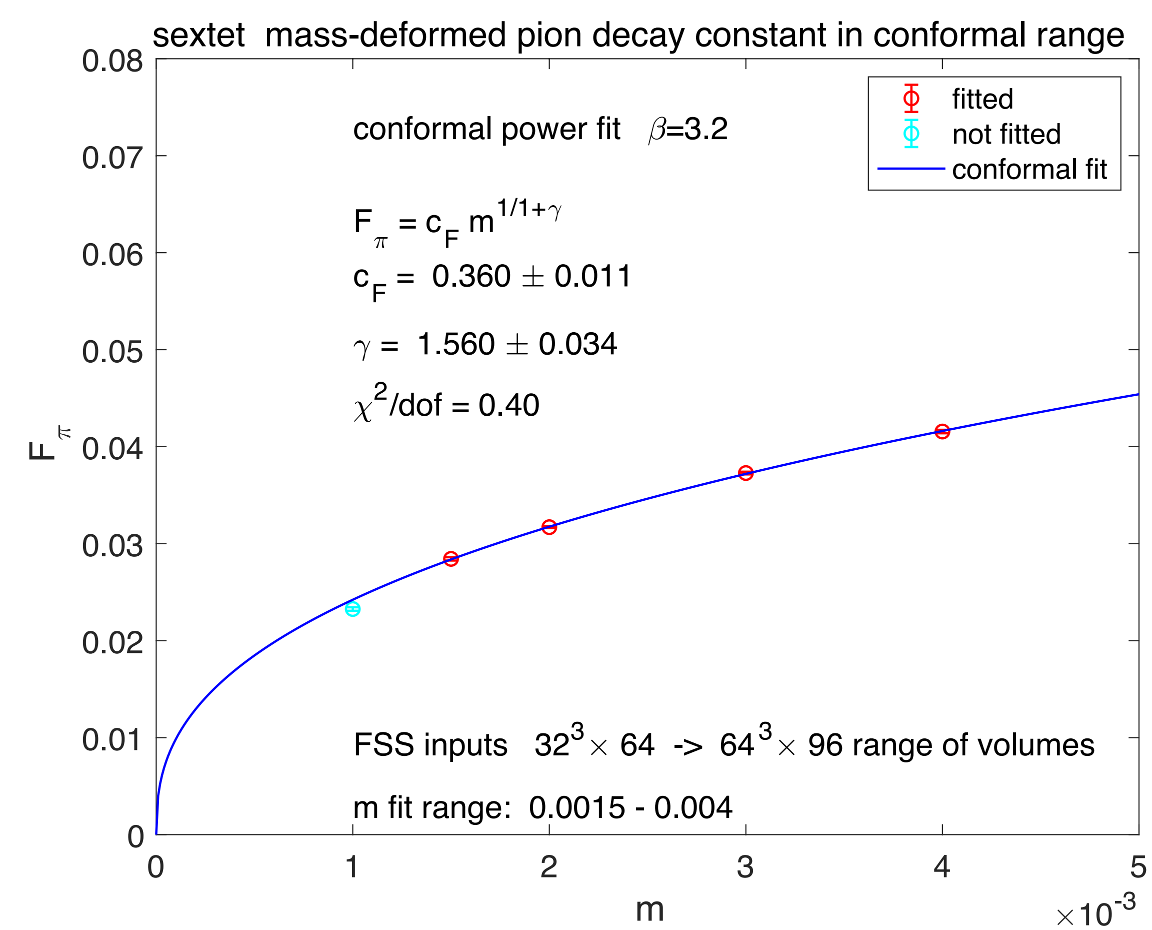
γ is inconsistent between M and F fits!

Not chiPT either

dilaton scaling?







scaling test (independent of the shape of the dilaton potential):

$(aM_\pi)^2 \cdot (aF_\pi)^{2-y} = C \cdot am \quad \text{with } C = 2aB_\pi (af_\pi)^{2-y}$

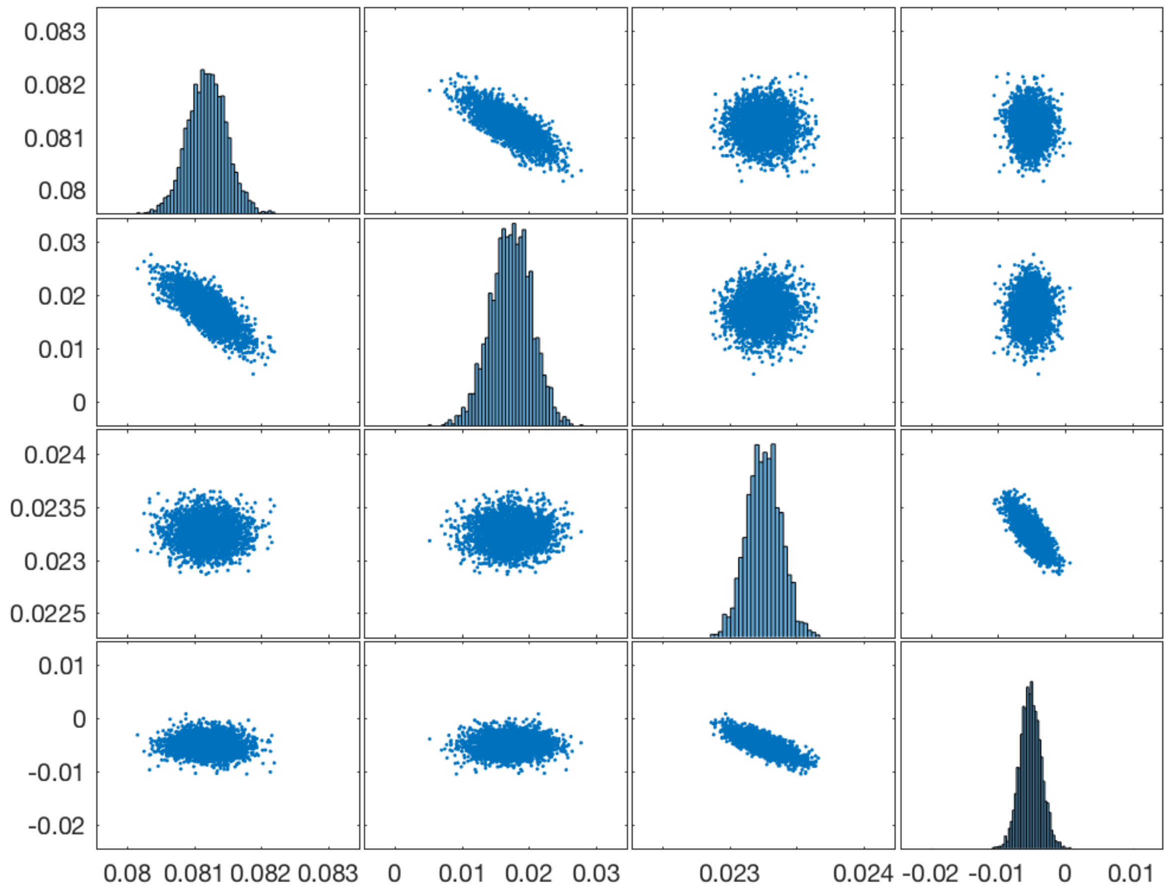
$\gamma = 3 - y$ mass anomalous dimension (what scale?)

FSS and covariance matrix are used in the fits

for checks: Bayesian posterior y distribution is determined from

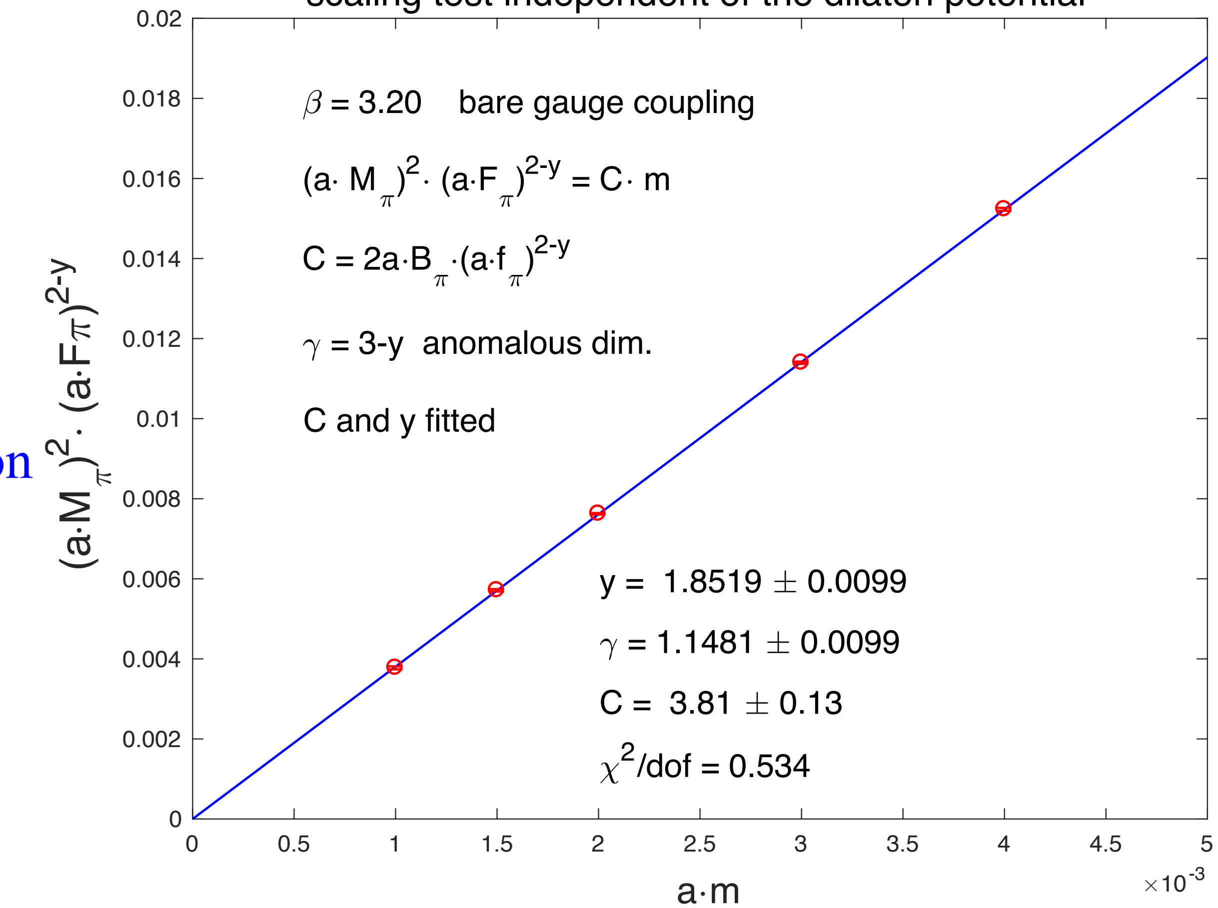
Markov Chain Monte Carlo on the Maximum Likelihood Function

Mpi, Fpi, C_M, C_F from MCMC Bayesian posterior distribution of the parameters

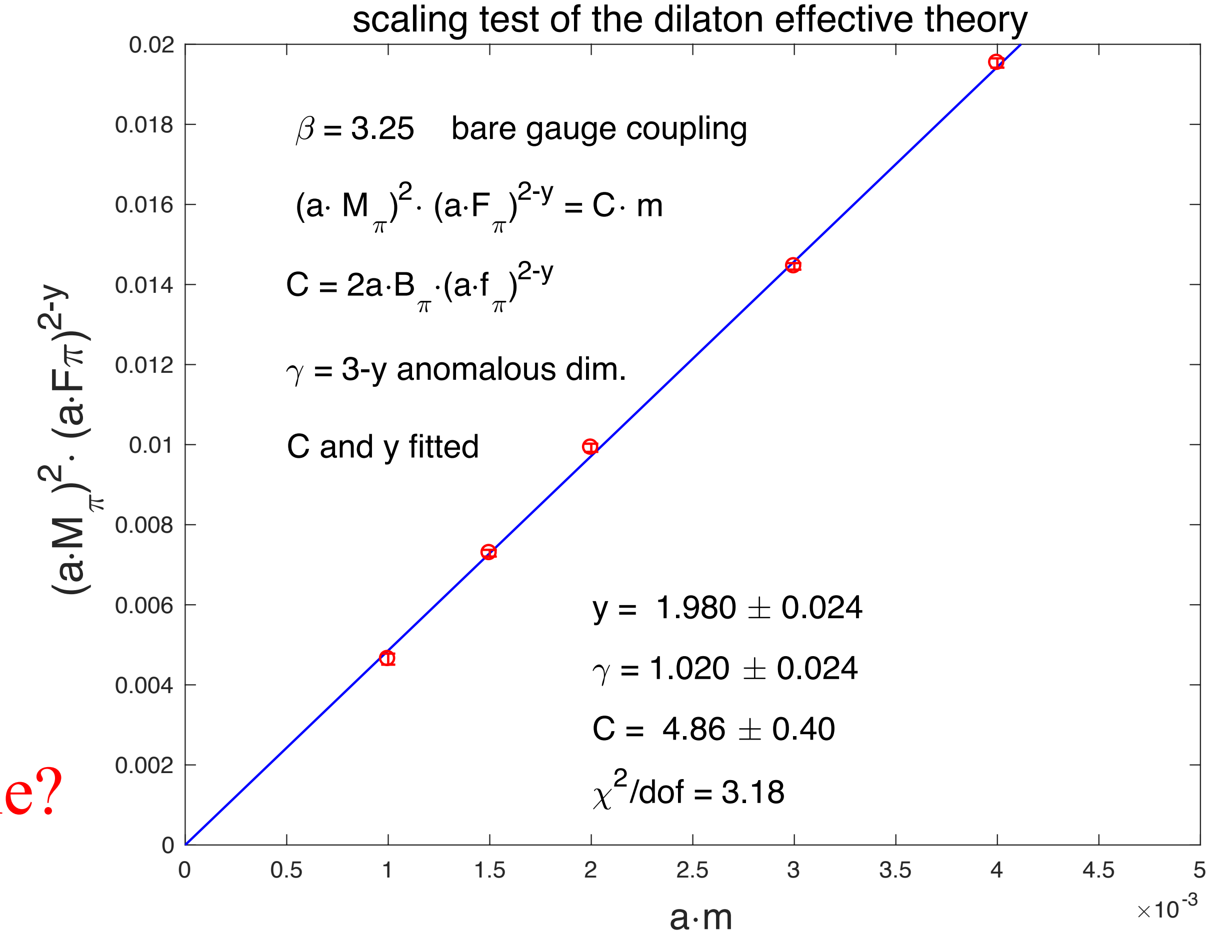


all is well? cutoff-dependent γ^*? if not, what γ scale?

scaling test independent of the dilaton potential

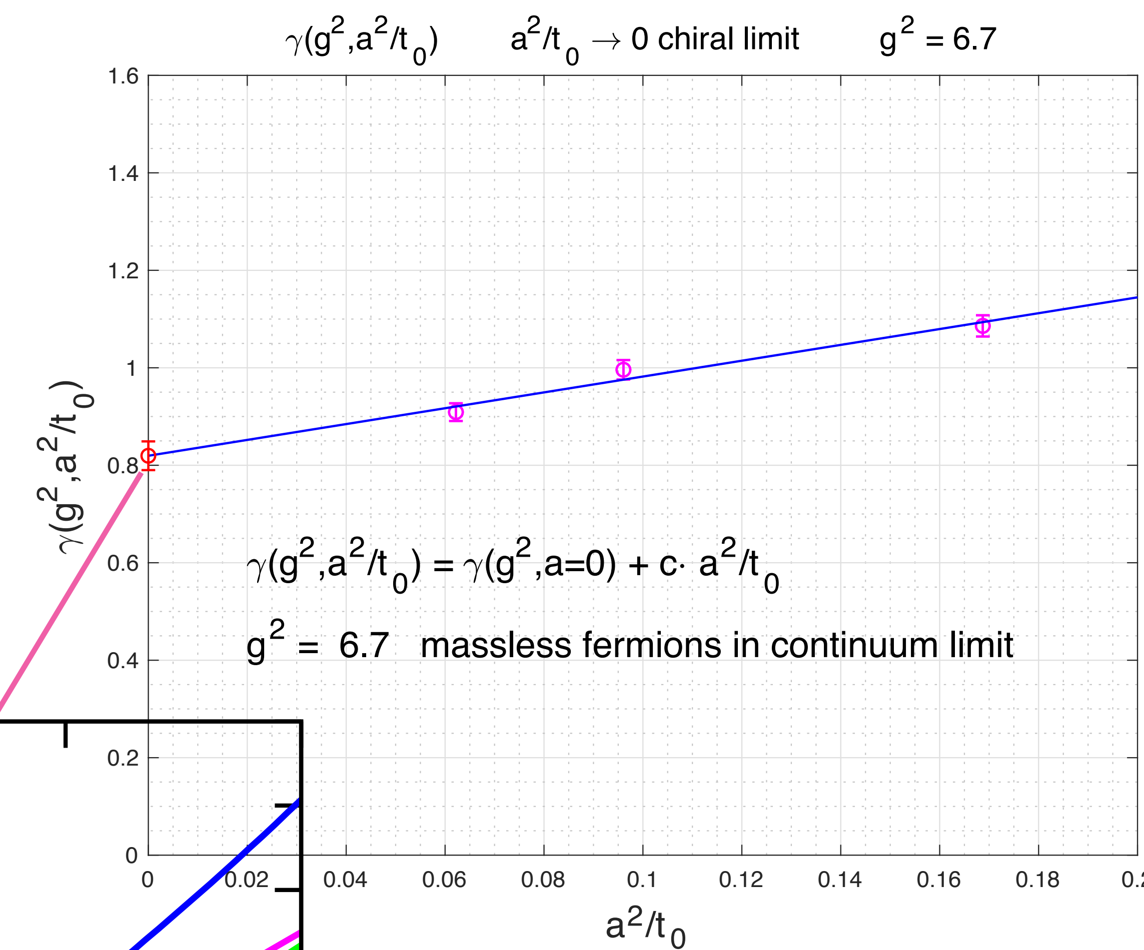


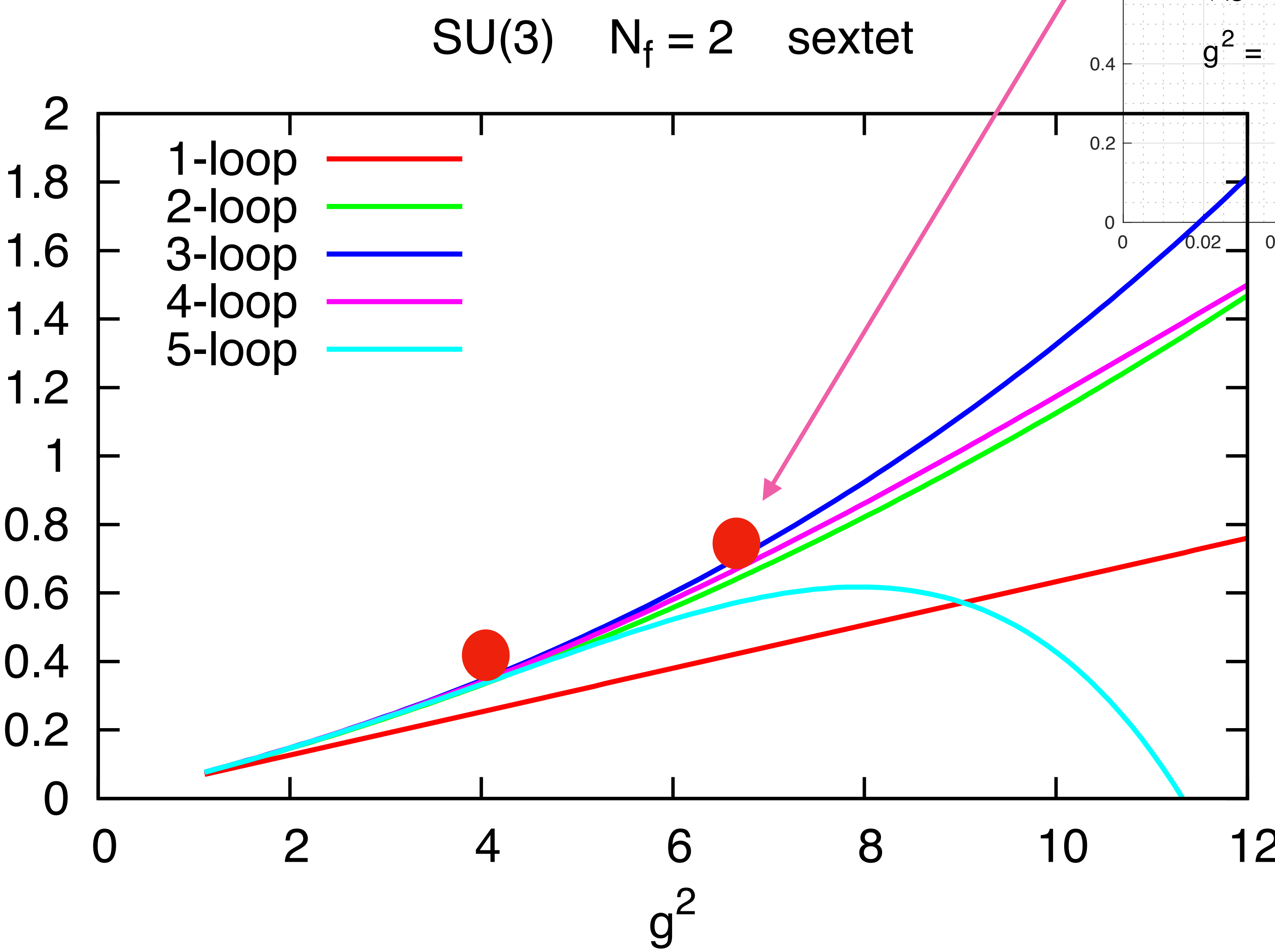
scaling test of the dilaton effective theory



mass anomalous dimension γ from Dirac spectrum: sextet data

- Chebyshev expansion of mode number
- infinite volume limit from FSS
- m -> 0 chiral limit at fixed a
- a -> 0 continuum limit





- Chebyshev expansion of mode number
- infinite volume limit from FSS
- m -> 0 chiral limit at fixed a
- a -> 0 continuum limit

scaling test II: from $V(\chi) \approx \chi^p$ large χ asymptotic shape of the dilaton potential: $(aM_\pi)^2 \cdot (aF_\pi)^{2-p} = B$

covariance matrix is used in the fits shown

cross check: p and B generated from ensembles of Fpi and Mpi at each m in Markov Chain Monte Carlo of the exact Maximum Likelihood Function without covariance matrix approx.

full analysis in Ricky Wong talk

these fits are failing (what did "the other sextet analysis" do?)

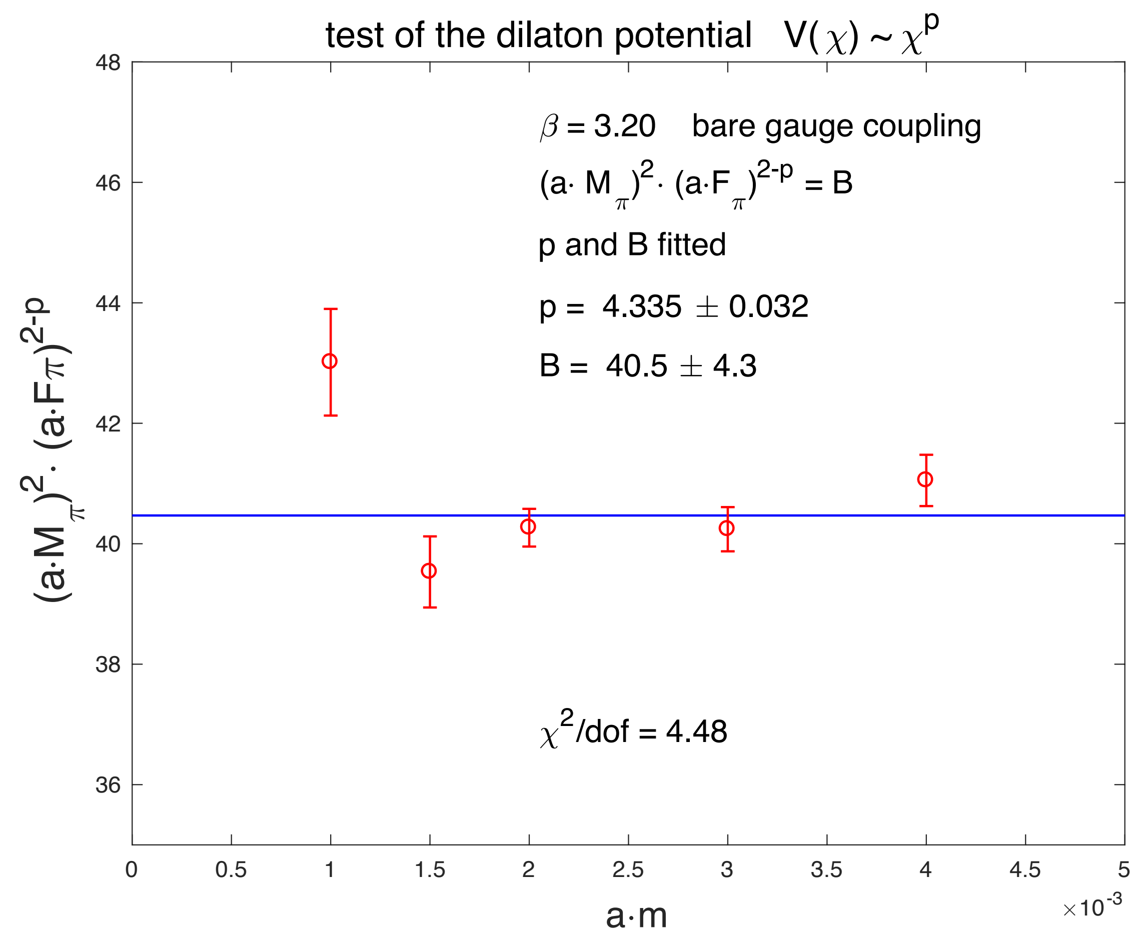
control of loop effects?

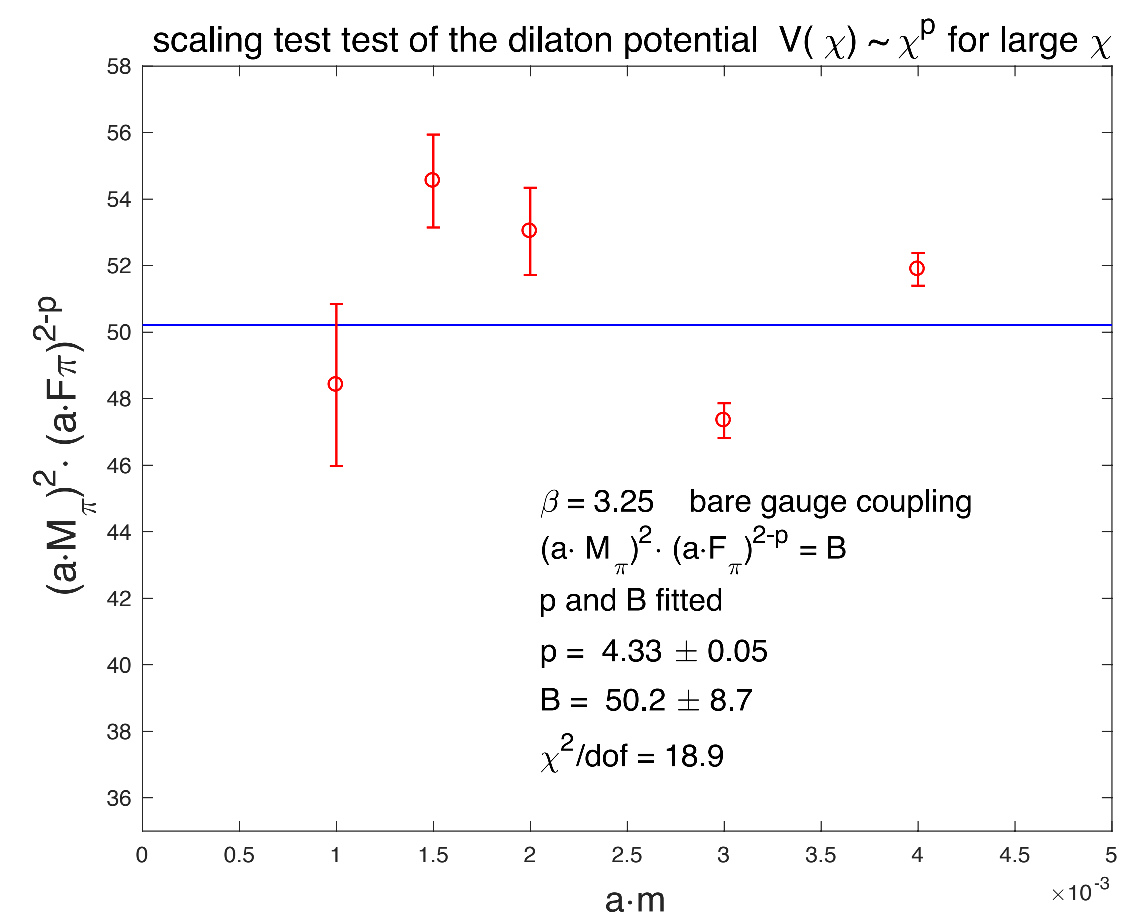
cutoff effects? not prime suspect

missing dilaton potential terms?

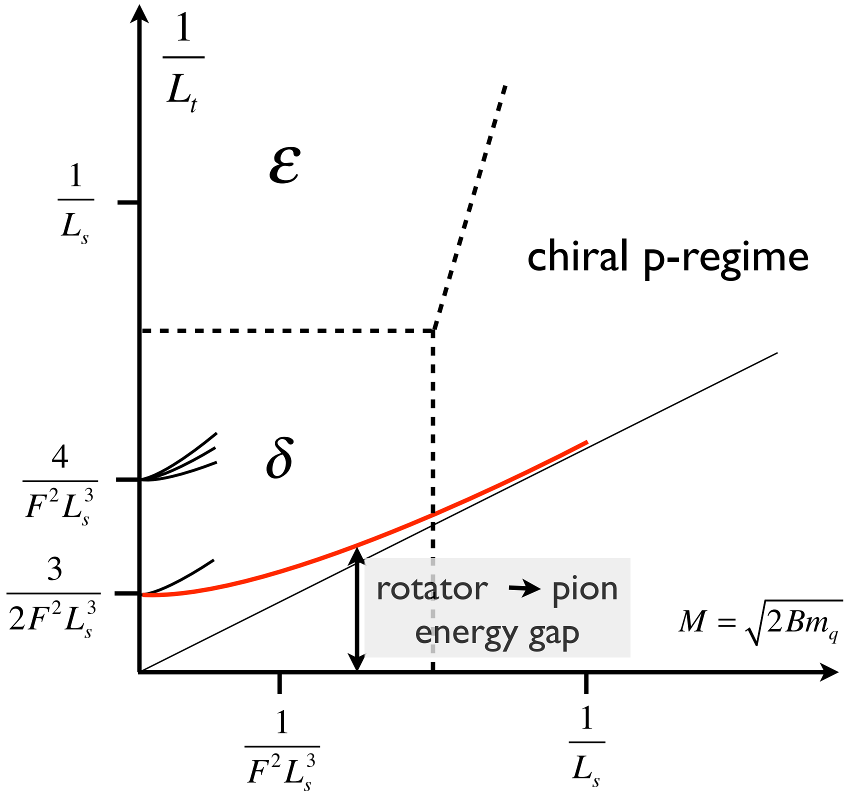
limited FSS at Q=0? not prime suspect

what is the definition and fit consistency of y and γ?



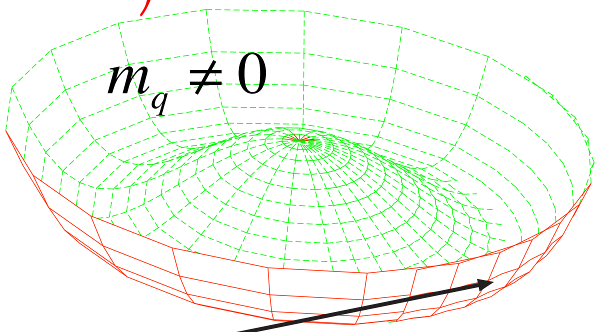


dilaton "decoupling" in the ε-regime



$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi-V(\chi)+\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right]-\frac{f_\pi^2m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi-V(\chi)\;-\frac{f_\pi^2m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

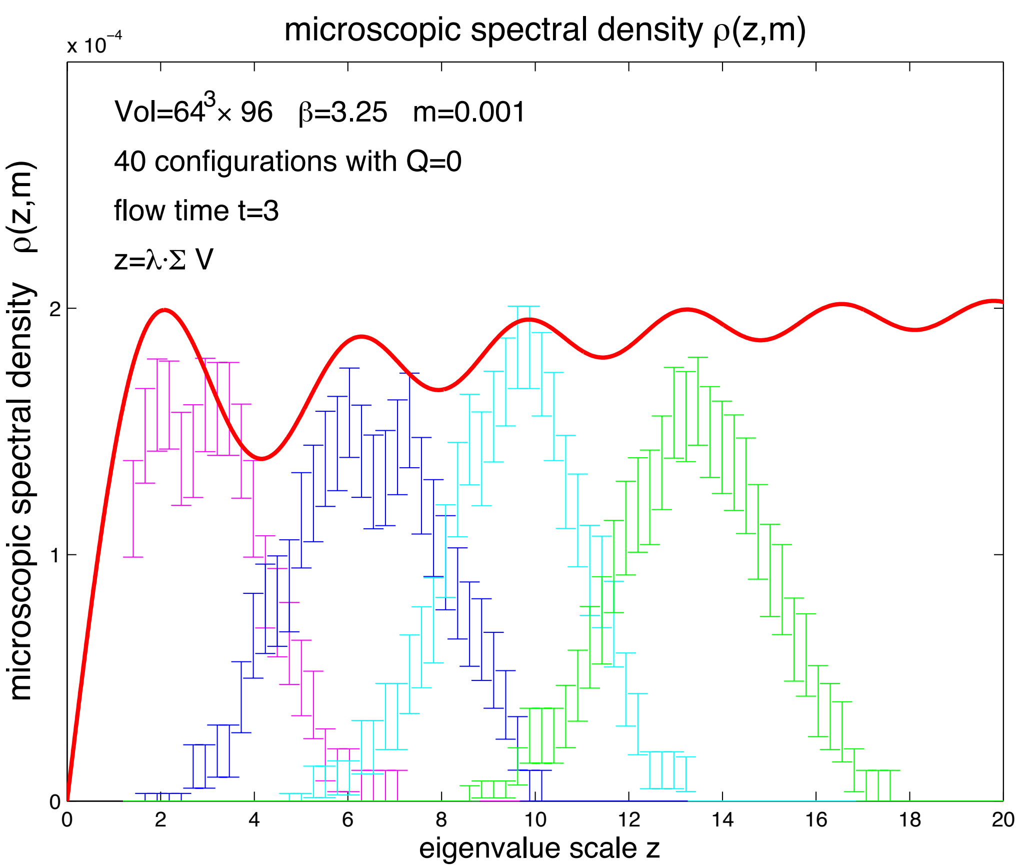


in RMT dynamics:

$m_\pi\neq 0$ mixed $\varepsilon-$regime (Damgaard-Fukaya)

pion light and dilaton "heavy"

tilted condensate



$m_\pi=0$ limit: dilaton is decoupled from pion zero mode in ε regime

- Mixed regime RMT analysis:
sea quarks earlier in p-regime this is changed now
spectrum in ϵ-regime

- taste breaking is handled in same framework
James Osborne worked out

- thanks to James for the discussions and the opportunity for checking our software on the output of his code

dilaton “decoupling” in the ε-regime

- simulations of the ε-regime are set up and running:
- staggered stout fermions at our medium fine lattice spacing
- dropping down from our lightest pion mass m*a ~ 0.07 in the p-regime
- one order of magnitude (2 orders of magnitude in the fermion mass)
- 64^4 lattice size running in the m=0.001- 0.00001 fermion mass range
- Mpi*L ~ 0.5 !!
- F*L ~ 1 is our projection - important for ϵ-regime expansion

constraint effective potential

$$\exp(-\Omega U_\Omega(\Phi)) = \prod_x \int d\phi(x)\delta\left(\Phi - \frac{1}{\Omega}\sum_x \phi(x)\right)\exp(-S[\phi])$$

scalar field ϕ(x) elementary, or source of composite operator

$$P(\Phi) = \frac{1}{Z}\exp(-\Omega U_\Omega(\Phi)), \quad Z = \int d\Phi' \exp(-\Omega U_\Omega(\Phi'))$$

probability distribution of order parameter in finite volume Ω

implementation with fermion fields: jk, Lee Lin, Pietro Rossi, Yue Shen, **NPB Proc. Suppl. 9 (1989) 99-104**

$$\frac{dU_{\text{eff}}}{d\Phi} = m^2\Phi + \frac{1}{6}\lambda\langle\phi^3\rangle_\Phi - N_F y\langle\bar{\psi}\psi\rangle_\Phi, \quad \langle\bar{\psi}\psi\rangle_\Phi = \langle \mathrm{Tr}(D[\phi]^{-1})\rangle_\Phi$$

HMC at fixed zero momentum mode of the scalar field

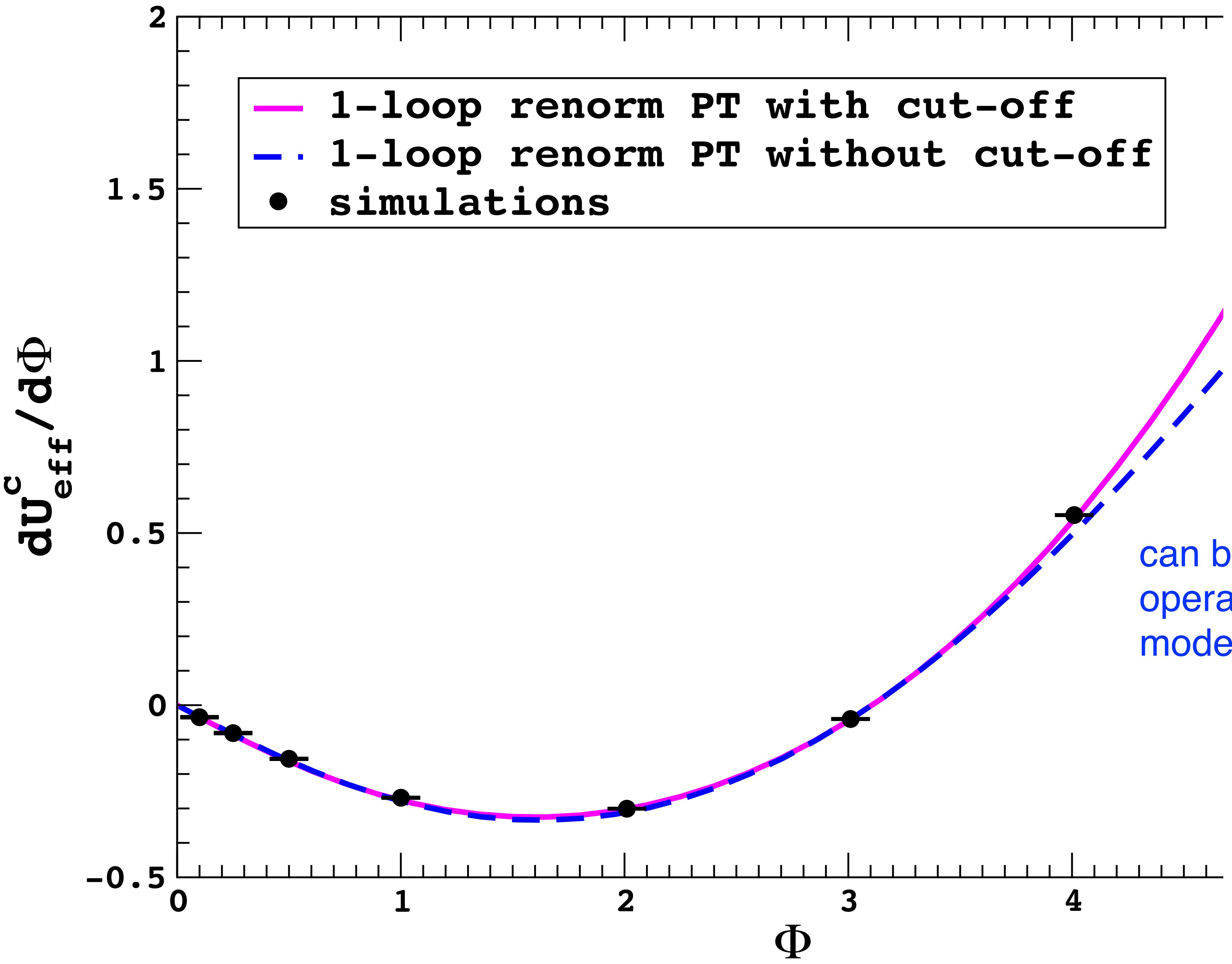
composite 0++ scalar emergent from NJL: equivalent Yukawa model

$$\dot{\phi}(x,t) = \pi(x,t)\ ,$$

$$\dot{\pi}(x,t) = -\left[\frac{\partial S_{\text{eff}}}{\partial\phi(x,t)} - \frac{1}{\Omega}\sum_y \frac{\partial S_{\text{eff}}}{\partial\phi(y,t)}\right]$$

$$\frac{1}{\Omega}\sum_y \phi(y,t) = \Phi, \quad \sum_y \pi(y,t) = 0$$

constraint effective potential



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LatHC

can be extended to four-fermion operator and more general BSM models

can we extend the analysis to gauge theories?

Summary:

- sextet model is consistent with χSB from all angles we looked at
- general EFT approach will change the χPT analysis
- dilaton EFT is a new fresh look
- dilaton signatures are problematic in sextet model
- sources of the problem?
- missing dilaton terms? scale dependent $\gamma(\lambda)$? loop control?
- the ε-regime (RMT) is new opportunity for general EFT signatures!
- constraint effective potential method
- Nf=12?