Linear Sigma EFT for Nearly Conformal Gauge Theories

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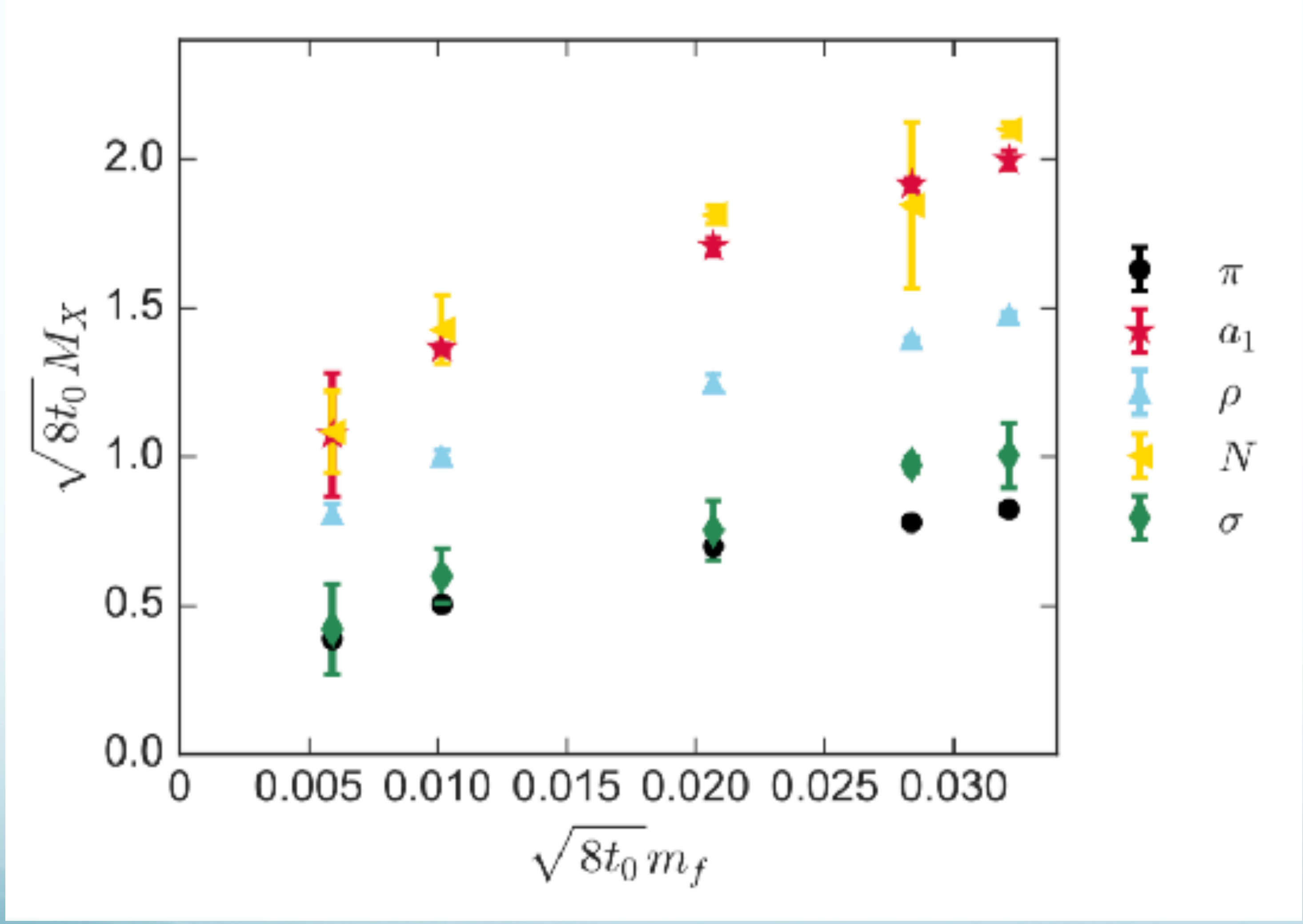
Outline

- Introduction: Fitting data from numerical lattice calculations in nearly conformal gauge theories using EFTs.
- The Linear Sigma EFT
- Chiral symmetry breaking and power counting
- Expressions for decay constant and particle masses
- Conclusions and future directions

Our Program

Nearly conformal gauge theories have N_f just below $N_{fc}(R)$

Example - Lattice Data for N_c=3, N_f=8



1807.08411 (LSD collaboration)

$$\mathscr{L} = ?$$

$$M_\sigma \ll M_\rho$$

- Assume light scalar has a weakly coupled EFT description.
- May be UV completion for EW sector.

Motivation

- If the correct EFT is known, then predictions for other observables can be made - e.g form factors, scattering lengths.
- Enables chiral extrapolation.
- Different EFT approaches should be compared with lattice data to see what provides the best fit - e.g [Golterman, Shamir ('15), Hansen, Langæble, Sannino ('16)]
- Linear Sigma EFT naturally incorporates light flavored scalars - [Shrock, Kurachi ('06)].

The Linear Sigma EFT

$SU_L(N_f) \times SU_R(N_f)$ chiral symmetry, spontaneously broken to $SU_V(N_f)$

$$M_a^{\bar{b}} \to L_a^c M_c^{\bar{d}} \left(R^\dagger\right)_{\bar{d}}^{\bar{b}}$$

$L, R \in SU_{L,R}(N_f)$

For $N_f=2$ only, the linear multiplet can be chosen to be real

$$M(x) = \frac{\sigma(x)}{\sqrt{2}}\mathbf{1} + i\pi(x)_i T_i$$

For all N_f, a complex linear multiplet can be constructed

$$M(x) = \frac{\sigma(x) + i\eta'(x)}{\sqrt{N_f}}\mathbf{1} + (a_i(x) + i\pi_i(x))T_i$$

$2N_f^2$ degrees of freedom total

Symmetry spontaneously broken when

$$\langle\sigma(x)\rangle = F$$

The Heavy η'

- In the underlying gauge theory, the η' state is made heavy by mixing with topological fluctuations of the gluon field strength.
- Direct lattice calculations also indicate that $M_\eta \gg M_\rho$ [LatKMI 1710.06549 (N_f=8)].
- We therefore remove the η' state from the EFT.

$$M(x) = \exp\left[i\frac{\sqrt{N_f}}{F}\left(\frac{\eta'(x)}{\sqrt{N_f}} + \pi_i(x)T_i\right)\right]\left(\frac{\sigma(x)}{\sqrt{N_f}} + a_i(x)T_i\right)$$

- Reparametrize M(x) so that η' not mixed with other states under $SU_L(N_f) \times SU_R(N_f)$ transformations.
- Set η'=0.
- Removing η' in this way leaves chiral symmetry unbroken.

Leading-order Lagrangian

$$\mathscr{L}_{LO} = \frac{1}{2}\langle \partial_\mu M \partial^\mu M^\dagger \rangle - V_0(M) - V_{m_q}(M)$$

Chirally symmetric part includes all relevant and marginal operators:

$$V_0(M) = \frac{-m_\sigma^2}{4}\langle M^\dagger M \rangle + \frac{m_\sigma^2 - m_a^2}{8f^2}\langle M^\dagger M \rangle^2 + \frac{N_f m_a^2}{8f^2}\langle \left(M^\dagger M\right)^2 \rangle$$

For $N_f \leq 4$, determinant operators also become marginal/ relevant.

$V_{m_q}(M)$ incorporates the effect of the quark mass in the gauge theory. It explicitly breaks chiral symmetry.

Lower case letters refer to chiral limit masses and vevs.

Require that potential has symmetry breaking minimum.

Explicit Chiral Breaking

- Account for the effect of the quark masses using a spurion χ.

$$\chi_a^{\bar{b}} \to L_a^c \chi_c^{\bar{d}} \left(R^\dagger\right)_{\bar{d}}^{\bar{b}}$$

- Analogous to chiral perturbation theory in the nonlinear sigma model.
- Construct operators in the EFT out of ∂, M and χ.
- Chiral symmetry explicitly broken when

$$\chi \to B\mathscr{M} \qquad\qquad B \sim B_\pi = \langle 0|\bar{\psi}\psi|0\rangle/f_\pi^2$$

- The quark mass matrix is $\mathscr{M}$.
- Need a way to organize operators in terms of the size of their contribution to physical processes (power counting).

Power Counting

- EFT cutoff scale set by mass of lightest excluded state $\Lambda \sim M_\rho$.
- Particle masses and vevs are smaller than this cutoff:

$$M_\pi \lesssim M_\sigma \sim M_a \sim F \ll \Lambda$$

In chiral limit, $M_\pi = 0$

If instead $M_a \sim \Lambda$, we can remove the flavored scalars from the spectrum by taking the limit:

$m_a^2 \to \infty, \qquad m_\sigma^2, f^2$ held constant

$F \sim M_\sigma$ motivated by lattice data

Capitals denote physical quantities at any quark mass

When computing the size of an operator's contribution to a physical quantity:

for low momentum processes

$$\frac{\partial}{\Lambda} \rightarrow \frac{M_{\pi,\sigma,a}}{\Lambda} \lesssim \frac{M_\sigma}{\Lambda} \ll 1$$

$$\frac{M(x)}{\Lambda} \rightarrow \frac{F}{\Lambda}, \frac{M_{\pi,\sigma,a}}{\Lambda} \lesssim \frac{M_\sigma}{\Lambda} \ll 1$$

$$\frac{\chi}{\Lambda^2} \rightarrow \frac{m_q B}{\Lambda^2} \ll 1$$

The size of this quantity which we expand in is independent of the other 2

The Potential

The operators of the potential can be viewed as terms in a double expansion in two independent small quantities:

$$\frac{M(x)}{\Lambda} \sim \frac{M_\sigma}{\Lambda}, \qquad \frac{\chi}{\Lambda^2} \sim \frac{m_q B}{\Lambda^2}.$$

- Away from the chiral limit, as quark mass is increased, the spurion expansion parameter gets larger relative to the field expansion parameter.
- Operators which contain more powers of χ become more numerically important relative to operators which only contain powers of M(x).
- Include operators which contain more powers of χ in $V_{m_q}(M)$

$$V_{m_q}(M) = -\sum_{i=1}^{9} \tilde{c}_i \mathcal{O}_i$$

Include in $V_{m_q}(M)$ operators which contribute at the same level as $\langle M^\dagger M \rangle^2$ in V_0.

Relative size of spurion

$$\frac{\chi}{\Lambda^2} \sim \left(\frac{M(x)}{\Lambda} \right)^\alpha$$

Symbol	Operator	$\alpha \lesssim 1$	$1 < \alpha \lesssim 2$
O_1	$\langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	✓
O_2	$\langle M^\dagger M \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_3	$\langle (M^\dagger M)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X
O_4	$\langle \chi^\dagger M + M^\dagger \chi \rangle^2$	✓	X
O_5	$\langle \chi^\dagger \chi M^\dagger M \rangle$	✓	X
O_6	$\langle \chi^\dagger \chi \rangle \langle M^\dagger M \rangle$	✓	X
O_7	$\langle \chi^\dagger M \chi^\dagger M + M^\dagger \chi M^\dagger \chi \rangle$	✓	X
O_8	$\langle \chi^\dagger \chi \rangle \langle \chi^\dagger M + M^\dagger \chi \rangle$	✓	X
O_9	$\langle (\chi^\dagger \chi)(\chi^\dagger M + M^\dagger \chi) \rangle$	✓	X

Masses and Decay Constant

Working to tree level in the EFT

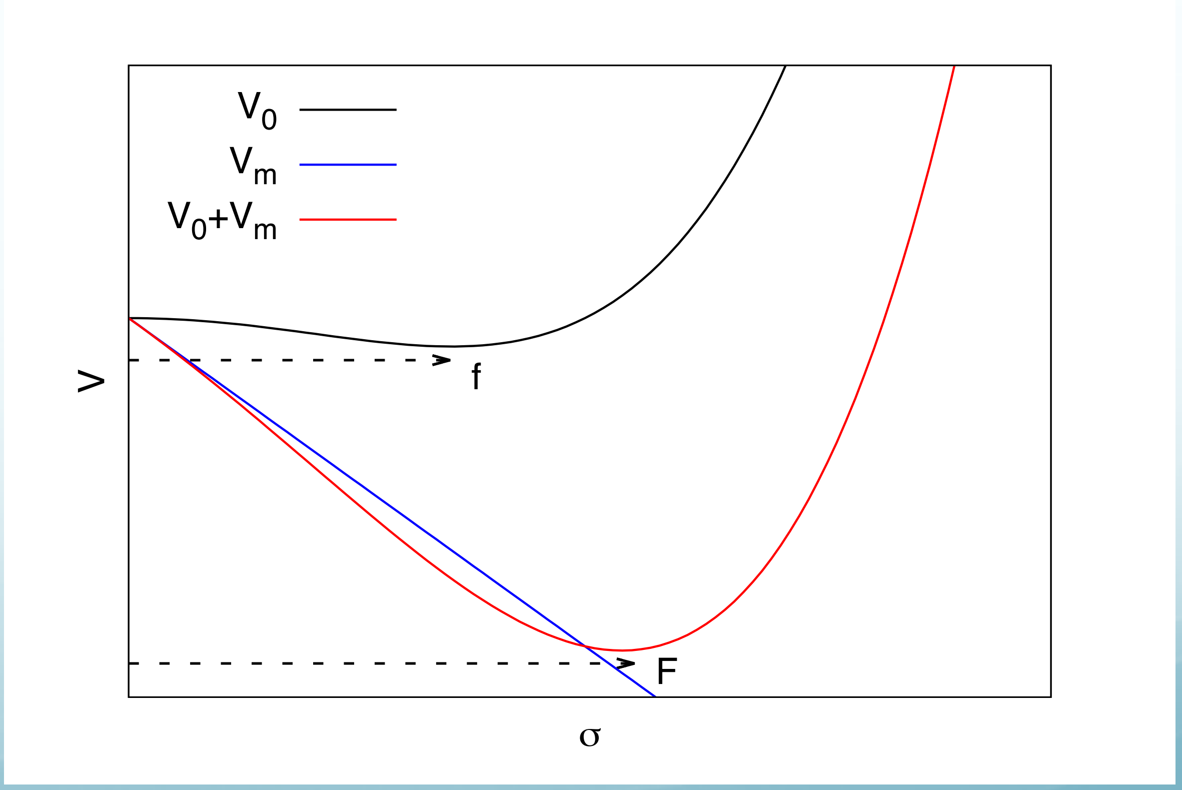
$$F^2 = f^2 + \frac{2f^2}{m_\sigma^2}\left[2Bm_q\frac{f}{F} + 6Bm_q(c_2+c_3)F + 2B^2m_q^2(4c_4+c_5+c_6+2c_7) + 2B^3m_q^3\frac{c_8+c_9}{F}\right],$$

$$M_\pi^2 = 2Bm_q\frac{f}{F} + 2Bm_q(c_2+c_3)F + 8B^2m_q^2(c_4+c_7) + 2B^3m_q^3\frac{c_8+c_9}{F},$$

$$M_\sigma^2 = m_\sigma^2 + 6Bm_q\frac{f}{F} + 6Bm_q(c_2+c_3)F + 4B^2m_q^2(4c_4+c_5+c_6+2c_7) + 6B^3m_q^3\frac{c_8+c_9}{F},$$

$$M_a^2 = m_a^2\frac{F^2}{f^2} + 4Bm_q\frac{f}{F} + 8Bm_qc_2F + 2B^2m_q^2(8c_4+c_5+c_6+2c_7) + 4B^3m_q^3\frac{c_8+c_9}{F}.$$

$$F_\pi = \sqrt{\frac{2}{N_f}}F$$



An Inequality

Without the O(χ²) terms in the potential, the inequality holds:

$$M_\sigma^2 \geq 3M_\pi^2$$

[Kuti, '17]

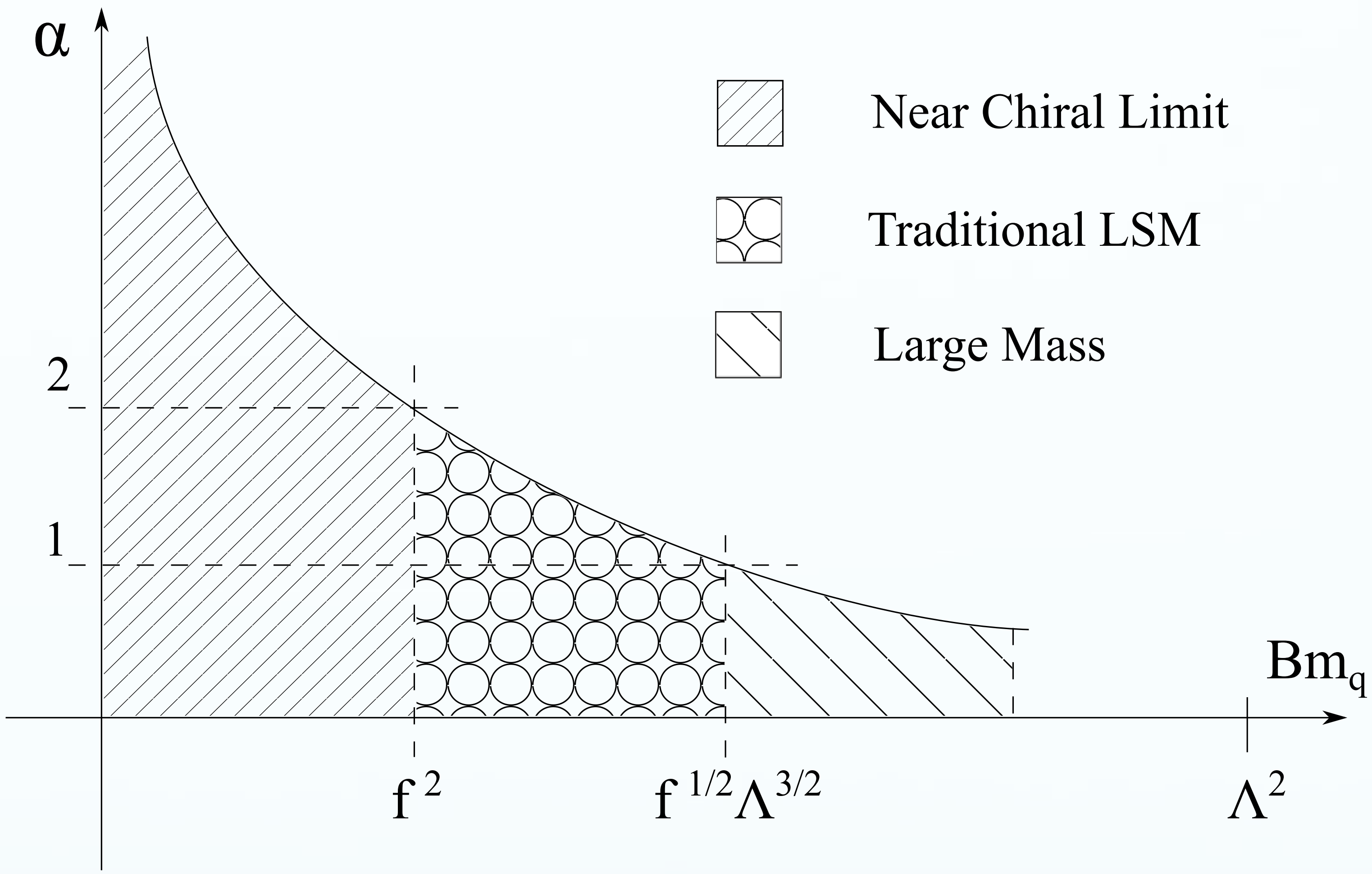
Inequality is not satisfied by lattice data for N_c=3 N_f=8 [LSD] and N_f=2 (sextet) [LatHC].

$$3M_\pi^2 - M_\sigma^2 + m_\sigma^2 = 4B^2 m_q^2(2c_4 - c_5 - c_6 + 4c_7)$$

Violated by terms with 2 powers of χ

Consistency with lattice data possible if extra terms are included in the leading-order potential.

Quark Mass Regimes



Regime	Relevant operators in potential that are most important	$M_\pi^2 \approx Bm_q$	$M_\sigma^2 \geq 3M_\pi^2$
$0 < Bm_q < f^2$	$\langle M^\dagger M \rangle$	Y	Y
$f^2 < Bm_q < f^{1/2}\Lambda^{3/2}$	$\mathcal{O}_1$	N	Y
$f^{1/2}\Lambda^{3/2} \lesssim Bm_q$	$\mathcal{O}_{2-9}$	N	N

Summary and Future Directions

- Want to find an accurate EFT description for low energy physics of nearly conformal gauge theories.

  -- Test against and make predictions for lattice data.

- Linear Sigma EFT can incorporate flavored scalars.

- For larger quark masses, extra terms in $V_{m_q}(M)$ can play a significant role. The EFT expansion can still be convergent in this regime.

  -- The inequality $M_\sigma^2 \geq 3M_\pi^2$ can be evaded.

- Need to perform fits.

- In future, consider explicit breaking of flavor symmetry within our framework e.g SU(8) → SU(2) x SU(6): $m_2 = 0$, $m_6 \neq 0$. See [Meurice '18]. More similar to real world Higgs.

Removing the Flavored Scalars

The potential V_0 can be rewritten as

$$V_0 = \frac{m_a^2 N_f}{8f^2}\left\langle \left[M^\dagger M - \frac{1}{N_f}\langle M^\dagger M\rangle \mathbf{1}\right]^2 \right\rangle + \frac{m_\sigma^2}{8f^2}\left[\langle M^\dagger M\rangle - f^2\right]^2$$

Taking the limit $m_a^2 \to \infty, \quad m_\sigma^2, f^2$ held constant imposes the constraints

$$M^\dagger M = \frac{1}{N_f}\langle M^\dagger M\rangle \mathbf{1}$$

One parametrization of M(x), which (after the removal of η') satisfies these constraints is

$$M(x) = \frac{\sigma(x)}{\sqrt{N_f}} \exp\left[i\frac{\sqrt{N_f}}{F}\pi_i(x) T_i \right]$$

Flavored scalars have been removed