ChPT with an Isosinglet Scalar

A phenomenological extension

Martin Hansen

Kasper Langæble, Francesco Sannino

CP³ Origins
Cosmology & Particle Physics

XIIIth Quark Confinement and the Hadron Spectrum

1-6 August 2018, Dublin, Ireland

Overview

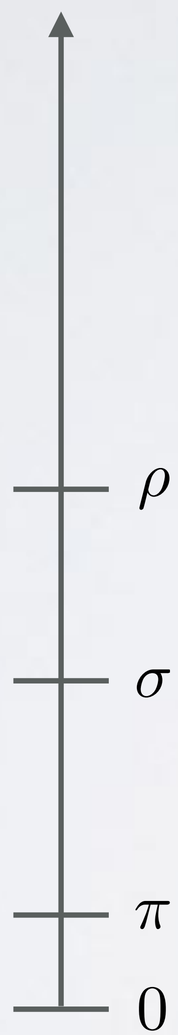
- Why ~~do~~ should we care about the scalar?
- The chiral Lagrangian
- The scalar extension
- One-loop results
- The different natures of the scalar

MH, K. Langæble, F. Sannino,
Extending chiral perturbation theory with an isosinglet scalar
arXiv:1610.02904 - Phys.Rev. D95 (2017) 036005

Energy scales

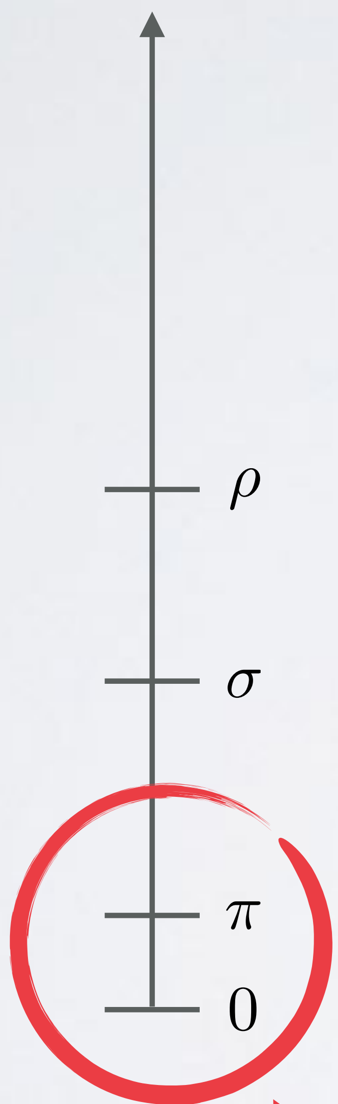
Scales with 2 flavours

QCD



Scales with 2 flavours

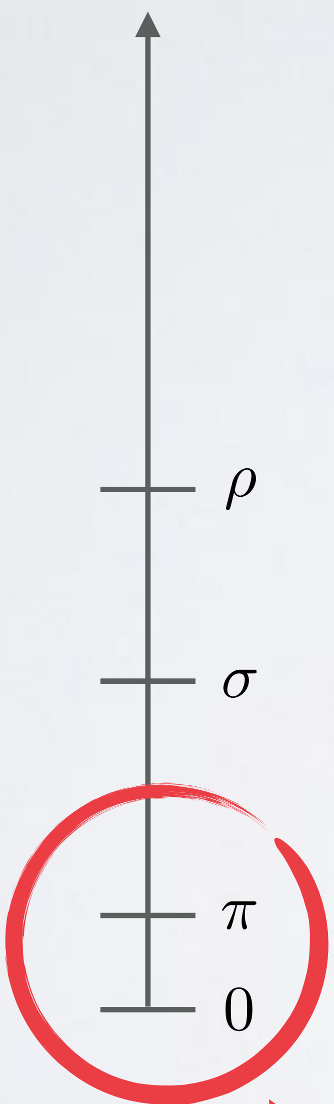
QCD



Region of validity

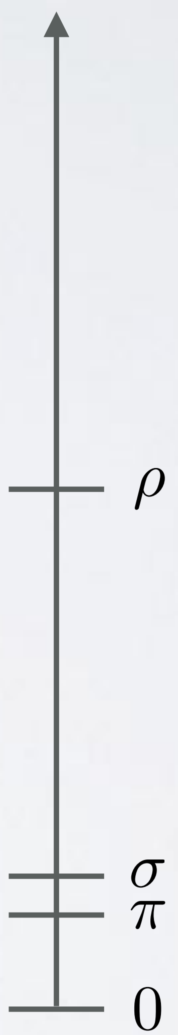
Scales with 2 flavours

QCD



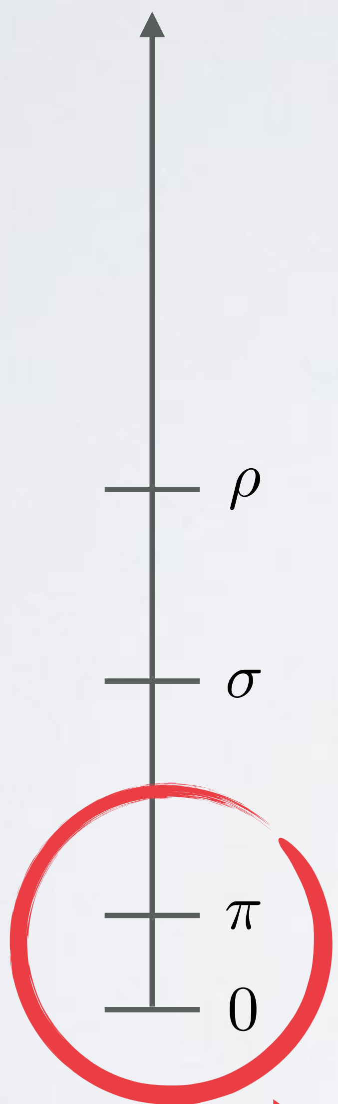
Region of validity

Near-conformal



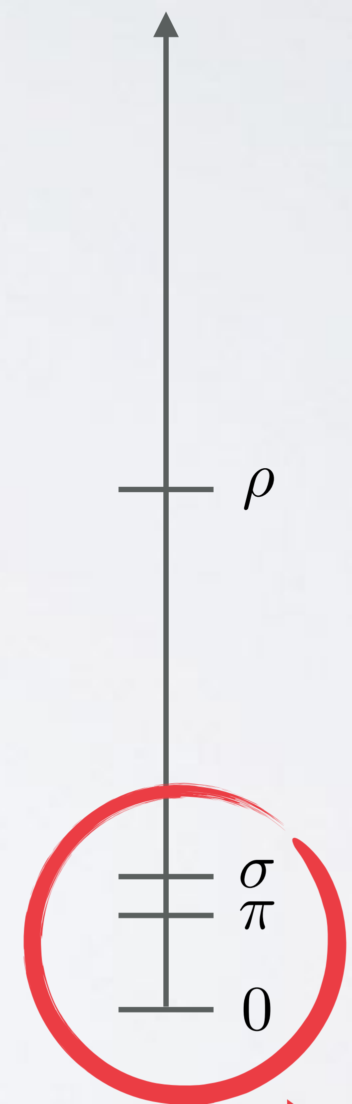
Scales with 2 flavours

QCD



Region of validity

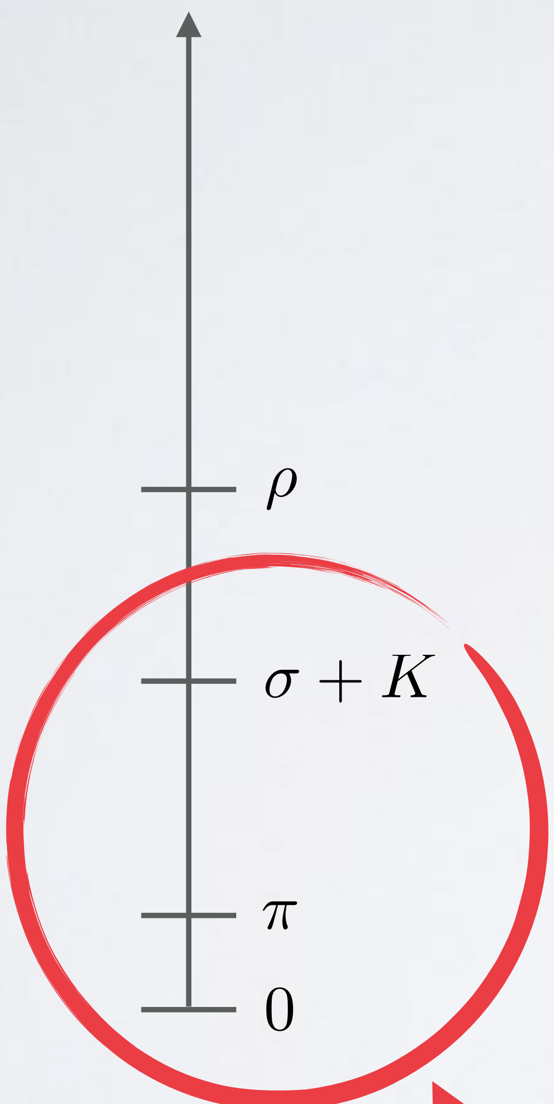
Near-conformal



Nearly generate states

Scales with 3 flavours

QCD



Kaons and $f_0(500)$ almost degenerate

Region of validity

Lessons

- In some BSM models, the pseudoscalar and scalar is almost degenerate (according to lattice investigations)
- In 3-flavour QCD, the Kaons are almost degenerate with the scalar

Lessons

- In some BSM models, the pseudoscalar and scalar is almost degenerate (according to lattice investigations)
- In 3-flavour QCD, the Kaons are almost degenerate with the scalar
- **Consequently, in some cases the scalar should probably not be ignored**

ChPT with scalar

Approach

- Adopt a counting scheme

$$\mathcal{O}(m_\pi^2) \sim \mathcal{O}(m_\sigma^2) \sim \mathcal{O}(p^2)$$

- Valid in some intermediate range of quark masses since

$$m_\pi^2 = A m_q \ , \qquad m_\sigma^2 = m_0 + B m_q$$

- Sufficiently close to the chiral limit, the scalar should be integrated out because $m_\sigma \gg m_\pi$
- Obviously, the construction only works in models with spontaneous chiral symmetry breaking

The chiral Lagrangian

- Non-linearly realized Lagrangian
- Let G be the flavor symmetry and let H be the stability group. The Goldstone boson manifold G/H is then parametrized by

$$u = \exp\left(\frac{i}{\sqrt{2}f_\pi} X^a \phi^a\right)$$

- Invariants

$$u_\mu = i(u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger)$$
$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

The chiral Lagrangian

- The chiral Lagrangian reads

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\langle u_\mu u^\mu + \tilde{\chi}_+\rangle$$
$$\mathcal{L}_4 = L_0\langle u_\mu u_\nu u^\mu u^\nu\rangle + L_1\langle u_\mu u^\mu\rangle\langle u_\nu u^\nu\rangle + L_2\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle + \cdots$$

- Many low-energy constants (unknown beyond LO)

$$B_0\ , \quad f_\pi\ , \quad L_0\ , \quad L_1\ , \quad \ldots$$

The chiral Lagrangian

- The LECs can be divided in contributions from various pieces, such as heavier resonances (as done in VMD)

$$L_i = \hat{L}_i + \sum_R L_i^R$$

- If the scalar is not considered heavy, there is a dynamical (non-constant) contribution to the LECs

The chiral Lagrangian

- Introduce the scalar as a non-trivial background field

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\left[1 + S_1\left(\frac{\sigma}{f_\pi}\right) + S_2\left(\frac{\sigma}{f_\pi}\right)^2 + \cdots\right]\langle u_\mu u^\mu\rangle$$
$$+ \frac{f_\pi^2}{4}\left[1 + S_3\left(\frac{\sigma}{f_\pi}\right) + S_4\left(\frac{\sigma}{f_\pi}\right)^2 + \cdots\right]\langle \tilde{\chi}_+\rangle$$

- Expansion of LO Lagrangian is sufficient at NLO
- Four additional low-energy constants !
- New counter terms are also needed

The scalar Lagrangian

- Standard **φ**^{4} theory

$$\mathcal{L}_\sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2\left[1 + S_5\left(\frac{\sigma}{f_\pi}\right) + S_6\left(\frac{\sigma}{f_\pi}\right)^2\right]$$

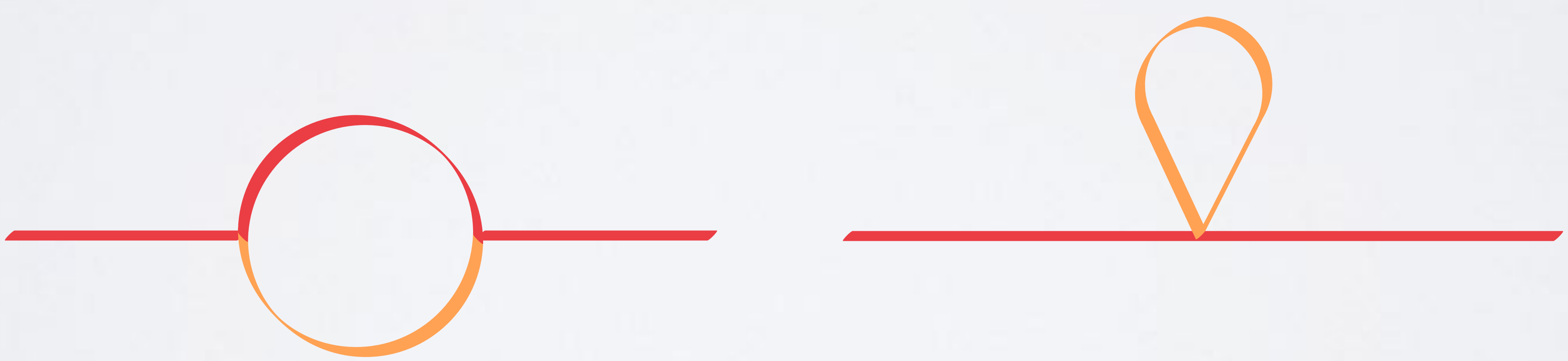
- Expansion of kinetic term unnecessary for on-shell quantities (related via EOM)
- Assume vanishing expectation value for scalar field

$$S_5 \geq -2\sqrt{S_6} \quad , \quad S_6 \geq 0$$

Pion self-energy

- Standard diagrams at NLO

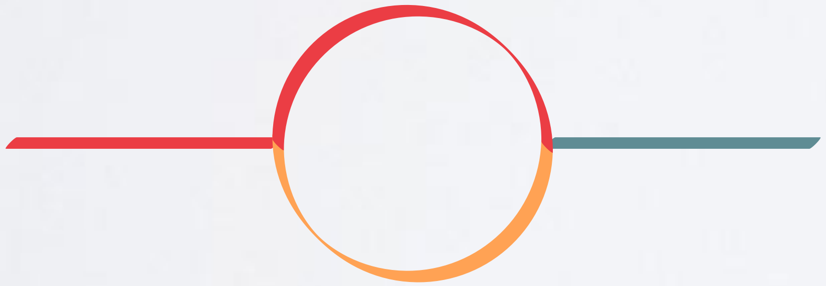
- New diagrams



Pion decay constant

- Standard diagrams at NLO

- New diagrams



Scalar self-energy

- Standard diagrams

- New diagrams

Pion mass at NLO

Polynomial

Logarithms

Unitarity corrections

$$\hat{m}_\pi^2 = m_\pi^2 + \frac{m_\pi^4}{f_\pi^2}(a_1 + a_2 L_\pi + a_3 J_{\pi\sigma\pi}) + \frac{m_\sigma^4}{f_\pi^2}(a_4 L_\sigma + a_5 J_{\pi\sigma\pi}) + \frac{m_\pi^2 m_\sigma^2}{f_\pi^2}(a_6 + a_7 L_\pi + a_8 L_\sigma + a_9 J_{\pi\sigma\pi})$$

Pion mass at NLO

- Coefficients are largely independent of the pattern of chiral symmetry breaking

	a_i	b_i	c_i
1	b_M	b_F	$6S_6$
2	$a_M - \frac{1}{2}(2S_1S_3 - S_1^2)$	$a_F - \frac{3}{8}S_1^2$	$-\frac{1}{4}n_\pi S_1^2$
3	$-(S_1 - S_3)^2$	$-\frac{1}{2}(S_1S_3 - S_1^2)$	$-9S_5^2$
4	$\frac{1}{4}S_1^2$	$-2K_1^r$	$2n_\pi(S_1S_3 - S_1^2) + n_\pi(S_4 - S_2)$
5	$-a_4$	$\frac{1}{8}(12S_2 + S_1^2)$	$-n_\pi(S_1 - S_3)^2$
6	$4(K_2^r - K_1^r)$	$-\frac{1}{4}S_1^2$	$\frac{1}{2}n_\pi S_1^2$
7	$-a_4$	$-\frac{1}{2}(S_1 - S_3)^2$	$n_\pi(S_1^2 - S_1S_3)$
8	$S_4 - S_2 - S_1^2 + S_1S_3$	$-\frac{1}{8}S_1^2$	–
9	$S_1^2 - S_1S_3$	b_3	–

Pion decay constant at NLO

Polynomial

Logarithms

Unitarity corrections

$$\hat{f}_\pi = f_\pi + \frac{m_\pi^2}{f_\pi}(b_1 + b_2 L_\pi + b_3 J_{\pi\sigma\pi}) + \frac{m_\sigma^2}{f_\pi}(b_4 + b_5 L_\sigma + b_6 J_{\pi\sigma\pi}) + \frac{H_{\pi\sigma\pi}}{f_\pi}(b_7 m_\pi^4 + b_8 m_\sigma^4 + b_9 m_\pi^2 m_\sigma^2).$$

Wave function renormalisation

Scalar mass at NLO

Logarithms

Unitarity corrections

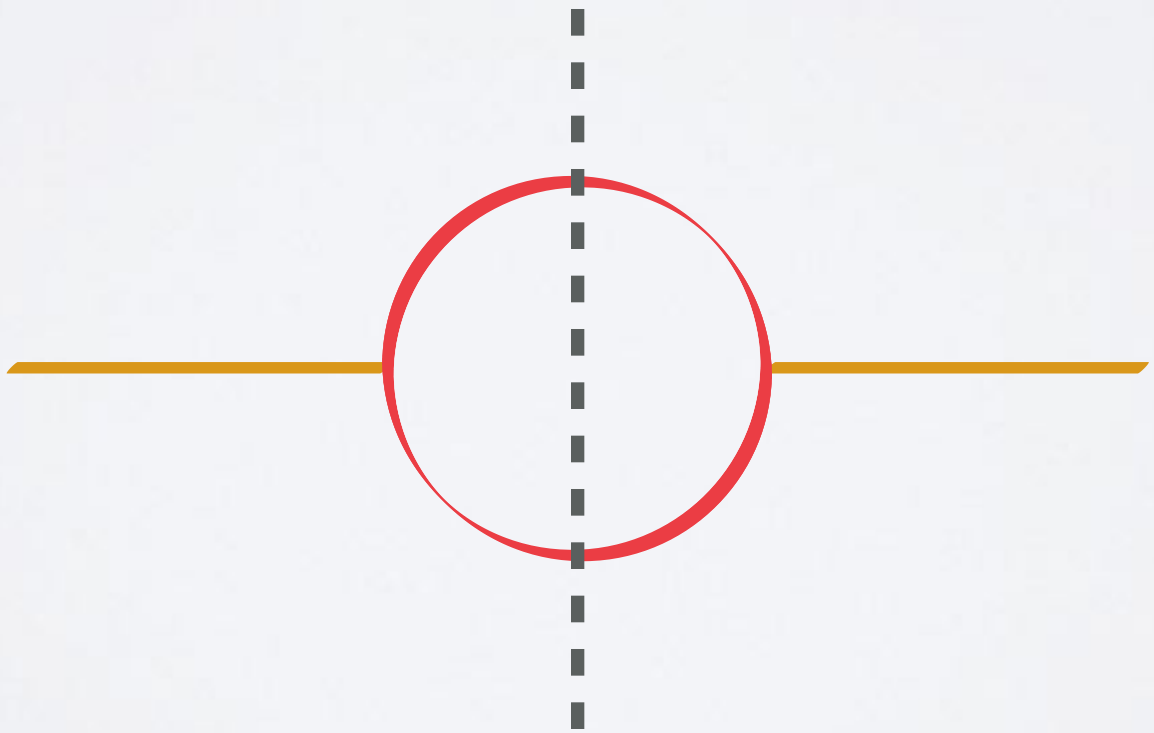
$$\hat{m}_\sigma^2 = m_\sigma^2 + \frac{m_\sigma^4}{f_\pi^2}(c_1 L_\sigma + c_2 J_{\pi\pi\sigma} + c_3 J_{\sigma\sigma\sigma} - 2K_3^r)$$
$$+ \frac{m_\pi^4}{f_\pi^2}(c_4 L_\pi + c_5 J_{\pi\pi\sigma} - 2K_5^r)$$
$$+ \frac{m_\pi^2 m_\sigma^2}{f_\pi^2}(c_6 L_\pi + c_7 J_{\pi\pi\sigma} - 2K_4^r),$$

Polynomial part from renormalisation / counter terms

Scalar decay width at NLO

- Decay channel $\sigma \to \pi\pi$

$$\Gamma = -\frac{c_2 m_\sigma^4 + c_5 m_\pi^4 + c_7 m_\sigma^2 m_\pi^2}{16\pi m_\sigma f_\pi^2}\sqrt{1-\frac{4m_\pi^2}{m_\sigma^2}}$$



Consistency checks

- In the chiral limit: $m_\pi^2 \to 0$
- Vanishing pion mass

$$\hat{m}_\pi^2 = 0$$

- Constant decay constant

$$\hat{f}_\pi = \underbrace{f_\pi + \frac{m_\sigma^2}{f_\pi}\left(b_4 + (b_5 + b_6)L_\sigma - \frac{b_8}{32\pi^2}\right)}_{= f_\pi \text{ in normal ChPT}}$$

- Results independent of renormalisation scale

Natures of the scalar

Natures of the scalar

- Many low-energy constants in normal ChPT
- … even more when including a scalar

- Different realisations of the scalar leads to different predictions for the constants $S_{1,2,3,4}$

Natures of the scalar

- Many low-energy constants in normal ChPT
- … even more when including a scalar

- Different realisations of the scalar leads to different predictions for the constants $S_{1,2,3,4}$

- **No guarantee that the counting scheme is appropriate for the various conceivable limits!**

Dilaton limit

- Introduce the dilaton as the conformal compensator

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\left[\langle u_\mu u^\mu \rangle \exp\left(\frac{2\sigma}{f_\pi}\right) + \langle \chi_+ \rangle \exp\left(\frac{y\sigma}{f_\pi}\right)\right]$$

- Here $y = 3 - \gamma^*$ with γ^* the anomalous dimension of the fermion mass

Dilaton limit

- Introduce the dilaton as the conformal compensator

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\left[\langle u_\mu u^\mu\rangle \exp\left(\frac{2\sigma}{f_\pi}\right) + \langle \chi_+\rangle \exp\left(\frac{y\sigma}{f_\pi}\right)\right]$$

- Here $y = 3 - \gamma^*$ with γ^* the anomalous dimension of the fermion mass
- Expanding the exponentials lead to the matching

$$S_1 = S_2 = 2 \quad , \quad S_3 = y \quad , \quad S_4 = \frac{y^2}{2}$$

Dilaton limit

- Very close to the conformal window one expects $\gamma^* \approx 1$

$$S_1 = S_2 = S_3 = S_4 = 2$$

- In this limit the results simplify considerably, e.g.

$$\hat{m}_\pi^2 = m_\pi^2 + \frac{m_\pi^4}{f_\pi^2}\left(b_M + (a_M - 2)L_\pi\right) + \frac{m_\sigma^4}{f_\pi^2}\left(L_\sigma - J_{\pi\sigma\pi}\right) + \frac{m_\pi^2 m_\sigma^2}{f_\pi^2}\left(a_6 - L_\pi\right)$$

Other limits

- **Linear sigma model** without explicit breaking term

$$f_\pi = v \quad , \quad S_1 = 2 \quad , \quad S_2 = 1 \quad , \quad m_\sigma^2 = 2\lambda v^2$$

- **Goldstone boson**
  - Scalar is invariant under at least a shift symmetry
  - Only derivative couplings are allowed
  - One can allow for a controllably small breaking by choosing

$$S_i \ll \mathcal{O}(1)$$

Conclusions

- Simple extension of ChPT to account for a dynamical scalar
- Calculated one-loop corrections to the pion mass and decay constant and the scalar mass
- Valid for different patterns of chiral symmetry breaking
- Generic approach that allows for different limits, corresponding to different realisations of the scalar

Thank you!