ChPT with an Isosinglet Scalar
A phenomenological extension

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Overview

- Why should we care about the scalar?
- The chiral Lagrangian
- The scalar extension
- One-loop results
- The different natures of the scalar

MH, K. Langæble, F. Sannino,
Extending chiral perturbation theory with an isosinglet scalar
Energy scales
Scales with 2 flavours

QCD

\[ \rho \]
\[ \sigma \]
\[ \pi \]
\[ 0 \]
Scales with 2 flavours

QCD

Region of validity
Scales with 2 flavours

QCD

\[ \rho \]
\[ \sigma \]
\[ \pi \]
\[ 0 \]

Region of validity

Near-conformal

\[ \rho \]
\[ \sigma \]
\[ \pi \]
\[ 0 \]
**Scales with 2 flavours**

**QCD**

- Region of validity

**Near-conformal**

- Nearly generate states
QCD

Region of validity

Kaons and $f_0(500)$ almost degenerate

Scales with 3 flavours
Lessons

- In some BSM models, the pseudoscalar and scalar is almost degenerate (according to lattice investigations).
- In 3-flavour QCD, the Kaons are almost degenerate with the scalar.
Lessons

- In some BSM models, the pseudoscalar and scalar is almost degenerate (according to lattice investigations)
- In 3-flavour QCD, the Kaons are almost degenerate with the scalar

Consequently, in some cases the scalar should probably not be ignored
ChPT with scalar
Approach

- Adopt a counting scheme

\[ \mathcal{O}(m^2_\pi) \sim \mathcal{O}(m^2_\sigma) \sim \mathcal{O}(p^2) \]

- Valid in some intermediate range of quark masses since

\[ m^2_\pi = A m_q , \quad m^2_\sigma = m_0 + B m_q \]

- Sufficiently close to the chiral limit, the scalar should be integrated out because \( m_\sigma \gg m_\pi \)

- Obviously, the construction only works in models with spontaneous chiral symmetry breaking
Non-linearly realized Lagrangian

Let $G$ be the flavor symmetry and let $H$ be the stability group. The Goldstone boson manifold $G/H$ is then parametrized by

$$u = \exp \left( \frac{i}{\sqrt{2} f_\pi} X^a \phi^a \right)$$

Invariants

$$u_\mu = i(u^\dagger (\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger)$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$
The chiral Lagrangian reads

\[ \mathcal{L}_2 = \frac{f_\pi^2}{4} \langle u_\mu u_\mu + \tilde{\chi}_+ \rangle \]

\[ \mathcal{L}_4 = L_0 \langle u_\mu u_\nu u_\mu u_\nu \rangle + L_1 \langle u_\mu u_\mu \rangle \langle u_\nu u_\nu \rangle + L_2 \langle u_\mu u_\nu \rangle \langle u_\nu u_\nu \rangle + \cdots \]

Many low-energy constants (unknown beyond LO)

\[ B_0 , \quad f_\pi , \quad L_0 , \quad L_1 , \quad \cdots \]
The LECs can be divided in contributions from various pieces, such as heavier resonances (as done in VMD)

\[ L_i = \hat{L}_i + \sum_R L^R_i \]

If the scalar is not considered heavy, there is a dynamical (non-constant) contribution to the LECs
The chiral Lagrangian

- Introduce the scalar as a non-trivial background field

\[ \mathcal{L}_2 = \frac{f_\pi^2}{4} \left[ 1 + S_1 \left( \frac{\sigma}{f_\pi} \right) + S_2 \left( \frac{\sigma}{f_\pi} \right)^2 + \cdots \right] \langle u_\mu u^\mu \rangle \]

\[ + \frac{f_\pi^2}{4} \left[ 1 + S_3 \left( \frac{\sigma}{f_\pi} \right) + S_4 \left( \frac{\sigma}{f_\pi} \right)^2 + \cdots \right] \langle \tilde{\chi}_+ \rangle \]

- Expansion of LO Lagrangian is sufficient at NLO
- Four additional low-energy constants!
- New counter terms are also needed
The scalar Lagrangian

- Standard $\varphi^4$ theory

$$\mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \left[ 1 + S_5 \left( \frac{\sigma}{f_\pi} \right) + S_6 \left( \frac{\sigma}{f_\pi} \right)^2 \right]$$

- Expansion of kinetic term unnecessary for on-shell quantities (related via EOM)

- Assume vanishing expectation value for scalar field

$$S_5 \geq -2 \sqrt{S_6} \quad , \quad S_6 \geq 0$$
Pion self-energy

- Standard diagrams at NLO

- New diagrams
Pion decay constant

- Standard diagrams at NLO
- New diagrams
Scalar self-energy

- Standard diagrams

- New diagrams
Pion mass at NLO

Polynomial

\[ \hat{m}_\pi^2 = m_\pi^2 + \frac{m_\pi^4}{f_\pi^2} \left( a_1 + a_2 L_\pi + a_3 J_{\pi\sigma\pi} \right) \]

\[ + \frac{m_\sigma^4}{f_\pi^2} \left( a_4 L_\sigma + a_5 J_{\pi\sigma\pi} \right) \]

\[ + \frac{m_\pi^2 m_\sigma^2}{f_\pi^2} \left( a_6 + a_7 L_\pi + a_8 L_\sigma + a_9 J_{\pi\sigma\pi} \right) \]

Logarithms

Unitarity corrections
Pion mass at NLO

- Coefficients are largely independent of the pattern of chiral symmetry breaking

<table>
<thead>
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<th>(i)</th>
<th>(a_i)</th>
<th>(b_i)</th>
<th>(c_i)</th>
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<tr>
<td>1</td>
<td>(b_M)</td>
<td>(b_F)</td>
<td>(6S_6)</td>
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<td>2</td>
<td>(a_M - \frac{1}{2}(2S_1S_3 - S_1^2))</td>
<td>(a_F - \frac{3}{8}S_1^2)</td>
<td>(-\frac{1}{4}n_\pi S_1^2)</td>
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<td>3</td>
<td>(- (S_1 - S_3)^2)</td>
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<td>(-9S_5^2)</td>
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<tr>
<td>4</td>
<td>(\frac{1}{4}S_1^2)</td>
<td>(-2K'_r)</td>
<td>(2n_\pi(S_1S_3 - S_1^2) + n_\pi(S_4 - S_2))</td>
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<tr>
<td>5</td>
<td>(-a_4)</td>
<td>(\frac{1}{8}(12S_2 + S_1^2))</td>
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<tr>
<td>6</td>
<td>(4(K'_r - K'_r))</td>
<td>(-\frac{1}{4}S_1^2)</td>
<td>(\frac{1}{2}n_\pi S_1^2)</td>
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<tr>
<td>7</td>
<td>(-a_4)</td>
<td>(-\frac{1}{2}(S_1 - S_3)^2)</td>
<td>(n_\pi(S_1^2 - S_1S_3))</td>
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<tr>
<td>8</td>
<td>(S_4 - S_2 - S_1^2 + S_1S_3)</td>
<td>(-\frac{1}{8}S_1^2)</td>
<td>(-)</td>
</tr>
<tr>
<td>9</td>
<td>(S_1^2 - S_1S_3)</td>
<td>(b_3)</td>
<td>(-)</td>
</tr>
</tbody>
</table>
Pion decay constant at NLO

Polynomial

\[
\hat{f}_\pi = f_\pi + \frac{m^2_\pi}{f_\pi} (b_1 + b_2 L_\pi + b_3 J_{\pi\sigma\pi}) \\
+ \frac{m^2_\sigma}{f_\pi} (b_4 + b_5 L_\sigma + b_6 J_{\pi\sigma\pi}) \\
+ \frac{H_{\pi\sigma\pi}}{f_\pi} (b_7 m^4_\pi + b_8 m^4_\sigma + b_9 m^2_\pi m^2_\sigma).
\]

Logarithms

Unitarity corrections

Wave function renormalisation
Scalar mass at NLO

Logarithms

\[ \hat{m}_\sigma^2 = m_\sigma^2 + \frac{m_\sigma^4}{f_\pi^2} (c_1 L_\sigma + c_2 J_{\pi\pi\sigma} + c_3 J_{\sigma\sigma\sigma} - 2K_3^r) \]

+ \frac{m_\pi^4}{f_\pi^2} (c_4 L_\pi + c_5 J_{\pi\pi\sigma} - 2K_5^r)

+ \frac{m_\pi^2 m_\sigma^2}{f_\pi^2} (c_6 L_\pi + c_7 J_{\pi\pi\sigma} - 2K_4^r),

Unitarity corrections

Polynomial part from renormalisation / counter terms
Scalar decay width at NLO

- Decay channel $\sigma \rightarrow \pi \pi$

\[
\Gamma = -\frac{c_2 m_\sigma^4 + c_5 m_\pi^4 + c_7 m_\sigma^2 m_\pi^2}{16\pi m_\sigma f_\pi^2} \sqrt{1 - \frac{4m_\pi^2}{m_\sigma^2}}
\]
Consistency checks

- In the chiral limit: $m^2_\pi \to 0$
- Vanishing pion mass
  
  $$\hat{m}^2_\pi = 0$$
- Constant decay constant
  
  $$\hat{f}_\pi = f_\pi + \frac{m^2_\sigma}{f_\pi} \left( b_4 + (b_5 + b_6) L_\sigma - \frac{b_8}{32\pi^2} \right)$$
  
  = $f_\pi$ in normal ChPT
- Results independent of renormalisation scale
Natures of the scalar
Many low-energy constants in normal ChPT

... even more when including a scalar

Different realisations of the scalar leads to different predictions for the constants $S_{1,2,3,4}$
Natures of the scalar

- Many low-energy constants in normal ChPT
- ... even more when including a scalar

- Different realisations of the scalar leads to different predictions for the constants $S_{1,2,3,4}$

- No guarantee that the counting scheme is appropriate for the various conceivable limits!
Dilaton limit

- Introduce the dilaton as the conformal compensator

\[ \mathcal{L}_2 = \frac{f_\pi^2}{4} \left[ \langle u_\mu u^\mu \rangle \exp \left( \frac{2\sigma}{f_\pi} \right) + \langle \chi_+ \rangle \exp \left( \frac{y\sigma}{f_\pi} \right) \right] \]

- Here \( y = 3 - \gamma^* \) with \( \gamma^* \) the anomalous dimension of the fermion mass
Dilaton limit

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- Expanding the exponentials lead to the matching

\[ S_1 = S_2 = 2 \quad , \quad S_3 = y \quad , \quad S_4 = \frac{y^2}{2} \]
Dilaton limit

- Very close to the conformal window one expects $\gamma^* \approx 1$

\[ S_1 = S_2 = S_3 = S_4 = 2 \]

- In this limit the results simplify considerably, e.g.

\[
\hat{m}_\pi^2 = m_\pi^2 + \frac{m_\pi^4}{f_\pi^2} \left( b_M + (a_M - 2)L_\pi \right) \\
+ \frac{m_\sigma^4}{f_\pi^2} \left( L_\sigma - J_{\pi\pi} \right) + \frac{m_\pi^2 m_\sigma^2}{f_\pi^2} \left( a_6 - L_\pi \right)
\]
Other limits

- **Linear sigma model** without explicit breaking term
  
  \[ f_\pi = v, \quad S_1 = 2, \quad S_2 = 1, \quad m^2_\sigma = 2\lambda v^2 \]

- **Goldstone boson**
  
  - Scalar is invariant under at least a shift symmetry
  - Only derivative couplings are allowed
  - One can allow for a controllably small breaking by choosing
    
    \[ S_i \ll \mathcal{O}(1) \]
Conclusions

- Simple extension of ChPT to account for a dynamical scalar
- Calculated one-loop corrections to the pion mass and decay constant and the scalar mass
- Valid for different patterns of chiral symmetry breaking
- Generic approach that allows for different limits, corresponding to different realisations of the scalar
Thank you!