(Color-)magnetic flux tubes in dense matter

A. Haber, A. Schmitt, PRD 95, 116016 (2017); EPJ Web Conf. 137, 09003 (2017)
A. Haber, A. Schmitt, JPG 45, 065001 (2018)

- two-component superconductors: unconventional type-I/type-II behavior (flux tube clusters)
- flux tubes and domain walls in superconducting quark matter
Reminder: type-I/type-II superconductivity

- **Ginzburg-Landau parameter**
  \[ \kappa = \frac{\lambda}{\xi} \]

- **type-II superconductivity**
  for \( \kappa > 1/\sqrt{2} \): flux tube lattice
  for \( H_{c1} < H < H_{c2} \)

A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)
Two-component superconductors

- Ginzburg-Landau potential for two complex scalar fields with electric charges $q_1, q_2$ (neutron/proton: $q_1 = 2e$, $q_2 = 0$)

\[
U = \frac{B^2}{2} + \sum_{i=1,2} \left[ |(\nabla + iq_i A)\phi_i|^2 - \mu_i^2|\phi_i|^2 + \lambda_i|\phi_i|^4 \right] + 2h|\phi_1|^2|\phi_2|^2
\]

- further extensions:
  - derivative coupling ("entainment")
    M. G. Alford, G. Good, PRB 78, 024510 (2008)
    A. Haber, A. Schmitt, PRD 95, 116016 (2017)
  - color superconductor: 3 scalar fields $\phi_1, \phi_2, \phi_3$ and 3 gauge fields
    A. Haber, A. Schmitt, JPG 45, 065001 (2018)
Two-component superconductors in neutron star cores

- density-dependent $\kappa$
- type-I/type-II transition in the core?
- effect of coupling to superfluid on type-I/type-II transition?
Critical magnetic fields in a two-component system

- compute flux tube profiles and flux tube interaction
  → attractive long-distance interaction in type-II regime

<table>
<thead>
<tr>
<th>isolated superconductor</th>
<th>superconductor coupled to a superfluid</th>
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<tbody>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
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<tr>
<td>flux tubes</td>
<td>flux tubes</td>
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<tr>
<td>$H_{c1}$</td>
<td>$H_{c1}'$</td>
</tr>
<tr>
<td>$H_{c2}$</td>
<td>$H_{c2}'$</td>
</tr>
<tr>
<td>$H_c$</td>
<td>$H_c$</td>
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</table>

small-$v$ approximation
(our calculation)

complete solution
(conjecture)

numerical calculation supports conjecture
A. Haber, D. Müller, preliminary results

- first-order phase transition allows for flux tube clusters
  see also "type-1.5" superconductors J. Carlström, J. Garaud, E. Babaev, PRB 84, 134515 (2011)
Quark matter in a magnetic field

- color-superconducting quark matter in a magnetic field
- use perturbative results for Ginzburg-Landau parameters $\mu, \lambda, h$

<table>
<thead>
<tr>
<th>$\lambda^{1/2}$</th>
<th>$\mu^2$</th>
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</thead>
<tbody>
<tr>
<td>unpaired</td>
<td>2SC</td>
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<tr>
<td>$h/\lambda = -0.5$</td>
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</tbody>
</table>

Color-flavor locked (CFL) phase

- $\Phi_1 = \Phi_2 = \Phi_3$

2SC phase

- $\Phi_1 = \Phi_2 = 0$
- $\Phi_3 \neq 0$

Diagram:

- Unpaired and paired quark states
- Color-flavor locked phase
- 2SC phase

Graph:

- $H [\mu^2/\lambda^{1/2}]$
- $\Phi_1 = \Phi_2 = \Phi_3$
- $\Phi_1 = \Phi_2 = 0$
- $\Phi_3 \neq 0$
<table>
<thead>
<tr>
<th>CFL line defects</th>
<th>$(n_1, n_2, n_3)$</th>
<th>$\Gamma \left[ \pi / 3\mu_q \right]$</th>
<th>$\Phi_3 \left[ \pi / g \right]$</th>
<th>$\tilde{\Phi}_8 \left[ \pi / g_8 \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global vortex</td>
<td>$(n, n, n)$</td>
<td>$-n$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Forbes, Zhitnitsky (2002)</td>
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<tr>
<td>”Semi-superfluid” vortex</td>
<td>$(0, 0, n)$</td>
<td>$\frac{-n}{3}$</td>
<td>0</td>
<td>$\frac{2n}{3}$</td>
</tr>
<tr>
<td>Balachandran, Digal, Matsuura (2006)</td>
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<tr>
<td>Magnetic flux tube $T_{112}$</td>
<td>$(n, n, -2n)$</td>
<td>0</td>
<td>0</td>
<td>$-2n$</td>
</tr>
<tr>
<td>Iida (2005)</td>
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</tr>
<tr>
<td>Magnetic flux tube $T_{101}$</td>
<td>$(n, 0, -n)$</td>
<td>0</td>
<td>$-n$</td>
<td>$-n$</td>
</tr>
<tr>
<td>Haber, Schmitt (2018)</td>
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- winding numbers $n_1$, $n_2$, $n_3$ for three scalar fields
  
  baryon circulation  $\Gamma \propto n_1 + n_2 + n_3$

  (color-)magnetic flux  $\tilde{\Phi}_8 \propto n_1 + n_2 - 2n_3$

- photon/gluon mixing: Meissner effect for $\tilde{B}_8 = B_8 \cos \theta + B \sin \theta$

- magnetic defects in CFL can be superfluid vortices and magnetic flux tubes at the same time
Flux tube profiles

- flux tube with ”unpaired core”
- flux tube with ”2SC core”
  → additional $B_3$ field
  (cost in free energy)
  → non-vanishing condensate in core (gain in free energy)
Phase diagram including flux tubes

- in neutron stars
  \[ \mu_q \simeq 400 \text{ MeV} \Rightarrow g \simeq 3.5 \]
- weak-coupling methods and phenomenological models:
  \[ T_c \simeq (10 - 50) \text{ MeV} \]
- type-II regime for sufficiently large \( T_c/\mu_q \)
- type-I/type-II transition complicated (multi-component structure!)
Phase diagram including flux tubes

- in neutron stars
  \( \mu_q \simeq 400 \text{ MeV} \Rightarrow g \simeq 3.5 \)
- weak-coupling methods and phenomenological models:
  \( T_c \simeq (10 - 50) \text{ MeV} \)

\[
\begin{align*}
\text{CFL} & \quad \text{2SC} \\
H_{c2} & \quad H_{c1}(T_1) \\
H_{c1}(T_{101}) & \quad H_{c1}(T_{112})
\end{align*}
\]

- CFL flux tubes with 2SC core \((T_{101})\) preferred
- critical fields \( H \sim 10^{19} \text{ G} \gg H_{\text{NS}}, \) however:
  creation of flux tubes through cooling into superconducting phase?
- 2SC domain walls \((D)\) preferred over ordinary 2SC flux tubes \((T_1)\)
  for sufficiently large \( T_c/\mu_q \)
Some open questions

Do flux tube clusters exist in (a layer of) a neutron star? Do they affect transport properties/deformability?

What happens if CFL is rotated and placed into a magnetic field? Are there vortices and flux tubes (misaligned), like in neutron/proton matter?

What happens to the flux tubes/vortices at a quark/hadron interface?

Does the (color-)flux tube array sustain magnetic mountains?
Continuous grav. wave emission?
K. Glampedakis et al., PRL 109, 081103 (2012)