



University of Science and  
Technology of Mazandaran

# **First determination of $D^*$ -meson fragmentation functions and their uncertainties at next-to-next-to-leading order**

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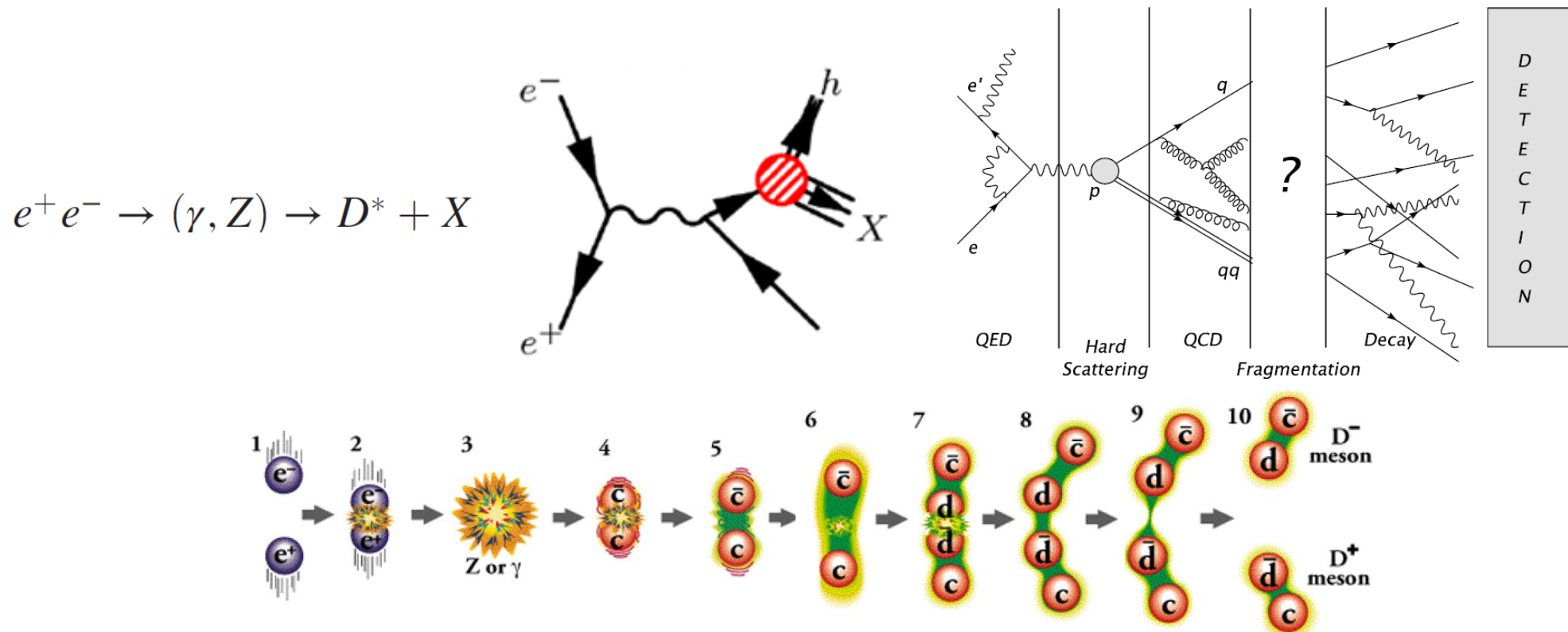
# Outline

- Fragmentation functions (FFs)
- QCD factorization theorem
- **Analysis strategy:**
  - The data sets
  - Fit settings
  - $\chi^2$  Minimizations and FFs uncertainties
- Numerical results for the global analysis of  $D^{*\pm}$  FFs
- Fit quality
- Summary and Conclusions

# Fragmentation functions

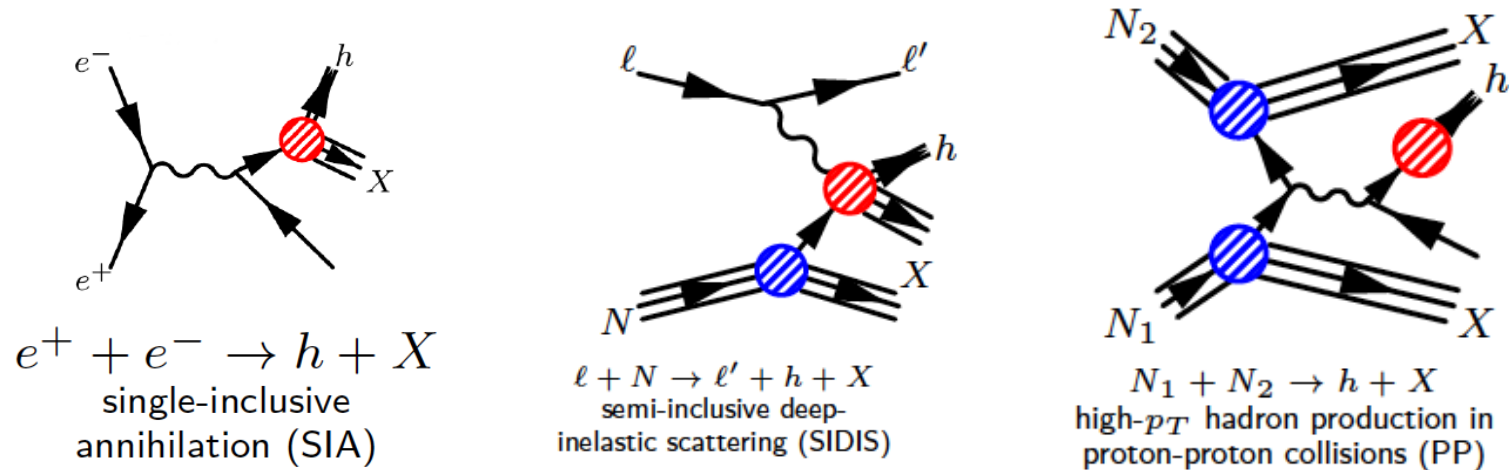
- Fragmentation functions describe the non-perturbative part of hard-scattering processes and along with the PDFs of initial hadrons (in hadron-hadron collision) and parton-level differential cross sections are three necessary ingredients to obtain theoretical predictions for hadroproduction cross sections.
- The process-independent FFs,  $D_i^H(z, \mu_F^2)$ , describe the probability for a parton  $i$  at the factorization scale  $\mu_F$  to fragment into a hadron  $H$  carrying away a fraction  $z$  of its momentum.
- The scaling violations of FFs are subject to the perturbatively computable DGLAP evolution equations.

- The optimal way to determine the  $D^{*\pm}$  FFs is to fit them to the experimental data extracted from the single-inclusive  $e^-e^+$  annihilation processes.



# QCD factorization theorem

- According to the factorization theorem, the differential cross section of process can be written as a convolutions of perturbatively calculable partonic cross sections with the FFs,



$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma} \otimes FF,$$

$$\sigma^{\ell N \rightarrow \ell h X} = \hat{\sigma} \otimes PDF \otimes FF,$$

$$\sigma^{pp \rightarrow hX} = \hat{\sigma} \otimes PDF \otimes PDF \otimes FF$$

- For the  $D^{*\pm}$  production in annihilations, the factorization theorem reads

$$\frac{1}{\sigma_{\text{tot}}} \frac{d}{dx_D} \sigma(e^+ e^- \rightarrow D^* X)$$

$$= \sum_i \int_{x_D}^1 \frac{dx_i}{x_i} D_i^{D^*} \left( \frac{x_D}{x_i}, \mu_F \right) \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_i}{dx_i} (x_i, \mu_R, \mu_F)$$

$D^{*\pm}$ -meson FFs Perturbatively calculable partonic cross sections

- The total cross section up to NNLO for  $e^+ e^-$  annihilation into hadrons

$$\sigma_{\text{tot}} = \frac{4\pi\alpha^2(Q)}{Q^2} \left( \sum_i^{n_f} \tilde{e}_i^2(Q) \right)$$

$$\times \left( 1 + \alpha_s K_{\text{QCD}}^{(1)} + \alpha_s^2 K_{\text{QCD}}^{(2)} + \dots \right)$$

# APFEL

- APFEL is a public PDF evolution library with QED corrections.
- APFEL evolution features:
  - Up to NNLO in QCD and LO in QED.
  - The time-like evolution for fragmentation functions in  $x$ -space.
  - FFNS and VFNS solution in  $x$ -space.
- The DIS module: Computation of DIS observables, Up to order  $\alpha_S^2$  (when possible), Heavy quark schemes: FONLL, FFNS and ZM-VFNS.



<http://apfel.hepforge.org>

# Available $D^{*\pm}$ FFs analysis

- **SKM18:** Maryam Soleymaninia, Hamzeh Khanpour, S. Mohammad Moosavi Nejad, First determination of  $D^{*\pm}$ -meson fragmentation functions and their uncertainties at next-to-next-to-leading order, Phys. Rev. D **97** (2018) no.7, 074014.

SIA data sets (ALEPH and OPAL), ZM-VFNS, NLO and NNLO accuracy

- **AKSRV17:** Daniele P. Anderle, Tom Kaufmann, Marco Stratmann, Felix Ringer, Ivan Vitev, Using hadron-in-jet data in a global analysis of  $D^*$  fragmentation functions, Phys.Rev. D96 (2017) no.3, 034028.

SIA data sets + hadron-hadron + jet fragmentaion in pp scattering, ZM-VFNS, NLO accuracy

- **KKKS08:** T. Kneesch, B.A. Kniehl, G. Kramer, I. Schienbein, Charmed-meson fragmentation functions with finite-mass corrections, Nucl.Phys. B799 (2008) 34-59.

SIA data sets (ALEPH, OPAL, CLEO and Belle), GM-VFNS, NLO accuracy



# Analysis strategy and results

Phys. Rev. D **97**, no. 7, 074014 (2018)

arXiv:1711.11344 [hep-ph]

# The data sets

- Most of experimental data for  $D^{*\pm}$  in  $e^+e^-$ -annihilation is reported by ALEPH, OPAL, CLEO and Belle Collaborations.
- We use the  $c$ -tagged and  $b$ -tagged SIA cross sections from ALEPH and OPAL Collaborations.

ALEPH: Eur. Phys. J. C 16, 597 (2000).

OPAL: Eur. Phys. J. C 1, 439 (1998).

Collaboration	Data properties	$\sqrt{s}$ GeV	Data points	$\mathcal{N}_i$	$\chi^2(\text{NLO})$	Collaboration	Data properties	$\sqrt{s}$ GeV	Data points	$\mathcal{N}_i$	$\chi^2(\text{NNLO})$
ALEPH	Inclusive	91.2	17	0.999 006	24.59	ALEPH	Inclusive	91.2	17	0.998 900	24.51
	$b$ -tagged	91.2	15	1.001 04	18.73		$b$ -tagged	91.2	15	1.000 990	17.99
OPAL	Inclusive	91.2	9	0.999 305	2.02	OPAL	Inclusive	91.2	9	0.999 099	1.92
	$b$ -tagged	91.2	9	0.999 672	8.01		$b$ -tagged	91.2	9	0.999 700	7.61
	$c$ -tagged	91.2	9	1.002 758	17.39		$c$ -tagged	91.2	9	1.002 699	16.94
<b>TOTAL:</b>			59		70.74	<b>TOTAL:</b>			59		68.97
<b>(<math>\chi^2/\text{d.o.f}</math>)</b>					1.31	<b>(<math>\chi^2/\text{d.o.f}</math>)</b>					1.27

# Fit settings

- We parametrize the  $z$  distributions of the  $c(\bar{c})$  and  $b(\bar{b})$  quark FFs at their starting scales  $\mu_0^2 = 18.5 \text{ GeV}^2$  as,

$$D_i^{D^{*\pm}}(z, \mu_0^2) = N_i z^{-(1+\alpha_i^2)} (1-z)^{\beta_i} e^{-\alpha_i^2/z}$$

- The FFs of gluon and light quarks are set to zero, i.e.

$$D_i^{D^{*\pm}}(z, \mu_0^2) = 0, \quad i = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

- According to the parton structure of  $D^{*-}$ , the FFs of  $D^{*-}$  can be obtained as

$$D_q^{D^{*-}}(z, \mu^2) = D_{\bar{q}}^{D^{*+}}(z, \mu^2) \quad D_g^{D^{*-}}(z, \mu^2) = D_g^{D^{*+}}(z, \mu^2)$$

# $\chi^2$ Minimizations and FFs uncertainties

- In our analysis, the total  $\chi^2$  is calculated in comparison with the experimental data for  $D^{*\pm}$  production in  $e^+e^-$  annihilation:

$$\chi_n^2(\{\eta_i\}) = \left(\frac{1 - \mathcal{N}_n}{\Delta \mathcal{N}_n}\right)^2 + \sum_{k=1}^{N_n^{\text{data}}} \left(\frac{(\mathcal{N}_n \mathcal{O}_k^{\text{data}} - T_k^{\text{theory}}(\{\eta_i\}))}{\mathcal{N}_n \delta D_k^{\text{data}}}\right)^2$$

- The *Hessian method* gives the uncertainties of a given observable  $\mathcal{O}$ :

$$[\Delta \mathcal{O}_i]^2 = \Delta \chi^2 \sum_{j,k} \left(\frac{\partial \mathcal{O}_i(\eta)}{\partial \eta_j}\right)_{\hat{\eta}} C_{j,k} \left(\frac{\partial \mathcal{O}_i(\eta)}{\partial \eta_k}\right)_{\hat{\eta}}$$

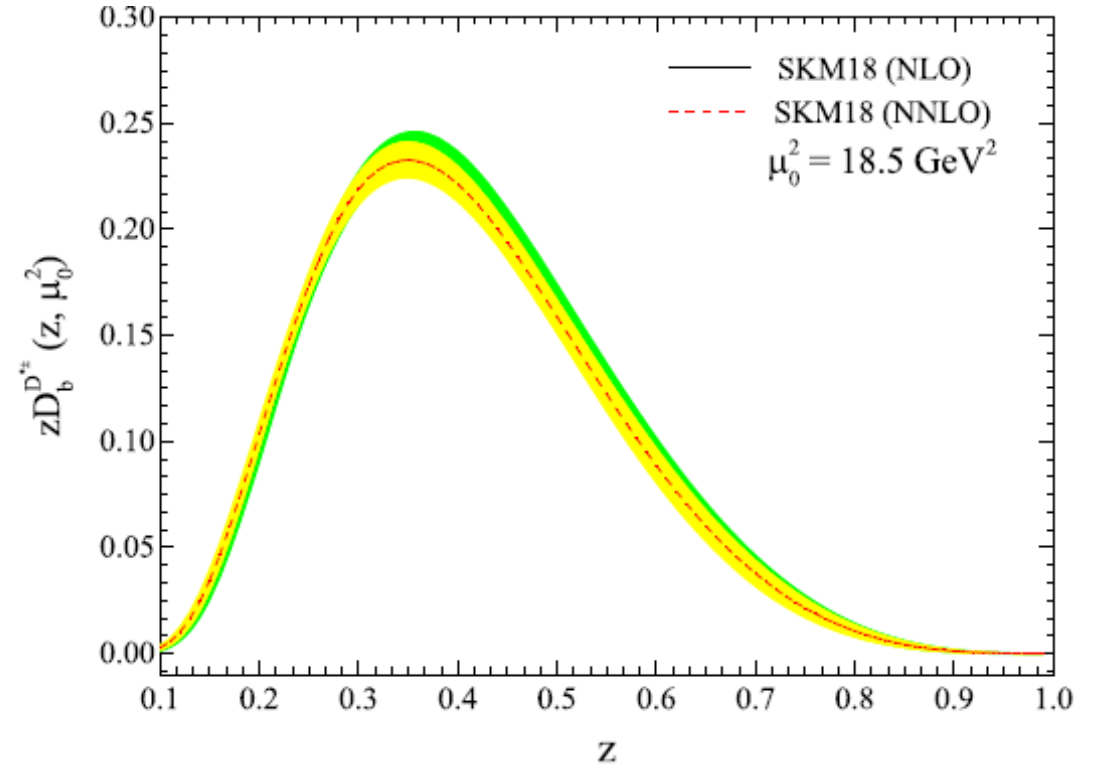
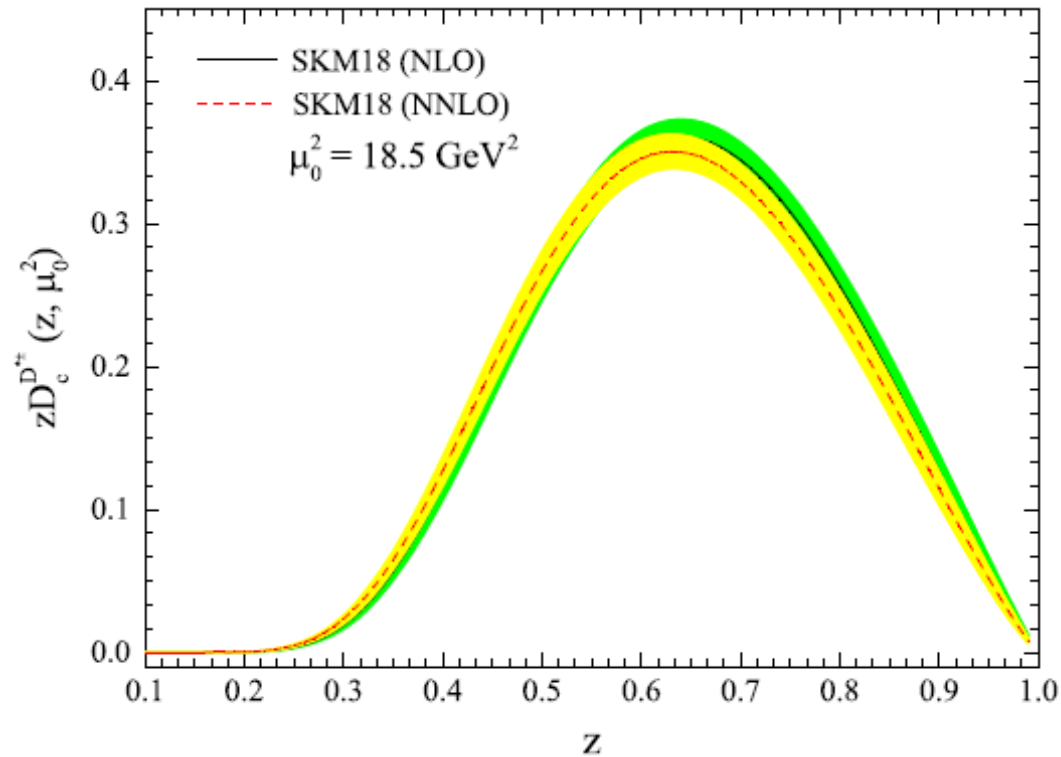
- The Hessian matrix is accessible by running the CERN program library MINUIT

F. James, Report No. CERN-D-506.

# Numerical results for the global analysis of $D^{*\pm}$ FFs

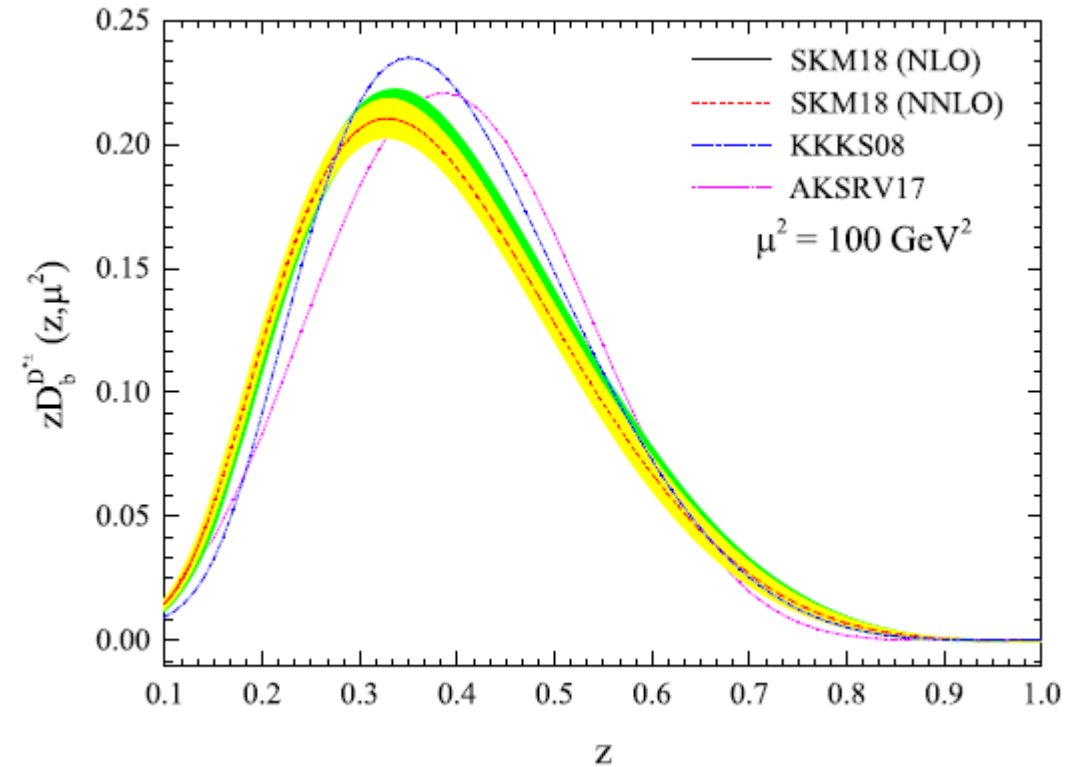
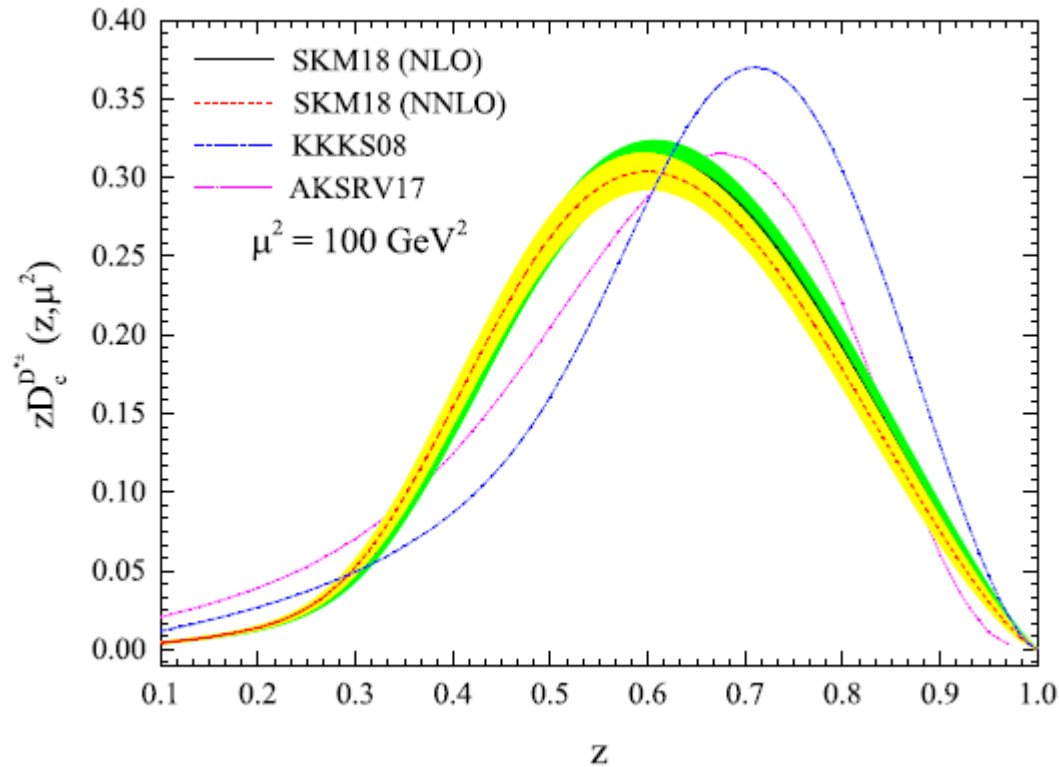
	Flavor $i$	$N_i$	$\alpha_i$	$\beta_i$
NLO	$c, \bar{c}$	67.031*	$1.908 \pm 0.019\ 4$	$1.133 \pm 0.070$
	$b, \bar{b}$	$5.742 \pm 1.574$	$0.994 \pm 0.038\ 5$	$3.249 \pm 0.279$
	Flavor $i$	$N_i$	$\alpha_i$	$\beta_i$
NNLO	$c, \bar{c}$	53.896*	$1.854 \pm 0.019\ 1$	$1.170 \pm 0.069$
	$b, \bar{b}$	$5.127 \pm 1.351$	$0.967 \pm 0.037\ 2$	$3.248 \pm 0.274$

# D\*-meson FFs at NLO and NNLO



SKM18 fragmentation densities and their uncertainties (shaded bands) are shown at the initial scale  $\mu_0^2 = 18.5 \text{ GeV}^2$  for c and b both at NLO (solid lines) and NNLO (dashed lines).

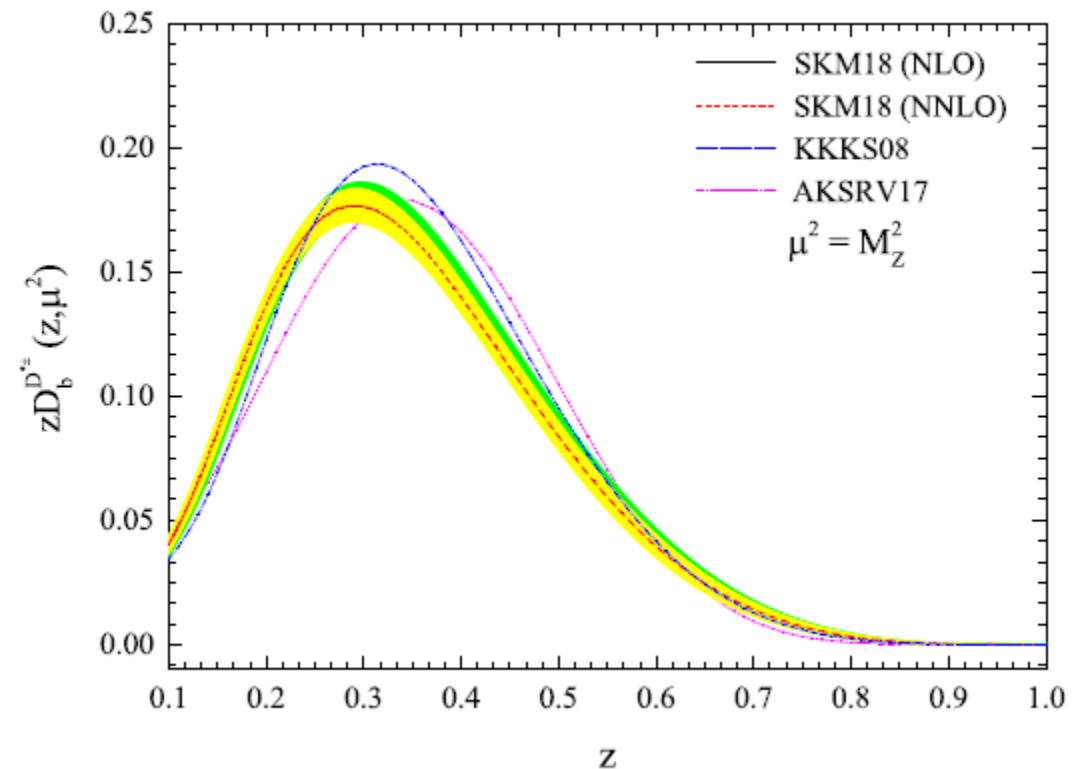
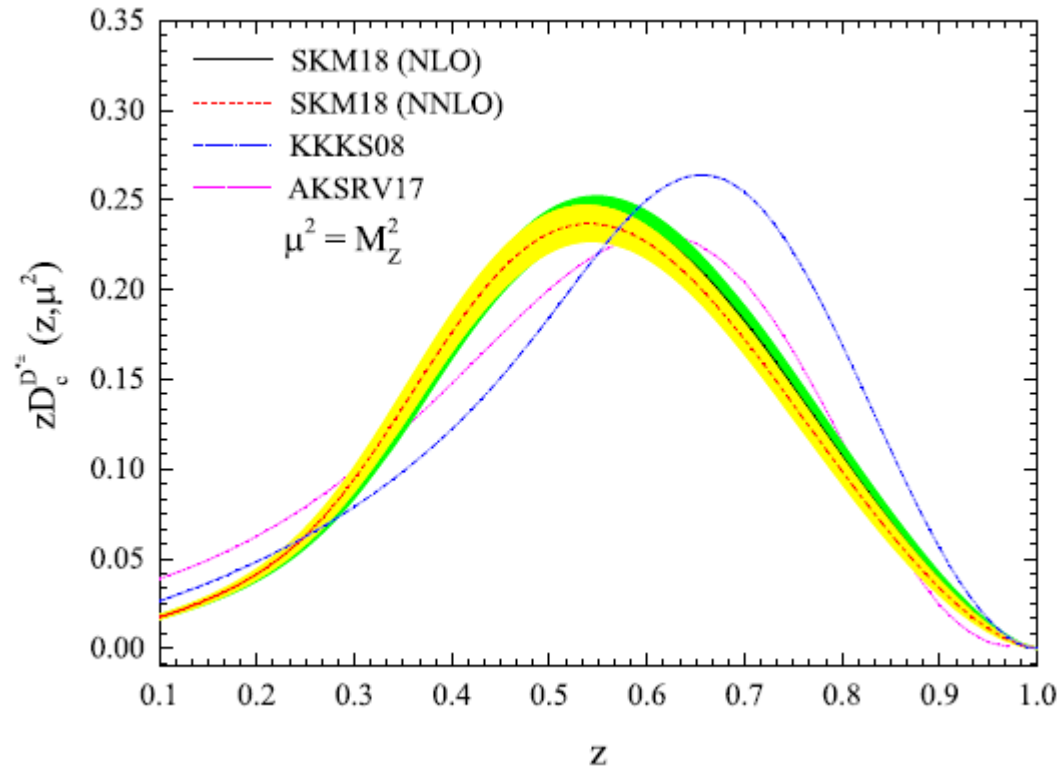
# Comparison with other QCD analyses



Fragmentation densities and their uncertainties (shaded bands) are shown  $\mu^2 = 100 \text{ GeV}^2$  for  $c$  and  $b$  both at NLO (solid lines) and NNLO (dashed lines). Our results are also compared with the KKKS08 (dot-dashed lines) and the AKSRV17 (short dashed lines) results at NLO.

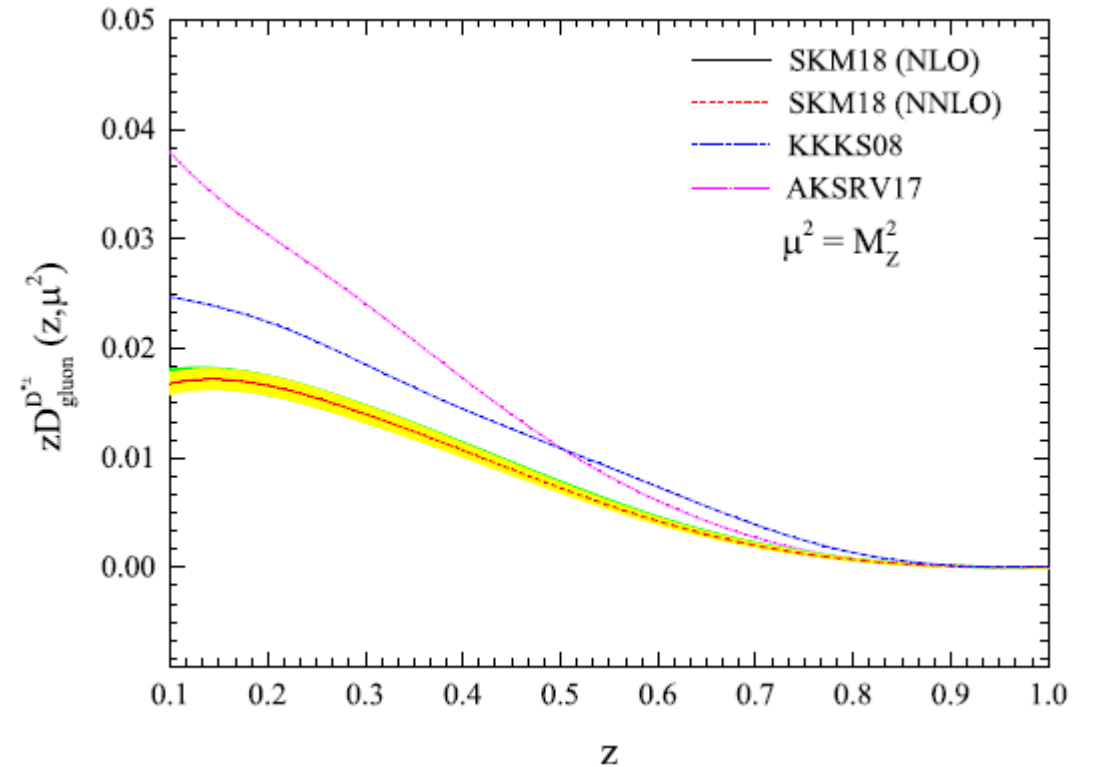
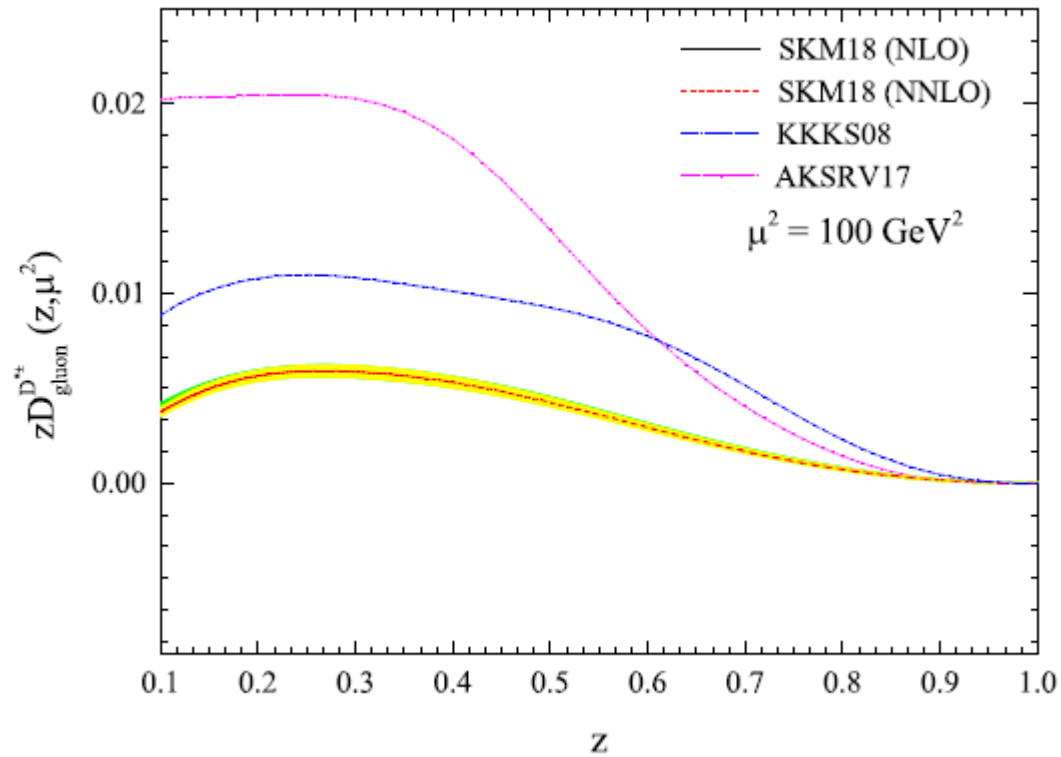
[D. P. Anderle, T. Kaufmann, M. Stratmann, F. Ringer, and I. Vitev, Phys. Rev. D 96, 034028 \(2017\).](#)

[T. Kneesch, B. A. Kniehl, G. Kramer, and I. Schienbein, Nucl. Phys. B799, 34 \(2008\).](#)



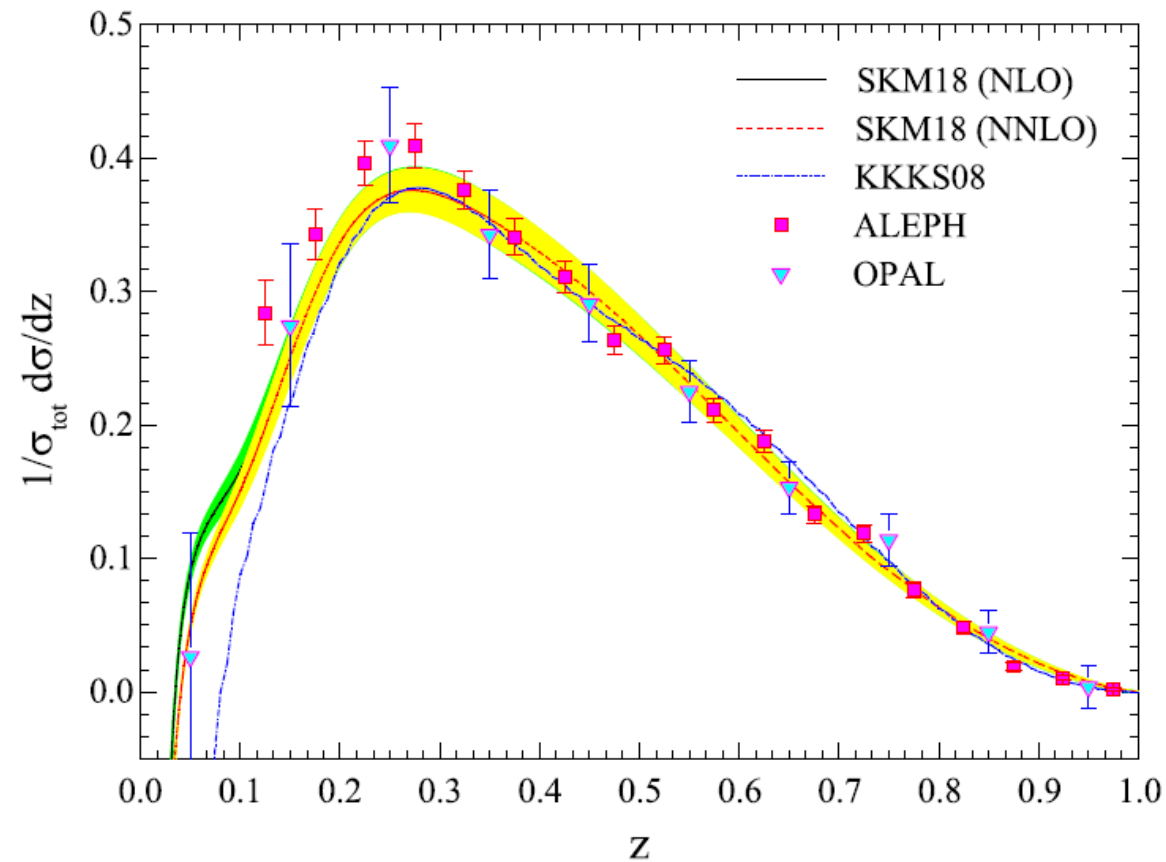
Fragmentation densities and their uncertainties (shaded bands) are shown  $\mu^2 = M_Z^2$  for  $c$  and  $b$  both at NLO (solid lines) and NNLO (dashed lines).



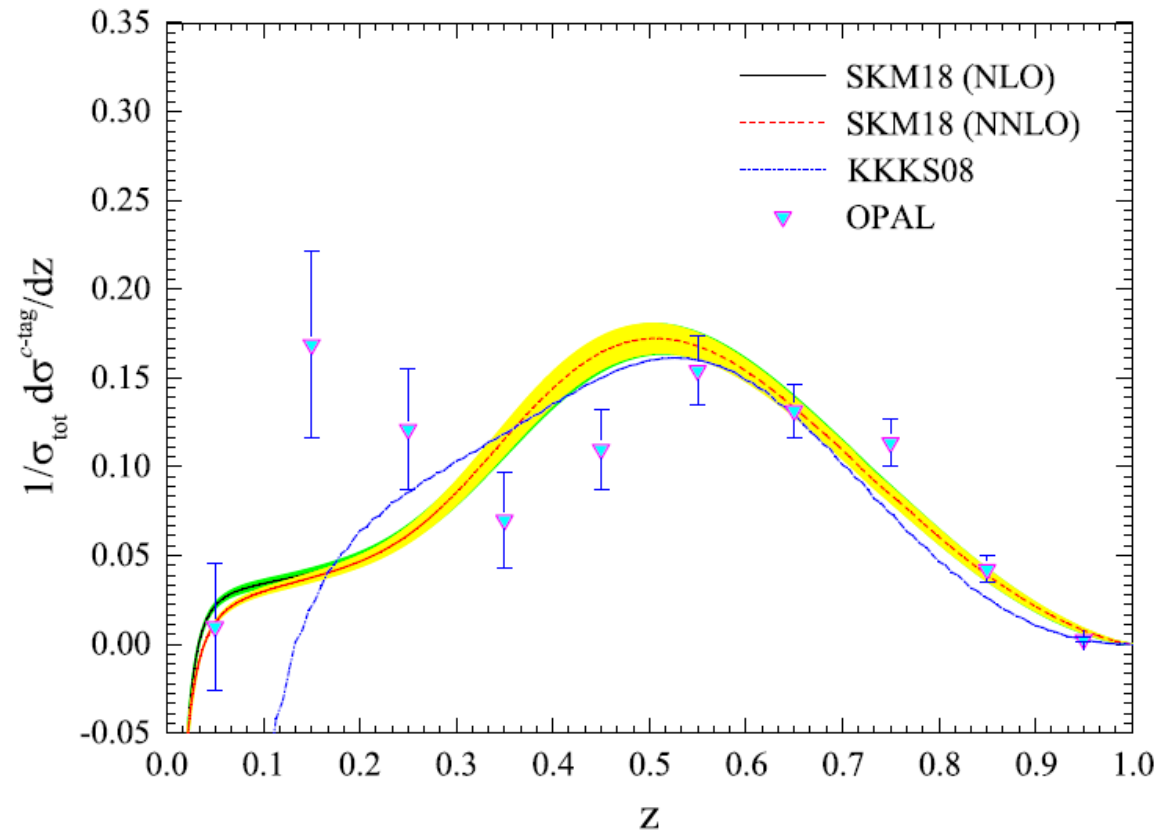


Fragmentation densities and their uncertainties (shaded bands) are shown  $\mu^2 = 100 \text{ GeV}^2$  and  $M_Z^2$  for the gluon densities both at NLO (solid lines) and NNLO (dashed lines).

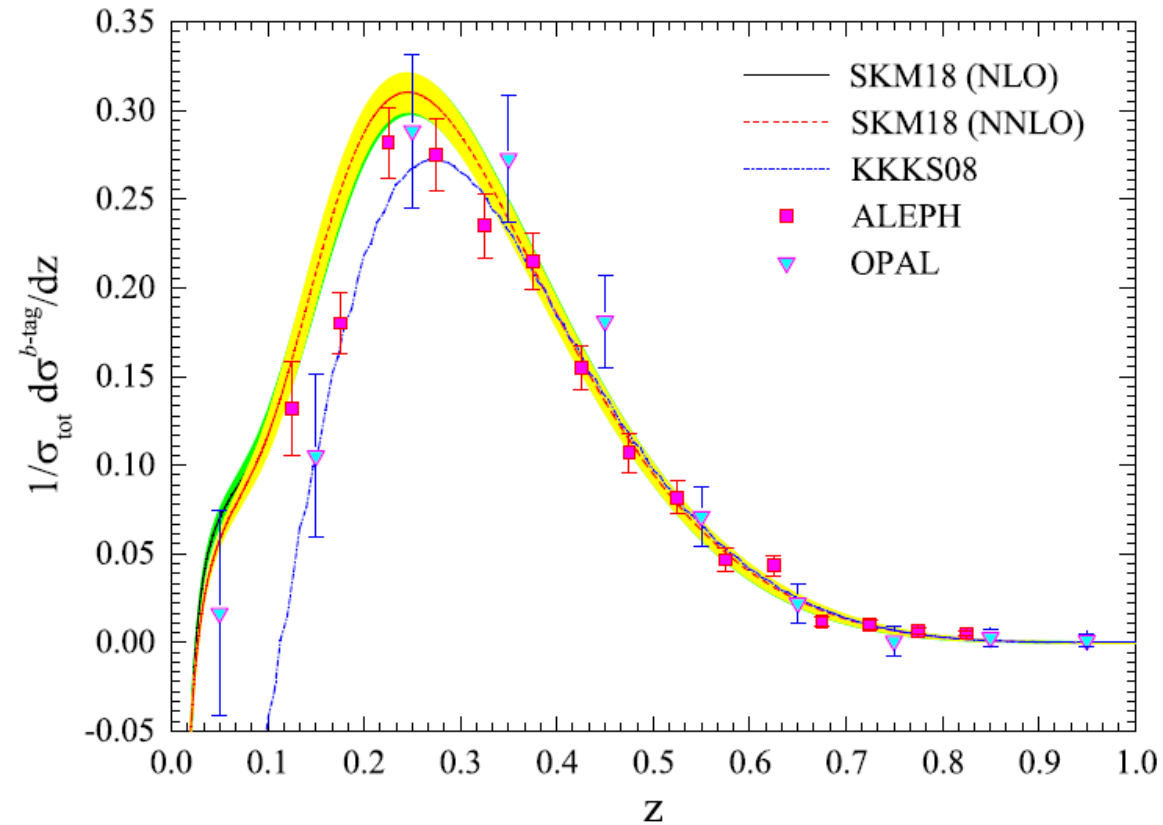
# Fit quality



Our NLO (solid line) and NNLO (dashed line) results for the normalized total cross sections of  $D^{*\pm}$ -production compared with the KKKS08 ones (dot-dashed line) at the scale  $Q = M_Z$ .

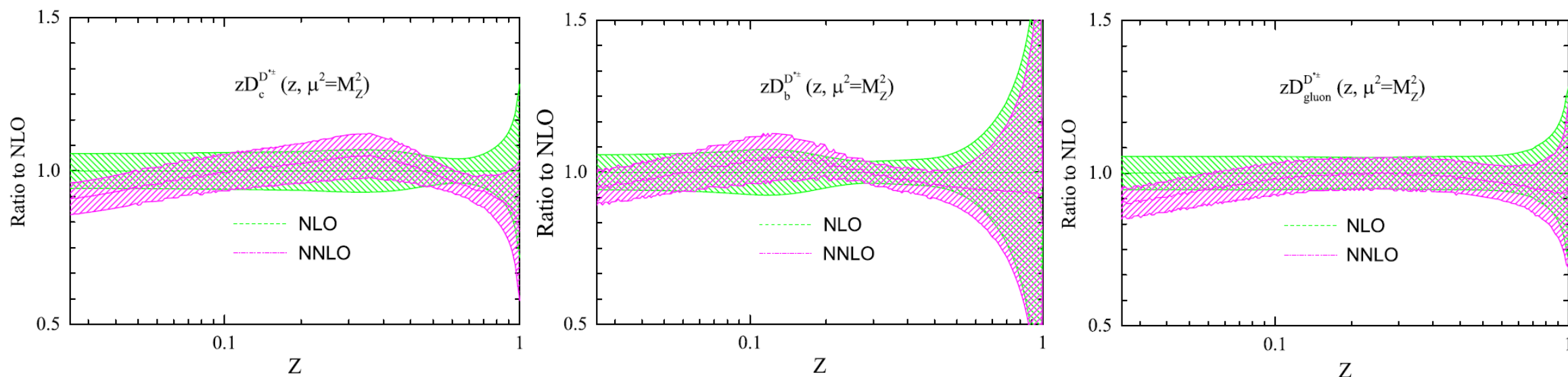


Our NLO (solid line) and NNLO (dashed line) results for the normalized charm-tagged cross sections of  $D^{*\pm}$ -production compared with the KKKS08 ones (dot-dashed line) at the scale  $Q = M_Z$ .



Our NLO (solid line) and NNLO (dashed line) results for the normalized bottom-tagged cross sections of  $D^{*\pm}$ -production compared with the KKKS08 ones (dot-dashed line) at the scale  $Q = M_Z$ .

# Uncertainties: NLO vs NNLO



The experimental uncertainties for the  $D^{*\pm}$ -meson FFs and SIA cross sections are similar in size both for the NLO and NNLO approximations.

# Summary and Conclusion

- We have determined the non-perturbative FFs of partons into the  $D^{*\pm}$ -meson at NLO perturbative QCD and, for the first time, at NNLO one from global analyses of single inclusive electron-positron annihilation.
- Our analyses are based on the ZM-VFN scheme in which all quarks are treated as massless partons.
- We applied all SIA experimental data as much as possible including most of the data from ALEPH and OPAL Collaborations.
- We considered the NNLO accuracy in our global fit using the public APFEL code.
- We found that the experimental uncertainties for the  $D^{*\pm}$ -meson FFs and SIA cross sections are similar in size both for the NLO and NNLO approximations.

*Thank  
you*

