NOVEL APPROACH TO HOLOGRAPHIC COMPOSITE HIGGS MODELS

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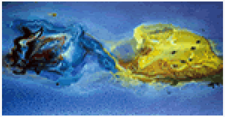
Quark Confinement and Hadron Sprectrum XIII



Maynooth University, Ireland
31 July - 6 August, 2018

Basic features of Composite Higgs models

I Symmetry breaking pattern: *D.B. Kaplan, H. Georgi, 1980's*

- a global symmetry group $\mathscr{G}$ is broken down to a subgroup $\mathscr{H}$;
- $\mathscr{H}$ necessarily contains $SU(2) \times U(1)$;
- the SM gauge group itself $(SU(2)' \times U(1)')$ lies in $\mathscr{H}'$;
- $\mathscr{H}'$ is rotated with respect to $\mathscr{H}$ by a certain angle θ $\hookrightarrow$ **misalignment**;

II Natural hierarchy:

- scale of the global symmetry breaking $\Lambda_{UV} = 4\pi F$;
- Fermi scale $\Lambda_{IR} = 4\pi v$, $v = F \sin\theta$;
- large scale separation $F >> v$ is possible but may demand some fine-tuning to keep the light states in the low energy part of the spectrum;

III The QCD-like strong dynamics doesn't need to be specified;

IV Higgs is associated with a Nambu-Goldstone Boson from the coset space $\mathscr{G}/\mathscr{H}$.

Minimal Composite Higgs model: *K. Agashe, R. Contino, A. Pomarol (2005)*

$$SO(5) \to SO(4) \simeq SU(2) \times SU(2)$$

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Strongly interacting and electroweak sectors

$$\mathscr{L} = \widetilde{\mathscr{L}}_{str.int.} + \mathscr{L}_{SM} + \widetilde{J}^{a_L\,\mu} W_\mu^{a_L} + \widetilde{J}^{Y\,\mu} B_\mu .$$

The currents of the strongly interacting sector $\widetilde{J}^{a_L\,\mu}$ and $\widetilde{J}^{Y\,\mu}$:

- are coupled to the SM gauge fields $W_\mu^{a_L}$ and B_μ
- belong to $SU(2)' \times U(1)'$ rotated with respect to the SM group (tildes)
- the misalignment is realized through the rotation of the generators:

$$T^A(\theta) = r(\theta)\, T^A(0)\, r^{-1}(\theta), \text{ with } r(\theta) = \begin{pmatrix} 1_{3\times 3} & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}.$$

Fundamental Lagrangian:
the $SO(5)$ invariant Lagrangian with rank 2 scalar fields ($s \to gsg^{-1}$, $g \in SO(5)$)

$$\mathscr{L}_{str.int.} = \frac{1}{2}\partial_\mu s_{\alpha\beta}\partial^\mu s^\top_{\beta\alpha} - \frac{1}{2} m^2 s_{\alpha\beta} s^\top_{\beta\alpha} + \text{higher order terms}$$

We may define the following composite operators:

A scalar operator, dimension $\Delta = 2$, spin $p = 0$: $\mathscr{O}_S^{\alpha\beta}(x) = s^{\alpha\gamma} s^{\gamma\beta}$;

A vector operator, dimension $\Delta = 3$, spin $p = 1$: $\mathscr{O}_V^{A\,\mu}(x) = i[T^A, s]_{\alpha\beta}\partial^\mu s^\top_{\beta\alpha}$:

$\Rightarrow$ for $A = a_L$: $J_\mu^{a_L} = \frac{g}{\sqrt{2}} \mathscr{O}_\mu^{a_L}(x)$ and for $A = 3_R$: $J_\mu^Y = \frac{g'}{\sqrt{2}} \mathscr{O}_\mu^{3_R}(x)$.

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Bottom-up holography for effective description

Motivated by AdS/CFT: *J.M. Maldacena, Adv. Theor. Math. Phys.* **2***, 231 (1998)*

IIB string theory on $AdS_5 \times S^5$ in low-energy approximation

$\Updownarrow$

$\mathcal{N} = 4$ SYM theory on ∂AdS_5 in $g_{YM} N_c \gg 1$ limit

Bottom-up: *J. Erlich, E. Katz, D.T. Son, M. A. Stephanov (2005), L. Da Rold, A. Pomarol (2005), A. Karch et al. (2006), S. J. Brodsky, G. F. de Teramond (2008)...* many more models for AdS/QCD

5D weakly coupled theories in a background of

$$AdS_5 \quad g_{MN} dx^M dx^N = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - d^2 z), \quad \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1).$$

$\Updownarrow$ generalized correspondence

4D strongly coupled models of interest (in large-'N_c' limit)

Methods: *S.S. Gubser, I.R. Klebanov, A.M. Polyakov (1998); E. Witten (1998)*

- $\mathcal{O}(x)$ in 4D theory $\Leftrightarrow$ $\phi(x,z)$ in 5D dual theory;
- dimension and spin of $\mathcal{O}(x)$ $\Leftrightarrow$ bulk mass $M^2R^2 = (\Delta - p)(\Delta + p - 4)$;
- source $\phi_{\mathcal{O}}(x)$ $\Leftrightarrow$ value on the boundary $\phi(x,\epsilon)$;
- global symmetry in 4D $\Leftrightarrow$ gauge symmetry in 5D ;

$\Uparrow$ enough to construct the effective $S_{5D}[\phi(x,z)]$ and derive its EOM:
- leading at small z mode: the bulk-to-boundary propagator,
- subleading mode: normalizable solutions providing z-profiles for KK decomposition & masses of physical states.

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BOTTOM-UP HOLOGRAPHY FOR EFFECTIVE DESCRIPTION

The point of holographic correspondence:

$$\mathcal{Z}_{4D}[\phi_{\mathcal{O}}] = \mathrm{Exp}\, i S_{5D}^{on-shell}|_{\phi(x,z)\to\phi(x,z=\varepsilon)}$$

$$\mathcal{Z}_{4D}[\phi_{\mathcal{O}}] = \int [\mathcal{D}s] e^{i\int d^4x [\mathcal{L}_{str.int.}(x) + \phi^A_{\mathcal{O}\mu}(x)\,\mathrm{Tr}\,\partial^\mu s[iT^A, s](x) + \phi^{\alpha\beta}_{\mathcal{O}}(x) s_{\beta\gamma} s_{\gamma\alpha}(x)]} =$$

$$= \mathrm{Exp} \sum_q \frac{1}{q!} \int \prod_{k=1}^{q} d^4x_k \langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_q(x_q)\rangle\, i\phi^1_{\mathcal{O}}(x_1)\ldots i\phi^q_{\mathcal{O}}(x_q)$$

$\Downarrow$

Variation of S_{5D} with respect to $\phi_{\mathcal{O}}$ gives n-pt correlation functions

$\Downarrow$

various phenomenological implications

$\Downarrow$ in particular

vacuum polarization amplitudes of the SM gauge fields

$$W^\mu \langle \tilde{J}^L_\mu(q) \tilde{J}^L_\nu(-q)\rangle W^\nu,\ W^\mu \langle \tilde{J}^L_\mu(q) \tilde{J}^R_\nu(-q)\rangle B^\nu,\ B^\mu \langle \tilde{J}^R_\mu(q) \tilde{J}^R_\nu(-q)\rangle B^\nu$$

We use these to provide estimations for:

- SM gauge boson masses
- EW oblique parameters
- Weinberg-like sum rules

Specific 5D Lagrangian for MCHM

The $SO(5)$ invariant action of $5D$ fields corresponding to $\mathcal{O}_S$ and $\mathcal{O}_V^{A\,\mu}$:

$$S_{5D} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}e^{-\Phi(z)}\,\mathrm{Tr}\,F_{MN}F_{KL}g^{MK}g^{LN} + \\ + \frac{1}{k_s}\int d^5x\sqrt{-g}e^{-\Phi(z)}\Big[\mathrm{Tr}\,g^{MN}(D_M H)^\top(D_N H) - M^2\,\mathrm{Tr}\,HH^\top - \\ - M^2\,\mathrm{Tr}(HD^\top + H^\top D)\Big]$$

- $D_M H = \partial_M H - i[A_M, H]$, $F_{MN} = (\partial_M A_N^A - \partial_N A_M^A + C^{ABC}A_M^B A_N^C)T^A$;
- Scalar fields parametrized via an $SO(5)$ tensor H:

$$H = \xi\Sigma\xi^\top, \quad \Sigma = \begin{pmatrix} 0_{4\times4} & 0 \\ 0 & f(z)\end{pmatrix} + iT^a\sigma^a(x,z), \quad \xi = \exp\left(\frac{i\Pi^i(x,z)\widehat{T}^i}{\sqrt{2}f(z)}\right);$$

  - index $A = 1,\ldots,10$ defines different $SO(5)$ generators;
  - fields with $a = 1,\ldots,6$ – unbroken sector (parametrize $SO(4)$);
  - fields with $i = 1,\ldots,4$ – broken sector (parametrize the coset $SO(5)/SO(4)$);
- $H \to H' = gHg^{-1}$ provided that $\xi \to \xi' = g\xi h^{-1}$, $\Sigma \to \Sigma' = h\Sigma h^{-1}$;
- D – additional 5×5 matrix, $D \to D' = gDg^{-1}$ (shift $H \to H + D$);
- Soft explicit $SO(5)$ breaking via $D = \begin{pmatrix} 0_{4\times4} & 0 \\ 0 & b(z)\end{pmatrix}$.

5D Lagrangian, quadratic part

$$S_{5D} = \int d^5x\sqrt{-g}e^{-\Phi(z)}\left(-\frac{1}{4g_5^2}\,\mathrm{Tr}\left(F_{\mu\nu}F_{\lambda\rho}g^{\mu\lambda}g^{\nu\rho} - 2\frac{z^2}{R^2}g^{\mu\nu}\partial_z A_\mu \partial_z A_\nu\right) + \frac{1}{k_s}g^{\mu\nu}f^2(z)A^i_\mu A^i_\nu\right) +$$
$$+\frac{1}{k_s}\int d^5x\sqrt{-g}e^{-\Phi(z)}\Big[g^{\mu\nu}\partial_\mu\sigma^a\partial_\nu\sigma^a - \frac{z^2}{R^2}\partial_z\sigma^a\partial_z\sigma^a - M^2\sigma^a\sigma^a +$$
$$+\frac{1}{2}g^{\mu\nu}\partial_\mu\Pi^i\partial_\nu\Pi^i - \frac{1}{2}\frac{z^2}{R^2}\partial_z\Pi^i\partial_z\Pi^i - M^2 f(z)b(z)\cos\frac{\sqrt{\Pi^i\Pi^i}}{f(z)} - \sqrt{2}f(z)g^{\mu\nu}A^i_\mu\partial_\nu\Pi^i\Big].$$

- ◊ The consequences of the symmetry breaking are now evident:
  - the subgroup $SO(4)$ – unbroken sector (A^a_μ and σ^a fields);
  - the coset $SO(5)/SO(4)$ – broken sector (A^i_μ and Π^i fields).
- ◊ The standard gauge for the bottom-up holography: $A_z = 0$ + enough gauge freedom to set $\partial^\mu A^i_\mu = 0$;
- ◊ The dilaton $\Phi(z) = \kappa^2 z^2$ ⇐ **standard SW model**;
- ◊ The ansätze for the background functions $f(z)$ and $b(z)$ to be defined:
- $\mathbf{f(z) = f\cdot\kappa z}$ – setting the global symmetry breaking scale, $\Lambda_{UV} \sim f$;
- $\mathbf{b(z)/f(z) = \mu_1 + \mu_2\cdot\kappa z}$, $\mu_1 = -1$, $\mu_2 \neq 0$ – adjusting the masses of the Goldstone bosons: $M^2_\Pi(n) = 4\kappa^2\left(n+1+\mu_2\right)$, $\mathbf{\mu_2 = -1 \Rightarrow M^2_\Pi(0) = 0}$

VECTOR CORRELATORS

Definitions:

$$\langle \mathcal{O}_\mu^{a/i}(q)\mathcal{O}_\nu^{b/j}(p)\rangle = \delta(p+q)\int d^4x e^{iqx}\langle \mathcal{O}_\mu^{a/i}(x)\mathcal{O}_\nu^{b/j}(0)\rangle = \frac{\delta^2 i S_{5D}^{on-shell}}{\delta i A_{\mathcal{O}\mu}^{a/i}(q)\delta i A_{\mathcal{O}\nu}^{b/j}(p)},$$

$$i\int d^4x e^{iqx}\langle \mathcal{O}_\mu^{a/i}(x)\mathcal{O}_\nu^{b/j}(0)\rangle = \delta^{ab/ij}\left(\frac{q_\mu q_\nu}{q^2}-\eta_{\mu\nu}\right)\Pi_{unbr/br}(q^2).$$

From the on-shell holographic Lagrangian:

$$\Pi_{unbr}(q^2) = -\frac{R}{2g_5^2}q^2\left[\ln\kappa^2\varepsilon^2 + 2\gamma_E + \psi\left(-\frac{q^2}{4\kappa^2}+1\right)\right],$$

$$\Pi_{br}(q^2) = -\frac{R}{2g_5^2}q^2\left(1-\frac{2(g_5Rf\kappa)^2}{q^2k_s}\right)\left[\ln\kappa^2\varepsilon^2 + 2\gamma_E + \psi\left(-\frac{q^2}{4\kappa^2}+1+\frac{(g_5Rf)^2}{2k_s}\right)\right],$$

where γ_E is the Euler-Mascheroni constant and ψ is the digamma function.
Matching the large-Q^2 logarithms: $\frac{g_5^2}{R} = \frac{4\pi^2}{5N_{tc}}$, $\frac{k_s}{R} = \frac{64\pi^2}{5N_{tc}}$.
A 'pion' pole term in $q^2 \to 0$ expansion of $\Pi_{br}(q^2)$, defining $\Lambda_{UV} = 4\pi F$:

$$F^2 = -\frac{\kappa^2 f^2 R^3}{k_s}\left(\ln\kappa^2\varepsilon^2 + 2\gamma_E + \psi\left(1+\frac{(g_5Rf)^2}{2k_s}\right)\right),$$

VECTOR CORRELATORS

Calculated 2-pt functions:

$$\Pi_{unbr}(q^2) = -\frac{R}{2g_5^2} q^2 \left[\ln\kappa^2\varepsilon^2 + 2\gamma_E + \psi\left(-\frac{q^2}{4\kappa^2}+1\right)\right],$$

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$$-\Pi_{br}(q^2)\big|_{q^2=0} = F^2 = -\frac{\kappa^2 f^2 R^3}{k_s}\left[\ln\kappa^2\varepsilon^2 + 2\gamma_E + \psi\left(1+\frac{(g_5 R f)^2}{2k_s}\right)\right].$$

- are subject to short distance ambiguities $C_0 + C_1 q^2$;
- contain **the UV regulator ε**;
  - the cut-off is assumed to be corresponding to the range of validity of the effective theory: $\varepsilon = \frac{1}{\Lambda_{\text{cut-off}}} \simeq \frac{1}{4\pi F}$.

The convergent correlators in an alternative representation ($n < \mathrm{N_{max}}$):

$$\widehat{\Pi}_{unbr}(Q^2) = \sum_n \frac{Q^4 F_V^2}{M_V^2(n)(Q^2+M_V^2(n))}, \quad \widehat{\Pi}_{br}(Q^2) = \sum_n \frac{Q^4 F_A^2(n)}{M_A^2(n)(Q^2+M_A^2(n))} - F^2$$

$$F_V^2 = \frac{2R\kappa^2}{g_5^2}, \quad F_A^2(n) = \frac{2R\kappa^2}{g_5^2}\frac{n+1}{n+1+\frac{(g_5 R f)^2}{2k_s}}, \quad F^2 = \frac{2R\kappa^2}{g_5^2}\sum_n \frac{\frac{(g_5 R f)^2}{2k_s}}{n+1+\frac{(g_5 R f)^2}{2k_s}}.$$

Correspondence between the cutting scales: $\ln N_{max} = -2\gamma_E - \ln\kappa^2\varepsilon^2$.

Vector correlators

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SM INCLUSION, GAUGE BOSON MASSES, AND THE S PARAMETER

As we have the couplings $\mathcal{L}_{eff} \supset \tilde{J}^{a_L\,\mu} W^{a_L}_\mu + \tilde{J}^{Y\,\mu} B_\mu$ we could include to the 4D partition function the following terms quadratic in natural sources W and B:

$$\mathcal{Z}_{4D}[\phi_{\mathcal{O}}] \supset \int d^4q\; W^\mu \frac{1}{2}\Pi^{\mu\nu}_{LL}(q^2) W^\nu + W^\mu \Pi^{\mu\nu}_{LR}(q^2) B^\nu + B^\mu \frac{1}{2}\Pi^{\mu\nu}_{RR}(q^2) B^\nu$$

Precisely, the relevant correlators are calculated from the 2-pt functions of rotated currents as $i\int d^4x e^{iqx}\langle \tilde{J}^{a_L}_\mu(x)\tilde{J}^{b_L}_\nu(0)\rangle = \delta^{a_L b_L}\frac{g^2}{2}\left(\frac{q_\mu q_\nu}{q^2} - \eta_{\mu\nu}\right)\Pi_{LL}(q^2)$, *etc.*

- $\boldsymbol{\Pi_{LL}(q^2)} = \boldsymbol{\Pi_{RR}(q^2)} = \frac{1+\cos^2\theta}{2}\Pi_{unbr}(q^2) + \frac{\sin^2\theta}{2}\Pi_{br}(q^2)$ provide in the basis of the physical SM gauge bosons the corresponding masses:

$$M^2_W = \frac{g^2}{4}\sin^2\theta F^2, \quad M^2_Z = \frac{g^2+g'^2}{4}\sin^2\theta F^2, \quad M^2_\gamma = 0.$$

- while the value of $\boldsymbol{\Pi_{LR}(q^2)} = \sin^2\theta\left(\Pi_{unbr}(q^2) - \Pi_{br}(q^2)\right)$ defines the S parameter of Peskin and Takeuchi:

$$\mathbf{S} = -4\pi\,\frac{d}{dQ^2}\Pi_{LR}(Q^2)\Big|_{Q^2=0} = \frac{2\pi R\sin^2\theta}{g_5^2}\left[\gamma_E + \psi\left(1+\frac{(g_5Rf)^2}{2k_s}\right) + \frac{(g_5Rf)^2}{2k_s}\psi_1\left(1+\frac{(g_5Rf)^2}{2k_s}\right)\right].$$

'Experimental' constraint: $-0.06 \le S \le 0.16$ *(Gfitter, 2014)*

LEFT-RIGHT CORRELATOR AND WEINBERG SUM RULES

$$\Pi_{LR}(q^2) = \sin^2\theta\left(\Pi_{unbr}(q^2) - \Pi_{br}(q^2)\right)$$

1st Weinberg sum rule:

$$\frac{1}{\pi}\int_0^{M^2(N_{max})}\frac{dt}{t}\mathrm{Im}\Pi_{LR}(t) = \sin^2\theta\sum_{n<N_{max}}(F_V^2(n) - F_A^2(n) - F^2(n)) = 0$$

2nd Weinberg sum rule:

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WSRs are formally valid, but the situation is different from the QCD:

- $n<\infty \Rightarrow$ the upper limit is $M^2(N_{max})$ instead of ∞;
  - the theory is endowed with a cut-off ε, or the number of the resonances has an upper bound N_{max};
- the integral of the imaginary part over the real axis and the sum over resonances are logarithmically divergent unless a cut-off is imposed;
- Sum rules are not saturated at all by just the first resonances;
- F^2 is actually $\sim \ln\kappa^2\varepsilon^2$, implying:

1WSR $\Rightarrow \sum_{n<N_{max}}(F_V^2(n) - F_A^2(n))$ is itself cut-off dependent if $N_{max}\to\infty$

$\Rightarrow$ symmetry restoration takes place very slowly in the UV.

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WSRs are formally valid, but the situation is different from the QCD:

- $n < \infty \Rightarrow$ the upper limit is $M^2(N_{max})$ instead of ∞;
  - the theory is endowed with a cut-off ε, or the number of the resonances has an upper bound N_{max};
- the integral of the imaginary part over the real axis and the sum over resonances are logarithmically divergent unless a cut-off is imposed;
- Sum rules are not saturated at all by just the first resonances;
- F^2 is actually $\sim \ln\kappa^2\varepsilon^2$, implying:

1WSR $\Rightarrow \sum_{n<N_{max}} (F_V^2(n) - F_A^2(n))$ is itself cut-off dependent if $N_{max} \to \infty$

$\Rightarrow$ symmetry restoration takes place very slowly in the UV.

Masses of composite states

We have degenerate vector and scalar states in the unbroken sector:

$$M_V^2(n) = M_\sigma^2(n) = 4\kappa^2(n+1), \quad n = 0,1,2...$$

(linear Regge trajectories – common feature of SW holographic models)

In the broken sector there are massless Goldstone bosons and their massive excitations, the vectorial fields have a constant shift in the intercept relatively to $M_V^2(n)$:

$$M_A^2(n) = 4\kappa^2\left(n+1+\frac{(g_5 R f)^2}{2k_s}\right), \quad M_\Pi^2(n) = 4\kappa^2 n, \quad n = 0,1,2...$$

The value of $\kappa^2 \implies$ predictions for new states in all channels

Taking the expression for F^2 of the current-algebra origin:

$$\frac{v^2}{\sin^2\theta} + \frac{5}{64\pi^2}\kappa^2 N_{tc}(fR)^2\left(\ln\frac{\kappa^2\sin^2\theta}{16\pi^2 v^2} + 2\gamma_E + \psi\left(1+\frac{(fR)^2}{32}\right)\right) = 0$$

$$\Downarrow \; v = 246 \text{ GeV}$$

$M_* = \sqrt{4\kappa^2}$ as a function of $(\sin\theta, fR, N_{tc})$

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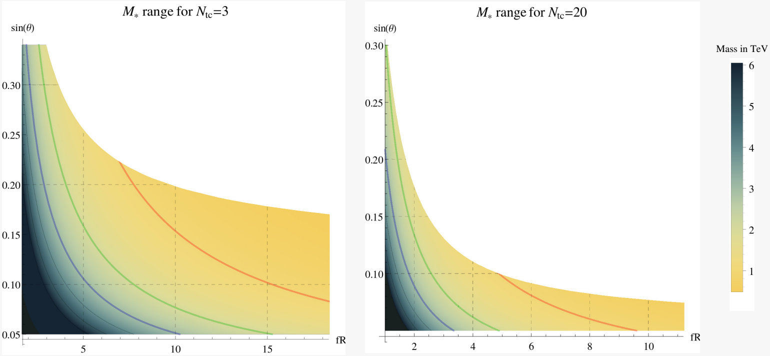


Figure 1: The density plots of M_* as a function of $(fR, \sin\theta)$ for $N_{tc} = 3,\ 20$.

- the coloured curves represent the lines of constant M_*:
  - $M_* = 1$ TeV,
  - $M_* = 2$ TeV,
  - $M_* = 3$ TeV,
  - and successive black curves for higher integer values.
- the colourless area – the sector prohibited by the S bound ($S \leq 0.16$).

TABLE 1: Different predictions of the minimal vector masses, $S \leq 0.16$.

$\sin\theta$	N_{tc}	fR	$M_* = M_V(0)$, TeV	$M_A(0)$, TeV	$\sim N_{max}$
0.25	2	9.1	0.89	2.20	244
0.25	3	5.2	1.21	1.99	133
0.25	4	3.9	1.37	1.92	102
0.25	10	2.0	1.66	1.86	70
0.30	2	5.5	1.26	2.14	85
0.30	3	3.7	1.50	2.03	60
0.30	4	2.9	1.61	1.99	51
0.30	10	1.6	1.81	1.96	41

Misalignment bound in MCHM:

$$\sin\theta \leq 0.34 \qquad \textit{(ATLAS, 2015)}$$

Smaller values of $\sin\theta$ (larger scale separation $F = \frac{v}{\sin\theta}$)

$$\Updownarrow$$

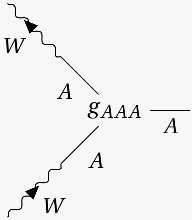
Larger fine tuning

Mixing with composite resonances

Due to the 2pt functions of a type $\langle \tilde{J}^L_\mu(x) J^{L/R/br}_\nu(0)\rangle$ the mixing terms appear:

$$\mathcal{L}_{4D} \supset \frac{g}{\sqrt{2}} \frac{1 \pm \cos\theta}{2} \sum_n W^a_\mu(x)(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu) A^a_{L/R\ \nu(n)}(x) \frac{F_V}{M_V(n)} -$$
$$+ \frac{g}{\sqrt{2}} \frac{\sin\theta}{\sqrt{2}} \sum_n W^a_\mu(x)(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu) A^a_{br\ \nu(n)}(x) \frac{F_A(n)}{M_A(n)}$$

For the parameter values of interest mixing is found to be small.
The largest component of A_L in W is not exceeding $2-7\%$.



For different resonances:

$$g_{AAA} \sim \frac{1}{\sqrt{N_{tc}}} \frac{1}{\sqrt{(n_1+1)(n_2+1)(n_3+1)}} \times \int dy e^{-y} y^2 L^1_{n_1}(y) L^1_{n_2}(y) L^1_{n_3}(y)$$

DIRECT COUPLINGS OF W TO COMPOSITE STATES

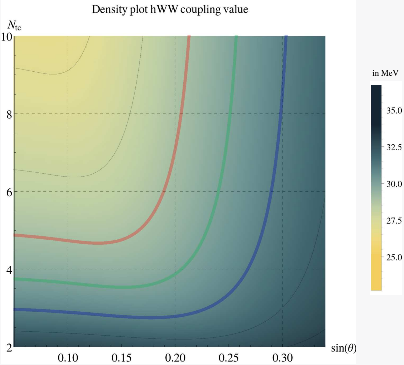


FIGURE 2: Colored lines mark integer values:
– 29 MeV, – 30 MeV, – 31 MeV.
SM result $g_{hWW}^{SM} \simeq 52$ MeV.

Extended $5D$ derivative:

$$D_\mu H = \partial_\mu H - i[A_\mu, H] - i[\widetilde{X}_\mu, H]$$

X_μ^A belong to $SO(4)'$ multiplet with only some non-zero components:
$X_\mu^{L\,\alpha} = \frac{g}{\sqrt{2}} W_\mu^\alpha$ and $X_\mu^{R\,3} = \frac{g'}{\sqrt{2}} B_\mu$.
Important: X do not depend on z.
$\hookrightarrow$ Extra terms from $\frac{1}{k_s} D_\mu H^\top D^\mu H$
For instance, with SM Higgs= $\Pi_4 = h$:

$$\mathcal{L}_{4D} \supset \frac{g_{hWW}}{2} h W_\mu W^\mu,$$

$$g_{hWW} = g_{hWW}^{SM} \cos\bar{\theta},$$

$$g_{hWW}^{SM} = \frac{g^2 F \sin\theta}{2},$$

$$\cos\bar{\theta} = \cos\theta \sqrt{\frac{\pi}{2}} \left(\sum_{n=0}^{N_{max}} \frac{1}{1 + n + \frac{(g_5 R f)^2}{2 k_s}} \right)^{-1/2}$$

CONCLUSIONS

The holographic composite Higgs model:

- $5D$ holographic setup;
- inspired by the effective models of QCD;
- a generalized sigma model coupled both to the composite resonances and to the SM gauge bosons;
- ansätze: the dilaton z-profile (common to all SW holographic models), and two functions $f(z)$ and $b(z)$.

Features:

- the Goldstone bosons can be made exactly massless;
- the vectors and scalars of the unbroken sector are degenerate in mass; not so for the states in the broken sector.
- the two Weinberg sum rules hold only in a formal sense as the sum over resonances has to be cut off (it is logarithmically divergent).

Predictions:

- S parameter and its restrictions on the model parameters;
- areas in parameter space where a resonance between 1 and 2 TeV is easily accommodated;
- possibilities: form factors, effective couplings and cross-sections, other symmetry breaking patterns, ...

Generalization to $SO(N) \to SO(N-1)$

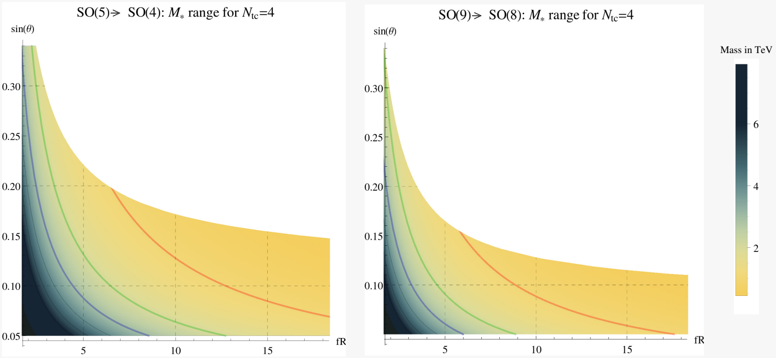


Figure 3: M_* as a function of $(fR, \sin\theta)$ for $N_{tc} = 4$ and different symmetry breaking patterns.
– $M_* = 1$ TeV, – $M_* = 2$ TeV, – $M_* = 3$ TeV, – higher integer values.

PDG(2018): $S \leq 0.12$

TABLE 2: Different predictions of the minimal vector masses, $S \leq 0.12$.

$\sin\theta$	N_{tc}	fR	$M_* = M_V(0)$, TeV	$M_A(0)$, TeV	$\sim N_{max}$
0.20	2	12.0	0.84	2.64	431
0.20	3	6.4	1.23	2.31	199
0.20	4	4.6	1.45	2.20	144
0.20	10	2.2	1.84	2.10	89
0.34	2	3.3	2.01	2.59	26
0.34	3	2.4	2.17	2.54	22
0.34	4	2.0	2.25	2.51	21
0.34	10	1.2	2.38	2.49	18