# Analysis of the b1 meson decay in local tensor bilinear representation

arXiv:1805:06559

XIIIth Quark Confinement and the Hadron Spectrum
3rd August 2018, Maynooth University, Ireland

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# Preliminary: field-current identity

- Example: charged vector meson ($\rho$) Lagrangian

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 - \frac{1}{2}m_0\rho^2 + \mathcal{L}_m(\psi, D_\nu\psi, f_{\mu\nu}) - \frac{1}{4}F_{\mu\nu}^2$$

$$f_{\mu\nu}^a = \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a + g_0 f^{abc}\rho_\mu^b \rho_\nu^c$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$\rho$ and $A_\mu$ in **(1,0)⊕(0,1)** representation

Equations of motion

$$\partial^\mu F_{\mu\nu} = -\frac{\delta\mathcal{L}}{\delta\hat{\rho}_\mu^0}\frac{\delta\hat{\rho}_\mu^0}{\delta A^\nu} = -\frac{e_0}{g_0}\frac{\delta\mathcal{L}}{\delta\hat{\rho}^{0\nu}} = \boxed{-e_0\left(\frac{m_0^2}{g_0}\right)\rho_\nu^0}$$

$$\partial^\mu \hat{f}_{\mu\nu}^a = g_0 J_\nu^{\rho,a} + m_0^2\hat{\rho}_\nu^a$$

$\hat{\rho}_\mu^0 \equiv \rho_\mu^3 + (e_0/g_0)A_\mu,\ \hat{\rho}_\mu^\pm \equiv \rho_\mu^\pm$

$\rho$ is coupled with *U(1)* gauge

by field redefinition $\xi_\mu^a \equiv (m_0^2/g_0)\hat{\rho}_\mu^a = (1/g_0)\partial^\nu \hat{f}_{\nu\mu}^a - J_\mu^{\rho,a}$

$$[\xi_i^a(r,t), \xi_j^b(r',t)] = 0,$$

$$[\xi_0^a(r,t), \xi_0^b(r',t)] = if^{abc}\delta^3(r-r')\xi_0^c(r,t),$$

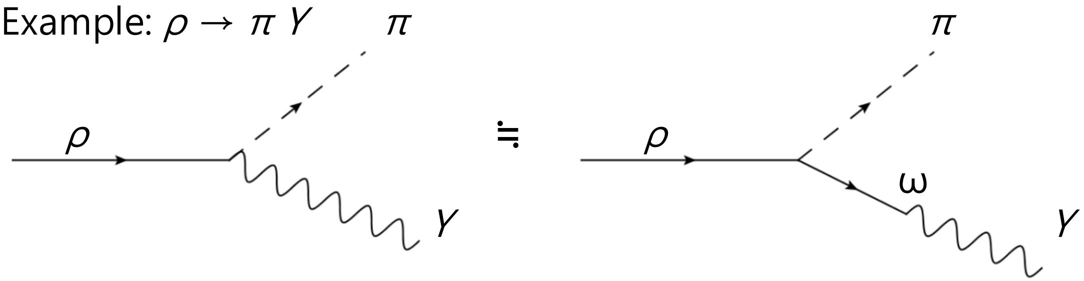
$$[\xi_0^a(r,t), \xi_j^b(r',t)] = if^{abc}\delta^3(r-r')\xi_j^c(r,t) + i\left(m_0^2/g_0^2\right)\delta^{ab}\partial_{r_j}\delta^3(r-r'),$$

$\rho$ can be regarded as external source of E.M. field
→ Photon can receive effective mass from intermediate $\rho$ states

# Vector meson dominance

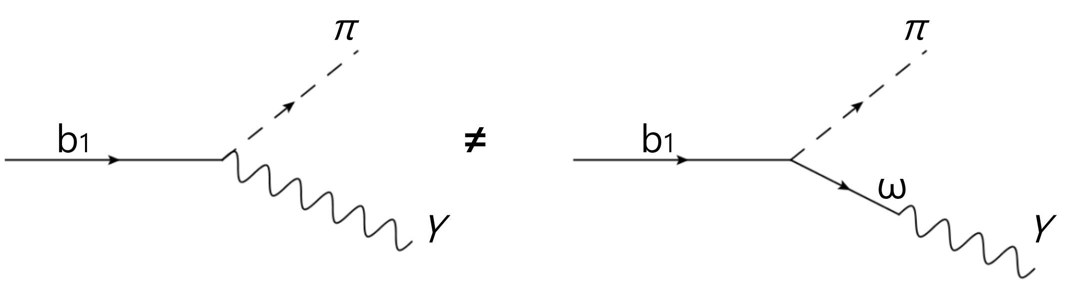
- External photon can be replaced with vector meson

Example: $\rho \rightarrow \pi\, \gamma$




Effective Lagrangian in VMD hypothesis explains well

- **Exceptional phenomenon :** $b_1 \rightarrow \pi\, \gamma$




$\Gamma(b_1 \rightarrow \pi\gamma)$ = 230 KeV (experiment) $\Gamma(b_1 \rightarrow \pi\gamma)$ = 30~160 KeV (VMD scenario)

For $b_1$ decay, VMD hypothesis does not work well

# b1 in $\left(\frac{1}{2},\frac{1}{2}\right) \oplus \left(\frac{1}{2},\frac{1}{2}\right)$ representation

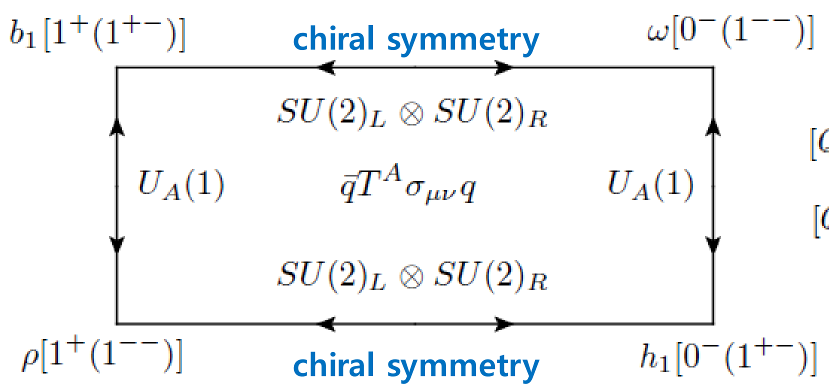
- Local bilinear in tensor representation [Cohen and Ji, PRD55(1997)6850]

$$\bar{b}_1: \ [I^G J^{PC} = 1^+(1^{+-})] \to \frac{1}{2}\epsilon_{ijk} \langle 0 \mid \bar{q} T^a \sigma_{ij} q \mid \bar{b}_1(p,\lambda)\rangle = i f^T_{b_1^a}(-\bar{\epsilon}^{(\lambda)}_k p_0)$$

The other vector mesons in tensor bilinear

$$\bar{\omega}: \ [I^G J^{PC} = 0^-(1^{--})] \to \langle 0 \mid \bar{q} T^0 \sigma_{0k} q \mid \bar{\omega}(p,\lambda)\rangle = i f^T_{\omega}(-\bar{\epsilon}^{(\lambda)}_k p_0),$$
$$\bar{\rho}: \ [I^G J^{PC} = 1^+(1^{--})] \to \langle 0 \mid \bar{q} T^a \sigma_{0k} q \mid \bar{\rho}(p,\lambda)\rangle = i f^T_{\rho^a}(-\bar{\epsilon}^{(\lambda)}_k p_0),$$
$$\bar{h}_1: \ [I^G J^{PC} = 0^-(1^{+-})] \to \frac{1}{2}\epsilon_{ijk} \langle 0 \mid \bar{q} T^0 \sigma_{ij} q \mid \bar{h}_1(p,\lambda)\rangle = i f^T_{h_1}(-\bar{\epsilon}^{(\lambda)}_k p_0),$$

Under ***$U_A(1)$*** and ***$SU(2)_L \times SU(2)_R$*** symmetries, the vector mesons reside in ***$U(2)_L \times U(2)_R$***




$$[Q_5^a, J_k^{b_1,b}(x)] = -i\delta^{ab} J_k^{\omega}(x), \ [Q_5^a, J_k^{\omega}(x)] = iJ_k^{b_1,a}(x),$$
$$[Q_5^0, J_k^{\omega}(x)] = -iJ_k^{h_1}(x), \ [Q_5^a, J_k^{\rho,b}(x)] = i\delta^{ab} J_k^{h_1}(x),$$

# Soft pion breaking

- Field transformation and leaking charge

$$\psi_A(\vec{x},t) \to \psi_A(\vec{x},t) - i\Lambda_i[Q^i(t),\psi_A(\vec{x},t)]$$
$$= \psi_A(\vec{x},t) - i\Lambda_i M^i_{AB}\psi_B(x,t),$$

$$\mathcal{L} \to \mathcal{L} - (\partial^\alpha \Lambda_i)J^i_\alpha(\vec{x},t) - \Lambda_i(\partial^\alpha J^i_\alpha(\vec{x},t))$$

If the symmetry is broken

$$\partial^\alpha J^i_\alpha(\vec{x},t) = i[Q^i(t),u(\vec{x},t)] = -i[Q^i(t),\mathcal{H}(\vec{x},t)]$$

Leaking charge flow

$$\frac{dQ^i(t)}{dt} = \int d^3x \partial^\alpha J^i_\alpha(\vec{x},t) = -i[Q^i(t),\int d^3x \mathcal{H}(\vec{x},t)]$$

- Pion corresponds to leaking chiral charge flow

$$\langle \pi^a(q)B|C(z)|A\rangle = i\int d^4x e^{iqx}(\partial^2+m_\pi^2)\langle B|\mathrm{T}[\pi^a(x)C(z)]|A\rangle$$
$$= i\left(\frac{q^2-m_\pi^2}{m_\pi^2 f_\pi}\right)\int d^4x e^{iqx}\langle B|\mathrm{T}[\partial^\mu J^a_{5\mu}(x)C(z)]|A\rangle$$

$$\pi^a(x) \simeq -(1/m_\pi^2 f_\pi)\partial^\alpha J^a_{5\alpha}(x)$$

$$\lim_{q\to 0}\langle \pi^a(q)B|C(z)|A\rangle \simeq \frac{i}{f_\pi}\langle B|[Q^a_5,C(z)]|A\rangle$$

→ $b_1$ decay process can be regarded as the consequence of chiral symmetry breaking

$[Q_5^a, J_k^{b_1,b}(x)] = -i\delta^{ab}J_k^\omega(x)$, $[Q_5^a, J_k^\omega(x)] = iJ_k^{b_1,a}(x)$, (**chiral partners in tensor representation**)

# $b_1$ decay process

- Correlation function from $b_1$ to $\omega$ via pion breaking

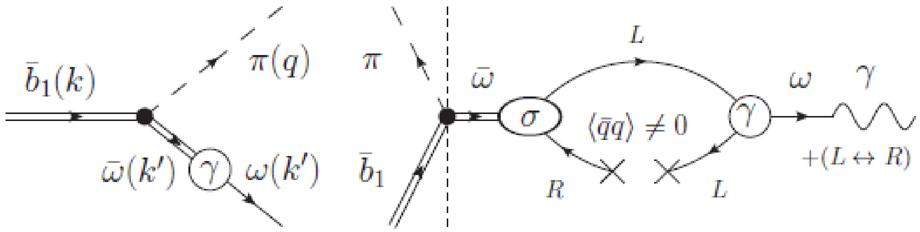
$$\left\langle \pi^a(q)\omega(k') \left| i\tilde{J}^a_{\mu\bar{\mu}}(k) \right| 0 \right\rangle \simeq \boxed{f^T_{b_1} \sum_\lambda \langle \pi^a(q)\omega(k') | \bar{b}^a_1(k,\lambda) \rangle} \left( \bar{\epsilon}^{(\lambda)^*}_{\mu} k_{\bar{\mu}} - \bar{\epsilon}^{(\lambda)^*}_{\bar{\mu}} k_{\mu} \right) - f^T_\rho \sum_\lambda \langle \pi^a(q)\omega(k') | \bar{\rho}(k,\lambda) \rangle \frac{\bar{\epsilon}_{\mu\bar{\mu}\alpha\bar{\alpha}}}{2} \left( \bar{\epsilon}^{(\lambda)^*\alpha} k^{\bar{\alpha}} - \bar{\epsilon}^{(\lambda)^*\bar{\alpha}} k^{\alpha} \right) + \cdots$$

b1 → $\pi\omega$ decay

where $\tilde{J}^a_{\mu\bar{\mu}} \equiv -(\epsilon_{\mu\bar{\mu}\alpha\bar{\alpha}}/2)\bar{q}T^a\sigma^{\alpha\bar{\alpha}}q$

- In soft pion limit ($\omega$ in **(1,0)⊕(0,1)** and $\bar{\omega}$ in **(1/2,1/2)⊕(1/2,1/2)** representation)

$$\left\langle \pi^a(q)\omega(k') \left| i\tilde{J}^a_{\mu\bar{\mu}}(k) \right| 0 \right\rangle = \frac{i}{f_\pi} \int d^3x e^{-i\vec{q}\cdot\vec{x}} \left\langle \omega(k') \left| \left[ J^a_{50}(x), i\tilde{J}^a_{\mu\bar{\mu}}(k) \right] \right| 0 \right\rangle$$
$$= \frac{i}{f_\pi} \left[ -3i \left\langle \omega(k) \left| iJ^0_{\mu\bar{\mu}}(k) \right| 0 \right\rangle + R_1(q) \right] \simeq \frac{3f^T_\omega}{f_\pi} \boxed{\sum_\lambda \langle \omega(k) | \bar{\omega}(k,\lambda) \rangle} \left( \bar{\epsilon}^{(\lambda)^*}_{\mu} k_{\bar{\mu}} - \bar{\epsilon}^{(\lambda)^*}_{\bar{\mu}} k_{\mu} \right) + \cdots$$

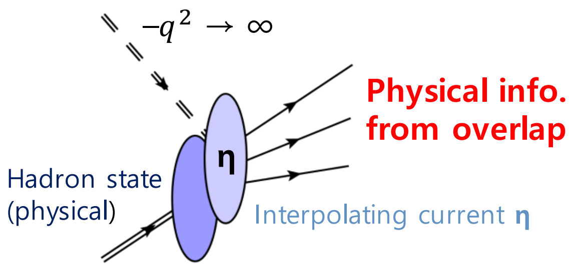



$\omega$ and $\bar{\omega}$ overlaps
when chiral symmetry is broken

Thus, one should identify whether $\bar{\omega}$ state is $\omega$(782) or not – we are using QCD sum rules

# Interpolation via tensor current

- To obtain physical information




a. Quasi-particle state will be extracted from the overlap
b. We need to construct proper interpolating current which can be strongly overlapped with object hadron state
c. Our object: **ω meson in tensor representation**

- Projection operator

Covariant interpolation

$$\omega[0^-(1^{--})] \to \langle 0 \left| \bar{q} T^0 \sigma_{\mu\nu} q \right| \omega(p,\lambda) \rangle = i f_\omega^T \left( \epsilon_\mu^{(\lambda)} p_\nu - \epsilon_\nu^{(\lambda)} p_\mu \right)$$

$$b_1[1^+(1^{+-})] \to -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} \langle 0 \left| \bar{q} T^a \sigma^{\alpha\beta} q \right| b_1(p,\lambda) \rangle = i f_{b_1^a}^T \left( \epsilon_\mu^{(\lambda)} p_\nu - \epsilon_\nu^{(\lambda)} p_\mu \right)$$

Projection of parity eigenmodes

$$\sum_\lambda \langle 0 \mid \bar{q} T^A \sigma_{\mu\bar{\mu}} q \mid [1^{--}](p,\lambda) \rangle \langle [1^{--}](p,\lambda) \mid \bar{q} T^B \sigma_{\nu\bar{\nu}} q \mid 0 \rangle = -\delta^{AB} \left( f_-^T \right)^2 p^2 P_{\mu\bar{\mu},\nu\bar{\nu}}^{(-)}$$

$$\sum_\lambda \langle 0 \mid \bar{q} T^A \sigma_{\mu\bar{\mu}} q \mid [1^{+-}](p,\lambda) \rangle \langle [1^{+-}](p,\lambda) \mid \bar{q} T^B \sigma_{\nu\bar{\nu}} q \mid 0 \rangle = -\delta^{AB} \left( f_+^T \right)^2 p^2 P_{\mu\bar{\mu},\nu\bar{\nu}}^{(+)}$$

$$P_{\mu\bar{\mu};\nu\bar{\nu}}^{(-)} = g_{\mu\nu} \frac{p_{\bar{\mu}} p_{\bar{\nu}}}{p^2} + g_{\bar{\mu}\bar{\nu}} \frac{p_\mu p_\nu}{p^2} - g_{\bar{\mu}\nu} \frac{p_\mu p_{\bar{\nu}}}{p^2} - g_{\mu\bar{\nu}} \frac{p_{\bar{\mu}} p_\nu}{p^2}, \quad P_{\mu\bar{\mu};\nu\bar{\nu}}^{(+)} = P_{\mu\bar{\mu},\nu\bar{\nu}}^{(-)} + (g_{\mu\bar{\nu}} g_{\bar{\mu}\nu} - g_{\mu\nu} g_{\bar{\mu}\bar{\nu}}).$$

# QCD Sum Rules: Overview

- Correlator of the tensor current

$$\Pi^{AB}_{\mu\bar{\mu};\nu\bar{\nu}}(k) = i\int d^4x e^{ikx}\,\langle \mathcal{T}[\bar{q}(x)T^A\sigma_{\mu\bar{\mu}}q(x)\,\bar{q}(0)T^B\sigma_{\nu\bar{\nu}}q(0)]\rangle$$
$$= \Pi^{\rm ope}_-(k^2)P^{(-)}_{\mu\bar{\mu};\nu\bar{\nu}} + \Pi^{\rm ope}_+(k^2)P^{(+)}_{\mu\bar{\mu};\nu\bar{\nu}},$$

- Energy dispersion relation and OPE (in **QCD degree of freedom**)

$$\Pi_i(q^2) = \frac{1}{2\pi i}\int_0^\infty ds\frac{\Delta\Pi_i(s)}{s-q^2} + P_n(q^2)$$

- Phenomenological Ansatz (in **hadronic degree of freedom**)

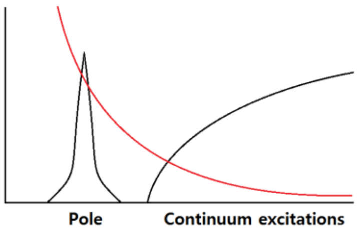
$$\Pi^{\rm ph.pole}_{\mp}(k^2) = \mathcal{P}\frac{(f^T_{\mp})^2k^2}{(k^2-m^2_{\mp})} - i\pi(f^T_{\mp})^2k^2\delta(k^2-m^2_{\mp})$$

Equating both sides, hadronic quantum number can be expressed in QCD degree of freedom

- Weighting - Borel transformation

$$\mathcal{B}[\Pi_i(q_0,|\vec{q}|)] \equiv \lim_{\substack{-q_0^2,n\to\infty\\ -q_0^2/n=M^2}} \frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0,|\vec{q}|)$$

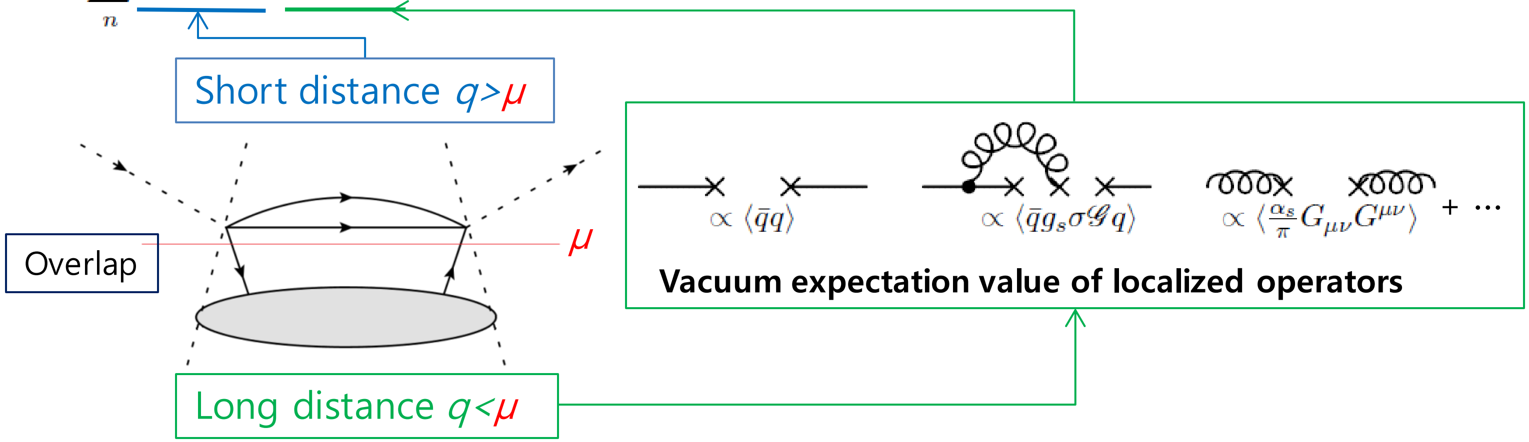
If the sum rule is well constructed, the physical numb er has weak dependence on Borel mass ***M***



# QCD SR: operator product expansion

- Operator product expansion (Example: 2-quark condensate in baryon correlation)

$$\Pi_i(q^2) = \sum_n C_n^i(q^2,\mu)\langle \hat{O}_n(\mu)\rangle_0$$




- Separation scale is set to be hadronic scale (≤ 0.5 GeV)
  - **Wilson coefficient** contains perturbative contribution above separation scale – short-ranged partonic propagation in hadron
  - **Condensate** contains non-perturbative contribution below separation scale – long ranged correlation in low energy part of hadron
  - Quark confinement inside hadron is low energy QCD phenomenon
  - Genuine properties of hadron are reflected in **the condensates**

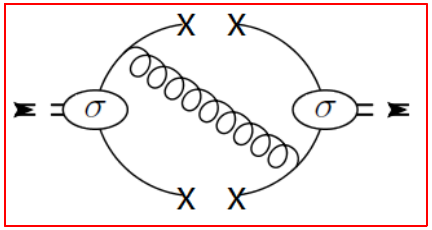
# Four-quark condensates

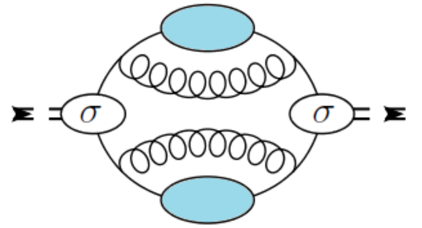
- Four-quark pieces determine spectral structure of invariant

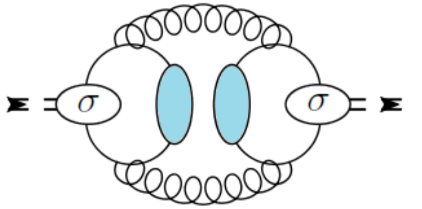
$$\mathcal{W}_M^{\text{subt.}}\left[\Pi_{\mp}^{\text{ope}}(k^2)\right] = \frac{1}{\pi}\int_0^{s_0} ds e^{-s/M^2}\text{Im}\left[\Pi_{\mp}^{\text{ope}}(s)\right]$$

$$= -\frac{1}{16\pi^2}\left[\left(1+\frac{7\alpha_s}{9\pi}\right)(M^2)^2 E_1(s_0) + \frac{\alpha_s}{3\pi}L(s_0)\right] - \frac{4\pi\alpha_s}{9M^2}\langle\bar{q}T^0\tau^{\bar{a}}\gamma_\eta q\,\bar{q}T^0\tau^{\bar{a}}\gamma^\eta q\rangle - \frac{1}{48}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$$

$$\mp\frac{16\pi\alpha_s}{M^2}\left(\langle\bar{q}T^A\tau^{\bar{a}}q\,\bar{q}T^A\tau^{\bar{a}}q\rangle + \langle\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\,\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\rangle\right)$$




Usual vacuum saturation hypothesis gives same factorization

$$\langle\bar{q}T^A\tau^{\bar{a}}q\bar{q}T^A\tau^{\bar{a}}q\rangle \to -\frac{a_A}{18}\langle\bar{q}T^0q\rangle^2$$

$$\langle\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\rangle \to -\frac{b_A}{18}\langle\bar{q}T^0q\rangle^2$$

In Bank-Casher formula, only Dirac zero-mode contributes

$$\langle\bar{q}q\rangle = -\int d^4x\left\langle\sum_\lambda\frac{\psi_\lambda(x)^\dagger\psi_\lambda(x)}{V}\frac{1}{m-i\lambda}\right\rangle = -\pi\langle\text{Tr}[J_{\lambda=0}(0,0)]\rangle$$

Dirac zero-mode correlation is on the gauge orbit
→ colored pieces can make non-zero contribution

In vacuum, all the topological configuration is possible
→ parity-odd pieces ($b_A$ type) can contribute as alternating s
eries of winding number

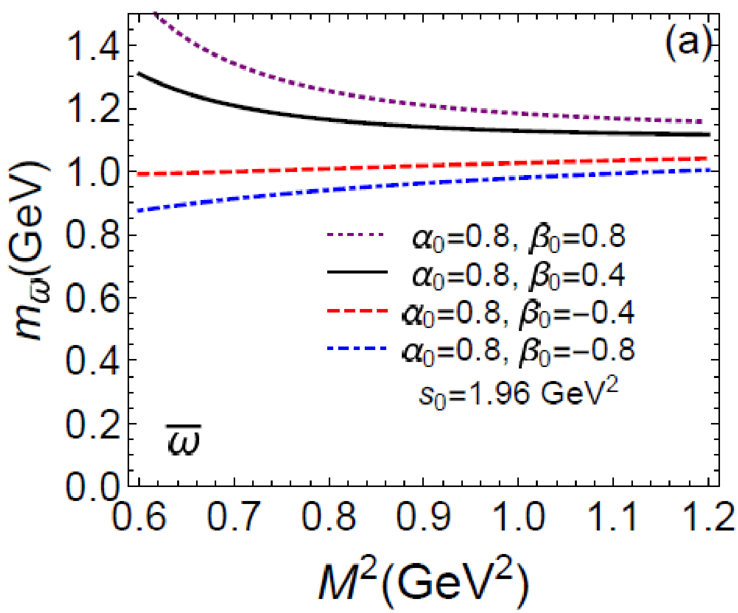
**[$a_0$=0.8, $b_0$=0.4]** has been used for the isoscalar mode

# Borel sum rules for $\left(\frac{1}{2},\frac{1}{2}\right)\oplus\left(\frac{1}{2},\frac{1}{2}\right)$ states

- **One particle pole ansatz**

$$\mathcal{W}_M^{\text{subt.}}\left[\Pi_{\mp}^{\text{ope}}(k^2)\right] = \frac{1}{\pi}\int_0^{s_0} ds e^{-s/M^2}\,\mathrm{Im}\left[\frac{f_{\mp}^{T\,2}s}{s-m_{\mp}^2+i\epsilon}\right] = -f_{\mp}^{T\,2}m_{\mp}^2 e^{-m_{\mp}^2/M^2}$$

$$= -\frac{1}{16\pi^2}\left[\left(1+\frac{7\alpha_s}{9\pi}\right)(M^2)^2E_1(s_0)+\frac{\alpha_s}{3\pi}L(s_0)\right]-\frac{4\pi\alpha_s}{9M^2}\left\langle\bar{q}T^0\tau^{\bar{a}}\gamma_\eta q\,\bar{q}T^0\tau^{\bar{a}}\gamma^\eta q\right\rangle-\frac{1}{48}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$$
$$\mp\frac{16\pi\alpha_s}{M^2}\left(\left\langle\bar{q}T^A\tau^{\bar{a}}q\,\bar{q}T^A\tau^{\bar{a}}q\right\rangle+\left\langle\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\,\bar{q}T^A\tau^{\bar{a}}\gamma_5 q\right\rangle\right)$$




$$m_{\mp}^2 = (M^2)^2\left(\frac{\partial}{\partial M^2}\mathcal{W}_M^{\text{subt.}}[\Pi^{(\mp)}_{\bar{q}T^A\sigma q}(k^2)]\right)\Big/\mathcal{W}_M^{\text{subt.}}[\Pi^{(\mp)}_{\bar{q}\sigma q}(k^2)]$$

Mass number ranges from **900** MeV ~ **1200** MeV
→ mass curve itself is not stable
→ higher than mass of ω(782)
→ there is no ω resonance in mass ~ 1 GeV

consider anomalous coupling from W-Z term

$$\mathcal{L}^{\epsilon 1}_{\omega\pi\rho} = \frac{g_{\omega\pi\rho}}{2}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}\omega_{\mu\bar{\mu}}\partial_\alpha\pi^a\rho^a_{\bar{\alpha}}$$

π-ρ hybrid state can be suggested

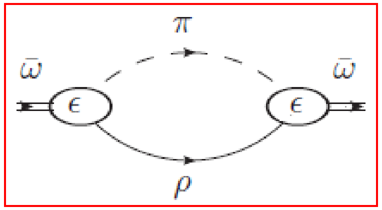
# Borel sum rules for $\bar{\omega}[0^-(1^{--})]$ state

- **Anomalous hybrid state**

$J^{\bar{\omega}}_{\mu\bar{\mu}}(x) \equiv \epsilon_{\mu\bar{\mu}\alpha\bar{\alpha}} \mathrm{Tr}\left[\partial^\alpha \pi(x) \rho^{\bar{\alpha}}(x)\right]$ inferred from Wess-Zumino term [Physics Letter 37B(1971)95]

Anomalous current correlator

$$\Pi^{\bar{\omega}}_{\mu\bar{\mu};\nu\bar{\nu}}(k) = i \int d^4x e^{ikx} \left\langle \mathcal{T}\left[\epsilon_{\mu\bar{\mu}\alpha\bar{\alpha}} \mathrm{Tr}[\partial^\alpha \pi(x)\rho^{\bar{\alpha}}(x)] \epsilon_{\nu\bar{\nu}\beta\bar{\beta}} \mathrm{Tr}[\partial^\beta \pi(0)\rho^{\bar{\beta}}(0)]\right]\right\rangle$$
$$= \frac{3i}{4}\epsilon_{\mu\bar{\mu}\alpha\bar{\alpha}}\epsilon_{\nu\bar{\nu}\beta\bar{\beta}} \int \frac{d^4p}{(2\pi)^4} \frac{(p+k)^\alpha (p+k)^\beta}{(p+k)^2 - m_\pi^2} \frac{1}{p^2 - m_\rho^2}\left(g^{\bar{\alpha}\bar{\beta}} - \frac{p^{\bar{\alpha}} p^{\bar{\beta}}}{m_\rho^2}\right)$$
$$= \Pi^{\bar{\omega}}_{-}(k^2) P^{(-)}_{\mu\bar{\mu};\nu\bar{\nu}} + \Pi_{+}(k^2) P^{(+)}_{\mu\bar{\mu};\nu\bar{\nu}},$$



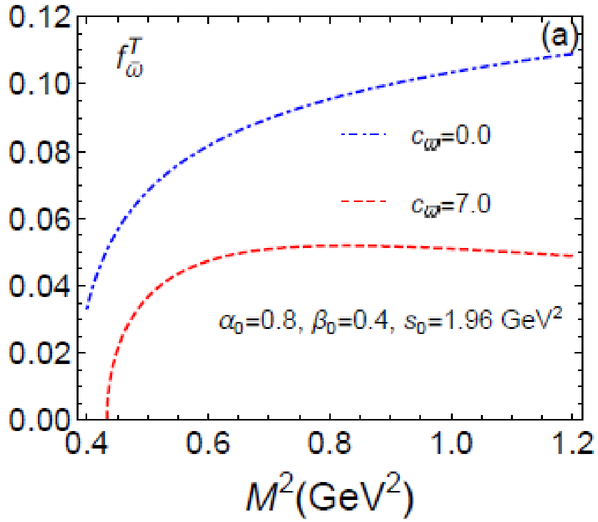

Weighted invariant for parity-odd mode

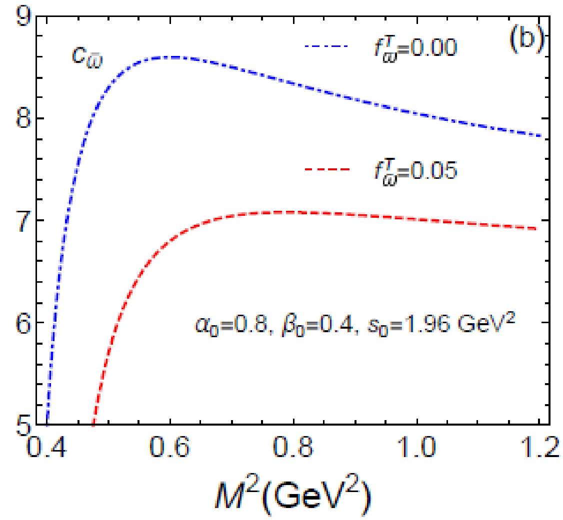
$$\mathcal{W}^{\mathrm{subt.}}_M[\Pi^{\bar{\omega}}_{(-)}(k^2)] = \frac{3}{4(4\pi)^2} \int_{m_\rho^2}^{s_0} ds e^{-s/M^2} \left[\left(-\frac{s}{6} + \frac{m_\rho^2}{2} - \frac{1}{2}\frac{(m_\rho^2)^2}{s} + \frac{1}{6}\frac{(m_\rho^2)^3}{s^2}\right)\right]$$
$$= \frac{3}{4(4\pi)^2}\Bigg[ -\frac{1}{6}\left[-M^2\left(s_0 e^{-s_0/M^2} - m_\rho^2 e^{-m_\rho^2/M^2}\right) - (M^2)^2\left(e^{-s_0/M^2} - e^{-m_\rho^2/M^2}\right)\right]$$
$$+ \frac{m_\rho^2}{2}\left[-M^2\left(e^{-s_0/M^2} - e^{-m_\rho^2/M^2}\right)\right] - \frac{(m_\rho^2)^2}{2}\left[\Gamma\left(0, m_\rho^2/M^2\right) - \Gamma\left(0, s_0/M^2\right)\right]$$
$$+ \frac{(m_\rho^2)^3}{6}\left[-\frac{1}{s_0}e^{-s_0/M^2} + \frac{1}{m_\rho^2}e^{-m_\rho^2/M^2} - \frac{1}{M^2}\left(\Gamma\left(0, m_\rho^2/M^2\right) - \Gamma\left(0, s_0/M^2\right)\right)\right]\Bigg]$$

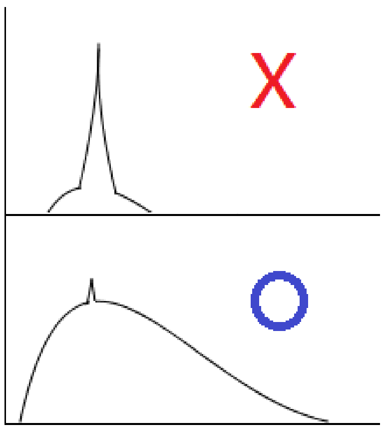
# Borel sum rules for $\bar{\omega}[0^-(1^{--})]$ state

- Generalized spectral sum rules for $\bar{\omega}$ state

$$f_\mp^T = \left\{ \frac{1}{m_\mp^2} e^{m_\mp^2/M^2} \left( -\mathcal{W}_M^{\text{subt.}} \left[\Pi_\mp^{\text{ope}}(k^2)\right] + \left(c_\mp^{\text{h}\bar{\text{y}}.}\right)^2 \mathcal{W}_M^{\text{subt.}} \left[\Pi_\mp^{\text{h}\bar{\text{y}}.}(k^2)\right] \right) \right\}^{\frac{1}{2}} \qquad c_\mp^{\text{h}\bar{\text{y}}.} = \left\{ \left( \mathcal{W}_M^{\text{subt.}} \left[\Pi_\mp^{\text{ope}}(k^2)\right] + \left(f_\mp^T\right)^2 m_\mp^2 e^{-m_\mp^2/M^2} \right) \Big/ \mathcal{W}_M^{\text{subt.}} \left[\Pi_\mp^{\text{h}\bar{\text{y}}.}(k^2)\right] \right\}^{\frac{1}{2}}$$




Borel curve for coupling is analyzed in proper parameter set **[α₀=0.8 β₀=0.4]**

$\bar{\omega}$ pole residue $f_{\overline{\omega}}$ is unstable until $c_{\overline{\omega}} = 7.0$ (reduces to 50%)

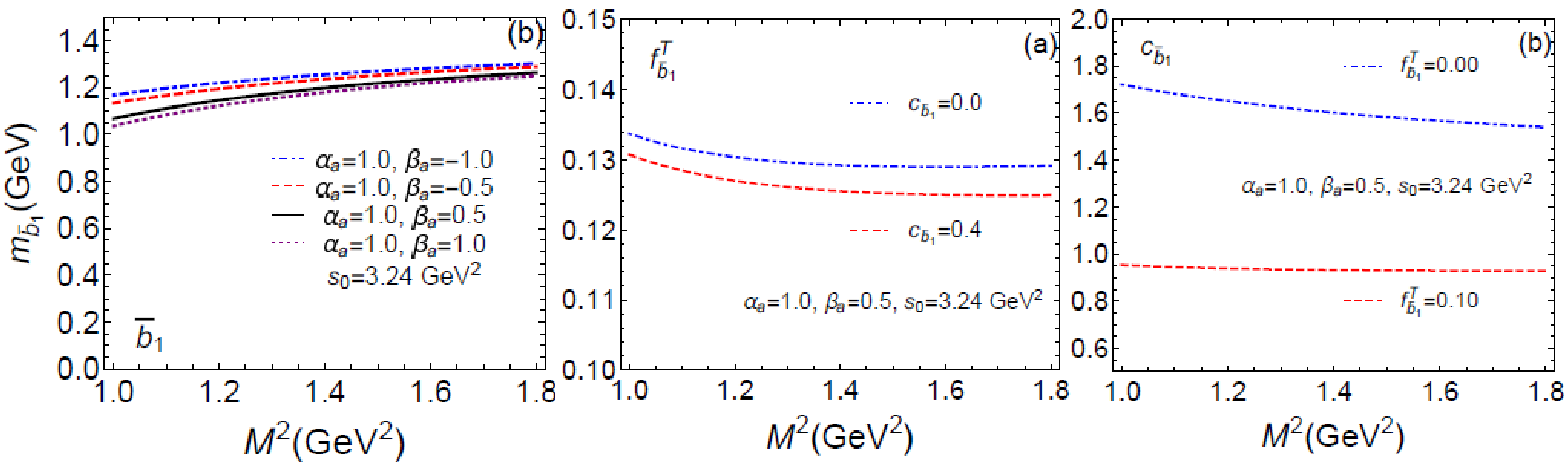
Hybrid coupling $c_{\overline{\omega}}$ is relatively stable and the stability is optimized at $f_{\overline{\omega}}$=0.05 (85%)

Anomalous hybrid coupling dominates → off-shell contribution can be important

# Borel sum rules for **b1**$[0^-(1^{--})]$ state

- Generalized spectral sum rules for $\bar{\text{b}}_1$ state

$$f^T_\mp = \left\{ \frac{1}{m^2_\mp} e^{m^2_\mp/M^2} \left( -\mathcal{W}^{\rm subt.}_M \left[\Pi^{\rm ope}_\mp(k^2)\right] + \left(c^{\rm h\bar{y}.}_\mp\right)^2 \mathcal{W}^{\rm subt.}_M \left[\Pi^{\rm h\bar{y}.}_\mp(k^2)\right] \right) \right\}^{\frac{1}{2}} \quad c^{\rm h\bar{y}.}_\mp = \left\{ \left( \mathcal{W}^{\rm subt.}_M \left[\Pi^{\rm ope}_\mp(k^2)\right] + \left(f^T_\mp\right)^2 m^2_\mp e^{-m^2_\mp/M^2} \right) \Big/ \mathcal{W}^{\rm subt.}_M \left[\Pi^{\rm h\bar{y}.}_\mp(k^2)\right] \right\}^{\frac{1}{2}}$$




Borel curve for coupling is analyzed in proper parameter set **[αa=1.0 βb=0.5]**

$\bar{\text{b}}_1$ pole residue $f_{\bar{\text{b}}_1}$ is stable even at $c_{\bar{\text{b}}_1} = 0.0$

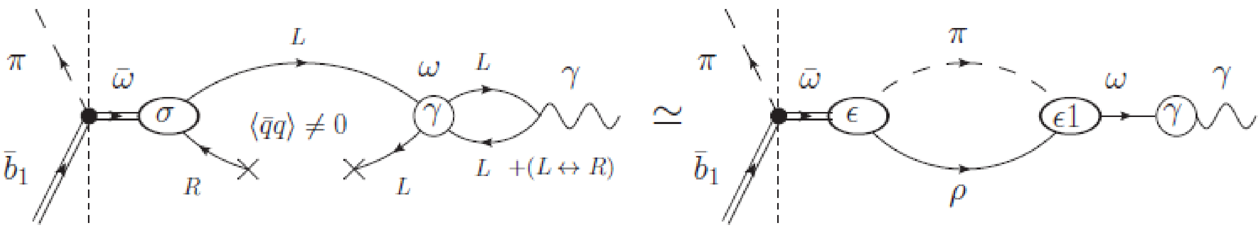
Hybrid coupling $c_{\bar{\text{b}}_1}$ is unstable and the accidental stability is obtained at $f_{\bar{\text{b}}_1}$=0.1 (60%)

Pole residue dominates and mass number agrees → **$\bar{\text{b}}_1$ state can be identified as b1(1235)**

# $\Gamma(\mathbf{b_1} \rightarrow \pi\bar{\omega} \rightarrow \pi\gamma)$

- Two possible channel for final $\gamma$ state

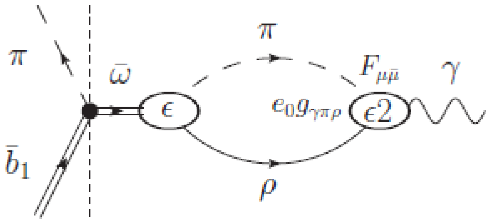
Final photon state via ω(782) (VMD channel)




$$\mathcal{L}^{\epsilon 1}_{\omega\pi\rho} = \frac{g_{\omega\pi\rho}}{2}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}\omega_{\mu\bar{\mu}}\partial_\alpha\pi^a\rho^a_{\bar{\alpha}} = -g_{\omega\pi\rho}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}\omega_{\bar{\mu}}\partial_\alpha\pi^a\partial_\mu\rho^a_{\bar{\alpha}}$$

If the whole legs are on mass-shell, the vertex would become similar with ε1 (**VMD**)

Final photon state from the hybrid direct after pion breaking (direct channel)




$$\mathcal{L}^{\epsilon 2}_{\gamma\pi\rho} = \frac{e_0 g_{\gamma\pi\rho}}{2m_\rho}\epsilon^{\mu\bar{\mu}\alpha\bar{\alpha}}F_{\mu\bar{\mu}}\partial_\alpha\pi^a\rho^a_{\bar{\alpha}} \simeq \frac{e_0 g_{\gamma\pi\rho}}{m_\rho}F_{\mu\bar{\mu}}\bar{\omega}^{\mu\bar{\mu}}$$

The most of phase space in loop is off mass-shell
→ direct channel can be important

# Summary

- **b**$_1$ can mixes with ω In local tensor representation
- In chiral symmetry broken phase, $\omega[0^-(1^{--})]$ like state can be obtained from **b**$_1$ after pion breaking
- The $\omega[0^-(1^{--})]$ like intermediate state looks like the hybrid state of π-ρ mesons
- Loop structure of the intermediate state can allow direct photon coupling channel