Structure of pion and kaon from lattice QCD

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In collaboration with:

R. Briceno, R. Edwards, C. Egerer, J. Karpie,
K. Orginos, J. Qiu, D.Richards, R. Sufian

Quark Confinement and Hadron Spectrum
Maynooth, Ireland, 5th August, 2018

Topics

- Progress on our lattice calculation of pion and kaon PDFs

[work done in collaboration with: Colin Egerer, Joe Karpie, Kostas Orginos, Jianwei Qiu, David Richards, Raza Sufian]

- Calculation of pion electromagnetic form factor at high momentum transfer

[work done in collaboration with: Raul Briceno, Robert Edwards, David Richards]

Motivation

- “Light-like” separated hadonic tensor-

  K-F Liu et al Phys. Rev. Lett. 72 1790 (1994), Phys. Rev. D62 (2000) 074501

  A Chambers et.al (2017) 1703.01153

- Position-space correlators: V. M. Braun, D. Müller, EPJ (2008)

- Quasi PDF using LaMET:

  X. Ji, Phys.Rev.Lett. 110, (2013), C Alexandrou et.al. (2018) 1803.02685

  J.-W. Chen et.al. (2018) 1803.04393

- Pseudo PDF :

  A. Radyushkin Phys.Lett. B767 (2017), K. Orginos et. al. (2017) 1706.05373

- New concept based on QCD factorisation– “Good lattice cross-sections”

  Y.-Q. Ma J.-W. Qiu (2014) 1404.6860, Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018

PDFs from "Good" Lattice Cross-sections

"Good" lattice cross sections [Ma & Qiu, Phys.Rev.Lett. 120 (2018) no.2, 022003]

If we can form single hadron matrix elements of renormalizable nonlocal operators such that -

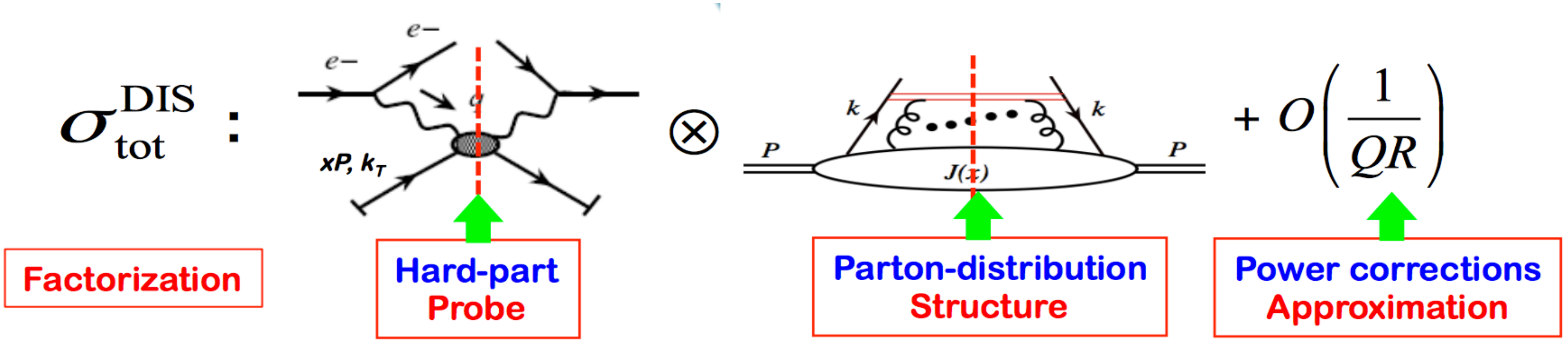
(1) it is calculable in lattice- QCD with an Euclidean time,

$$\sigma_n(\omega, \xi^2, P^2) = \langle P | T\{\mathcal{O}_n(\xi)\} | P \rangle$$

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}\, Z_{j_2} j_1(\xi)\, j_2(0)$$

(2) it has a well-defined continuum limit (UV finite) at the lattice spacing zero limit,

(3) it has the same and factorizable logarithmic collinear (CO) divergences as that of PDFs



the single hadron matrix element can be factorized into PDFs to all orders IR-safe hard coefficients with controllable power corrections, provided ξ^2 is sufficiently small,

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x}\, f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

Ioffe time: $\omega = P.\xi$

B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

$$f_{\bar a}(x,\mu^2) = -f_a(-x,\mu^2)$$

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}\, Z_{j_2} j_1(\xi)\, j_2(0)$$

What currents we can use?

$$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$$

$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\overline{\psi}_q \psi_q](\xi)\, [\overline{\psi}_q \psi_q](0)\,,$$

$$\mathcal{O}_V(\xi) = \xi^2 Z_V^2 [\overline{\psi}_q \not\xi \psi_q](\xi)\, [\overline{\psi}_q \not\xi \psi_q](0)\,,$$

$$\mathcal{O}_{\tilde{V}}(\xi) = -\frac{\xi^4}{2} Z_V^2 [\overline{\psi}_q \gamma_\nu \psi_q](\xi)\, [\overline{\psi}_q \gamma^\nu \psi_q](0)\,,$$

$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\overline{\psi}_q \not\xi \psi_{q'}](\xi)\, [\overline{\psi}_{q'} \not\xi \psi_q](0)\,, \ldots\,,$$

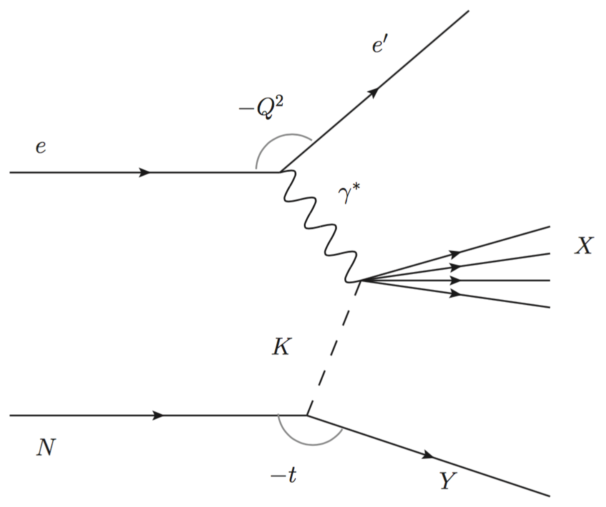
$$\mathcal{O}_{\mu\nu}(\xi) = \xi^4 Z_V^2 [\overline{\psi}_q \gamma_\mu \psi_q](\xi)\, [\overline{\psi}_q \gamma_\nu \psi_q](0)$$

Including flavor changing and gluonic currents with any single hadronic state

Four-point correlation function is numerically expensive

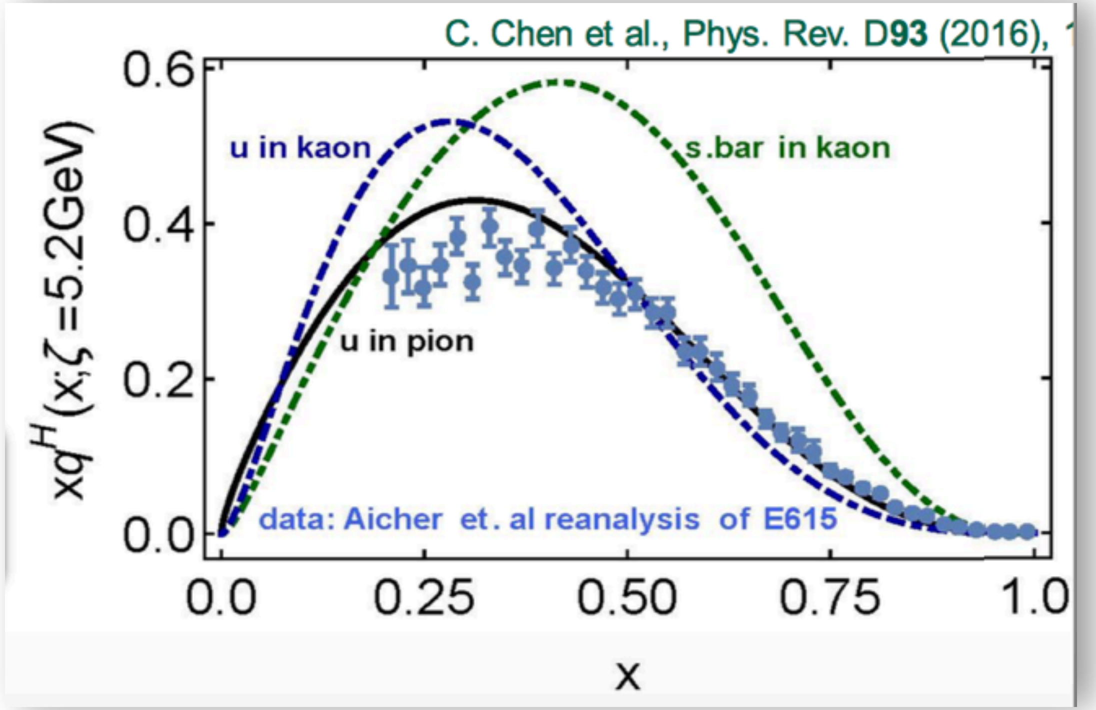
Motivation for pion and Kaon PDF calculation

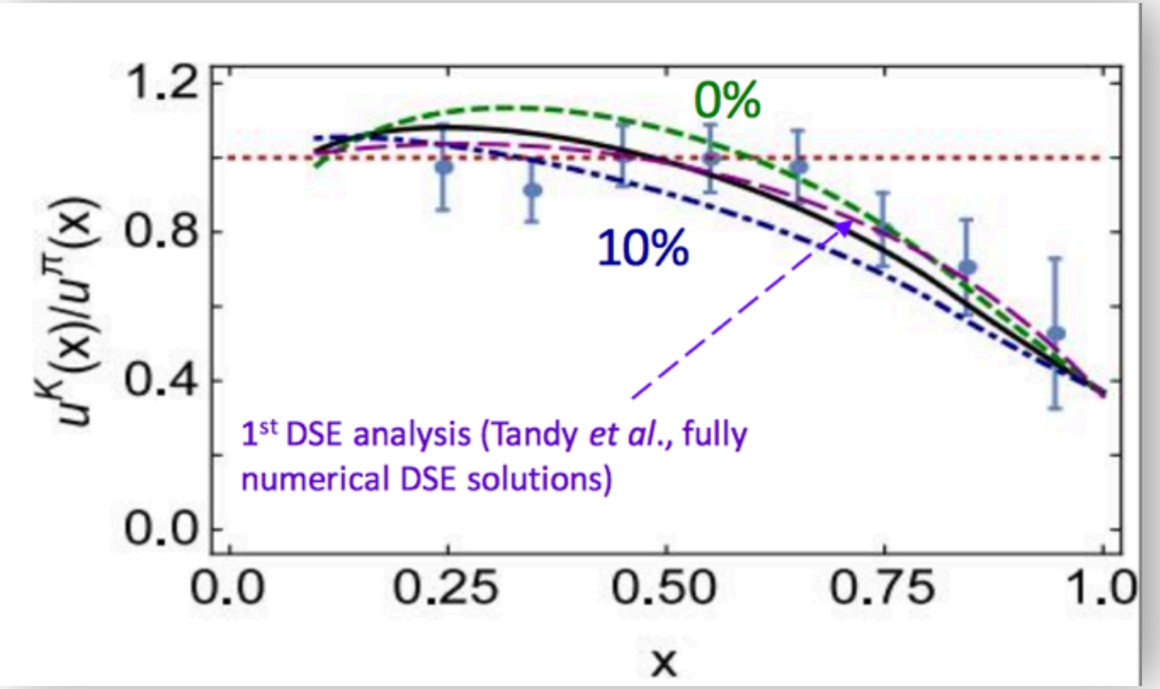
- Large-x behaviour of pion valence distribution an unresolved problem- $(1\text{-}x)^2$ or $(1\text{-}x)^1$?
- C12-15-006 experiment at JLab to explore large-x behavior
- Limited Old data from Drell-Yan for Kaon PDF
- JLab approved pion-TDIS will also measure Kaon structure functions for the first time ever at small x
- Long-searched-for Sullivan process for accessing the kaon structure function



EIC will add large (x,Q^2) landscape for both pion and kaon!

- Recent DSE calculations of pion and Kaon PDFs and ratios





First-principle lattice QCD calculation of needed –
First calculation of Kaon PDF on the lattice

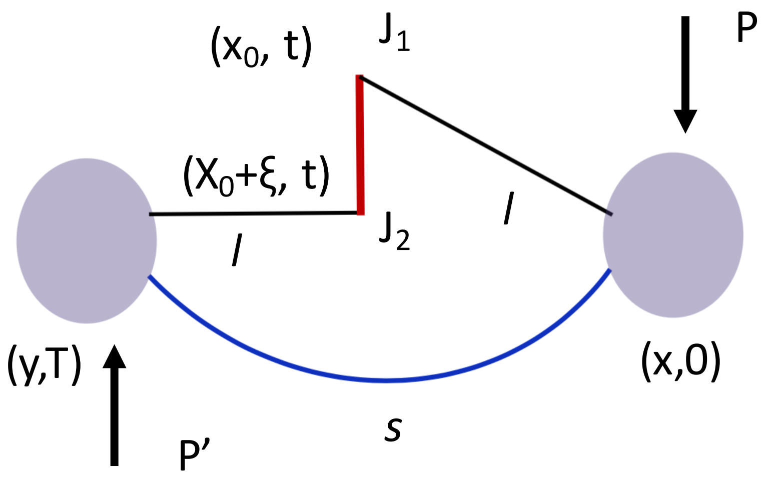
Lattice simulation

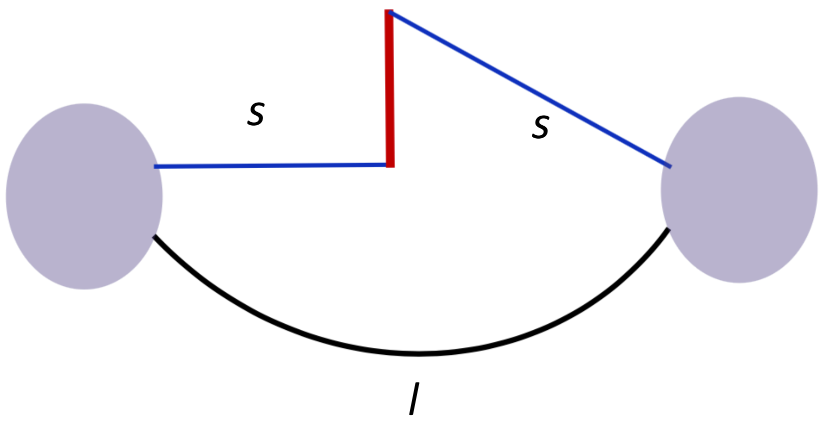
- Need calculation of four point functions, with 'red' being different heavier quarks [similar technique by Bali et. al. 1807.03073]

- Isoclover configurations 32^3 x 96, lattice spacing 0.127 fm, pion mass around 405 MeV

- Current renormalisations known from previous calculation, arXiv:1611.07452

- Two-exponential fits for ratios of 4pt and 2pt functions





ξ could be on/off axis

Preliminary lattice results for pion

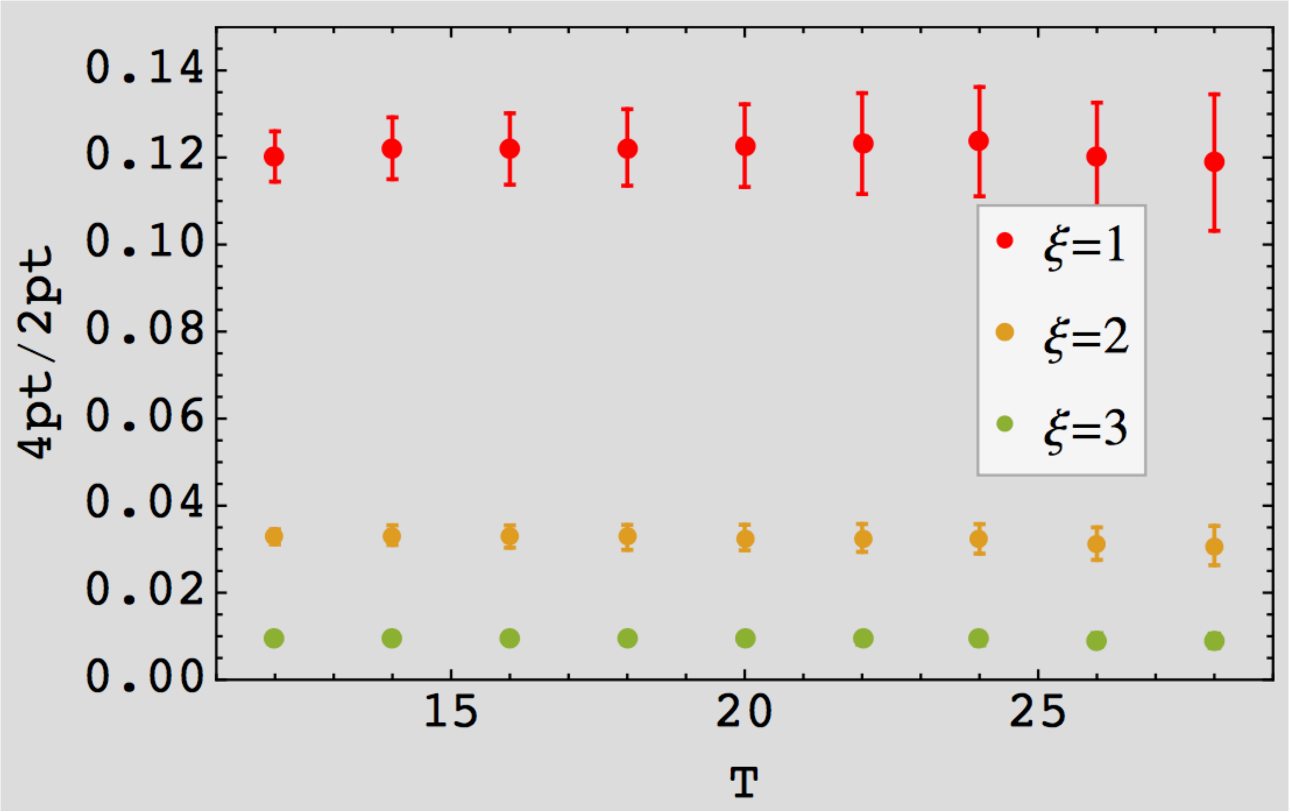
- Analysis shown here on isoClover with 450 Configurations

- Momentum smearing used for higher momentum

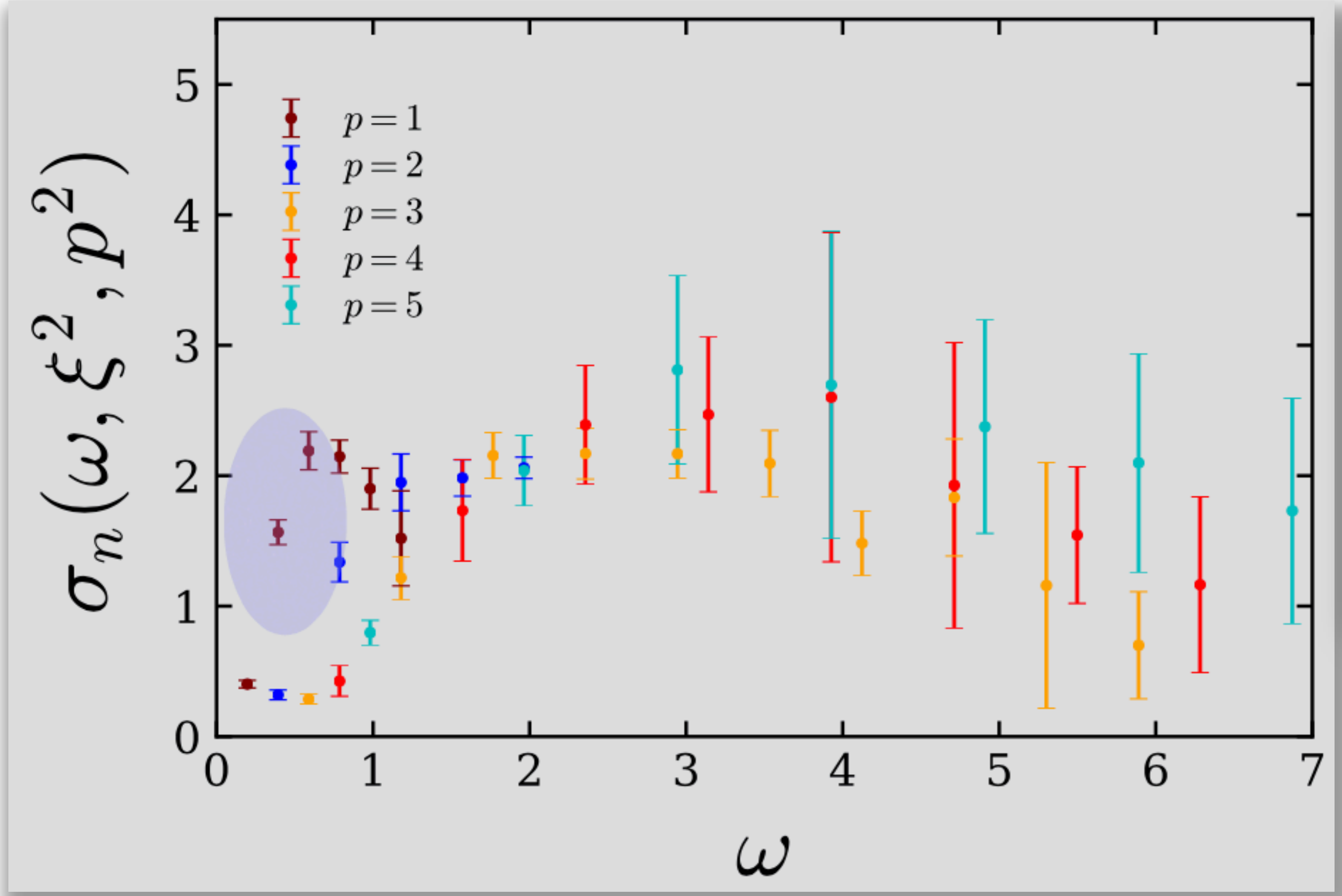
[Gunnar S. Bali, et al (PRD 2016)]

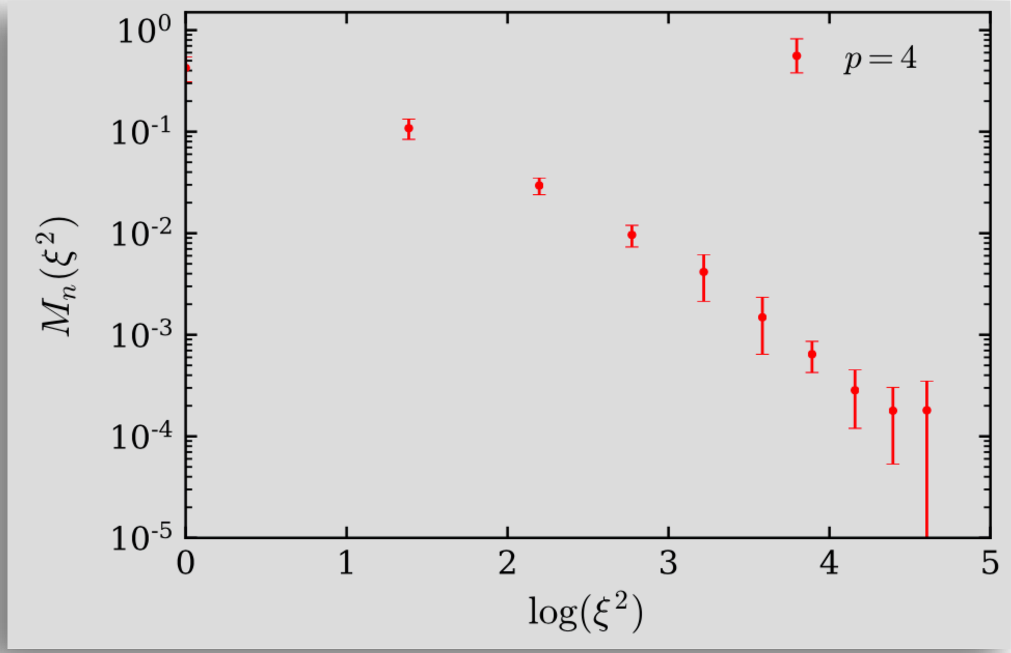
V-A matrix element

T is the source sink separation



Preliminary lattice results for pion





- p=1 (0.3 GeV) data deviates
- 1/3rd statistics shown here

V-V matrix element

"Global" fit for lattice data - technique

$$\sigma_n(\nu, z^2, p^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(x\nu, z^2, x^2p^2, \mu^2) + O(z^2\Lambda^2_{\rm QCD})$$

Many such lattice "cross-sections" To be calculated

We use

$$f_{\bar{a}}(x,\mu^2) = -f_a(-x,\mu^2)$$

to obtain valence quark and anti-quark distribution

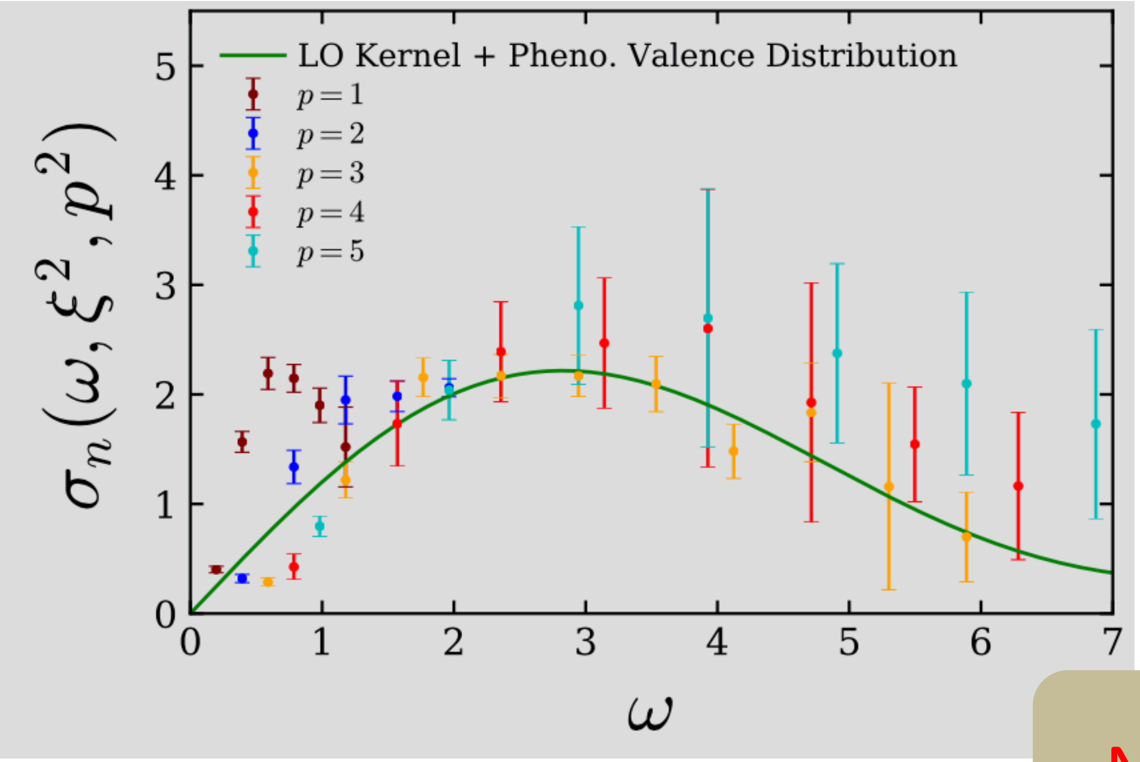
Perturbative Kernel known, Oscillatory [being calculated at LO and NLO]

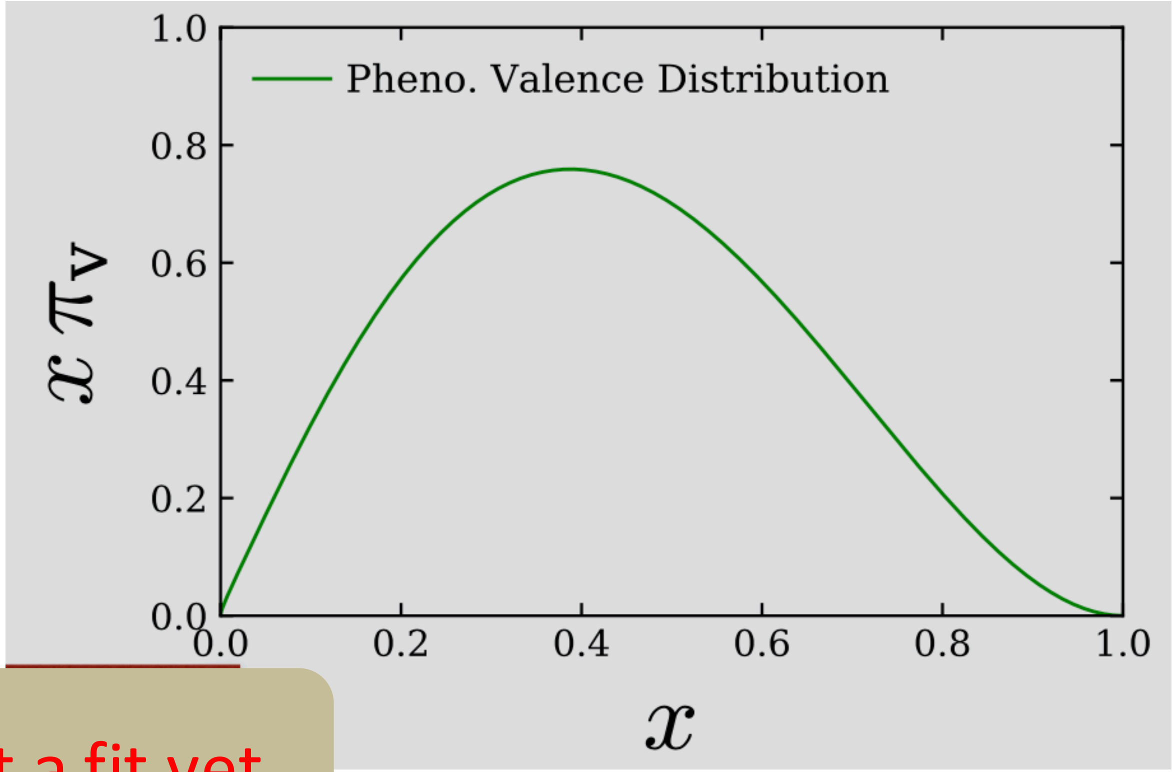
- Need many P.ξ data to have a good global fit for f(x)
- A model fit form [JAM Collaboration, arXiv:1804.01965]

- Regge : a = - 0.5
- Quark counting : b > 2

$$f_{abcd}(x) = N_{abcd}\; x^a(1-x)^b(1 + c\,\sqrt{x} + d\,x\,)$$

Preliminary lattice results for pion





Not a fit yet

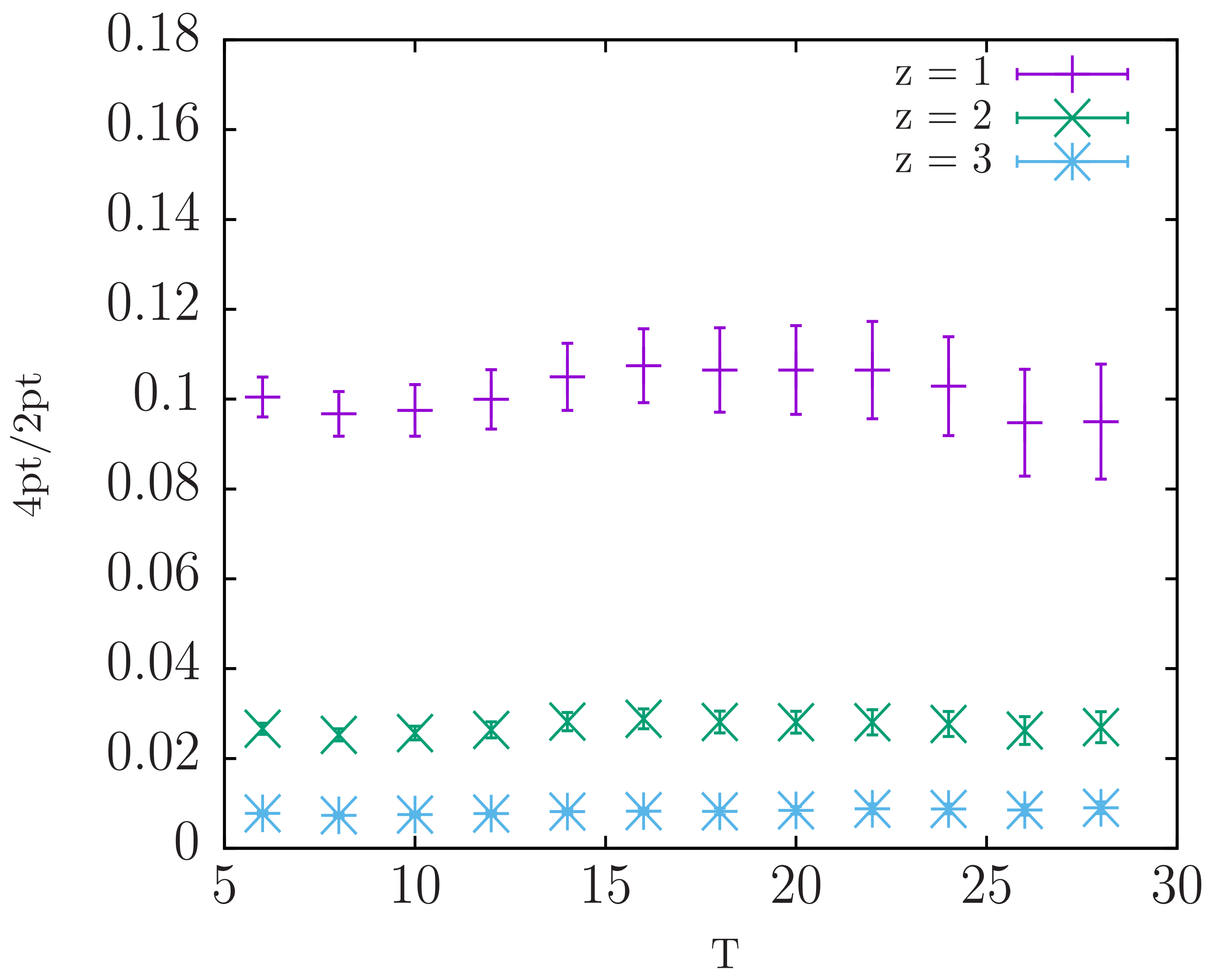
V-V matrix element

- p=1 (0.3 GeV) data deviates
- 1/3rd statistics shown here

For details: talk by Raza Sufian at Lattice 2018

(Very) preliminary lattice results for Kaon

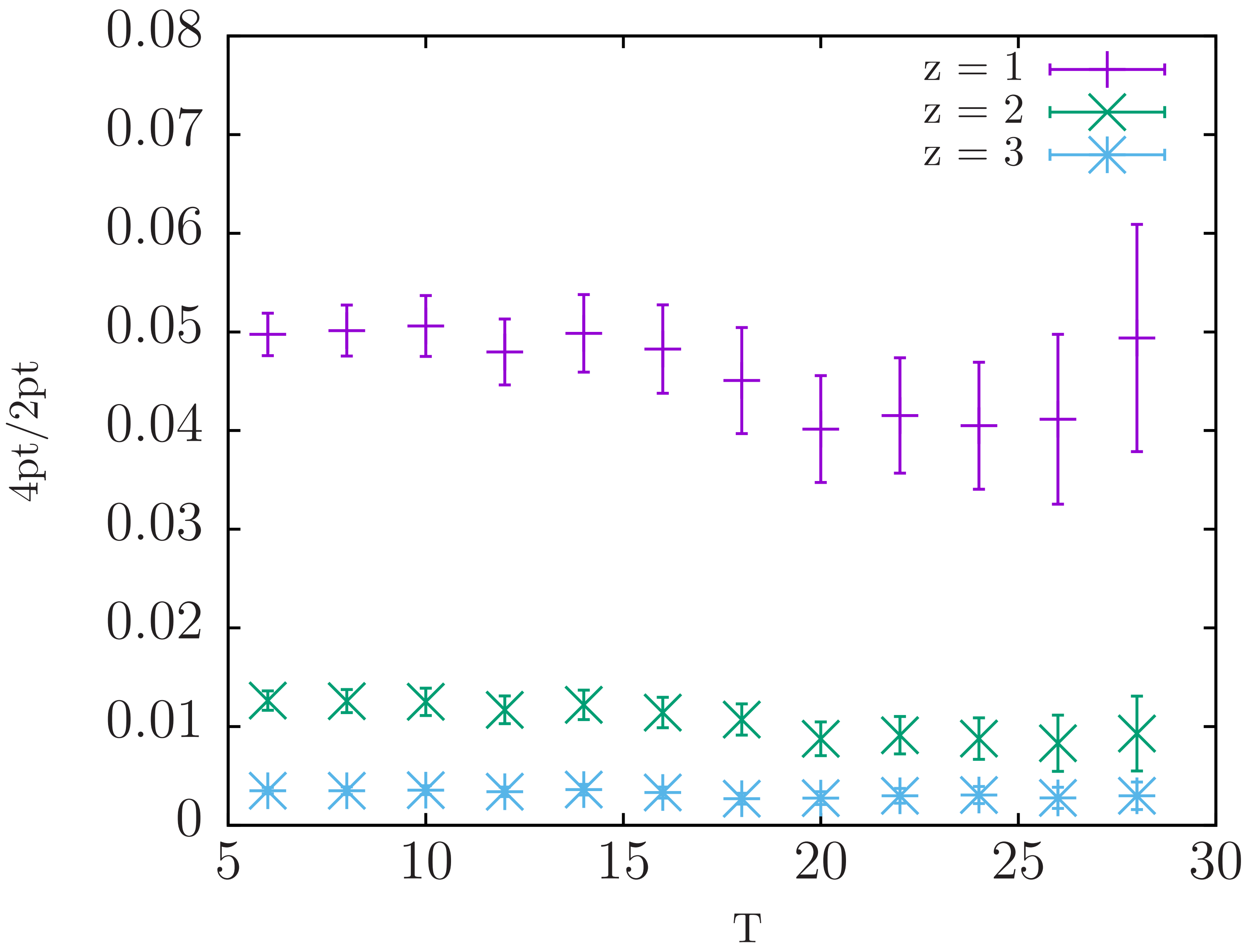
Calculation for Kaon PDF looks encouraging too



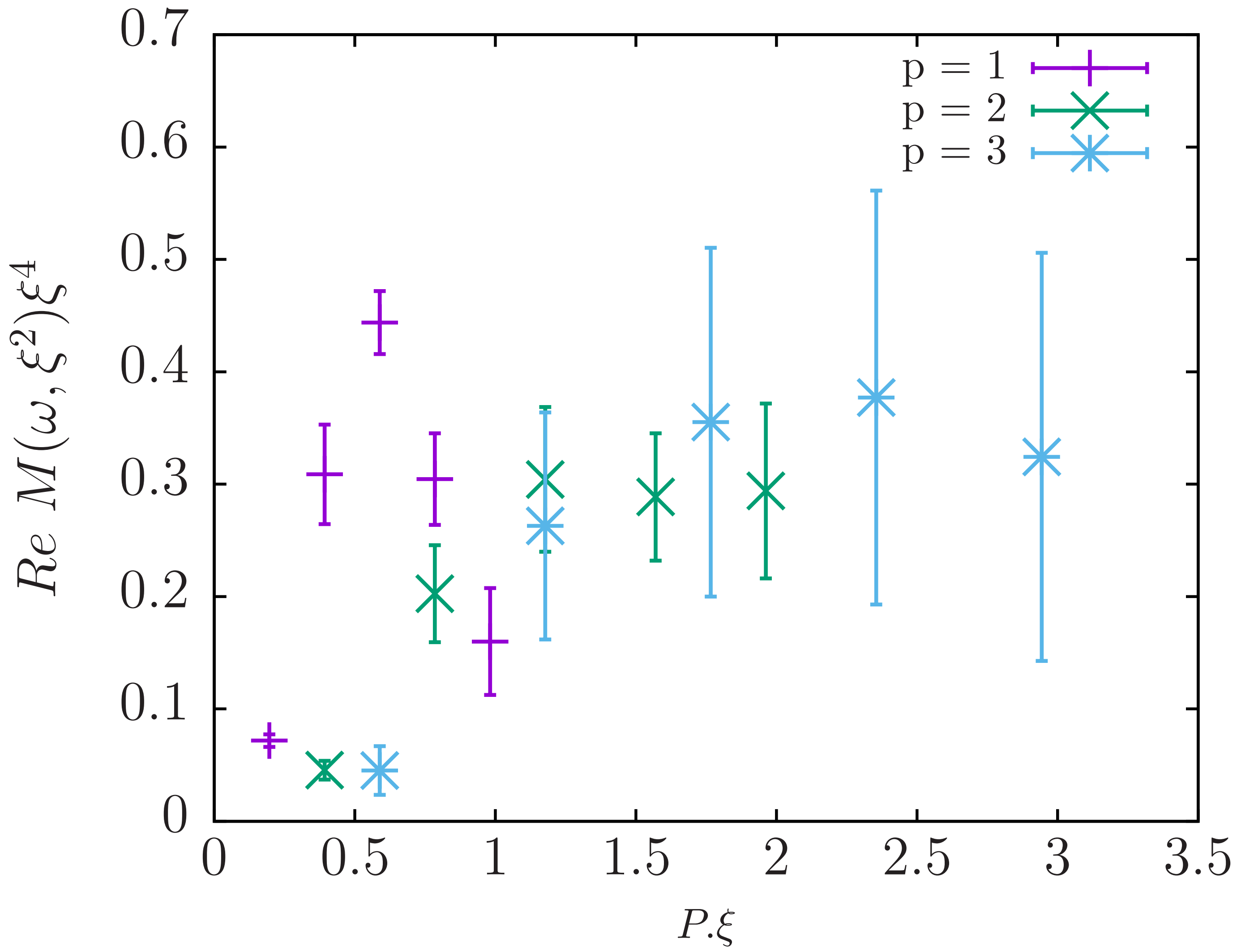
- Comparison between:

(J$_1$ = V, J$_2$ = A and vice versa) and different ξ = z

- $P_z = 1$ (~0.3 GeV)
- Real part of the 4pt correlator
- Strange quark is the active quark
- No signal after ξ = 6



- Comparison between: (J_1 = V, J_2 = A and vice versa) and different ξ = z
- *P_z= 1 (~0.3 GeV)*
- Real part of the 4pt correlator
- Light quark is the active quark
- No signal after ξ = 6



- Need lot more P.ξ data points for a realistic f(x) fit
- And with higher P values
- Real part of the V-A 4pt correlator multiplied by vector current renormalisation and a factor ξ^4
- Strange quark is the active quark

For details: talk by Bipasha Chakraborty at Lattice 2018

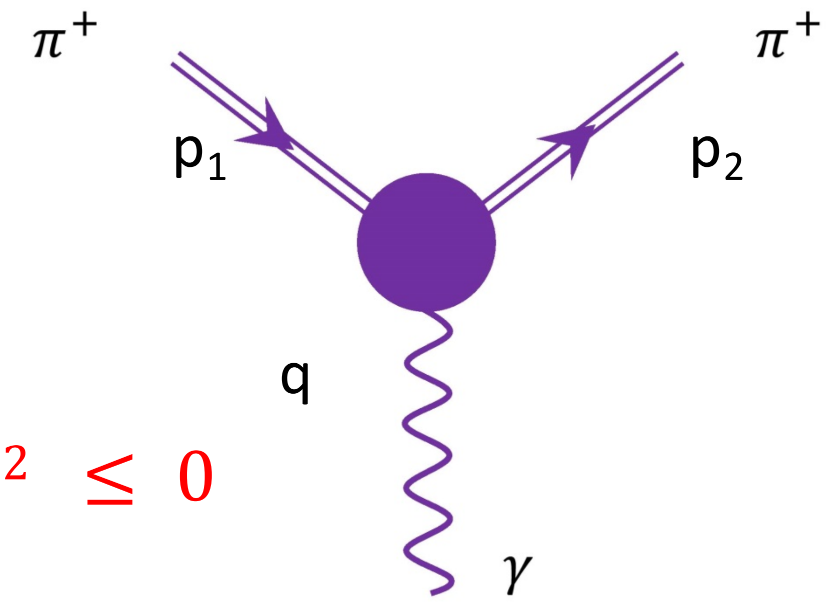
Ongoing…

- We have demonstrated that this method works
- Higher momenta, more statistics, calculation at different lattice spacings, quark masses and volumes - ongoing
- Methods to tackle ill posed inverse Fourier Transform being developed [ref. talk by Savvas Zafeiropoulos]
- In future, look into finite volume effects [ref. R. Briceno et. al. Phys. Rev. D 98, 014511 (2018)], although expect to be small
- Generalisation of other methods – pseudo, quasi, T33
- We can also get polarised PDFs, moments

Pion electromagnetic form factor at high Q^2 from lattice QCD

Definition

Simplest hadrons – pion, kaon



Space like "q":

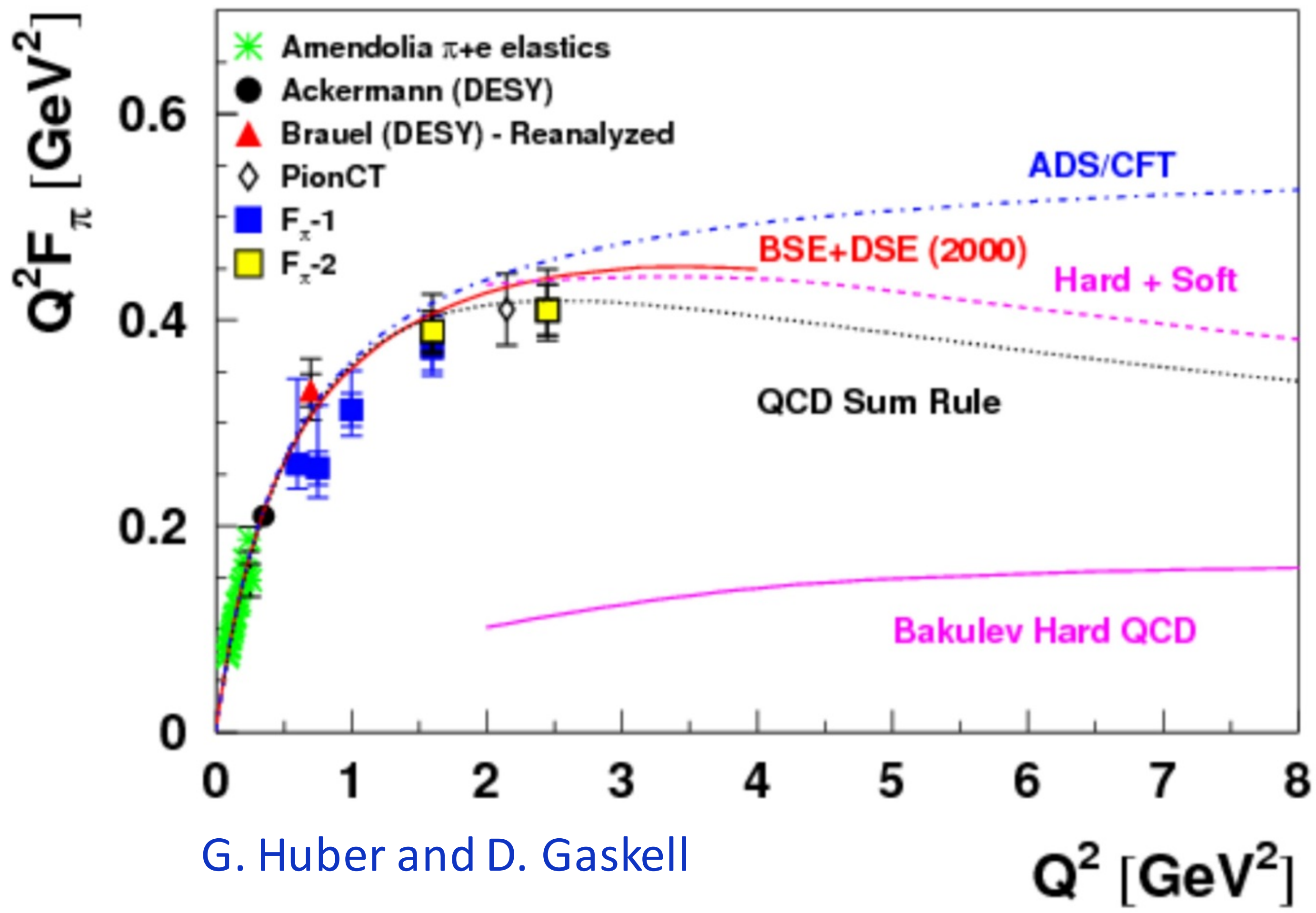
$$q^2 = (p_2 - p_1)^2 \leq 0$$

$$Q^2 = -q^2$$

$$\langle \pi^+(\vec{p}\,')|j^\mu|\pi^+(\vec{p})\rangle = (p + p')^\mu F_\pi(Q^2)$$

(in units of 'e')

Interplay between hard and soft scales



G. Huber and D. Gaskell

Hard tail ($Q^2 \rightarrow \infty$) from pQCD:

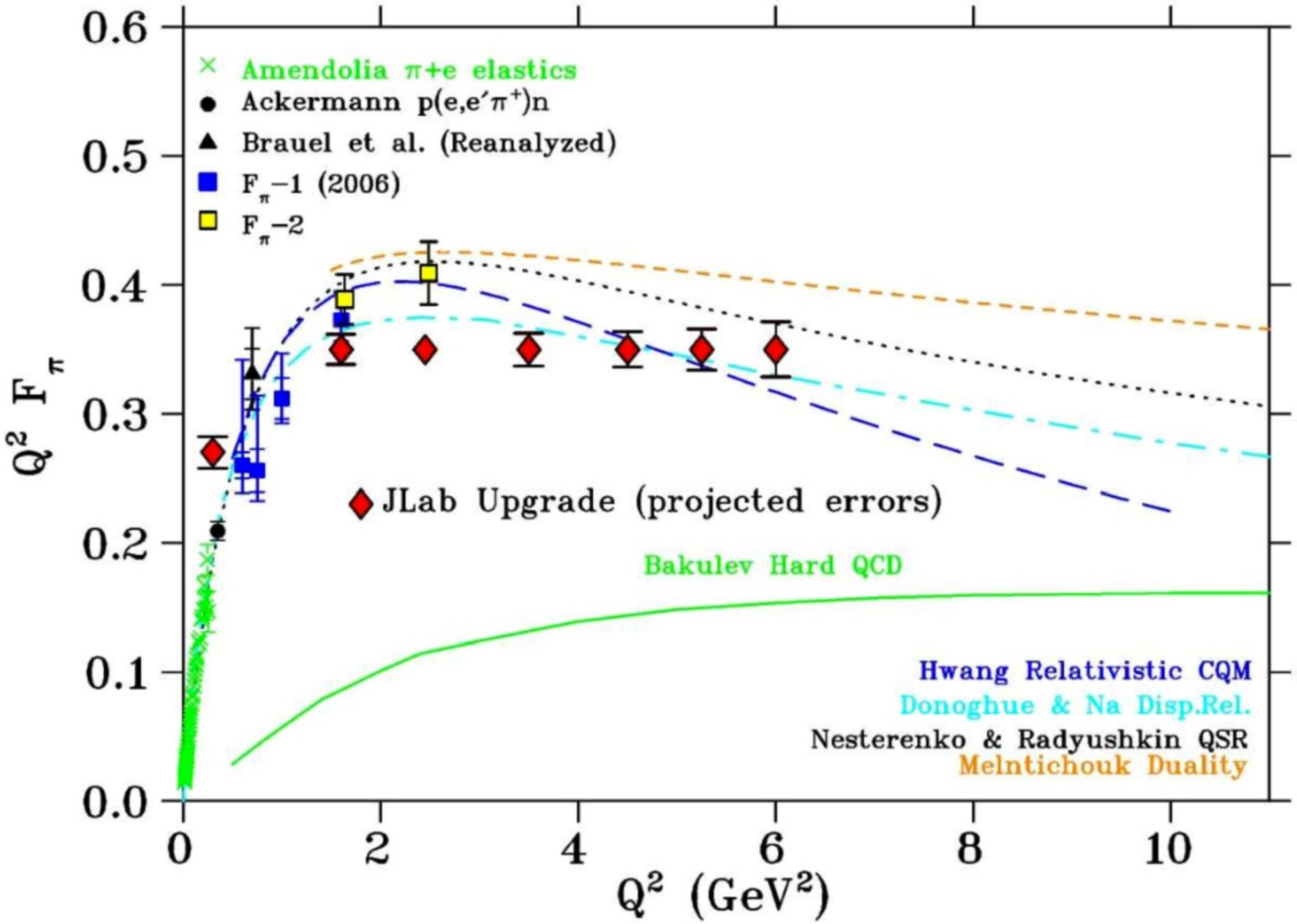
$$F_\pi(Q^2) \rightarrow \frac{16\pi\alpha_s(Q^2) f_\pi{}^2}{Q^2}$$

G. P. Lepage, S.J.Brodsky, Phys. Lett. 87B(1979)359

Soft part ($Q^2 < 1$ GeV^2):
vector meson dominance
with $F_\pi(0) = 1$,
data fits well

Need better understanding of the transition to the asymptotic region

JLAB 12 GeV upgrade

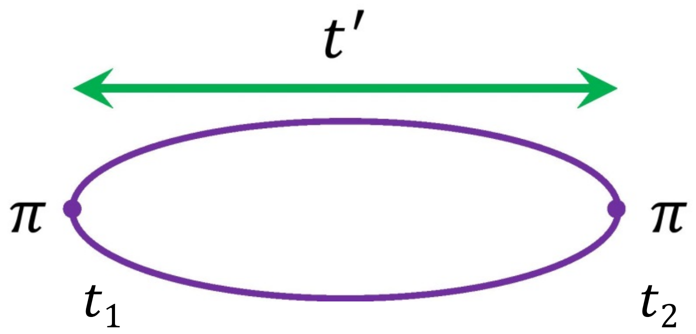


G. Huber and D. Gaskell

F_π measurements
at $Q^2 \sim 6-10$ GeV^2:
E12-06-101 at
JLAB Hall C,
For EIC, $Q^2 \sim 30$ GeV^2

Can we get some insight from first principles lattice QCD calculations to the question - where does the transition to pQCD happen?

Two-point correlator construction



$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

- Basis of operators

$$\mathcal{O} \sim \bar{\psi}\Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D}\psi$$

- Optimized operator for state $|n>$

$$\Omega_\mathfrak{n}^\dagger = \sum_i w_i^{(\mathfrak{n})}\mathcal{O}_i^\dagger$$

in a variational sense by solving generalized eigenvalue problem-

$$C(t)\, v^{(\mathfrak{n})} = \lambda_\mathfrak{n}(t)\, C(t_0)\, v^{(\mathfrak{n})}$$

- Diagonalize the correlation matrix – eigenvalues

$$\lambda_n(t) = \exp[-En(t - t_0)]$$

Two-point correlator construction

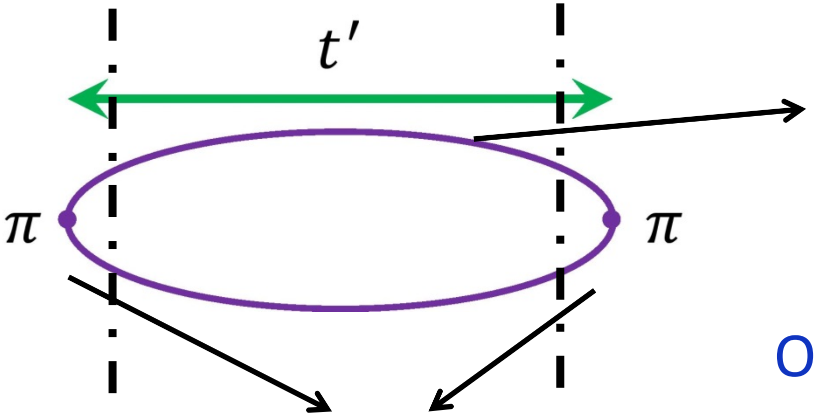
Correlator Construction: smearing of quark fields - 'distillation' with

$$\Box_{\vec{x}\vec{y}}(t) = \sum_{n=1}^{N_D} \xi^{(n)}_{\vec{x}}(t)\; \xi^{(n)\dagger}_{\vec{y}}(t)$$

Extraction of low lying hadron states

Meson creation operator :

$$\mathcal{O}^\dagger(\vec{p}) = \bar{\psi}_{\vec{x}} \Box_{\vec{x}\vec{y}}\, e^{-i\vec{p}\cdot\vec{y}}\, \mathbf{\Gamma}_{\vec{y}\vec{z}}\, \Box_{\vec{z}\vec{w}} \psi_{\vec{w}}$$



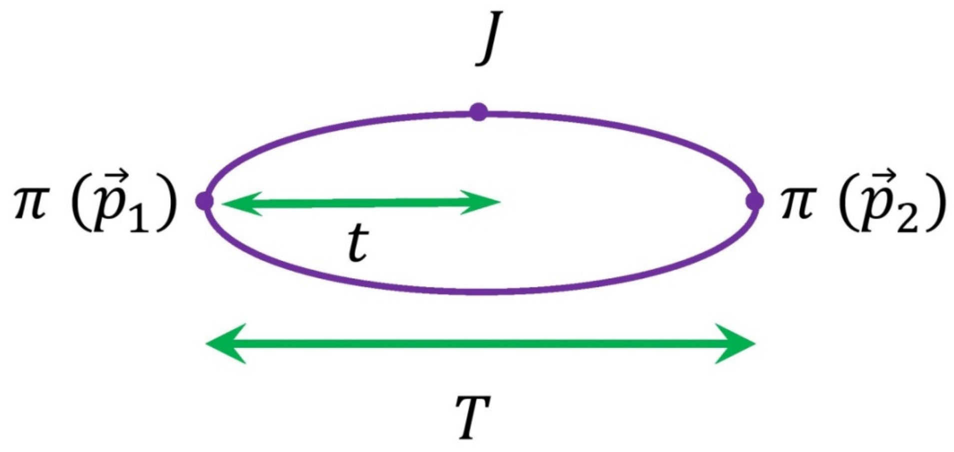
Parambulators by inverting the Dirac matrix

+

Operator construction with momentum projection

Form factor calculation

Need three-point correlator – calculated with Weighted operators



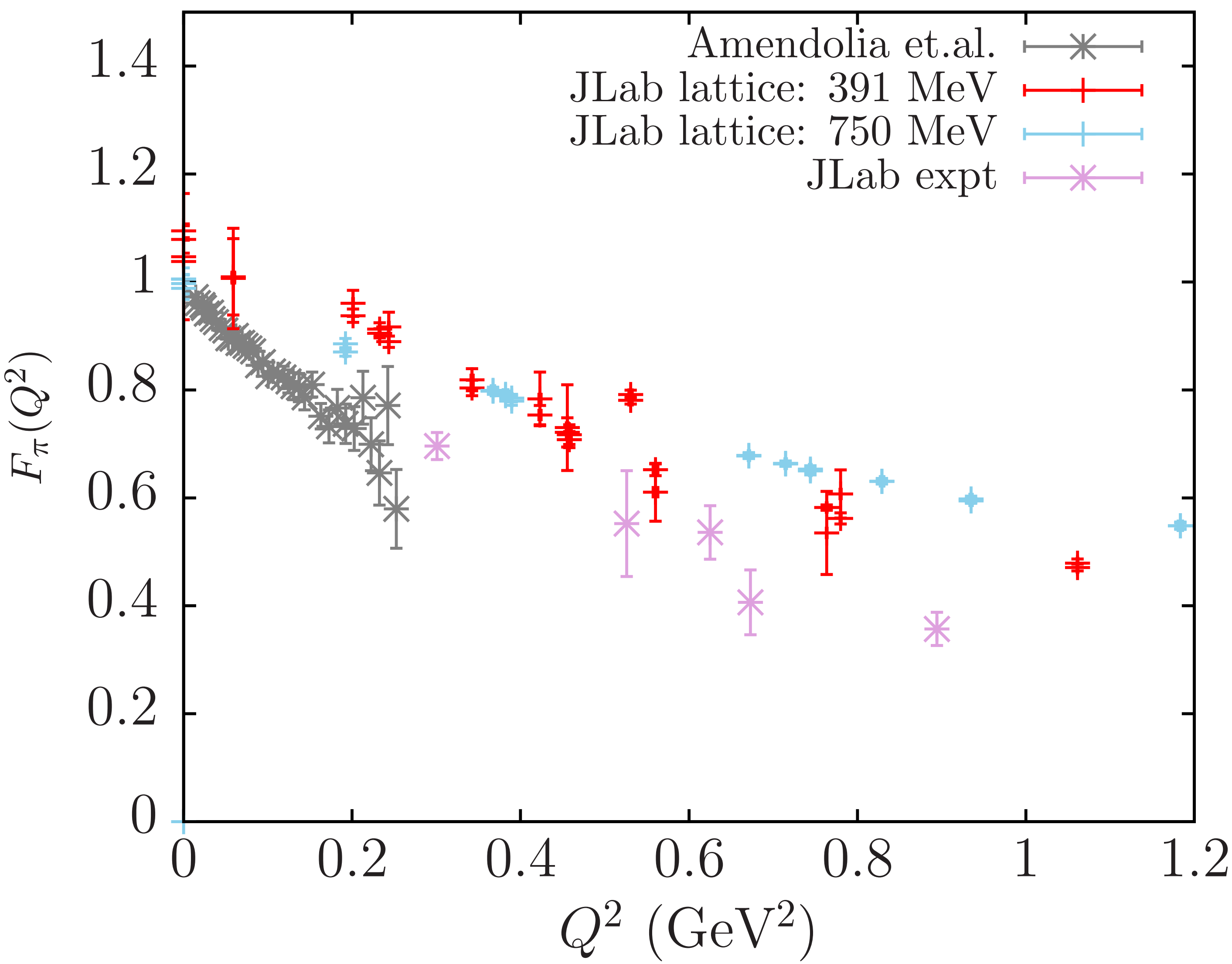
$$C_{\mathrm{f}\mu\mathrm{i}}(\Delta t, t) = \langle 0 | \mathcal{O}_{\mathrm{f}}(\Delta t)\, j_\mu(t)\, \mathcal{O}_{\mathrm{i}}^\dagger(0) | 0 \rangle$$

$$Z_V < \pi^+(p_2)|J^\mu{}_\pi(0)|\pi^+(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2)$$

Clover discretised fermion action

Z_V calculated using $F_\pi(q^2 = 0) = 1$

Pion electromagnetic form factor: up to $Q^2 = 1\ \text{GeV}^2$



JLAb 750 MeV: Phys.Rev. D91 (2015)

Anisotropy $\xi = 3.44$

$$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$$

$$a_s = 0.12\ \text{fm}$$

Pushing to higher Q^2 very challenging

Pion charge radius:

$$\langle r^2 \rangle \equiv -6 \frac{d}{dQ^2} F(Q^2) \Big|_{Q^2=0}$$

Parametrising Q^2 dependence $\quad Q^2 < 0.3\,\mathrm{GeV}^2$

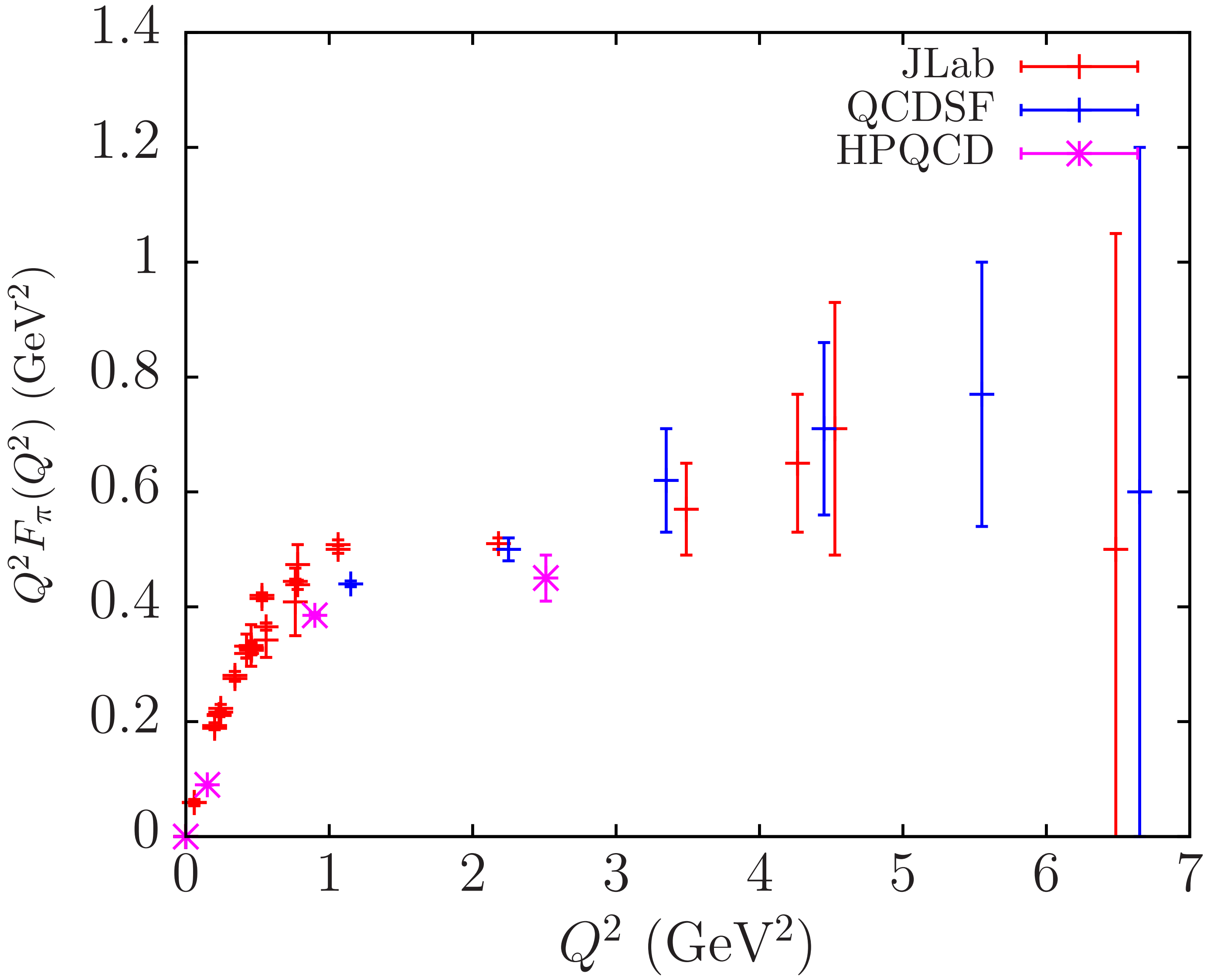
$$F_\pi(Q^2) = F(0)\, \frac{1}{1+Q^2/m^2}$$

Lightest vector meson mass

$m_\pi = 750$ MeV : 0.47(6) fm

$m_\pi = 390$ MeV : 0.55 (10) fm

Comparison of different lattice results:



Our result at 391 MeV

QCDSF, Phys.Rev. D96 (2017)

HPQCD, EPJ Web Conf. 175 (2018)

Z-expansion in progress…

Thank you!