

Studying the structure of few-hadron states

Raúl Briceño



Norfolk, VA [Home to ODU]



JLab, VA

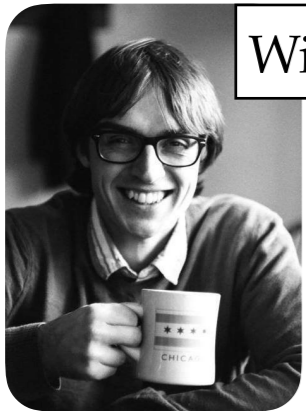
On-going work...



Hansen (CERN)



Baroni (South Carolina)



Wilson (Royal fellow / Trinity)



Ortega (W&M)

JLAB-TH

Relativistic, model-independent, multichannel $2 \rightarrow 2$ transition amplitudes in a finite volume

Raúl A. Briceño^{1,*} and Maxwell T. Hansen^{2,†}

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Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany*

(Dated: September 30, 2015)

We derive formalism for determining $2 + \mathcal{J} \rightarrow 2$ infinite-volume transition amplitudes from finite-volume matrix elements. Specifically, we present a relativistic, model-independent relation between finite-volume matrix elements of external currents and the physically observable infinite-volume matrix elements involving two-particle asymptotic states. The result presented holds for states composed of two scalar bosons. These can be identical or non-identical and, in the latter case, can be either degenerate or non-degenerate. We further accommodate any number of strongly-coupled two-scalar channels. This formalism will, for example, allow future lattice QCD calculations of the ρ -meson form factor, in which the unstable nature of the ρ is rigorously accommodated.

PACS numbers: 13.40.Gp,14.40.-n,12.38.Gc,11.80.Jy

Keywords: finite volume

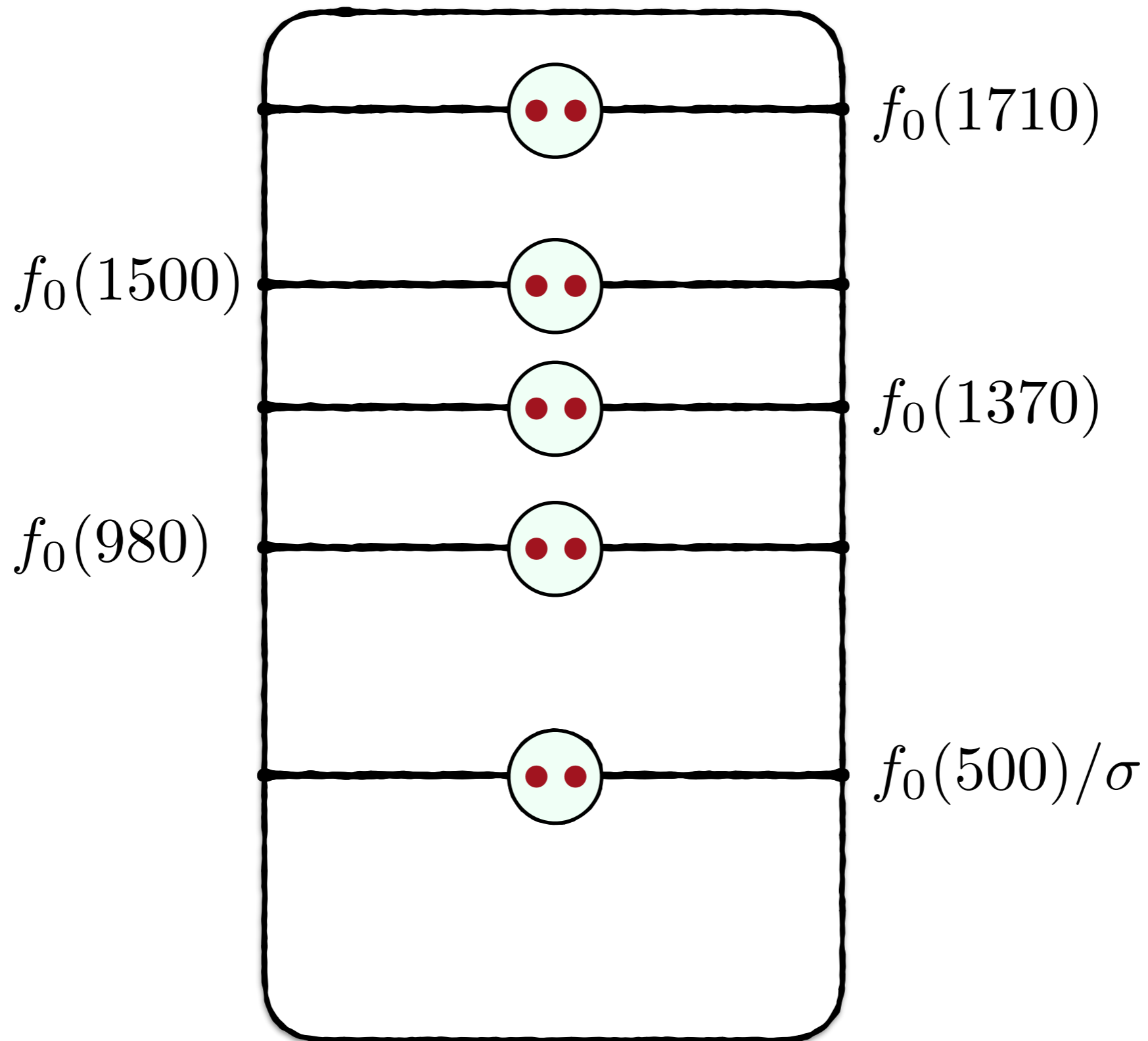
I. INTRODUCTION

Theoretical predictions of hadron structure are entering a new era. The precise determination of form factors for stable hadronic states is already well underway [1–4] and resonant form factor studies are not far behind. The first lattice QCD (LQCD) calculations of resonant $\mathcal{J} \rightarrow 2$ and $1 + \mathcal{J} \rightarrow 2$ transition processes appeared this year.¹ These studies considered $\gamma^* \rightarrow \pi\pi$ [5] and $\gamma^*\pi \rightarrow \pi\pi$ [6] transitions. In Ref. [6], the Hadron Collaboration determined the $\gamma^*\pi \rightarrow \pi\pi$ amplitude for a range of energies and for various virtualities of the photon. The resulting fit was analytically continued to the ρ -pole, thereby giving a first principles deter-

[hep-lat] 28 Sep 2015

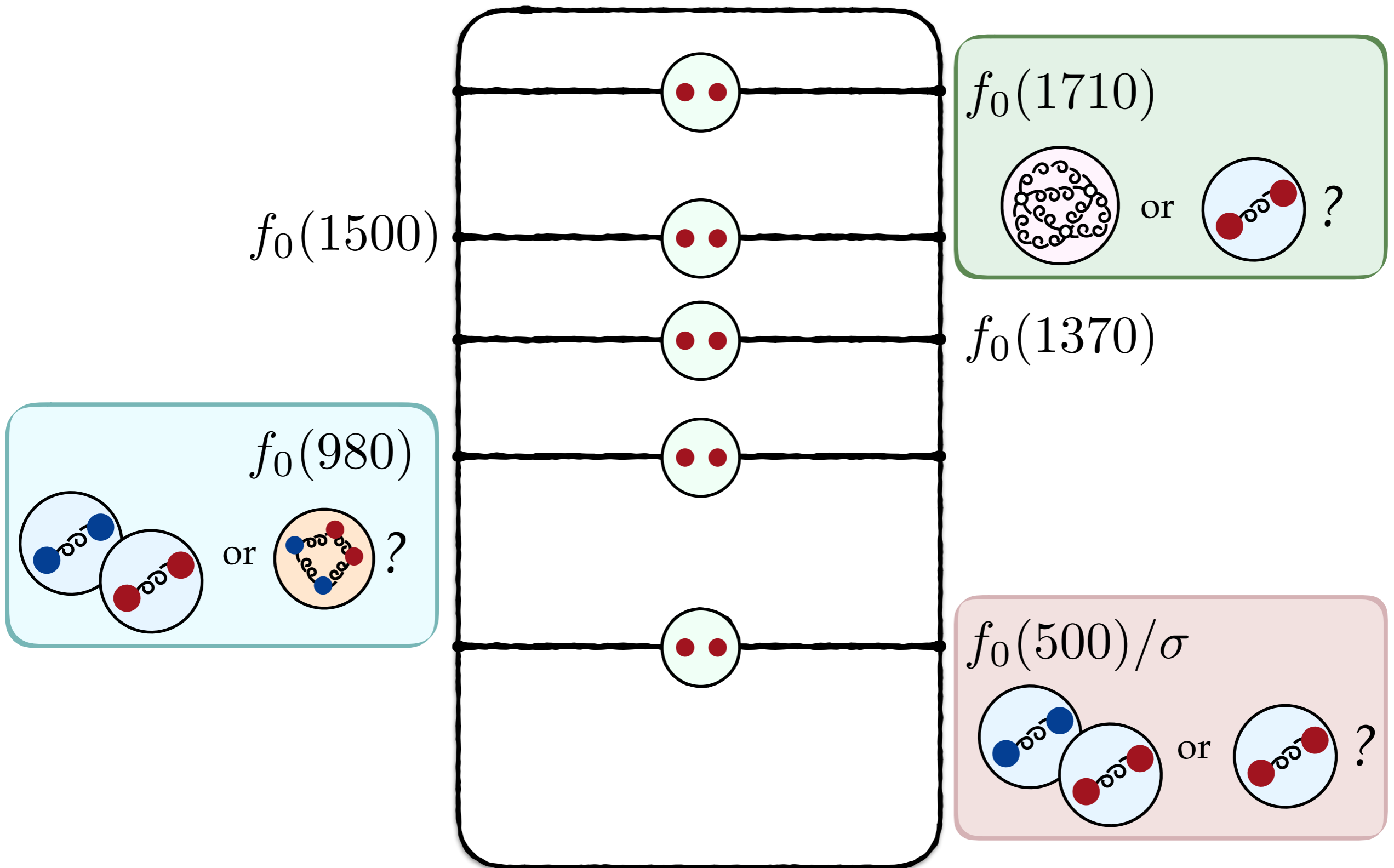
On the nature of states

Consider the 0^{++} channel...



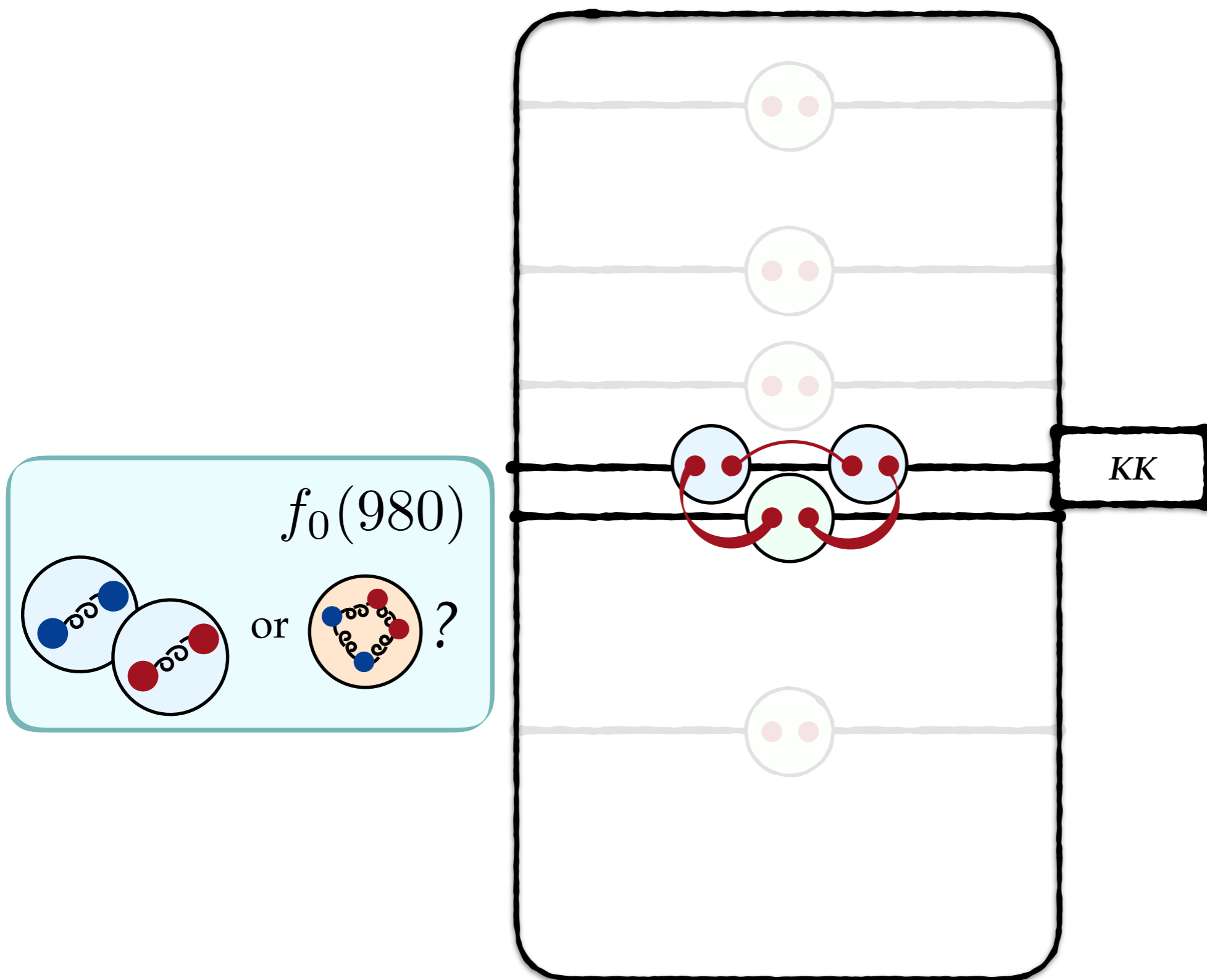
On the nature of states

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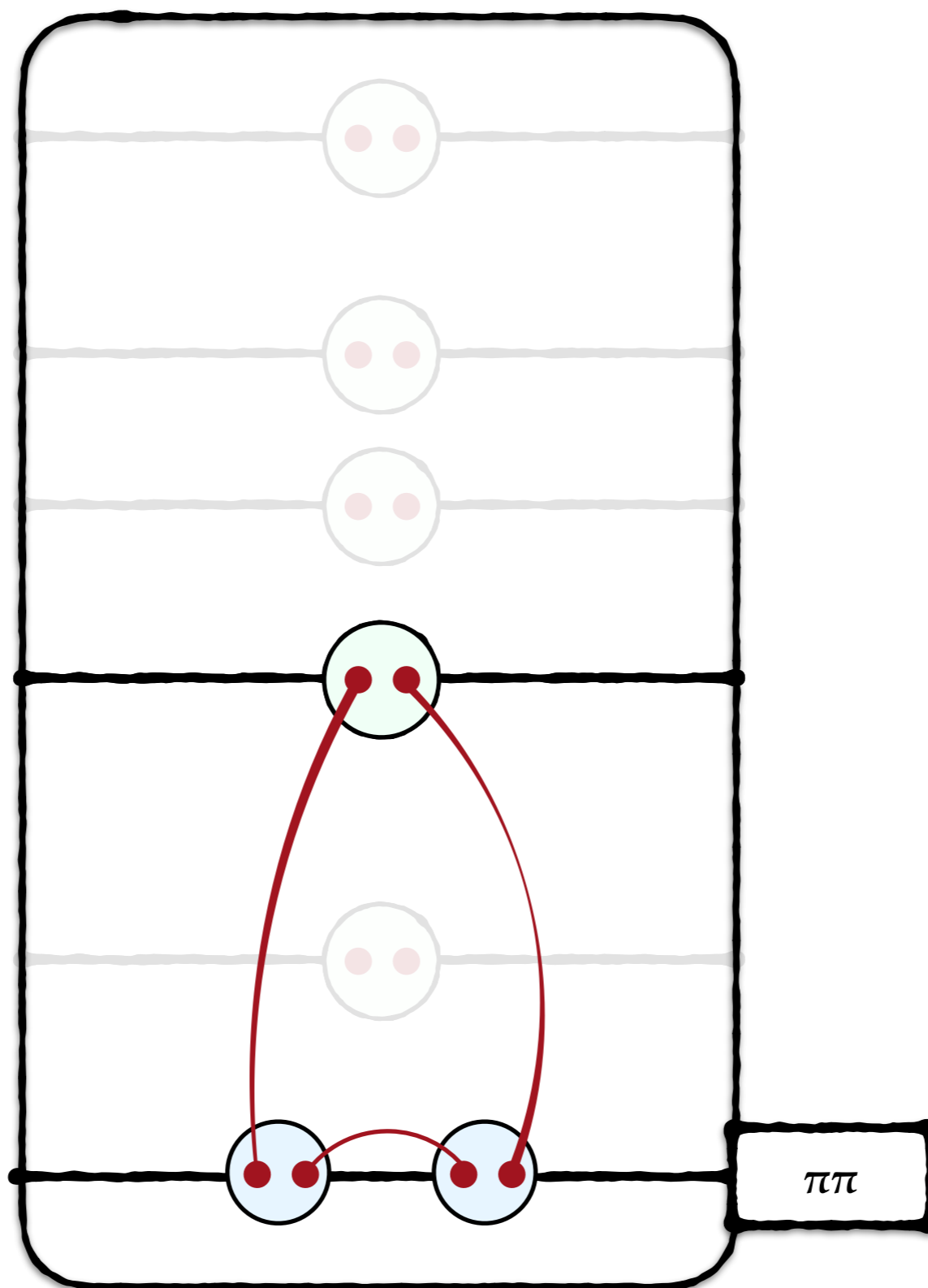
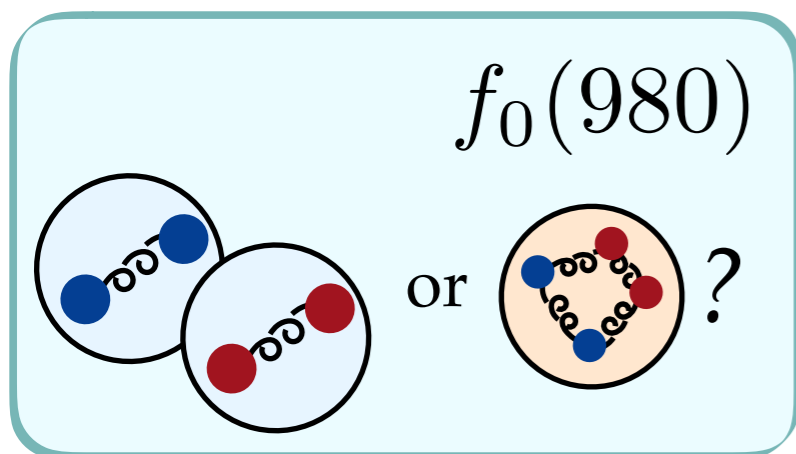
On the nature of states

Consider the 0^{++} channel...



On the nature of states

Consider the 0^{++} channel...



QCD spectroscopy

Amplitude analysis

Experiments

QCD

QCD spectroscopy

Amplitude analysis

GOAL:

Get insights to the governing patterns and rules of QCD from emergent phenomena

Observables to test our understanding:

- Production and decay
- Exotic states
- ...

Possible outcomes:

- Source of masses
- Role of glue
- Structure of excited states;
- ...

Experiments

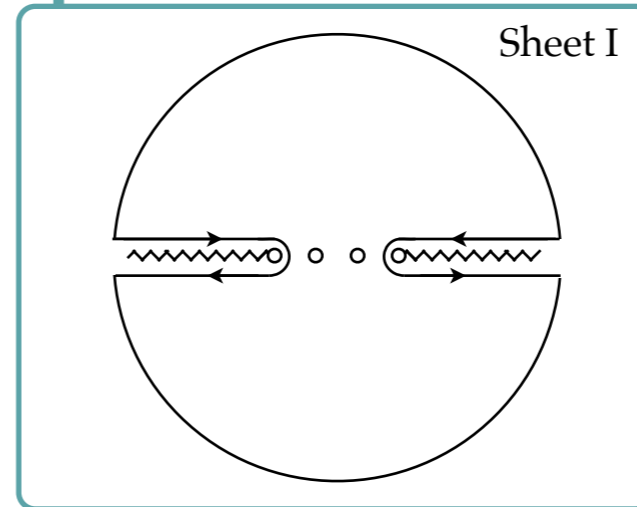
QCD

$$|n\rangle_{\text{QCD}} = c_0 \text{ (gluon ball) } + c_1 \text{ (quark-antiquark pair) } + c_2 \text{ (quark-gluon system) } + c_3 \text{ (quark-antiquark-gluon system) } + c_4 \text{ (quark-antiquark-gluon-gluon system) } + \dots$$

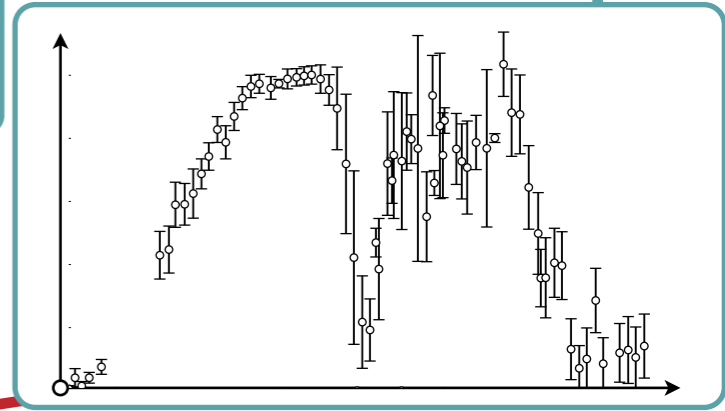
... perhaps there is a hierarchy [e.g. $c_0 > c_1 > c_2 > c_3 > c_4$]

QCD spectroscopy

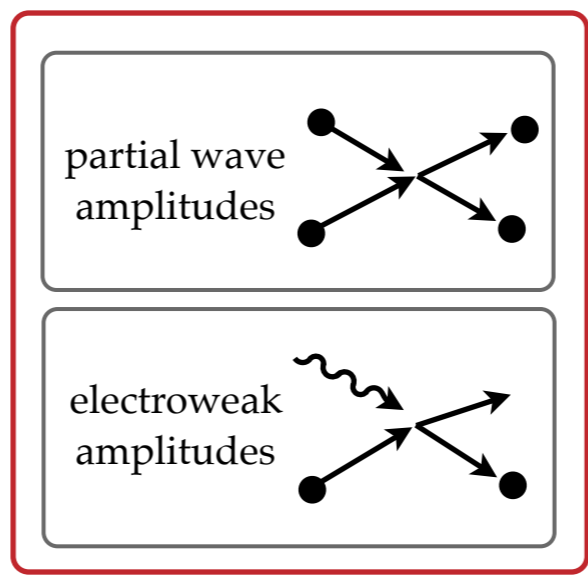
Amplitude analysis



Experiments



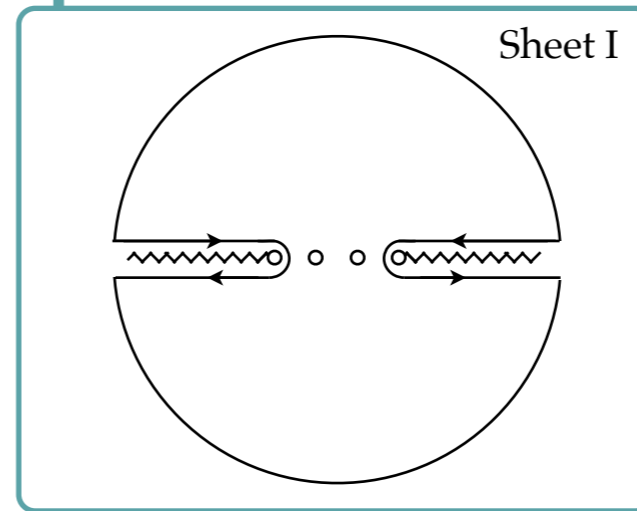
QCD



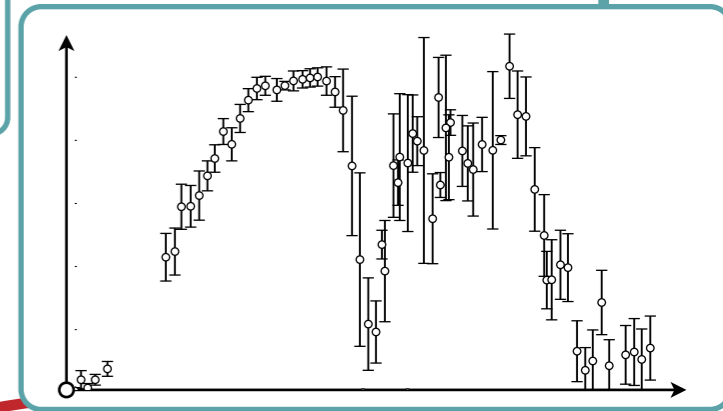
See Ryan Mitchell's talk for recent progress

QCD spectroscopy

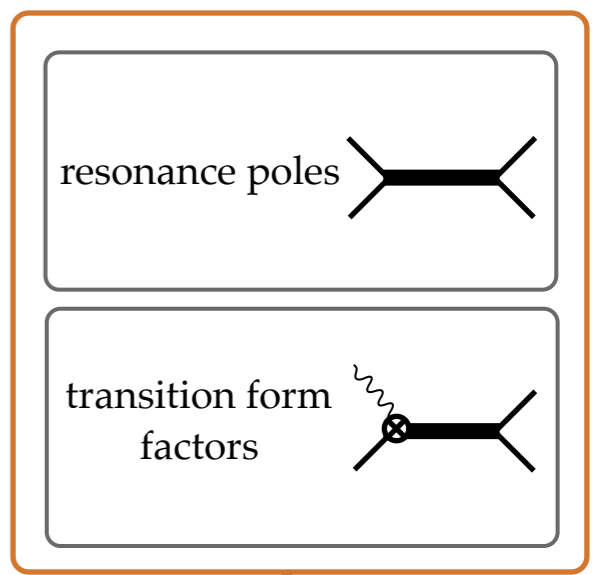
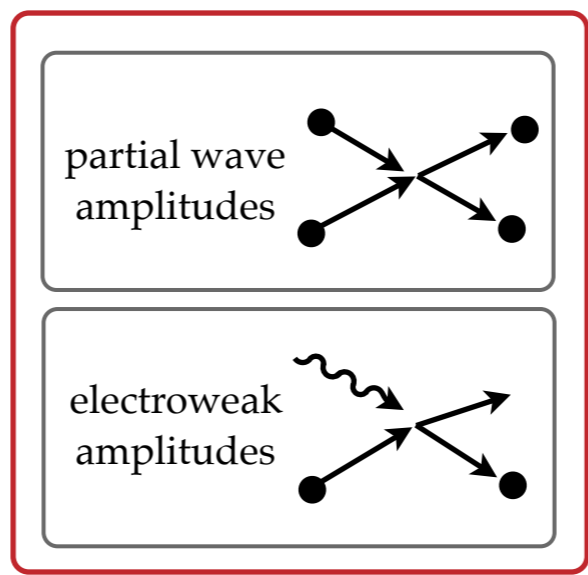
Amplitude analysis



Experiments



QCD



identification of states,
production/decay mechanisms

QCD spectroscopy

Amplitude analysis

Sheet I

Experiments

QCD

models & EFTs

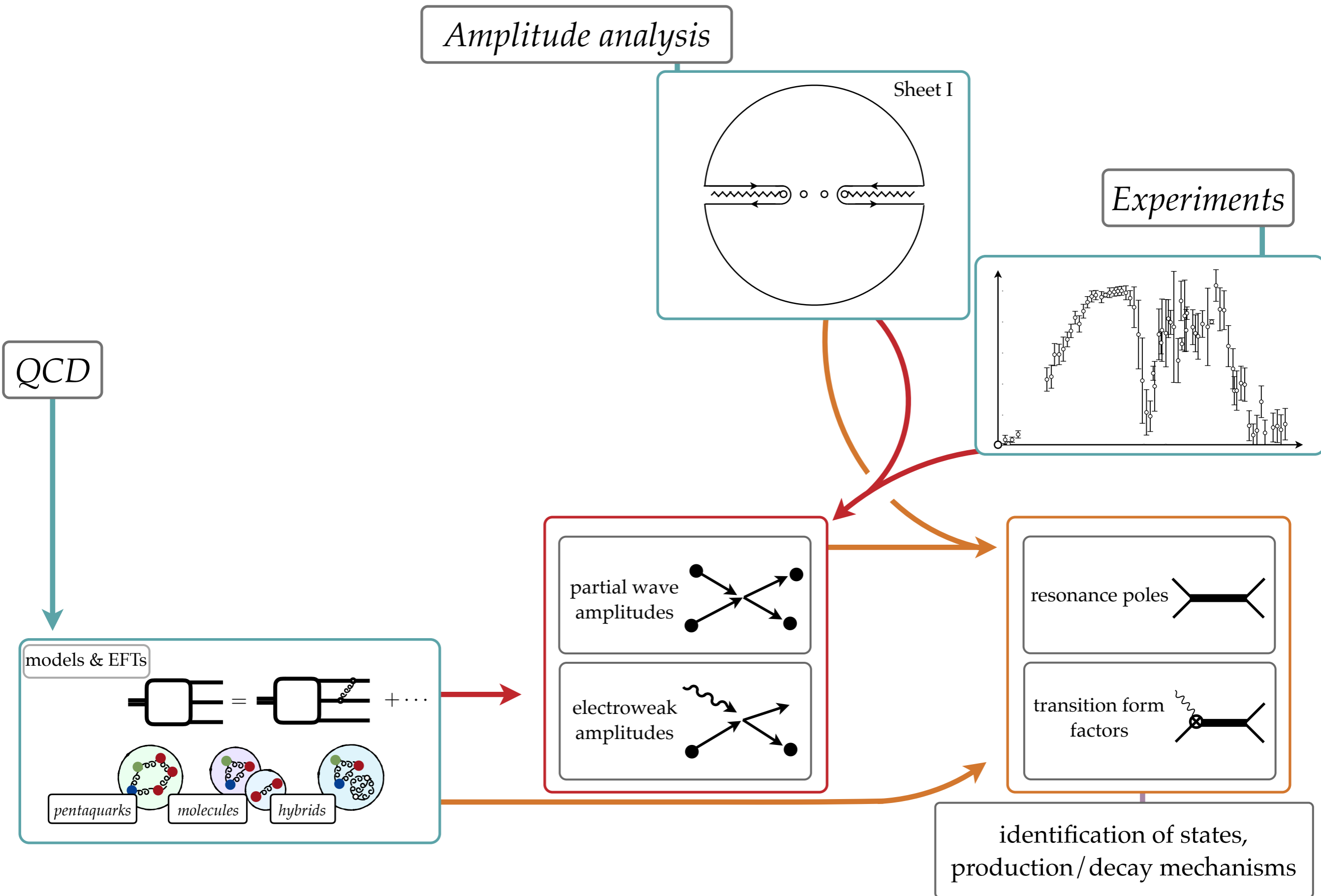
partial wave
amplitudes

electroweak
amplitudes

resonance poles

transition form
factors

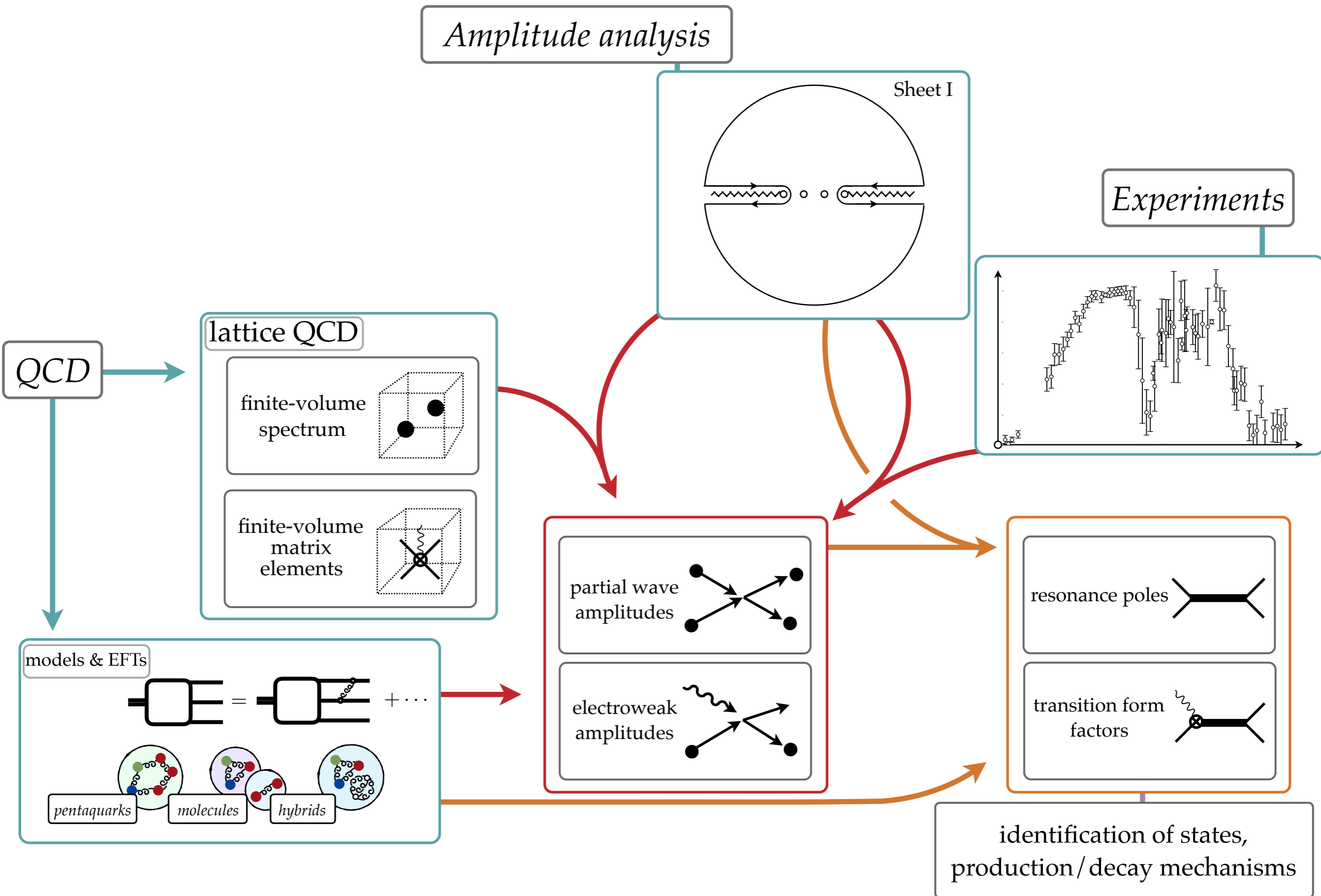
identification of states,
production/decay mechanisms



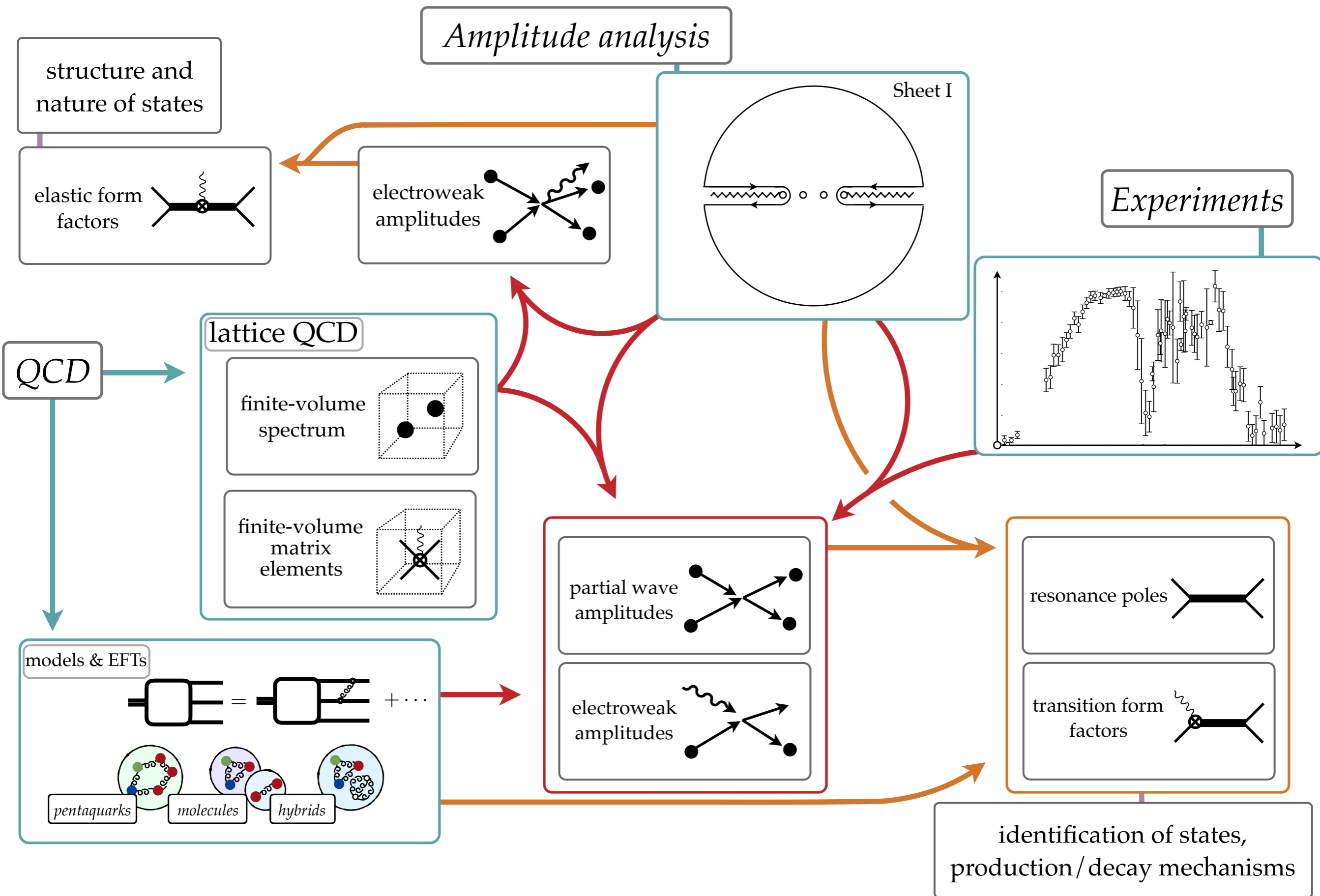
QCD spectroscopy

Amplitude analysis

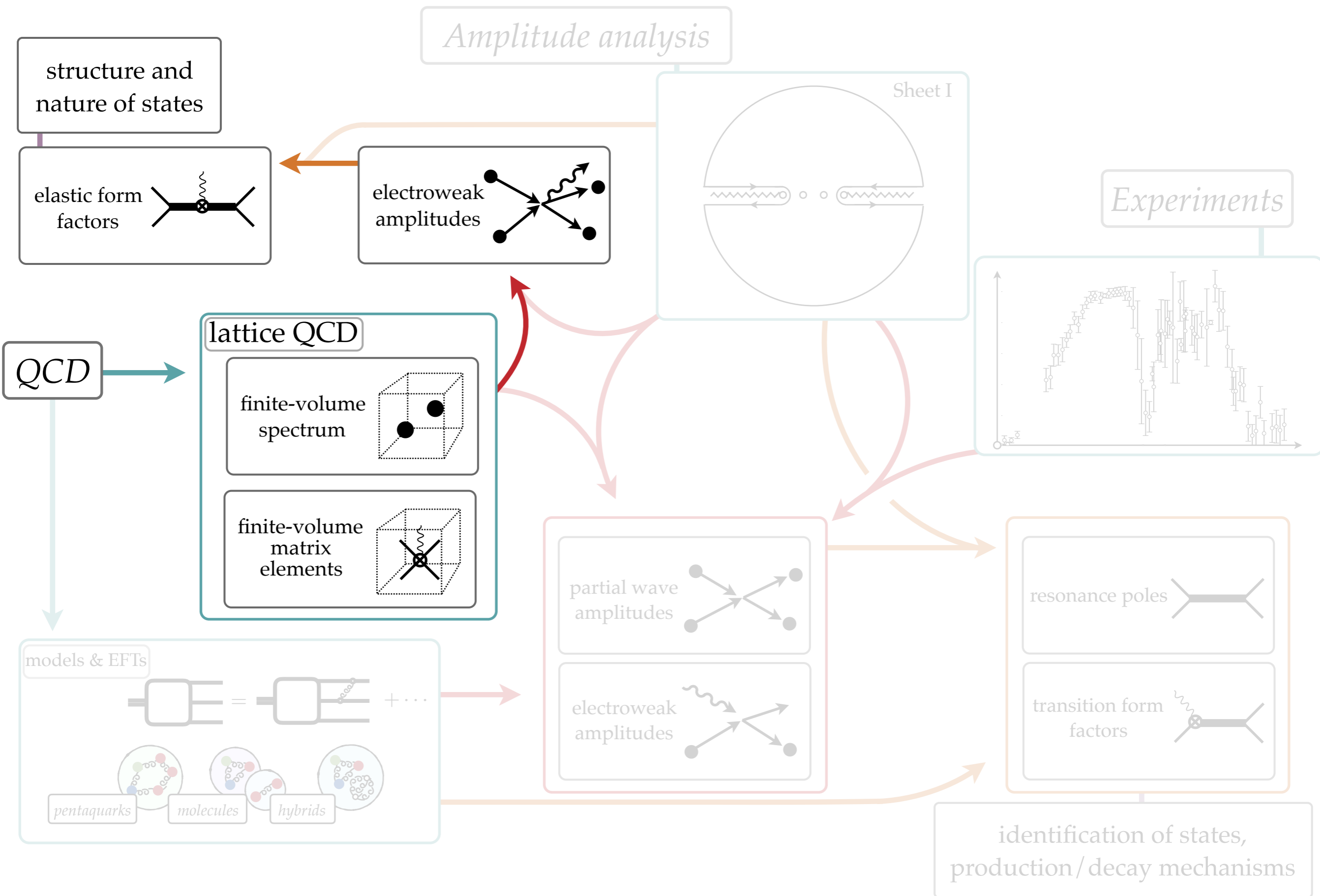
Experiments



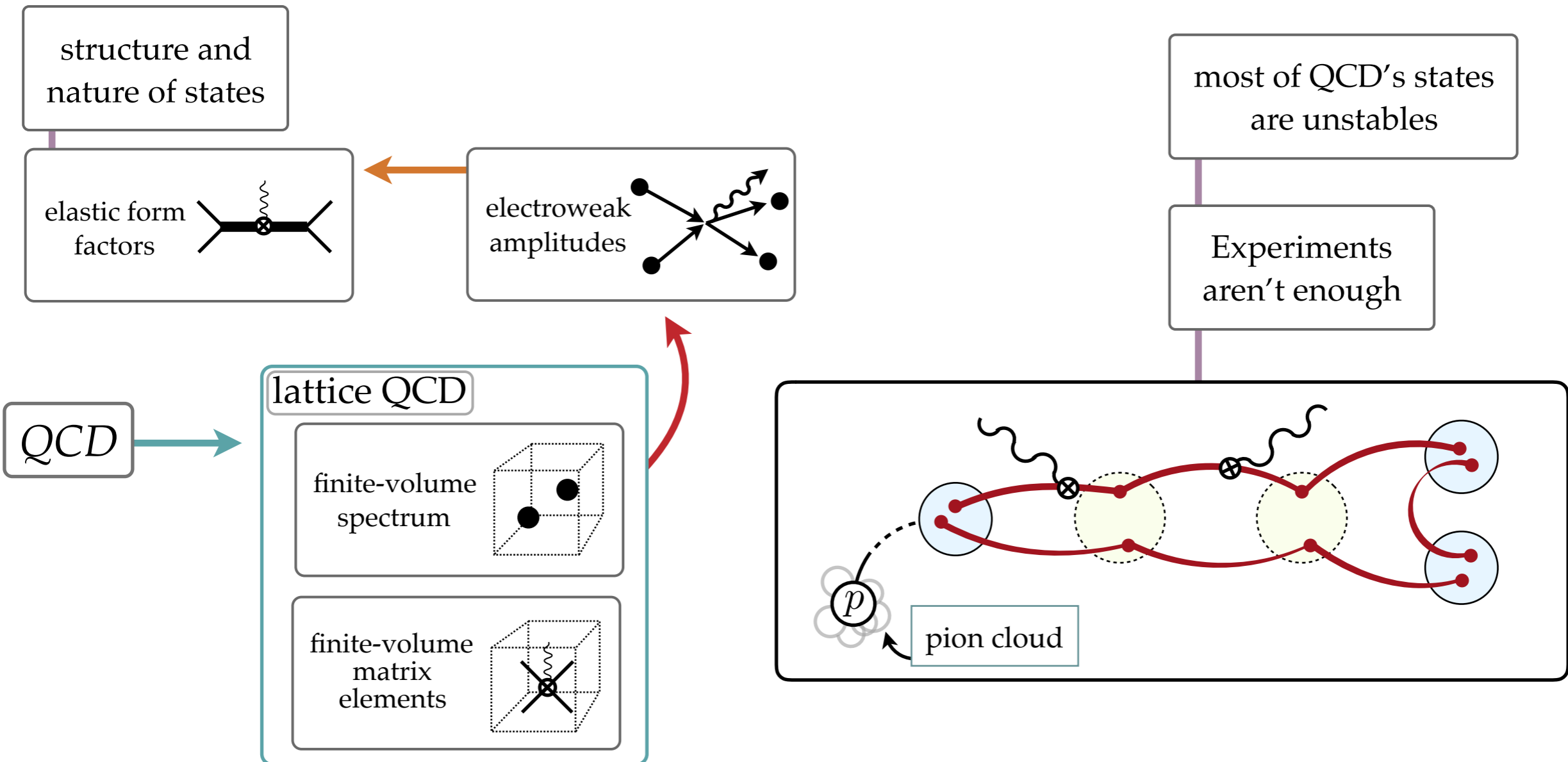
QCD spectroscopy



QCD spectroscopy

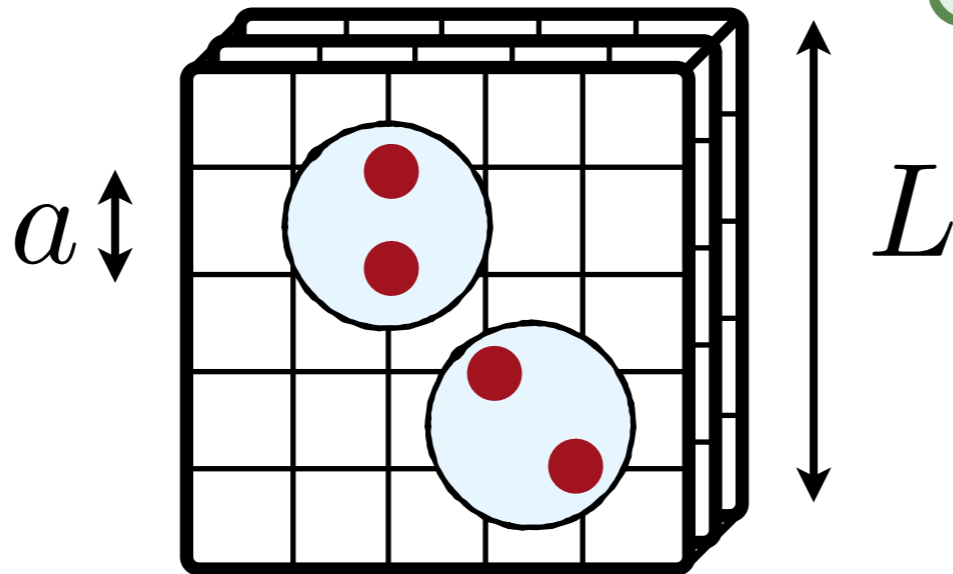


QCD spectroscopy



Lattice QCD

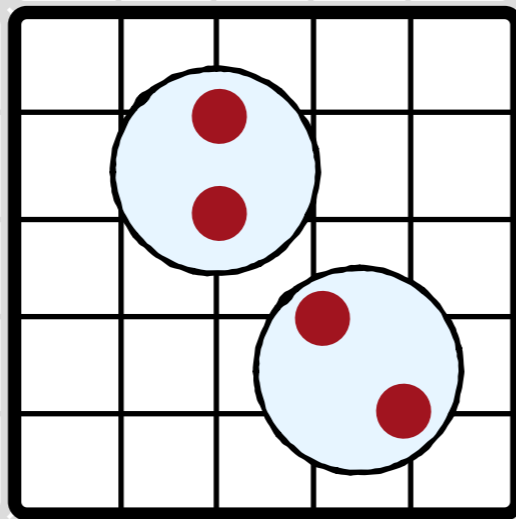
- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume



$$D_\mu = \left(\right) \updownarrow (L/a)^3 \times (T/a)$$

Lattice QCD

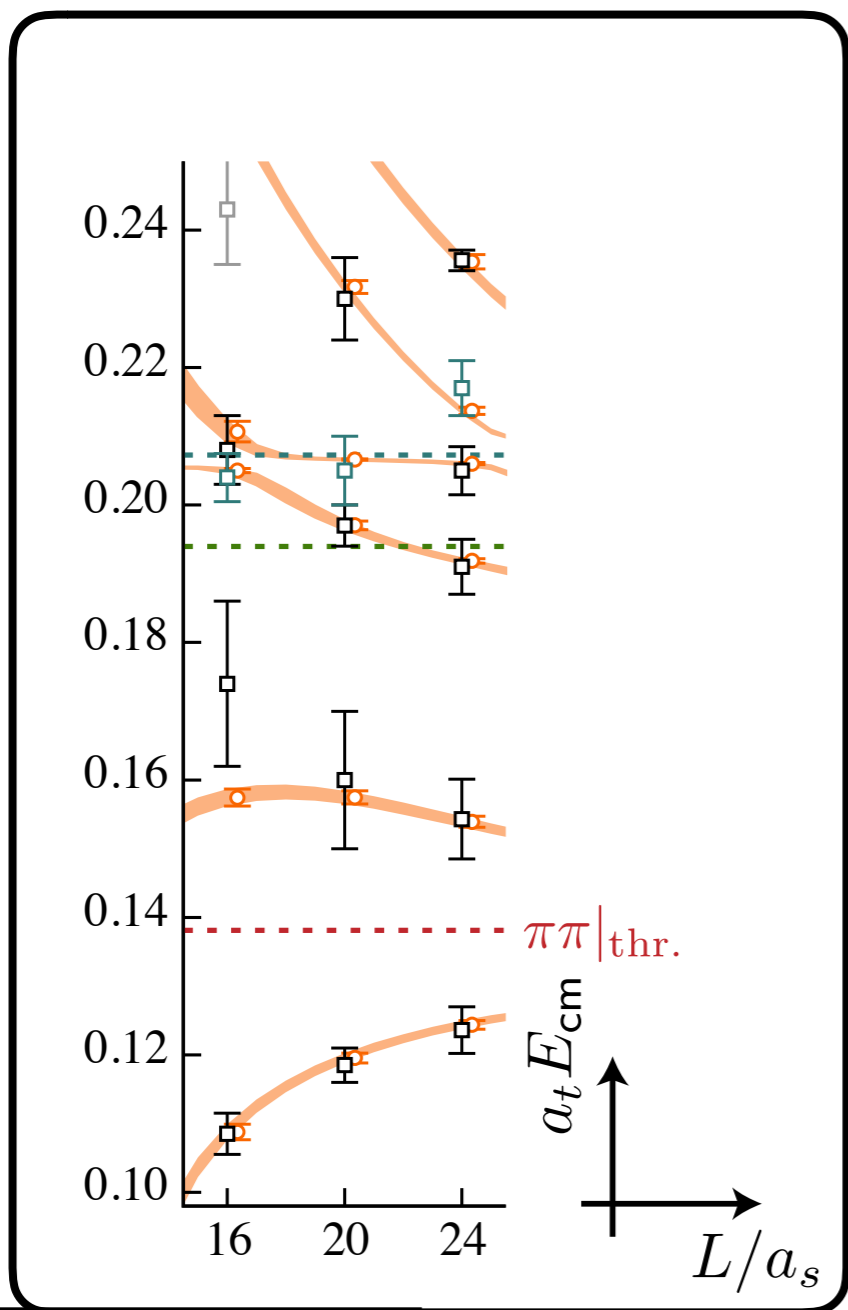
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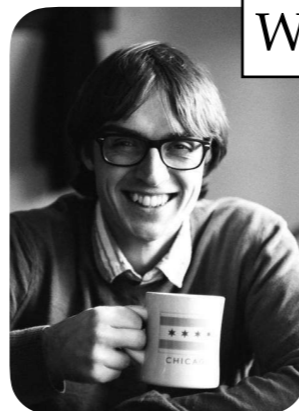
Never free!
No asymptotic states!
No scattering!

On the structure of states

Isoscalar 0^{++} channel:



$m_\pi \sim 400$ MeV



Wilson (Royal fellow / Trinity)



Dudek (W&M / JLab)

see J. Dudek's slides for details



Edwards (JLab)

002 (2017)

PHYSICAL REVIEW LETTERS

13

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,^{1,*} Jozef J. Dudek,^{1,2,†} Robert G. Edwards,^{1,‡} and David J. Wilson^{3,§}

(for the Hadron Spectrum Collaboration)

JLAB

Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,^{1,2,*} Jozef J. Dudek,^{1,3,†} Robert G. Edwards,^{1,‡} and David J. Wilson^{3,§}
(for the Hadron Spectrum Collaboration)

¹Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606

²Department of Physics, Old Dominion University, Norfolk, VA 23529, USA

³Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA

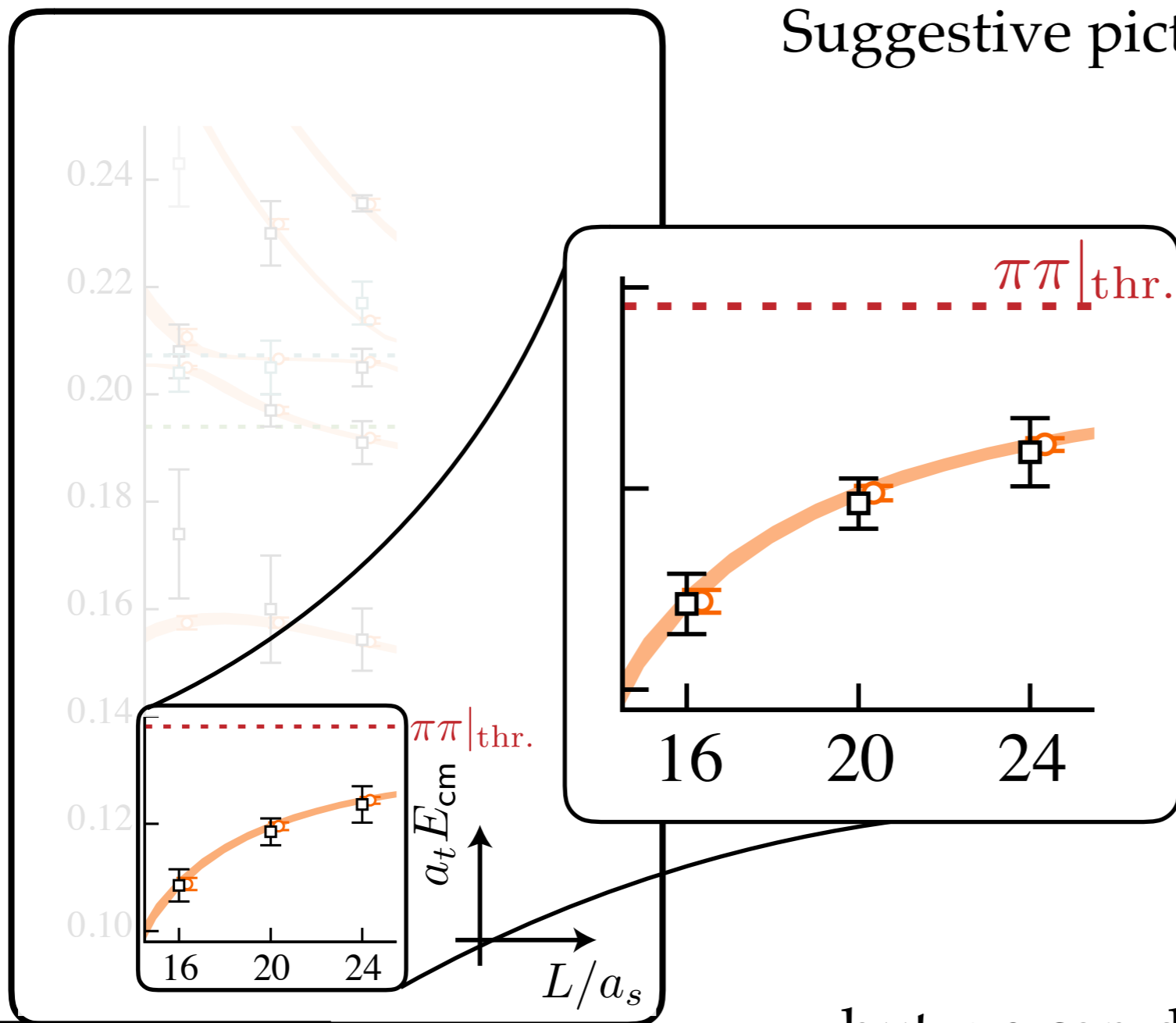
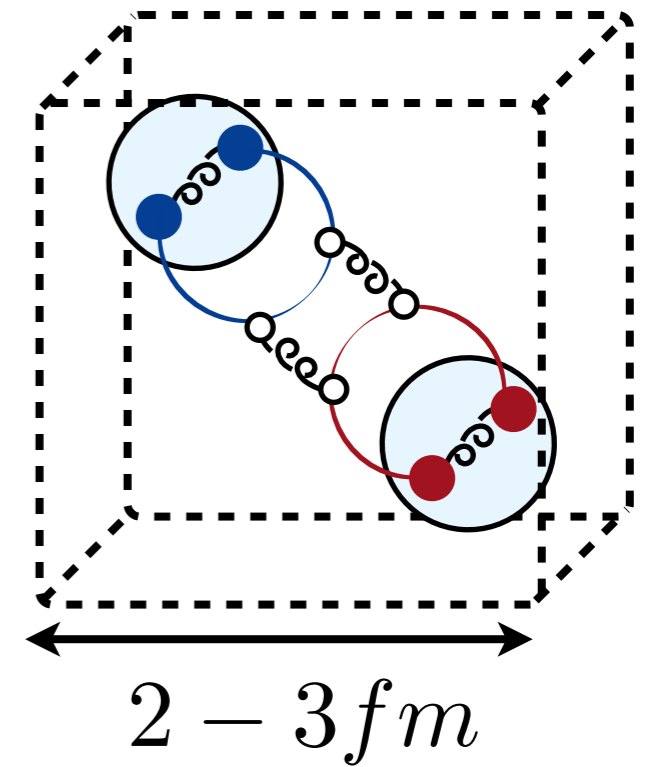
⁴School of Mathematics, Trinity College, Dublin 2, Ireland

(Date: August 23, 2017)

On the structure of states

Isoscalar 0^{++} channel:

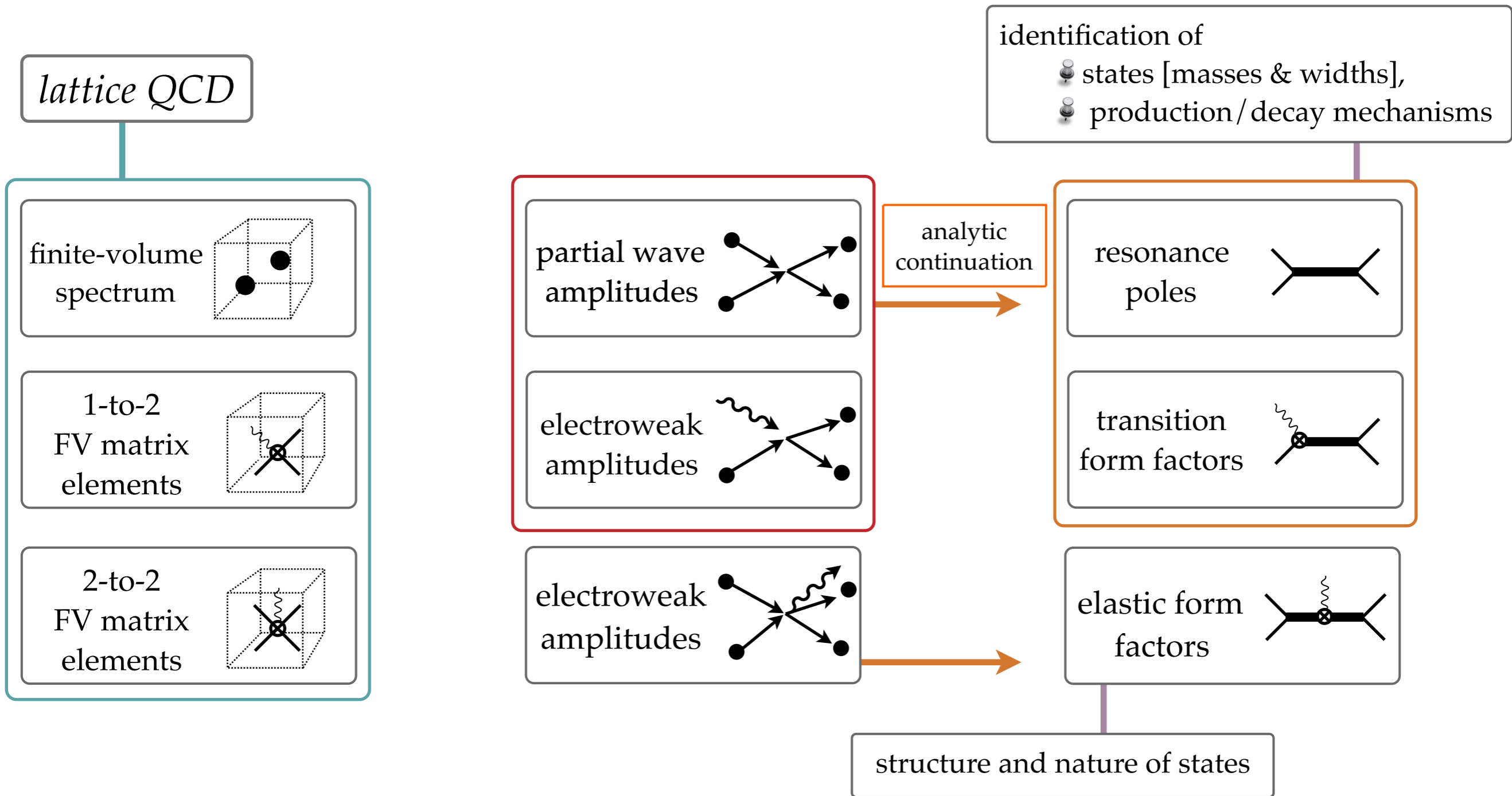
Suggestive picture:



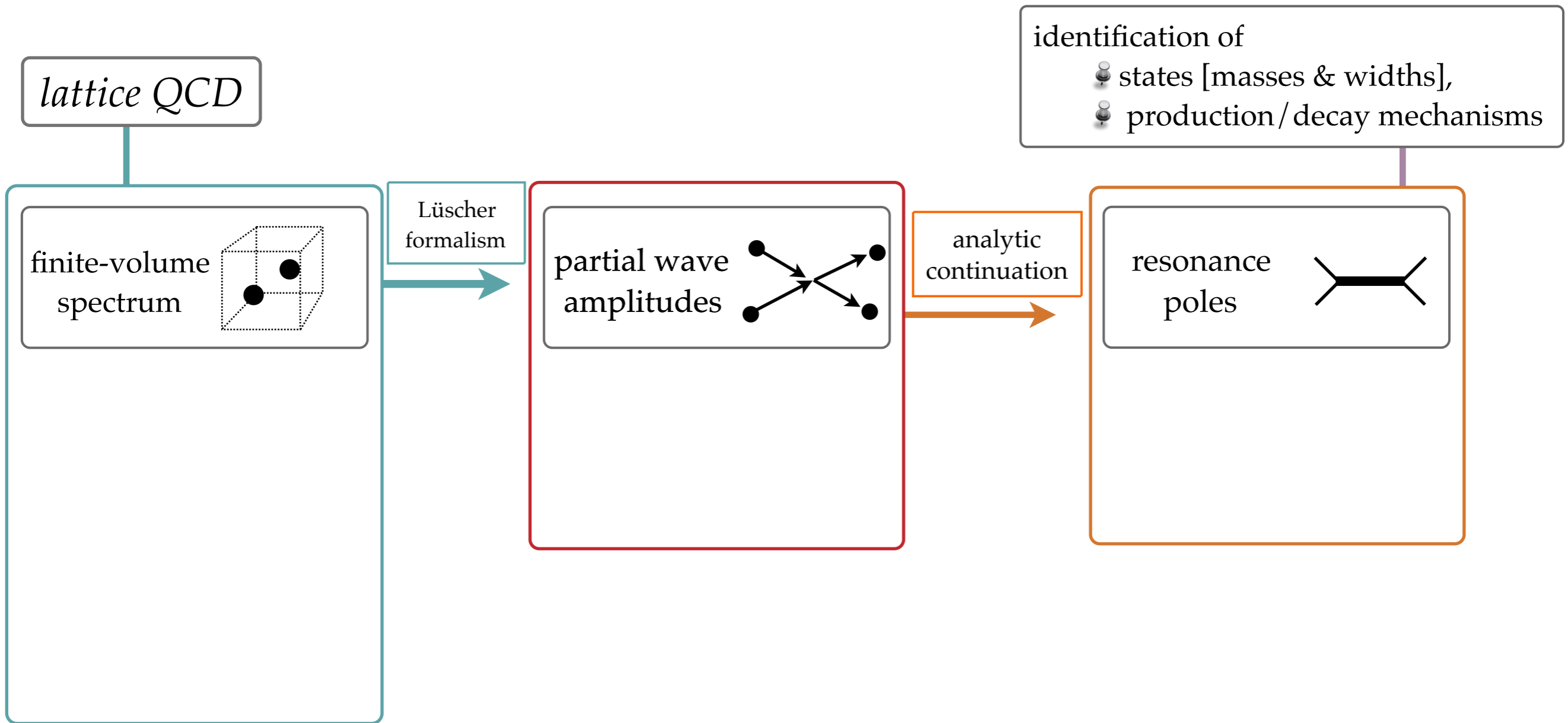
$m_\pi \sim 400 \text{ MeV}$

...but we can do much more...!

QCD spectroscopy



QCD spectroscopy



Two-body scattering

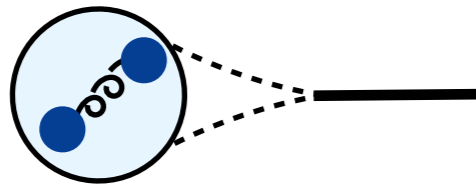
Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \underbrace{\text{tree} + \text{one-loop} + \text{two-loop} + \dots}_{\left\{ \text{non-perturbative kernel} \right\}}$$

*non-perturbative kernel including
all diagrams not shown...*

“yep, the left hand cut is there”

*IR limit of QCD, only interested in
hadronic d.o.f.*



Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

$$\begin{aligned} \text{one-loop} &= \int \frac{d^4 k}{(2\pi)^4} [iB(k, P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P - k)^2 - m^2 + i\epsilon} \\ &= \int \frac{d^3 k}{(2\pi)^3} \frac{[iB(k, P)]^2}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} + \text{“smooth”} \\ &= [iB_{on}] \rho [iB_{on}] + \text{“PV integral”} \\ &= \text{one-loop with cut} + \text{PV} \end{aligned}$$

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-body scattering

Unitarity using all orders perturbation theory:

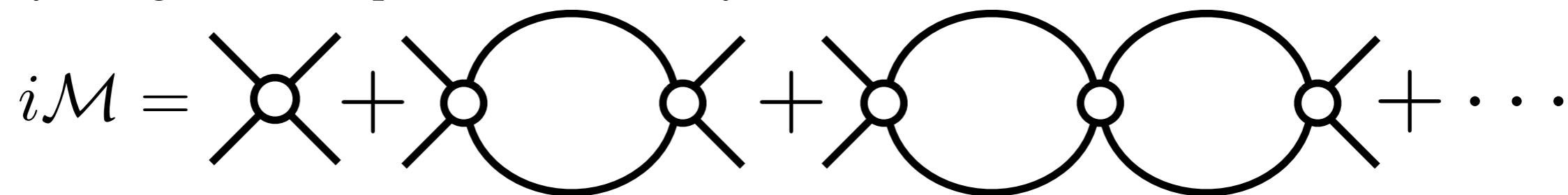
$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

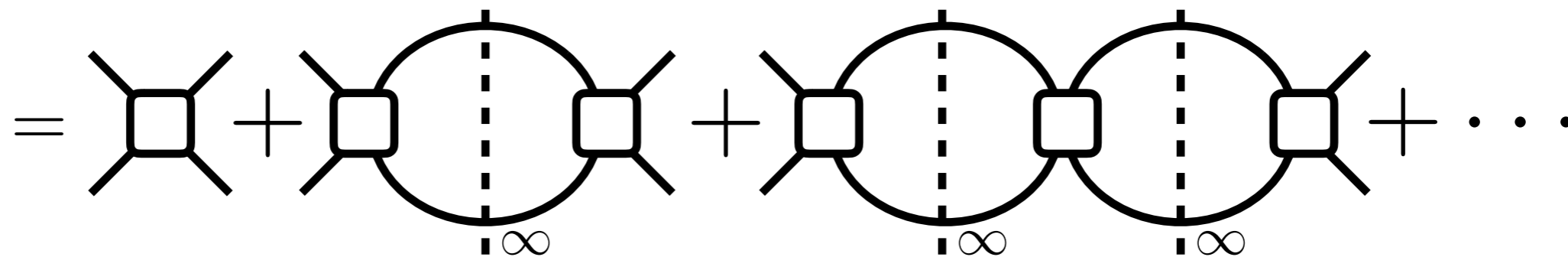
$$= \text{square} + \dots$$

$$= \underbrace{\text{square}}_{\text{K-matrix}} \left\{ \text{tree} + \text{one-loop PV} + \dots \right\}$$

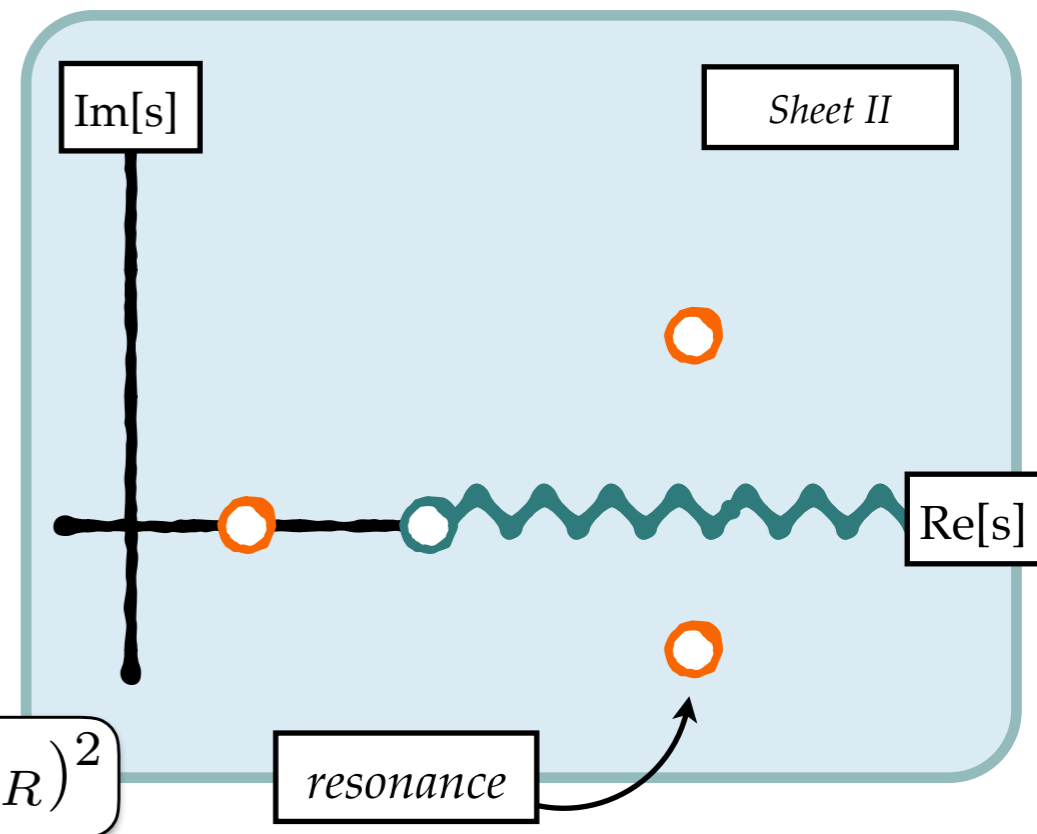
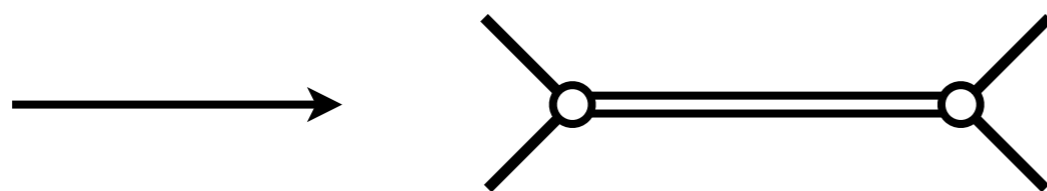
Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$


$$= \text{tree} + \text{one-loop with cut} + \text{two-loop with cut} + \dots$$


$$= i\mathcal{K} \frac{1}{1 - i\rho\mathcal{K}}$$

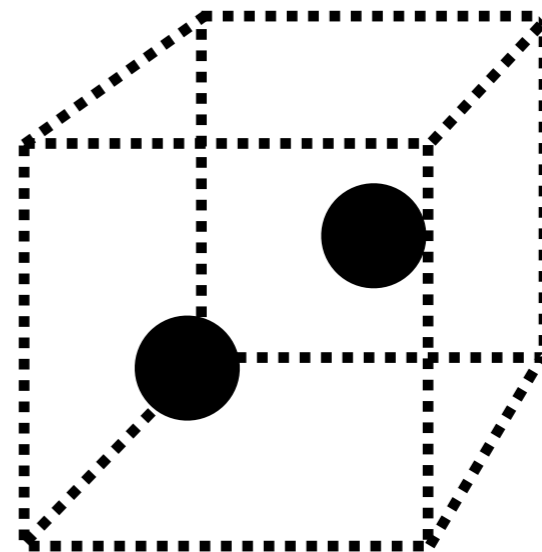


Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) = \text{[Diagram 1]} + \text{[Diagram 2]} + \dots$$

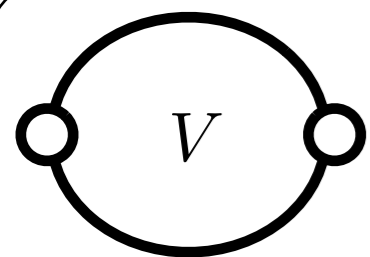
The diagram shows a series of terms in a sum. The first term is a circle with the letter V inside, and two small white circles on its left and right sides. The second term is two such circles connected at their right and left sides respectively, forming a chain of two circles. The third term is an ellipsis \dots .



Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) = \text{[Diagram: circle with V and two external lines]} + \text{[Diagram: two circles with V and two external lines]} + \dots$$



$$= \frac{1}{L^3} \sum_{\mathbf{k}} \frac{iB^2}{(2\omega_k)^2} \frac{i}{E - 2\omega_k} + \text{“smooth”}$$

$$= (iB) \left(\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} \right) (iB) + i\epsilon \text{ integral}$$

$$\equiv [iB] iF [iB] + i\epsilon \text{ integral}$$

$$= \text{[Diagram: dashed circle with V - \infty and two external lines]} + \text{[Diagram: solid circle with i\epsilon and two external lines]}$$

F replaces ρ

Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$\begin{aligned} C_L^{2pt.}(P) &= \text{[diagram: a circle with two external legs and a vertex labeled } V] + \text{[diagram: two circles with two external legs and two vertices labeled } V] + \dots \\ &= C_\infty(P) + \text{[diagram: a dashed circle with two external legs and two vertices labeled } V - \infty] + \text{[diagram: two dashed circles with two external legs and four vertices labeled } V - \infty] + \dots \\ &= \text{“smooth”} + A \frac{i}{F^{-1} + \mathcal{M}} B^\dagger \end{aligned}$$

poles satisfy: $\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$

• Lüscher (1986, 1991)

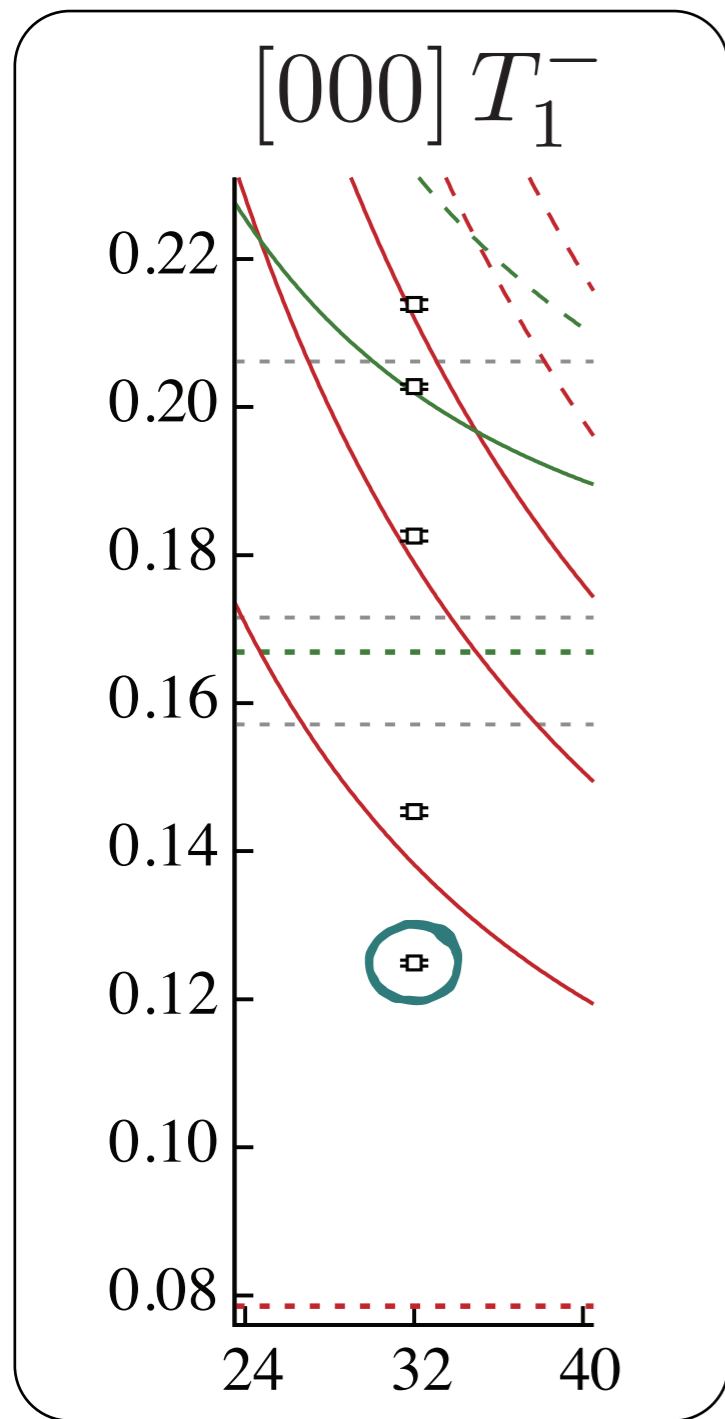
• Rummukainen & Gottlieb (1995)

• Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005)

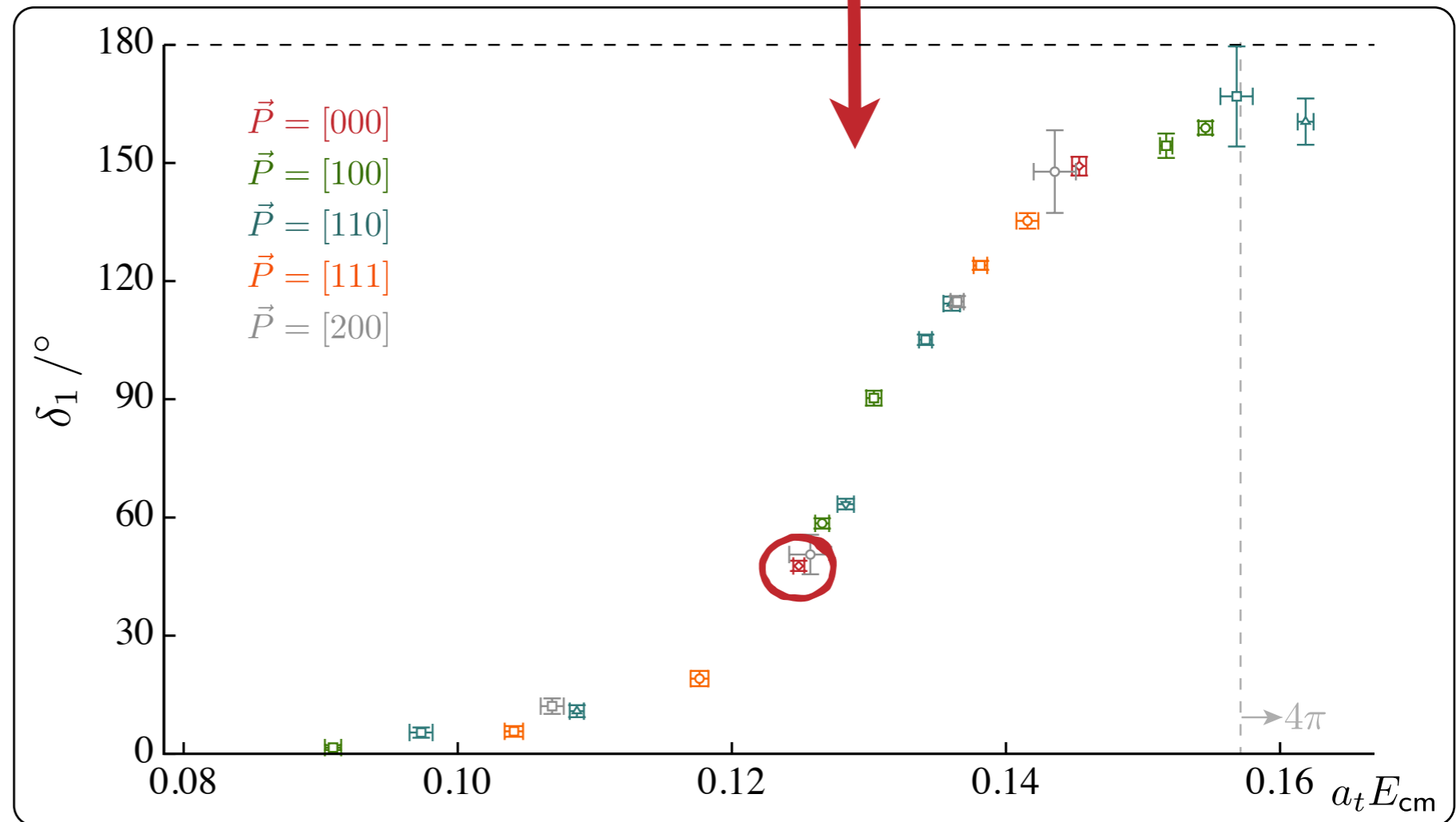
• Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)

• RB (2014)

$\pi\pi$ Spectrum - ($l=1$ channel)



$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$

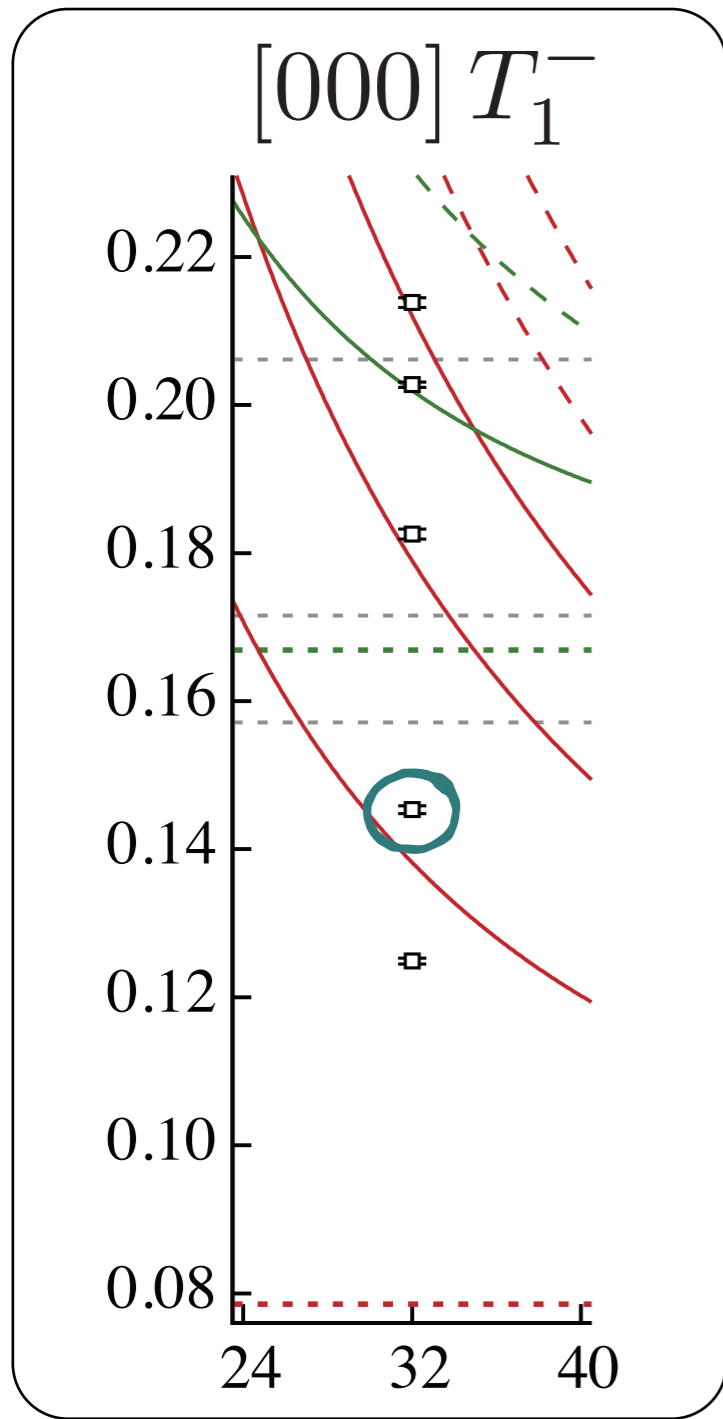


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

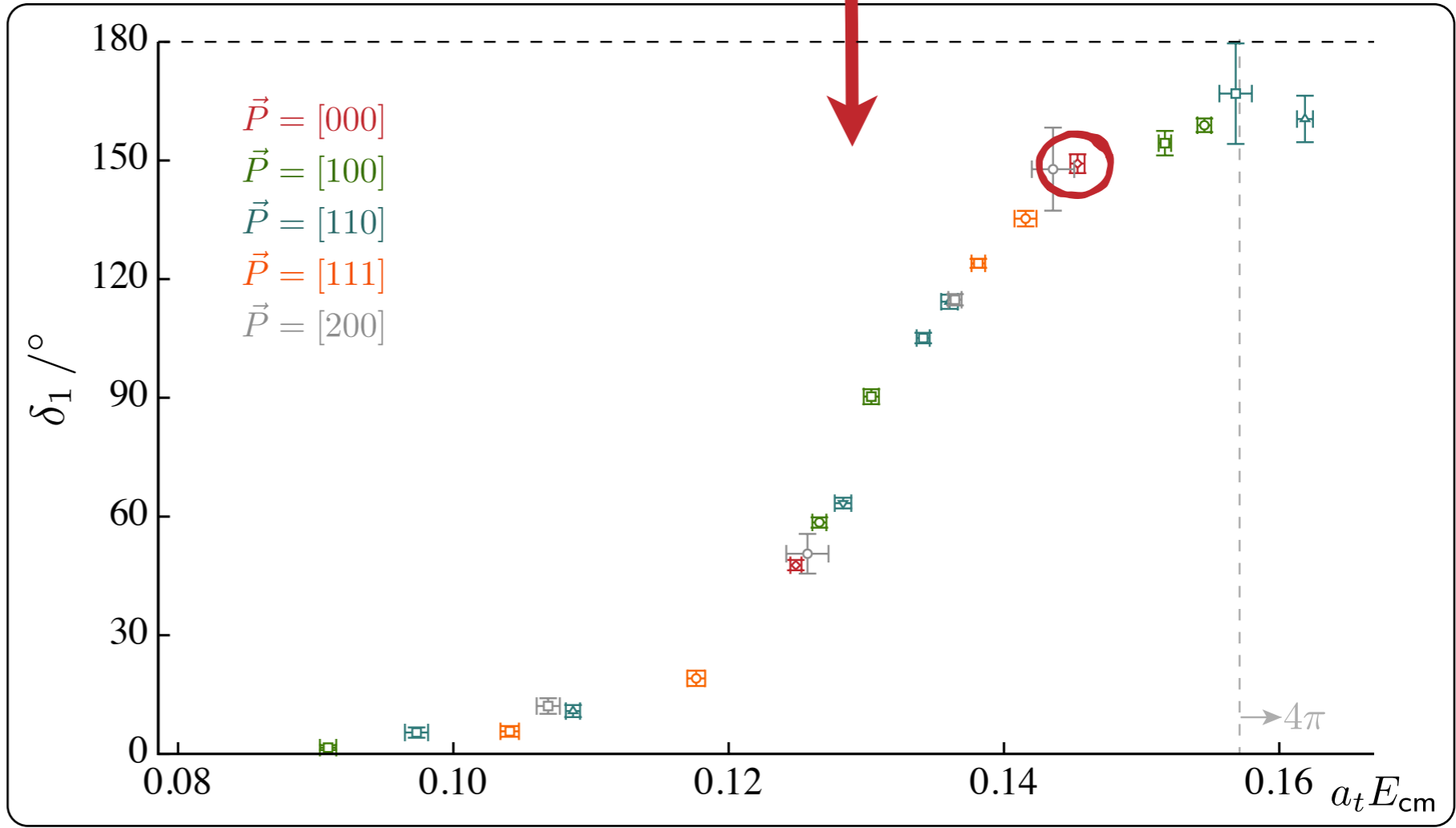
$$m_\pi \sim 240 \text{ MeV}$$

Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ Spectrum - ($l=1$ channel)



$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$

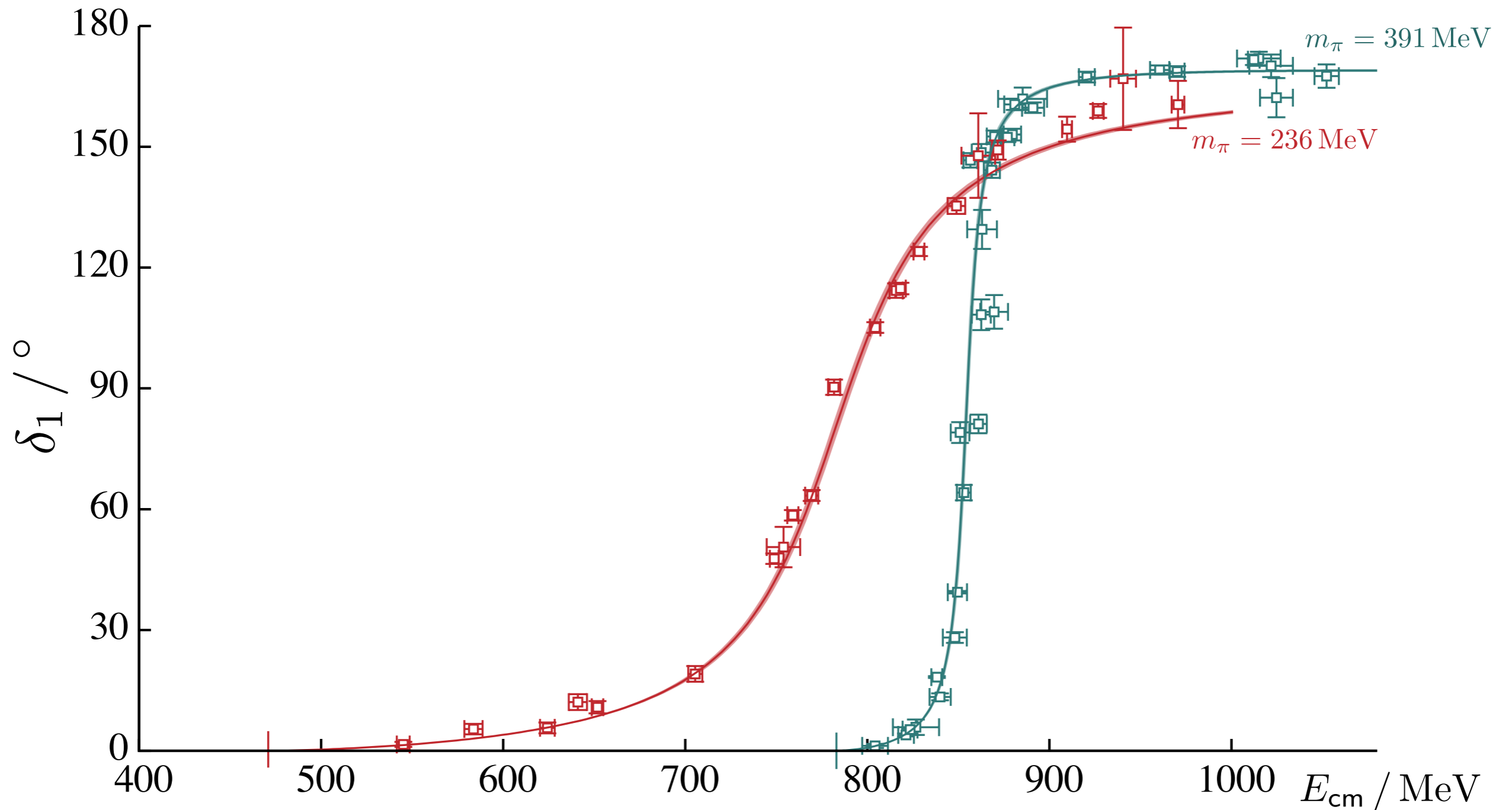


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Wilson, RB, Dudek, Edwards & Thomas (2015)

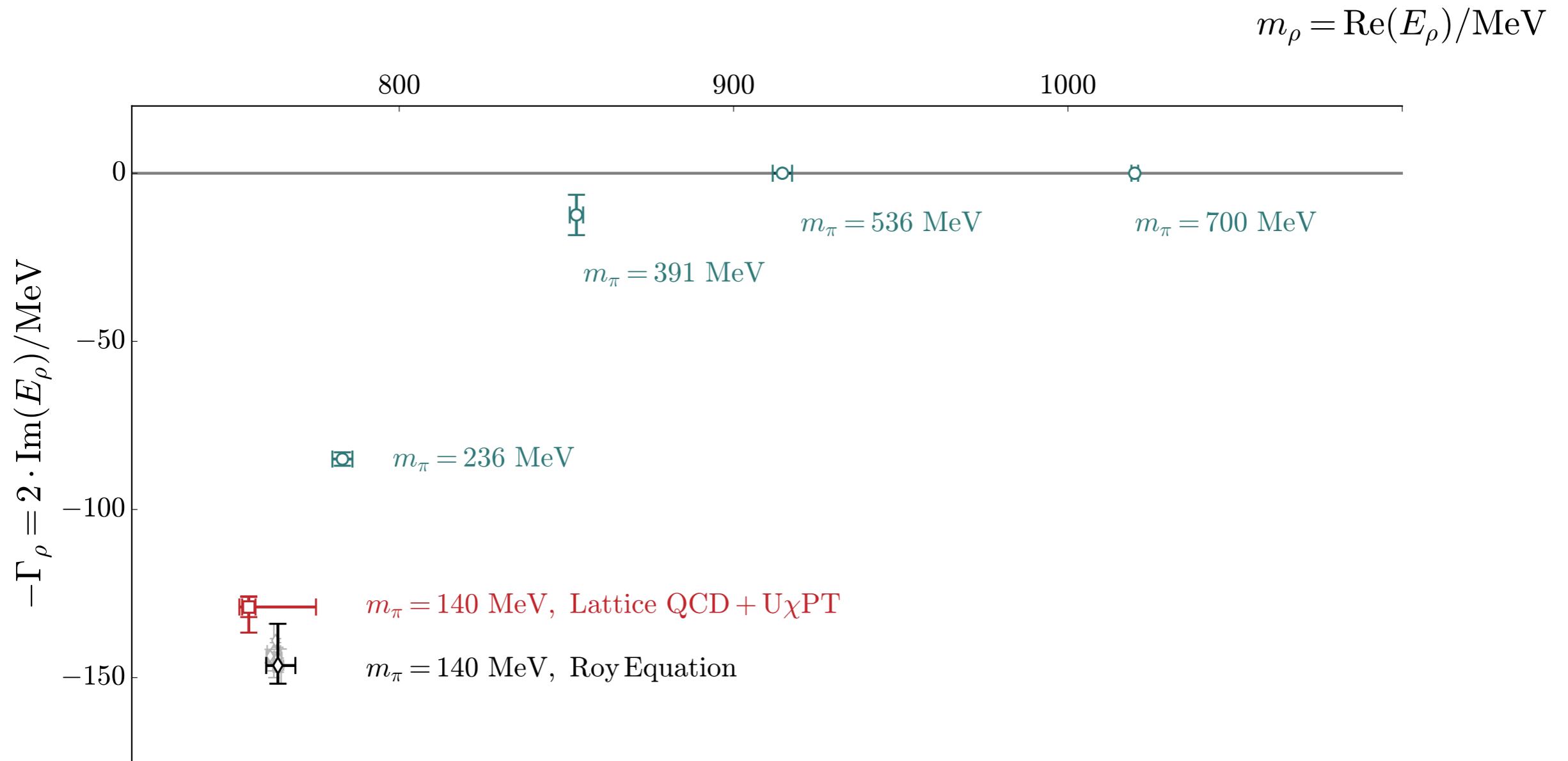
$\pi\pi$ Spectrum - ($l=1$ channel)



Dudek, Edwards & Thomas (2012)

Wilson, RB, Dudek, Edwards & Thomas (2015)

The ρ vs m_π



Lin *et al.* (2009)

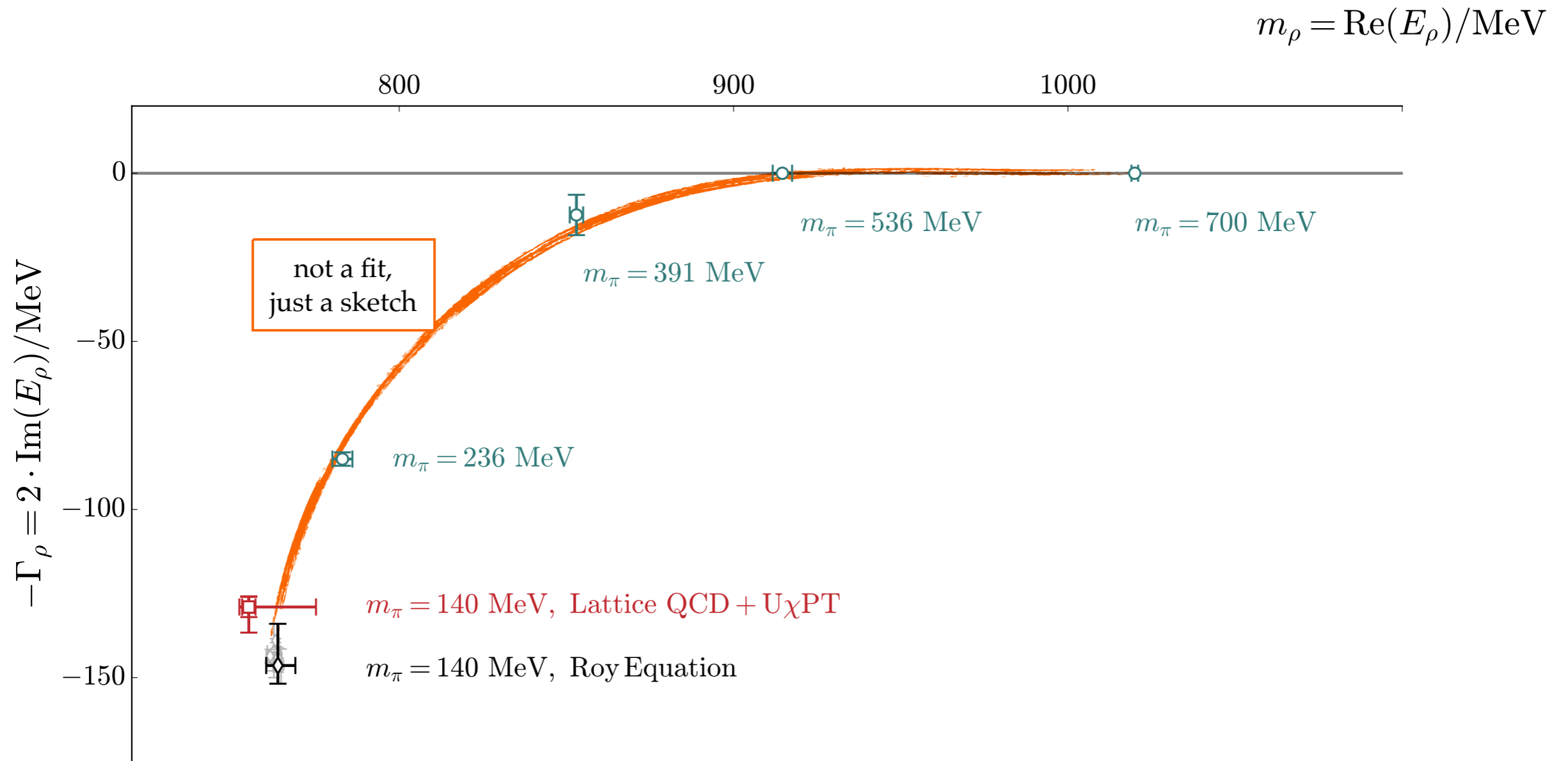
Dudek, Edwards, Guo & Thomas (2013)

Dudek, Edwards & Thomas (2012)

Wilson, RB, Dudek, Edwards & Thomas (2015)

Bolton, RB & Wilson (2015)

The ρ vs m_π



Lin *et al.* (2009)

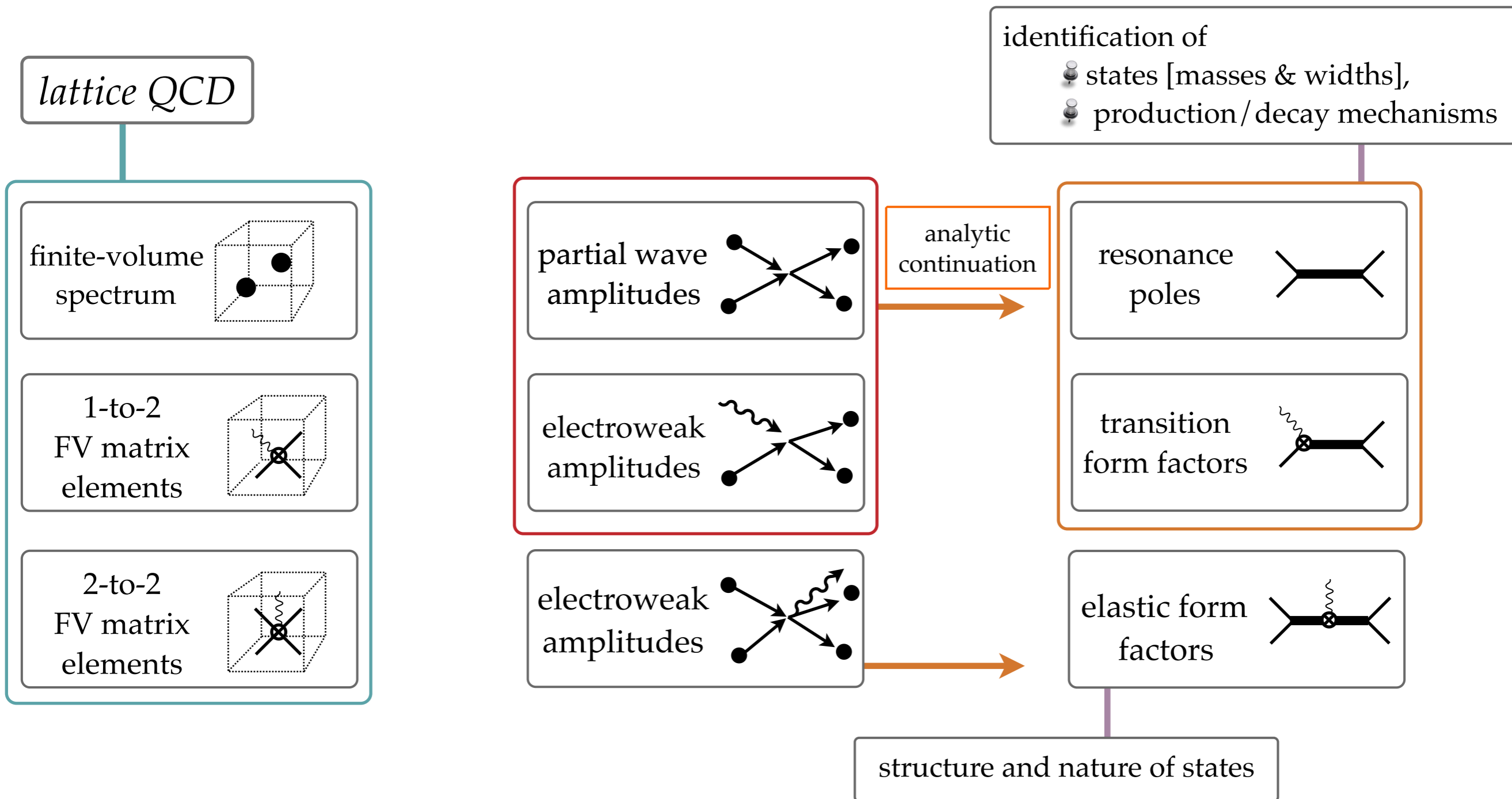
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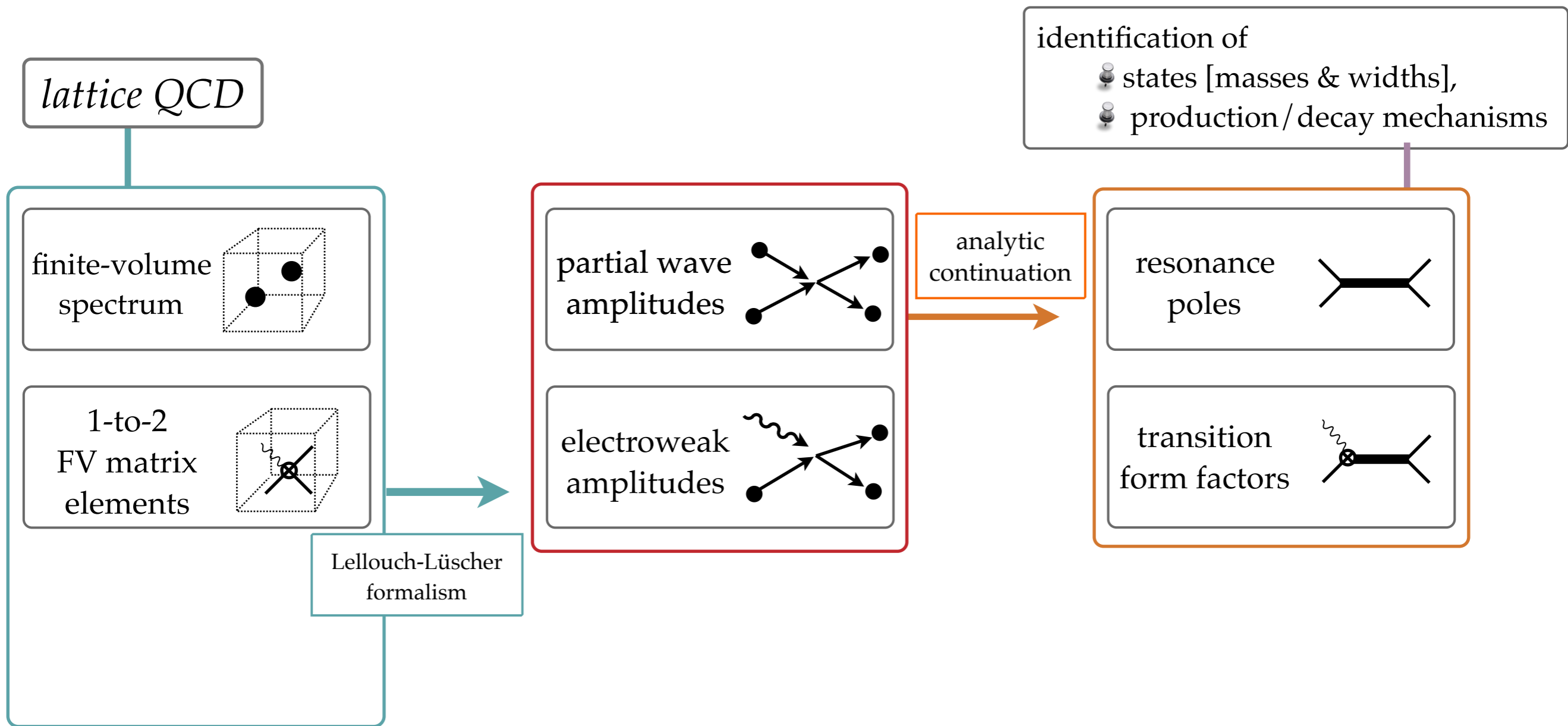
Wilson, RB, Dudek, Edwards & Thomas (2015)

Bolton, RB & Wilson (2015)

QCD spectroscopy



QCD spectroscopy



One-to-two transitions

Unitarity using all orders perturbation theory:

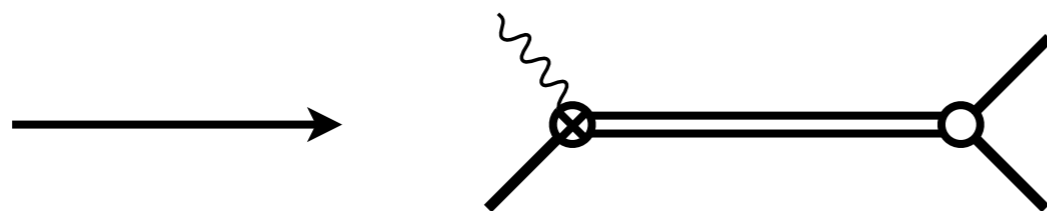
$$i\mathcal{A} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

The first row shows the expansion of $i\mathcal{A}$ as a sum of diagrams: a tree-level vertex with a wavy line, a one-loop diagram with a bubble, and a two-loop diagram with two bubbles, followed by an ellipsis.

$$= \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

The second row shows the same expansion as the first, but with unitarity cuts indicated by vertical dashed lines and infinity symbols (∞) at the bottom of each loop diagram.

$$= i\mathcal{H} \frac{1}{1 - i\rho\mathcal{K}}$$



Watson's theorem

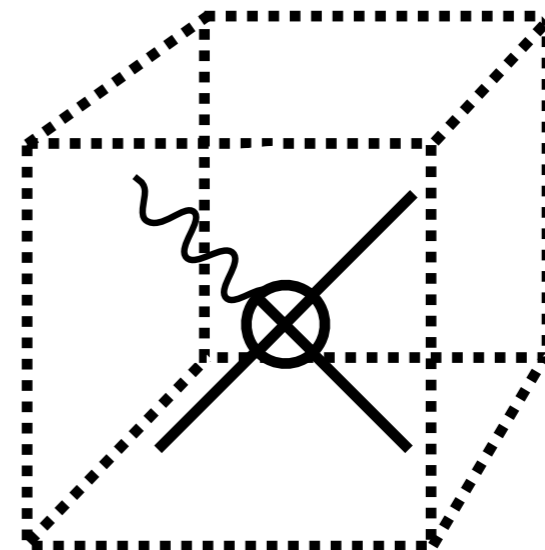
Same phase and analytic structure as the scattering amplitude

One-to-two in finite volume

Same as before...but with a current

$$C_L^{3pt.} = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$
The equation shows a sum of two Feynman diagrams. The first diagram consists of a circle labeled 'V' with a wavy line entering from the left and a solid line exiting from the bottom. The second diagram consists of two circles labeled 'V' connected at a central vertex, with a wavy line entering from the left and a solid line exiting from the right. Ellipses follow the second diagram.

same topologies that we considered before...



One-to-two in finite volume

Same as before...but with a current

$$\begin{aligned}
 C_L^{3pt.} &= \text{[Diagram 1]} + \text{[Diagram 2]} + \dots \\
 &= \text{[Diagram 3]} + \text{[Diagram 4]} + \dots \\
 &= \text{“smooth”} + \mathcal{A} \frac{i}{F^{-1} + \mathcal{M}} B^\dagger
 \end{aligned}$$

After lots of massaging...

$$|\langle \mathbf{2} | \mathcal{J} | \mathbf{1} \rangle_L| = \sqrt{\mathcal{A} \mathcal{R} \mathcal{A}}$$

Lellouch-Lüscher matrix:

$$\mathcal{R}(E_n, \mathbf{P}) \equiv \lim_{E \rightarrow E_n} \left[\frac{(E - E_n)}{F^{-1}(P, L) + \mathcal{M}(P)} \right]$$

📌 Lellouch & Lüscher (2000)

📌 Kim, Sachrajda, & Sharpe

📌 Christ, Kim & Yamazaki (2005)

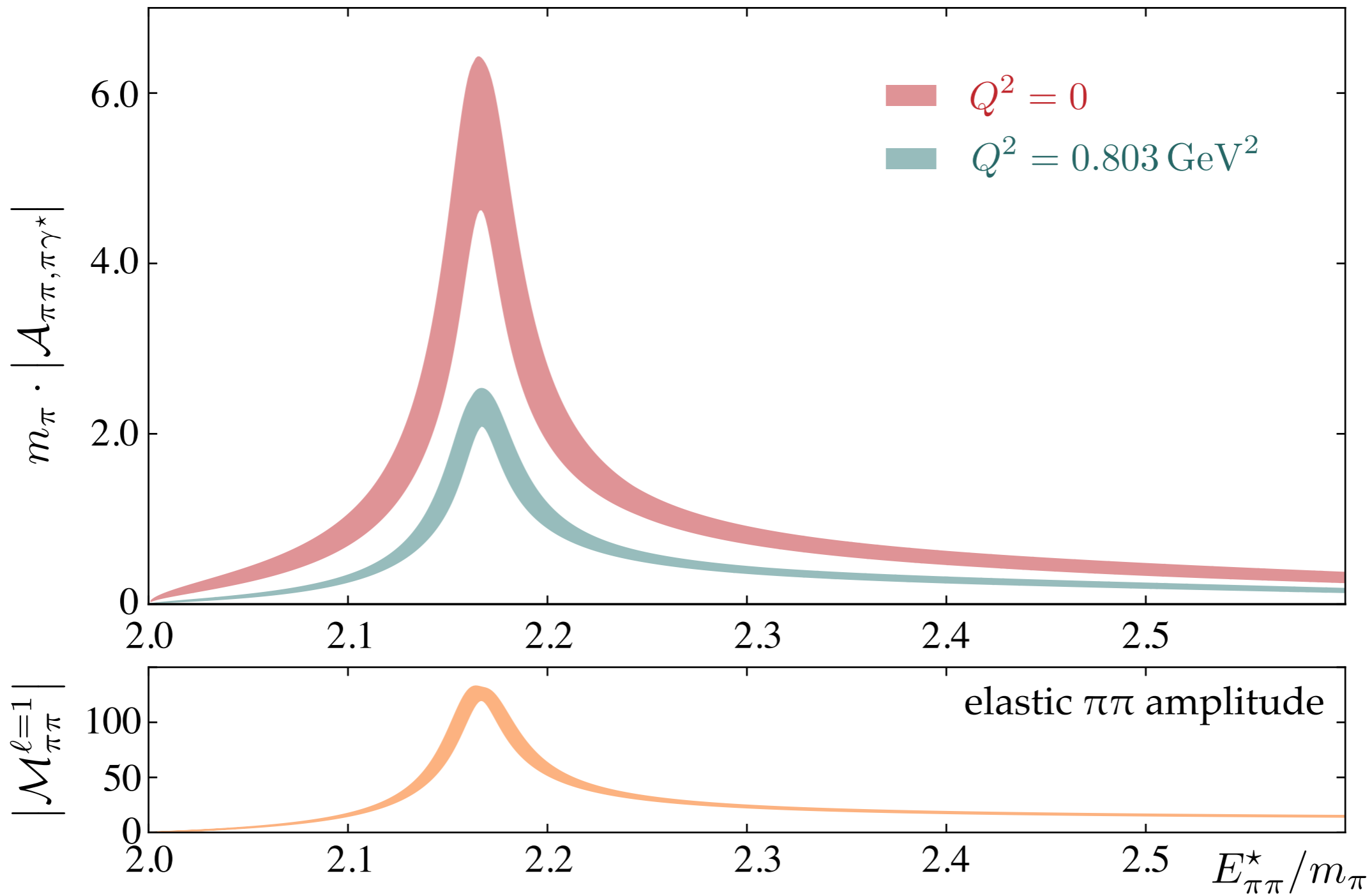
📌 ...

📌 Hansen & Sharpe (2012)

📌 RB, Hansen Walker-Loud (2014)

📌 RB & Hansen (2015)

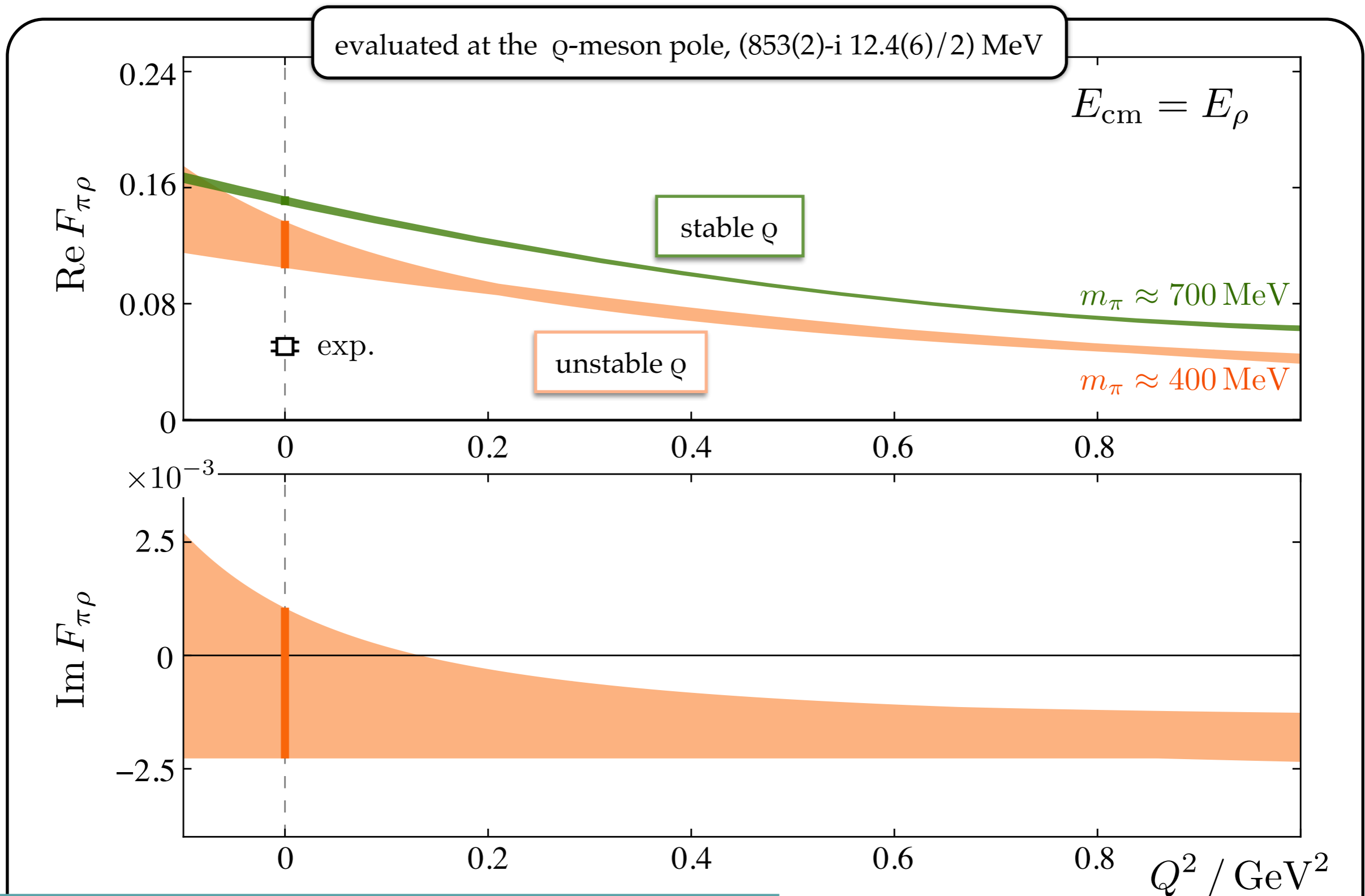
$\pi\gamma^*$ -to- $\pi\pi$ amplitude



$m_\pi \sim 400 \text{ MeV}$

RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

π -to- ρ form factor



Shultz, Dudek, & Edwards (2014)

RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

π -to- ρ form factor

evaluated at the ρ -meson pole, $(853(2)-i 12.4(6)/2)$ MeV

$$E_{\text{cm}} = E_{\rho}$$

0.24

$\text{Re } F_{\pi\rho}$

$\text{Im } F_{\pi\rho}$

The $\pi\gamma \rightarrow \pi\pi$ transition and the ρ radiative decay width from lattice QCD

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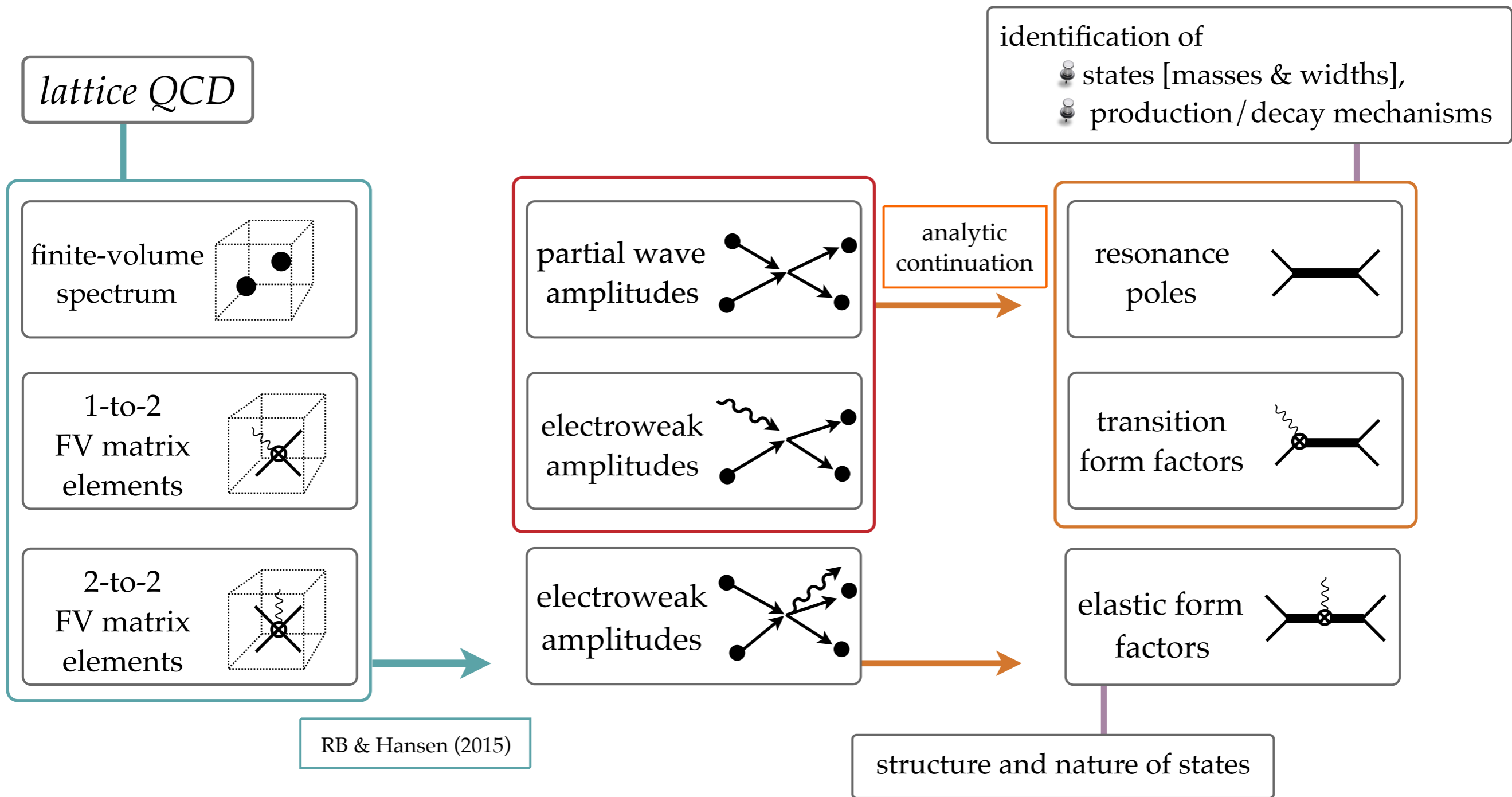
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(Dated: July 22, 2018)

We report a lattice QCD determination of the $\pi\gamma \rightarrow \pi\pi$ transition amplitude for the P -wave, $I = 1$ two-pion final state, as a function of the photon virtuality and $\pi\pi$ invariant mass. The calculation was performed with $2 + 1$ flavors of clover fermions at a pion mass of approximately 320 MeV, on a $32^3 \times 96$ lattice with $L \approx 3.6$ fm. We construct the necessary correlation functions using a combination of smeared forward, sequential and stochastic propagators, and determine the finite-volume matrix elements for all $\pi\pi$ momenta up to $|\vec{P}| = \sqrt{3} \frac{2\pi}{L}$ and all associated irreducible representations. In the mapping of the finite-volume to infinite-volume matrix elements using the Lellouch-Lüscher factor, we consider two different parametrizations of the $\pi\pi$ scattering phase shift. We fit the q^2 and s dependence of the infinite-volume transition amplitude in a model-independent way using series expansions, and compare multiple different truncations of this series. Through analytic continuation to the ρ resonance pole, we also determine the $\pi\gamma \rightarrow \rho$ resonant transition form factor and the ρ meson photocoupling, and obtain $|G_{\rho\pi\gamma}| = 0.0802(32)(20)$.

[lat] 22 Jul 2018

QCD spectroscopy



Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

$$i\mathcal{W}_{\mu_1 \cdots \mu_n} = \text{diagram} + \dots$$

$$\text{diagram} = i\mathcal{M} \frac{i}{k^2 - m^2 + i\epsilon} i\mathcal{W}_{\mu_1 \cdots \mu_n} + \text{“smooth”}$$

The diagrammatic expansion shows a vertex with four external lines and a wavy line. The first term is a pole term with a propagator denominator $k^2 - m^2 + i\epsilon$. The second term is a smooth function. Two circular insets illustrate the origin of the pole: the left inset shows a vertex with a small circle around it, and a dashed line connects it to the wavy line vertex, representing the pole; the right inset shows the vertex without the circle, representing the smooth part.

Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

$$i\mathcal{W}_{\mu_1 \cdots \mu_n} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]}$$
$$= \text{[diagram 7]} + \text{[diagram 8]}$$

$i\mathcal{W}_{\text{df}; \mu_1 \cdots \mu_n}$ ←

Two-to-two scattering **with current** - (df amp.)

Divergence-free amplitude

$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \text{[Diagram 5]}$$

New class of diagrams:

$$\begin{aligned}
 & \text{[Diagram 2]} \\
 &= \left\{ 1 + \text{[Diagram 2a]} + \text{[Diagram 2b]} + \dots \right\} \text{[Diagram 2]} \left\{ 1 + \text{[Diagram 2c]} + \text{[Diagram 2d]} + \dots \right\} \\
 &= \left[\frac{1}{1 - i\mathcal{K}\rho} \right] \text{[Diagram 2]} \left[\frac{1}{1 - \rho i\mathcal{K}} \right]
 \end{aligned}$$

same square-root (and possibly pole) singularities as two-body amplitudes

Two-to-two scattering **with current**

Divergence-free amplitude

$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{[diagrammatic expansion]} = \left[\frac{1}{1 - i\mathcal{K}\rho} \right] \left(\text{[diagrammatic terms]} \right) \left[\frac{1}{1 - i\mathcal{K}\rho} \right]$$

Complex function...depending on the one-body form factors

Naive Watson's theorem
does not apply!

finite-volume quantities **must**
be more complicated

Two-to-two scattering **with current**

Divergence-free amplitude


$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{[diagrammatic expansion]} = \left[\frac{1}{1 - i\mathcal{K}\rho} \right] \left(\text{[diagrammatic expansion]} \right) \left[\frac{1}{1 - i\mathcal{K}\rho} \right]$$

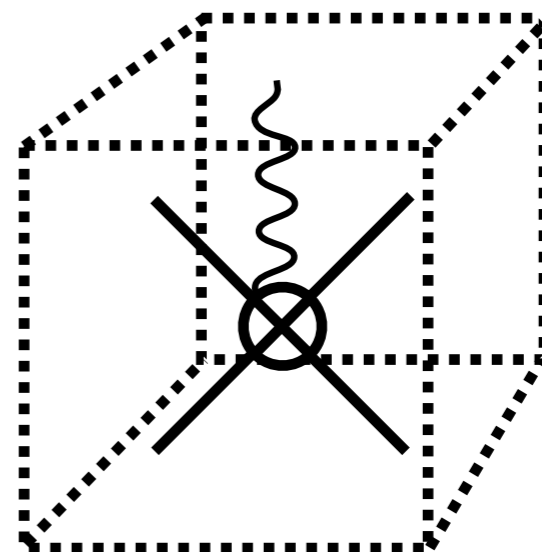
$$\xrightarrow{\lim s_i, s_f = s_R} \text{[diagrammatic expansion]} \sim \frac{g}{s_i - s_R} K_{\mu_1\cdots\mu_n} F_R(Q^2) \frac{g}{s_f - s_R}$$

$$|n\rangle_{\text{QCD}} = c_0 \text{[diagram]} + c_1 \text{[diagram]} + c_2 \text{[diagram]} + c_3 \text{[diagram]} + c_4 \text{[diagram]} + \dots$$

Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots$$




Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The equation shows a series of Feynman diagrams for a three-point correlation function. The first diagram is a circle with two external legs and a wavy line with a cross in a circle at the top. The second diagram is a circle with one external leg and a wavy line with a cross in a circle on the left. The third diagram consists of two circles connected at a vertex, with a wavy line with a cross in a circle at the top of the first circle. Ellipses follow the third diagram.

...everything is the same as before except for...

$$\text{[Diagram 1]} = \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \text{“smooth”}$$

The equation shows a decomposition of a diagram. The left side is a circle with two external legs and a wavy line with a cross in a circle at the top, labeled V/∞ . The right side is a sum of three diagrams, each labeled V/∞ , plus the word “smooth”. The first diagram on the right is a triangle with a dashed bottom edge, a wavy line with a cross in a circle at the top vertex, and two external legs at the bottom vertices. The second diagram on the right is a triangle with a dashed bottom edge, a wavy line with a cross in a circle at the top vertex, and two external legs at the bottom vertices, with a double line connecting the top vertex to the left bottom vertex. The third diagram on the right is a triangle with a dashed bottom edge, a wavy line with a cross in a circle at the top vertex, and two external legs at the bottom vertices, with a double line connecting the top vertex to the right bottom vertex.

Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

...everything is the same as before except for...

$$\text{[Diagram 1]} = \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \text{“smooth”}$$

leads to the presence of F-functions...

Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The diagrams show a series of terms in a sum. The first term is a circle with two external legs and a wavy line with a cross in a circle attached to the top. The second term is a circle with two external legs and a wavy line with a cross in a circle attached to the left. The third term is two circles connected by a vertical line, with a wavy line with a cross in a circle attached to the top of the left circle.

...everything is the same as before except for...

$$\text{[Diagram 4]} = \text{[Diagram 5]} + \text{[Diagram 6]} + \text{[Diagram 7]} + \text{“smooth”}$$

The diagrams show a decomposition of a circle with two external legs and a wavy line with a cross in a circle attached to the top. The first term on the right is a triangle with dashed lines and a wavy line with a cross in a circle attached to the top vertex. The second term is a triangle with solid lines and a wavy line with a cross in a circle attached to the top vertex. The third term is a triangle with dashed lines and a wavy line with a cross in a circle attached to the top vertex.

New finite-volume function:

$$G_{\mu_1 \dots \mu_n}(P_f, P_i, L) = \text{[Diagram 8]} - \text{[Diagram 9]}$$

The diagrams show two terms in a subtraction. Both terms are a semi-elliptical shape with dashed lines and a wavy line with a cross in a circle attached to the top. The first term has a solid line on the left side, and the second term has a solid line on the right side.

Two-particle in finite volume **with current**

After lots of massaging...

$$\left| \langle 2 | \mathcal{J} | 2 \rangle \right|_L^2 = \frac{1}{L^6} \text{Tr} [\mathcal{R} \mathcal{W}_{L,\text{df}} \mathcal{R} \mathcal{W}_{L,\text{df}}]$$

Building block #1) Lellouch-Luscher matrices:

$$\mathcal{R}(E_n, \mathbf{P}) \equiv \lim_{E \rightarrow E_n} \left[\frac{(E - E_n)}{F^{-1}(P, L) + \mathcal{M}(P)} \right]$$

“EASY”

-derivatives of amplitudes and F-function at the finite-volume spectra

Two-particle in finite volume **with current**

After lots of massaging...

$$\left| \langle 2 | \mathcal{J} | 2 \rangle \right|_L^2 = \frac{1}{L^6} \text{Tr} [\mathcal{R} \mathcal{W}_{L,\text{df}} \mathcal{R} \mathcal{W}_{L,\text{df}}]$$

Building block #2) stable particle form factor

$$\mathcal{W}_{L,\text{df}} - \mathcal{W}_{\text{df}}^{\mu_1 \cdots \mu_n} \sim \sum_{n'}^n \mathcal{M} [G^{(j)} \underbrace{f^{(j)}(-q^2)}] \mathcal{M}$$

“EASY”

- form factors of single-particle states

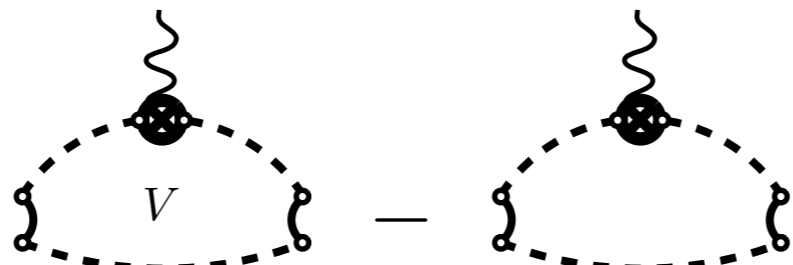
Two-particle in finite volume **with current**

After lots of massaging...

$$\left| \langle 2 | \mathcal{J} | 2 \rangle \right|_L^2 = \frac{1}{L^6} \text{Tr} [\mathcal{R} \mathcal{W}_{L,\text{df}} \mathcal{R} \mathcal{W}_{L,\text{df}}]$$

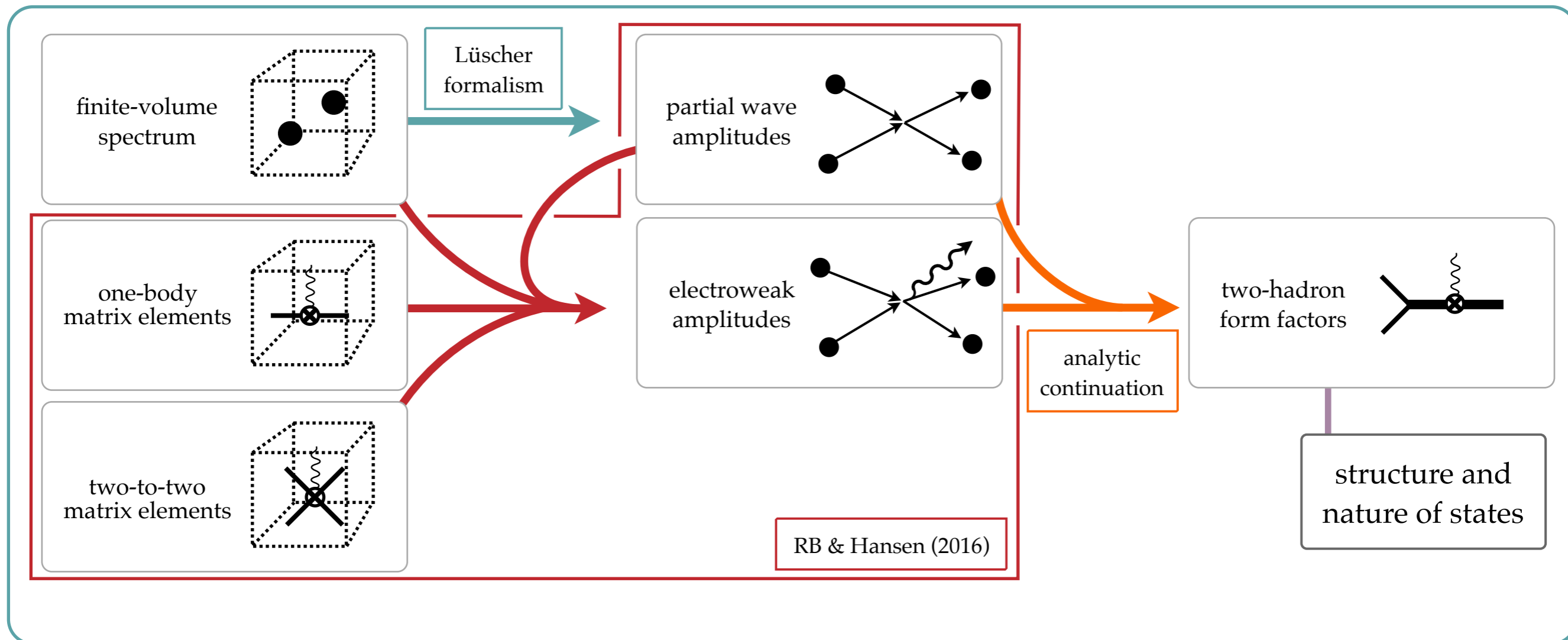
Building block #3) G-function

$$\mathcal{W}_{L,\text{df}} - \mathcal{W}_{\text{df}}^{\mu_1 \dots \mu_n} \sim \sum_{n'}^n \mathcal{M} [G^{(j)} f^{(j)}(-q^2)] \mathcal{M}$$

$$G_{\mu_1 \dots \mu_n}(P_f, P_i, L) = \text{Diagram 1} - \text{Diagram 2}$$








- Integral is complicated
- if $P_f = P_i$:
 - poles coincide...we found an analytic solution for the integral
- otherwise:
 - write the integral in terms of two integrals:
 - 4D, singular, covariant, dim-reg integral [semi-analytical]
 - 3D, smooth, evaluate numerically...

Take-home message

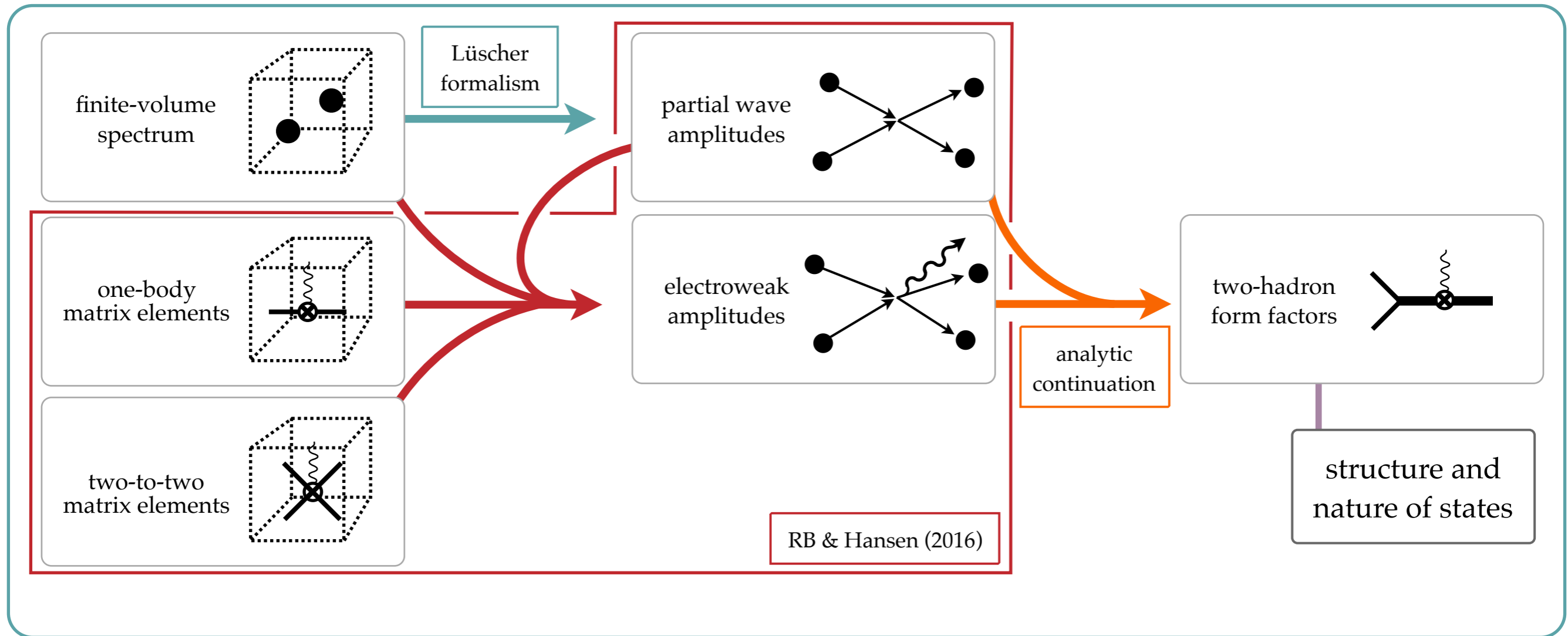


Multiple birds with one stone:

Access:

-  transition electroweak amplitudes
-  elastic electroweak amplitudes
-  structural information composite states:
 -  bound states
 -  resonance
-  remove all finite-volume systematics

Take-home message



Much harder than 0-to-2 and 1-to-2 transitions...but we have removed all major obstacles!

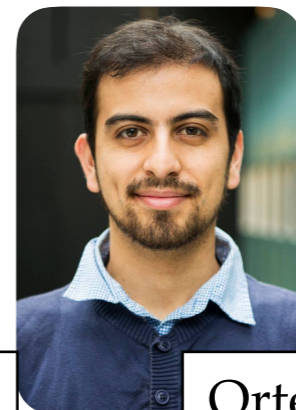
Ale's thumb of approval



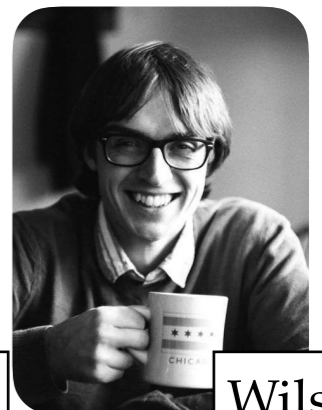
Baroni



Hansen



Ortega



Wilson