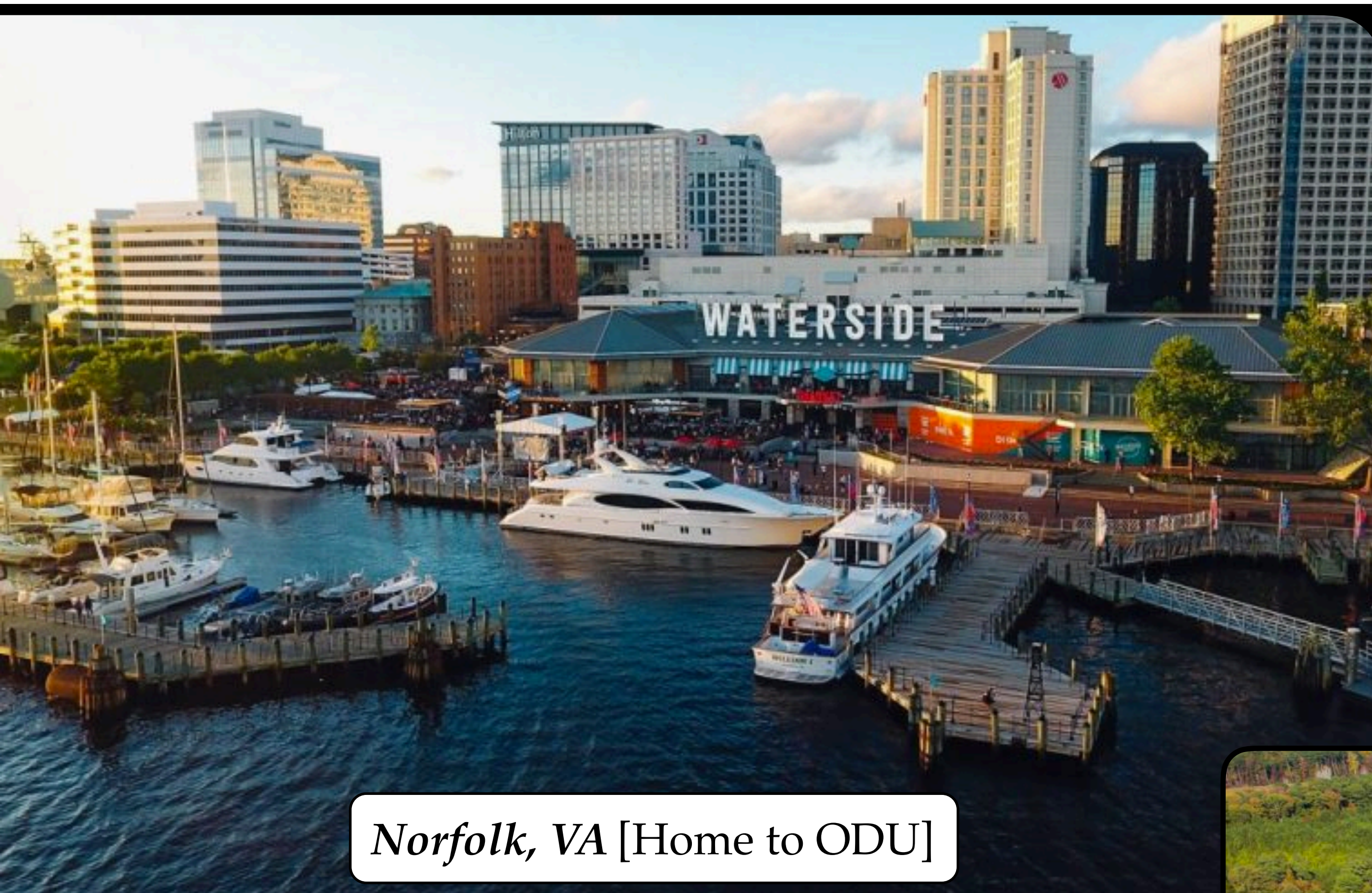
Studying the structure of few-hadron states

Raúl Briceño



Norfolk, VA [Home to ODU]

OLD DOMINION UNIVERSITY

Jefferson Lab

hadspec

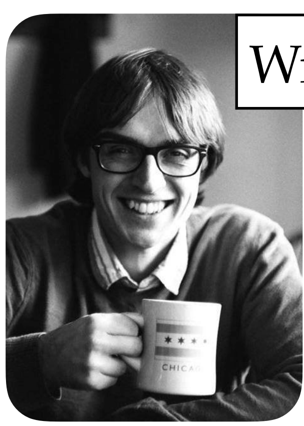
Confinement, 2018

JLab, VA

On-going work…

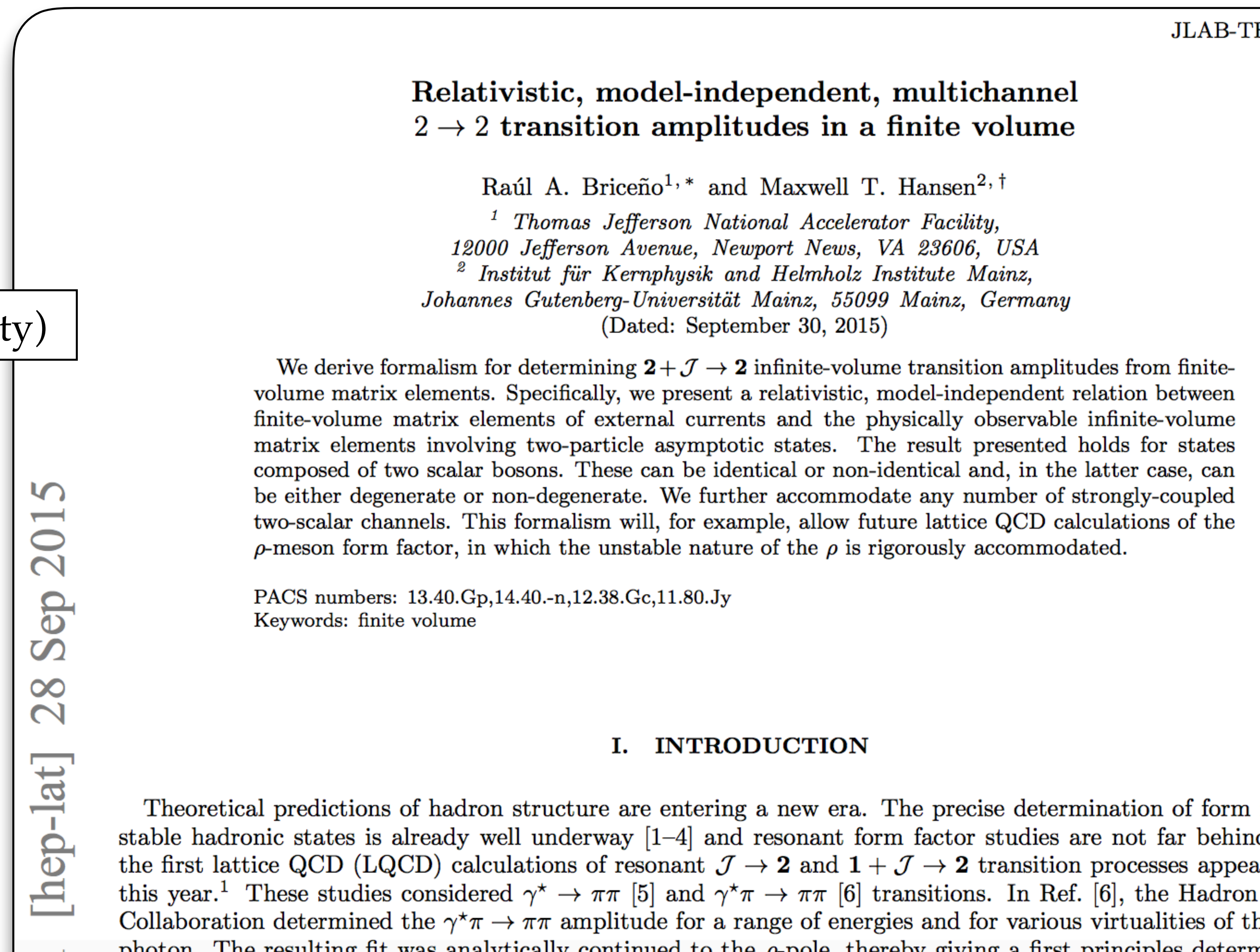
Hansen (CERN)

Baroni (South Carolina)



Wilson (Royal fellow / Trinity)

Ortega (W&M)



JLAB-TH

Relativistic, model-independent, multichannel $2 \to 2$ transition amplitudes in a finite volume

Raúl A. Briceño[1,*] and Maxwell T. Hansen[2,†]
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[2] *Institut für Kernphysik and Helmholz Institute Mainz, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany*
(Dated: September 30, 2015)

We derive formalism for determining $\mathbf{2}+\mathcal{J}\to\mathbf{2}$ infinite-volume transition amplitudes from finite-volume matrix elements. Specifically, we present a relativistic, model-independent relation between finite-volume matrix elements of external currents and the physically observable infinite-volume matrix elements involving two-particle asymptotic states. The result presented holds for states composed of two scalar bosons. These can be identical or non-identical and, in the latter case, can be either degenerate or non-degenerate. We further accommodate any number of strongly-coupled two-scalar channels. This formalism will, for example, allow future lattice QCD calculations of the ρ-meson form factor, in which the unstable nature of the ρ is rigorously accommodated.

PACS numbers: 13.40.Gp,14.40.-n,12.38.Gc,11.80.Jy
Keywords: finite volume

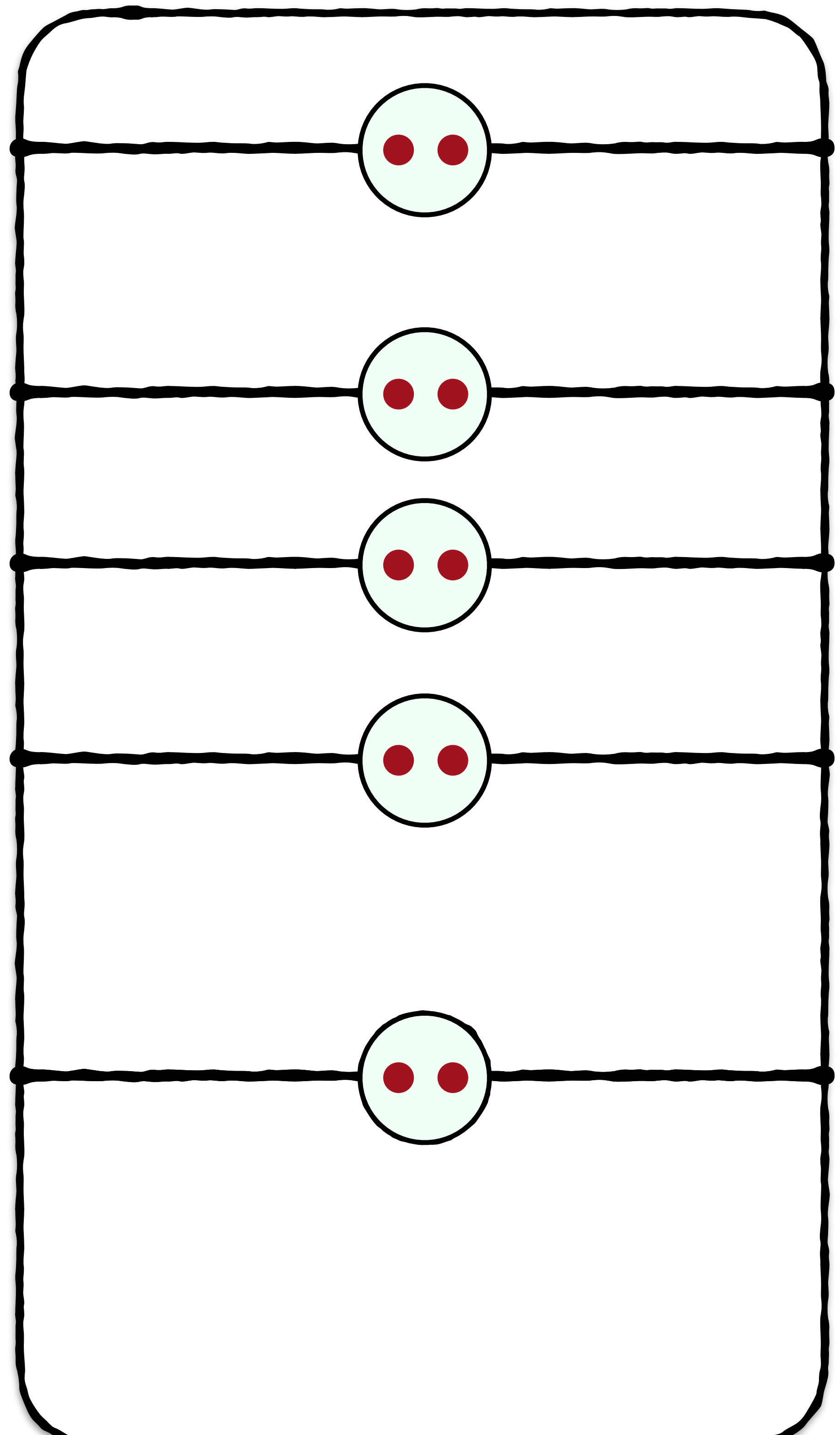
[hep-lat] 28 Sep 2015

I. INTRODUCTION

Theoretical predictions of hadron structure are entering a new era. The precise determination of form stable hadronic states is already well underway [1–4] and resonant form factor studies are not far behind the first lattice QCD (LQCD) calculations of resonant $\mathcal{J}\to\mathbf{2}$ and $\mathbf{1}+\mathcal{J}\to\mathbf{2}$ transition processes appea this year.[1] These studies considered $\gamma^\star\to\pi\pi$ [5] and $\gamma^\star\pi\to\pi\pi$ [6] transitions. In Ref. [6], the Hadron Collaboration determined the $\gamma^\star\pi\to\pi\pi$ amplitude for a range of energies and for various virtualities of th

On the nature of states

Consider the 0^{++} channel…



On the nature of states

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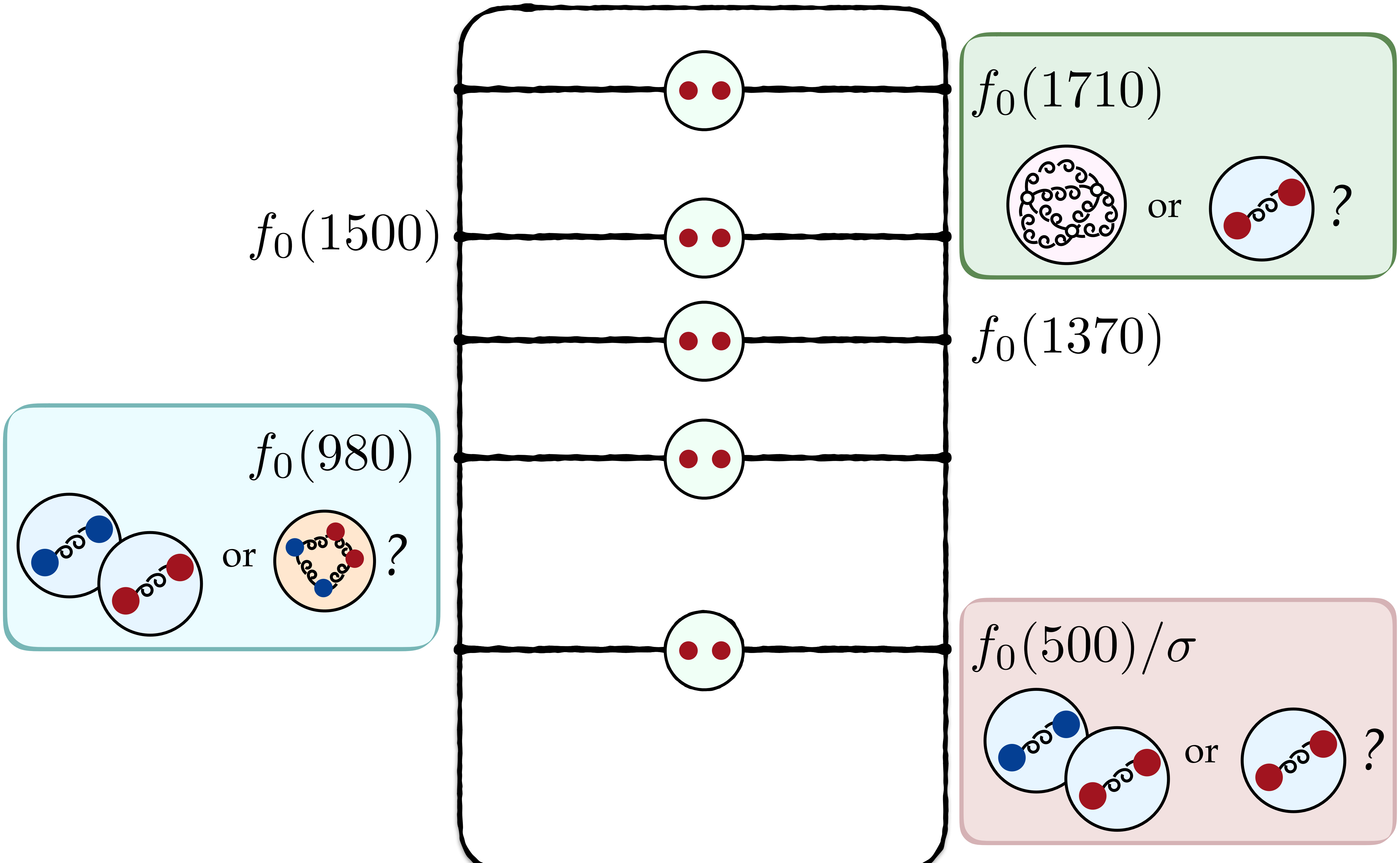

On the nature of states

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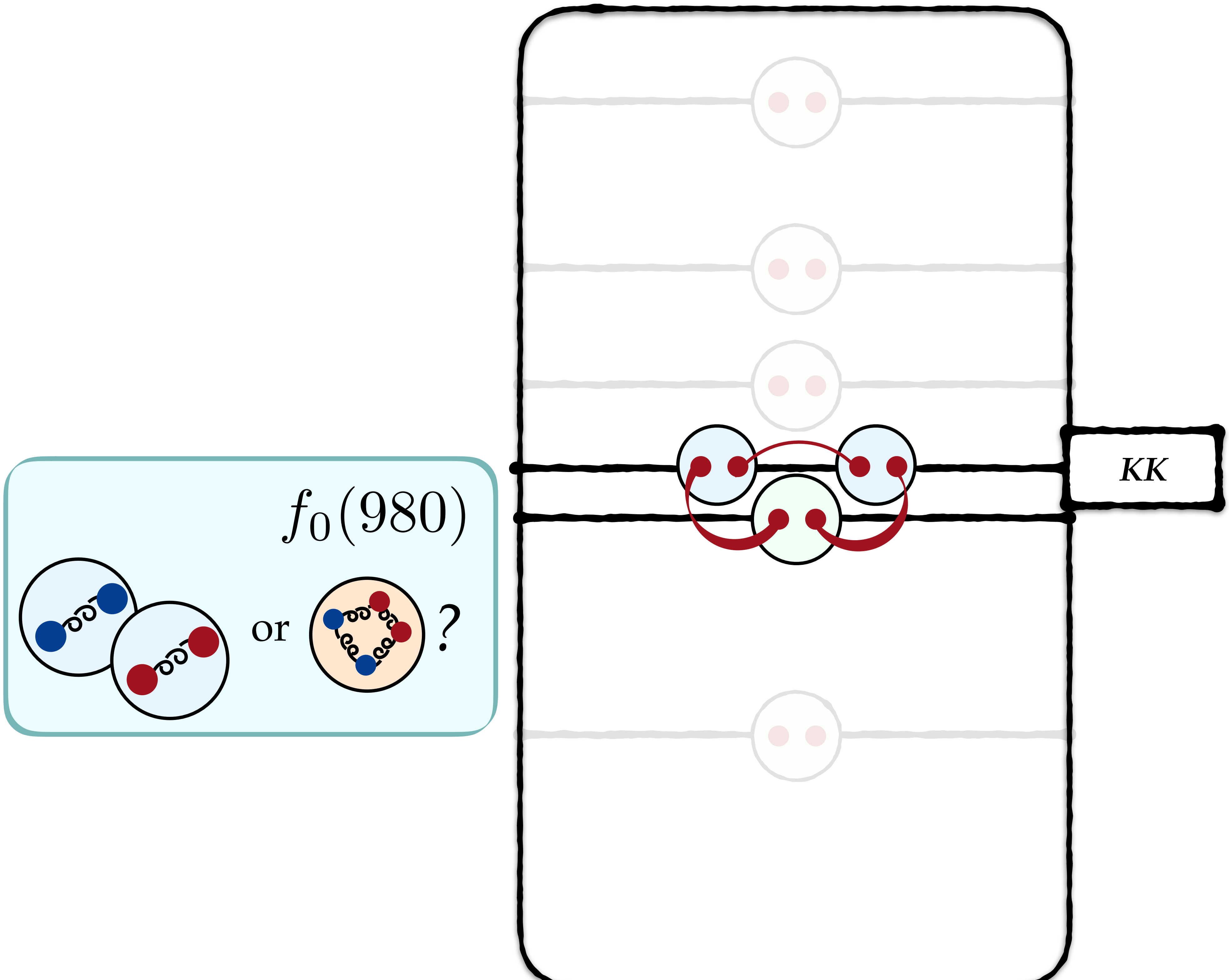

On the nature of states

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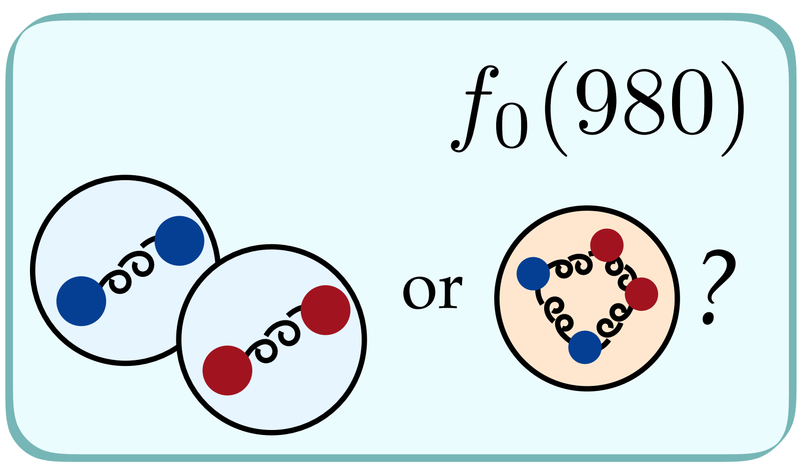
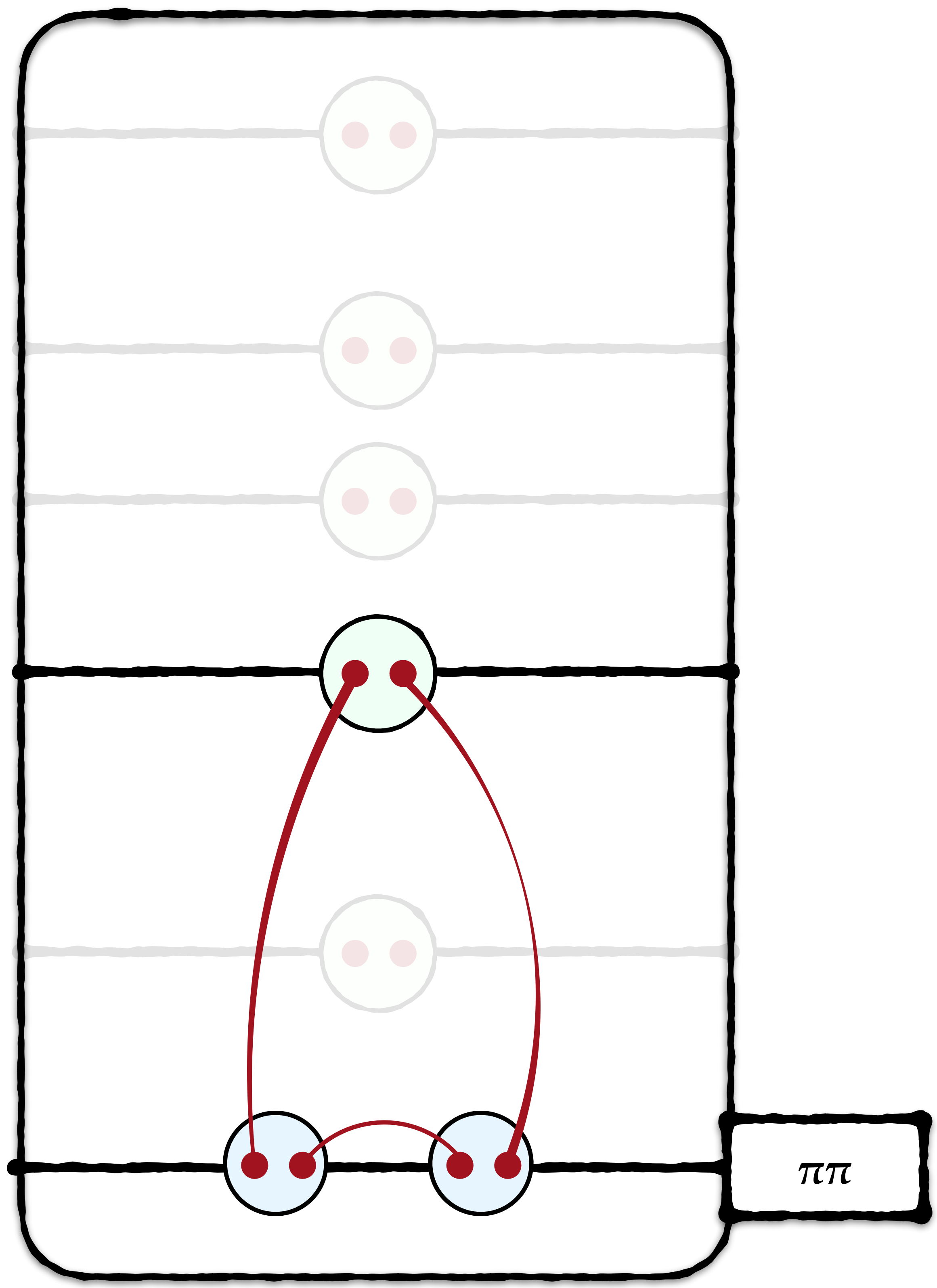




QCD spectroscopy

Amplitude analysis

Experiments

QCD

QCD spectroscopy

Amplitude analysis

Experiments

QCD

GOAL:

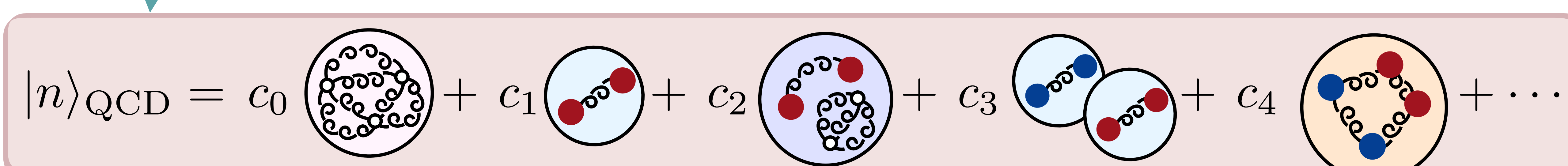
Get insights to the governing patterns and rules of QCD from emergent phenomena

Observables to test our understanding:

- Production and decay
- Exotic states
- …

Possible outcomes:

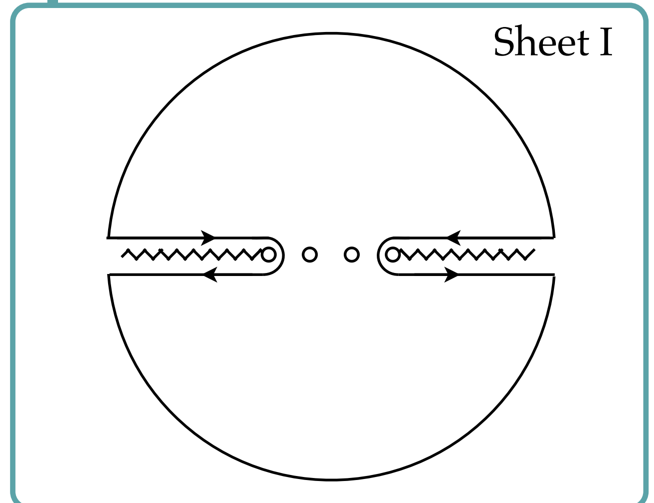
- Source of masses
- Role of glue
- Structure of excited states;
- …



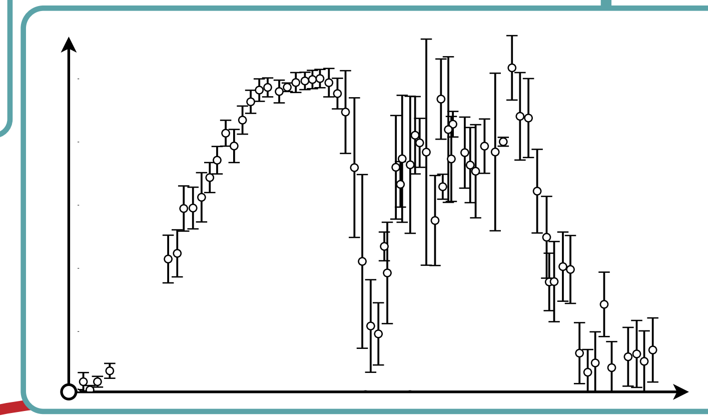
… perhaps there is a hierarchy [e.g. $c_0 > c_1 > c_2 > c_3 > c_4$]

QCD spectroscopy

Amplitude analysis

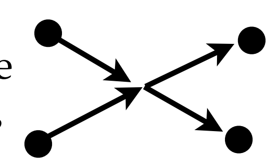


Experiments

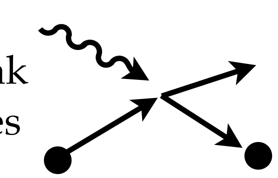


QCD

partial wave amplitudes

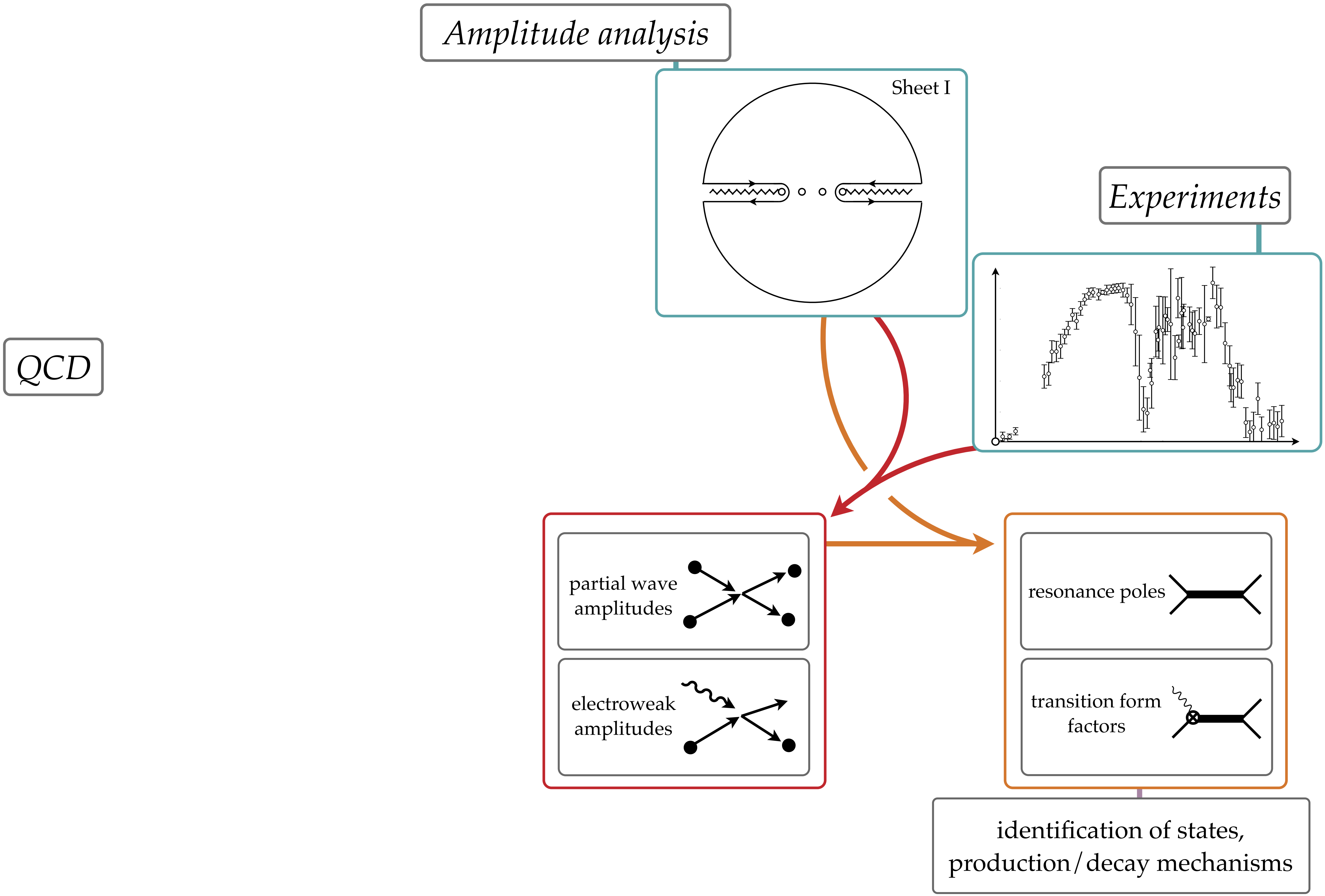


electroweak amplitudes



See Ryan Mitchell's talk for recent progress

QCD spectroscopy



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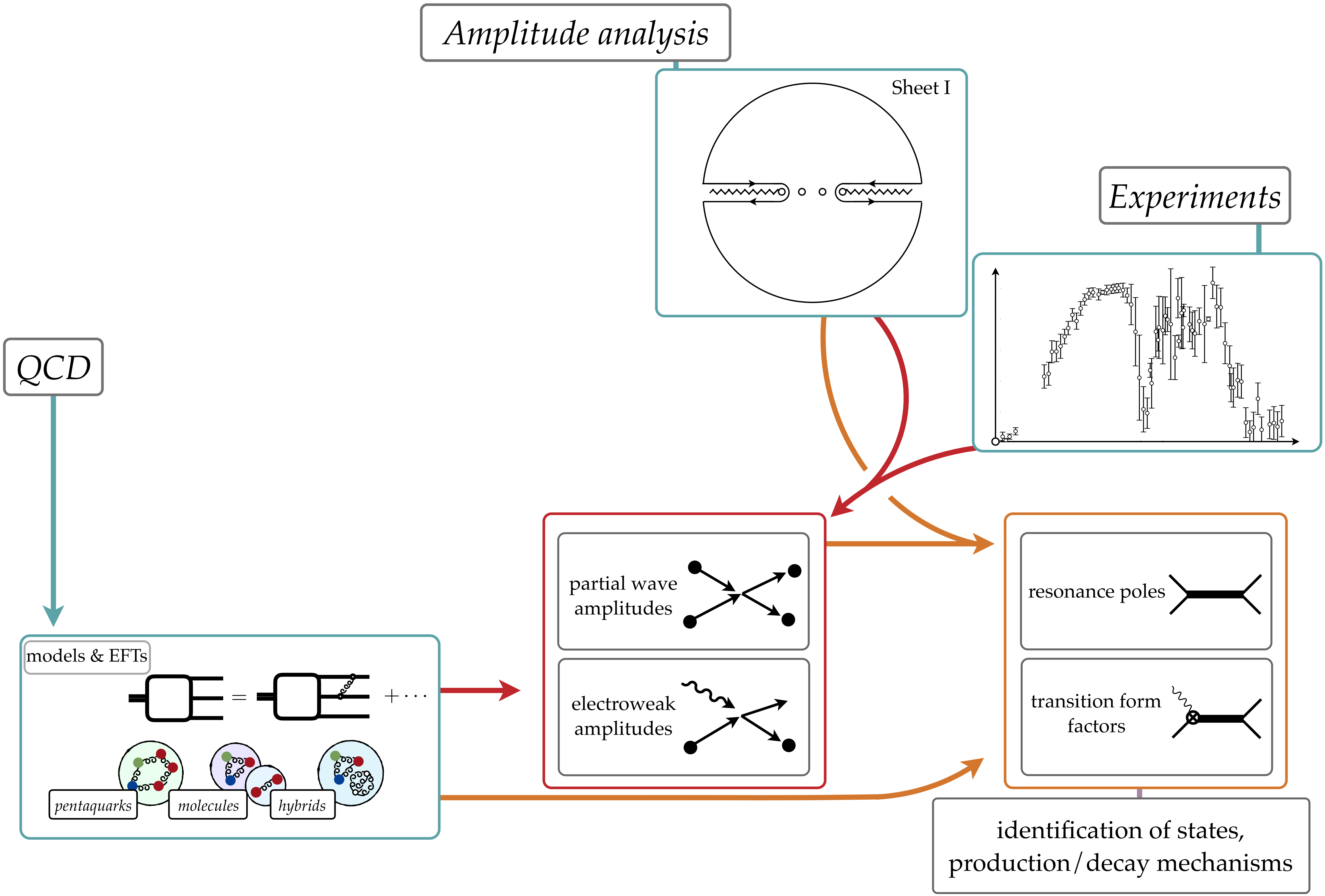
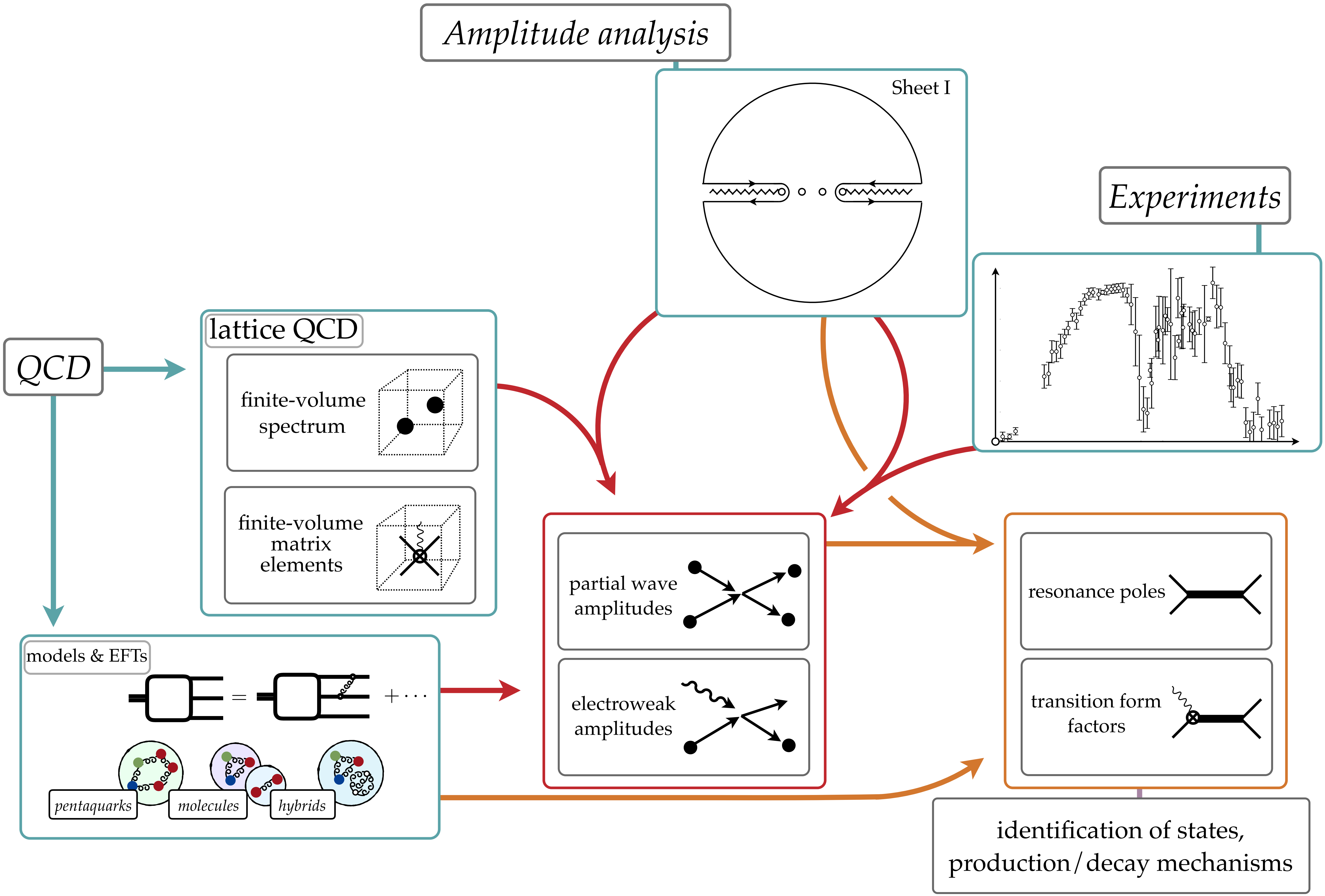
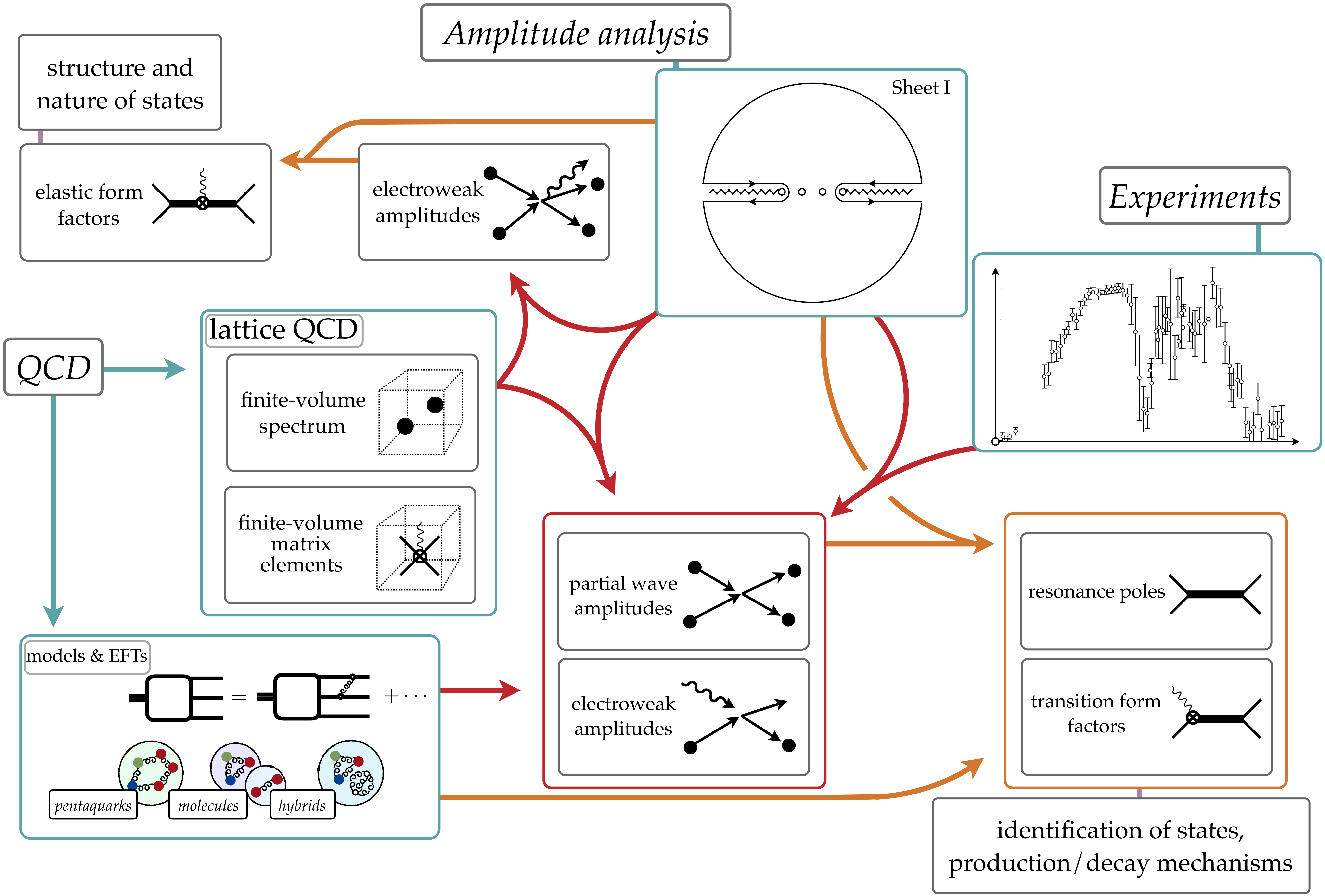


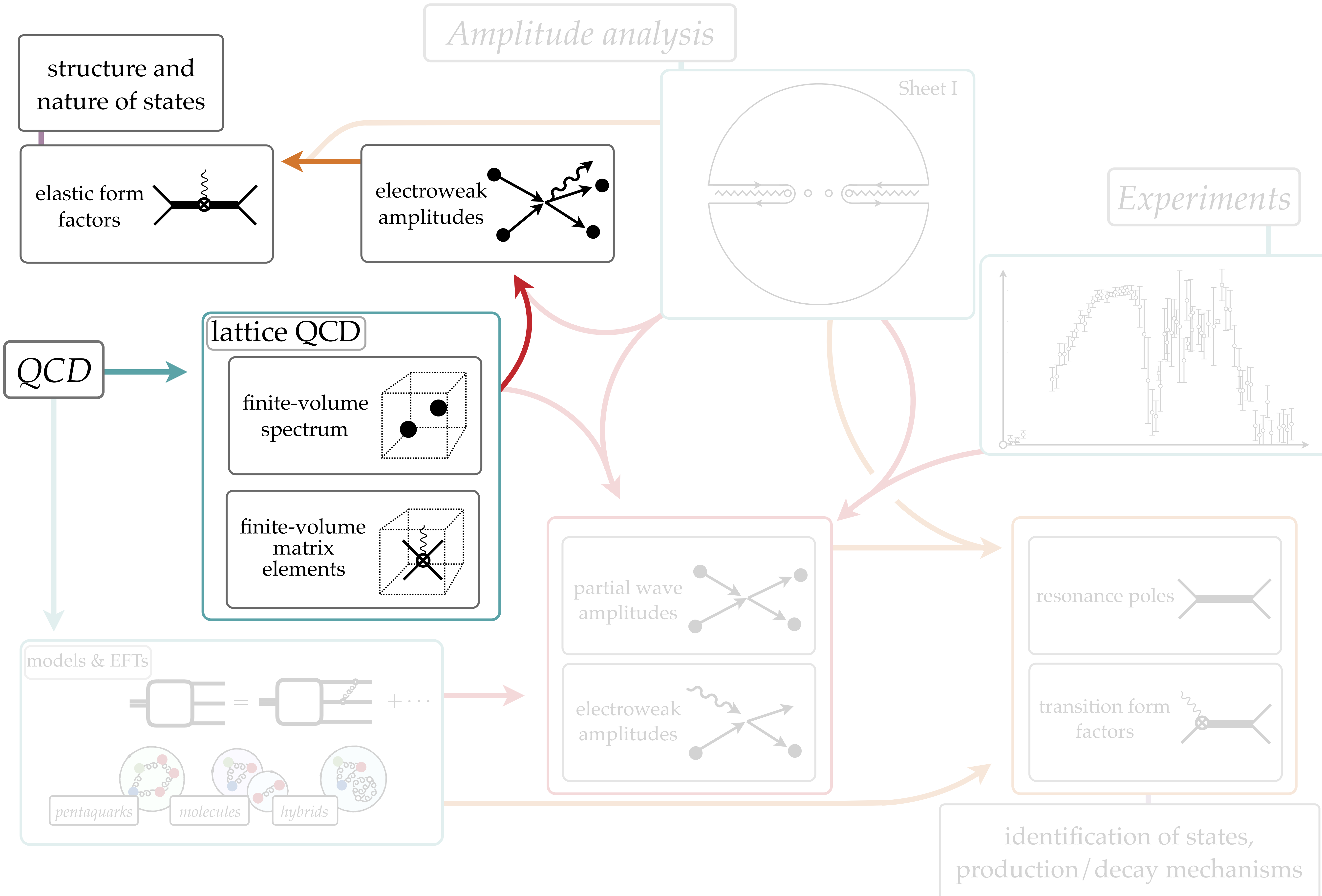
QCD spectroscopy



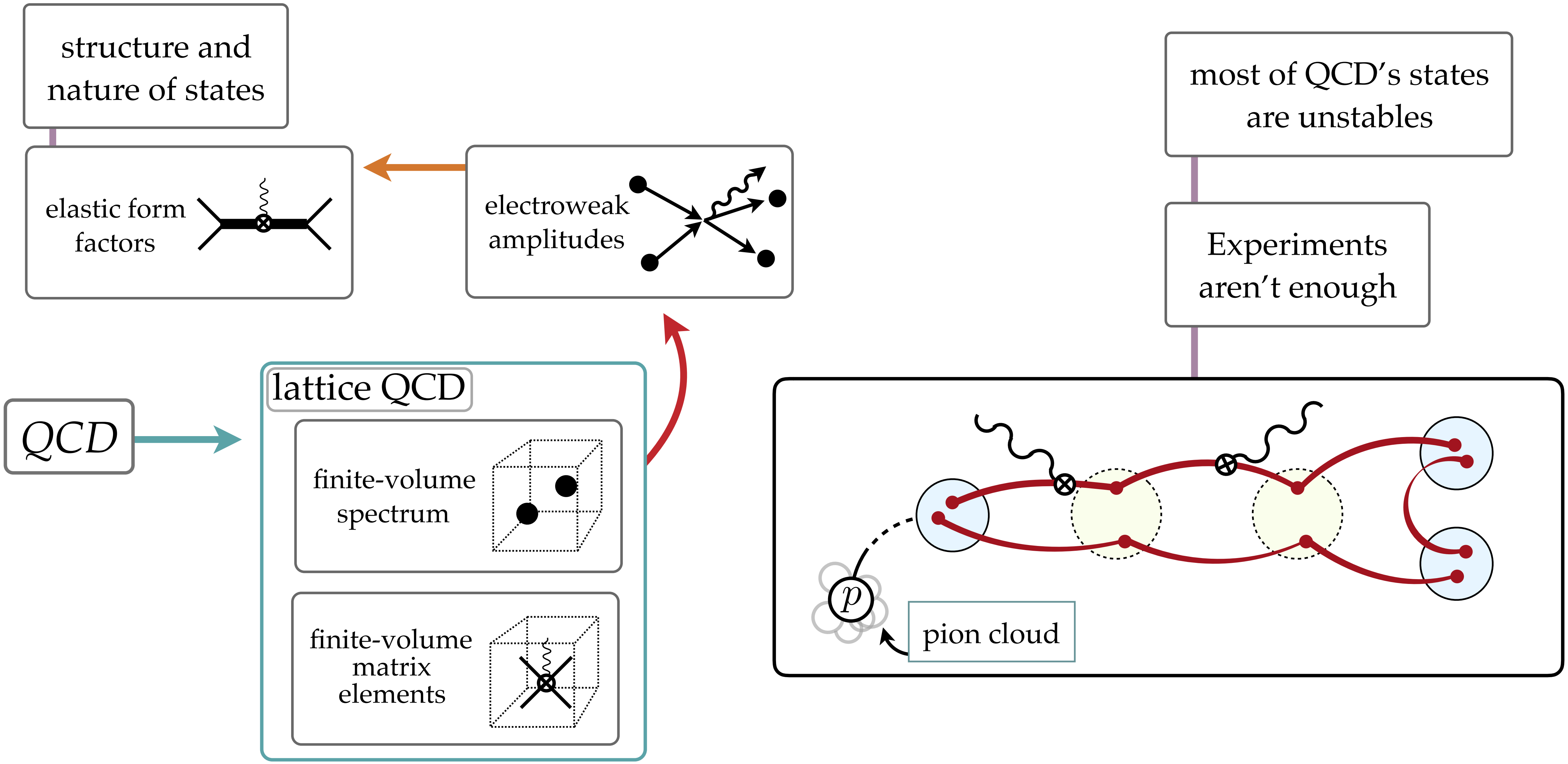
QCD spectroscopy



QCD spectroscopy



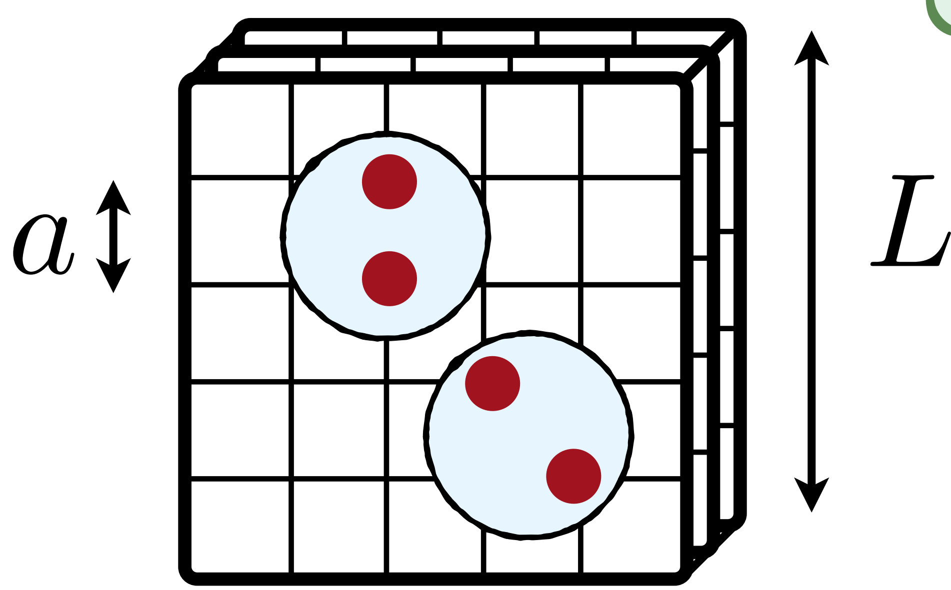
QCD spectroscopy



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- quark masses: $m_q \to m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume

$$D_\mu = \left(\quad \right) \updownarrow (L/a)^3 \times (T/a)$$



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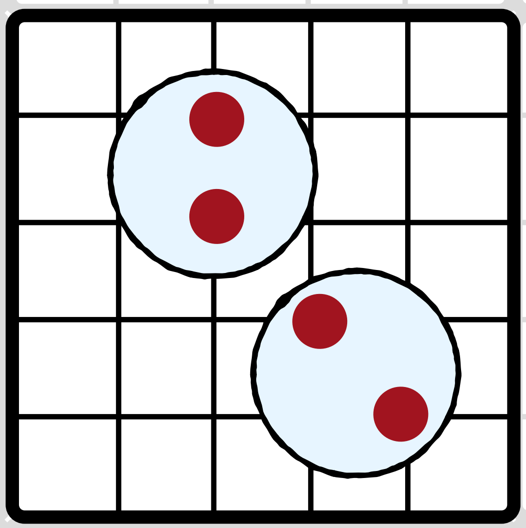
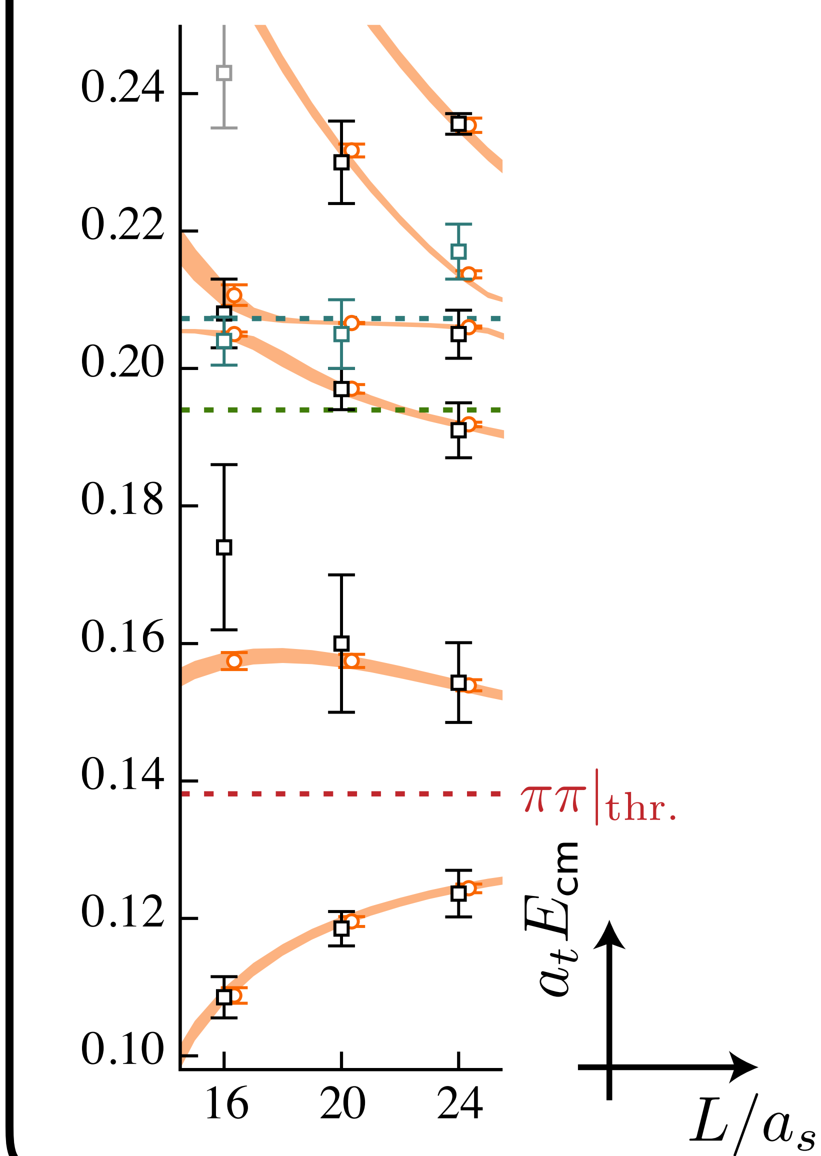


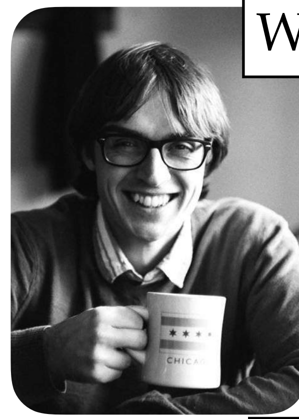
Never free!
No asymptotic states!
No scattering!

On the structure of states

Isoscalar 0^{++} channel:



$m_\pi \sim 400$ MeV



Wilson (Royal fellow / Trinity)

Dudek (W&M / JLab)

see J. Dudek's slides for details

hadspec

Edwards (JLab)

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Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

(for the Hadron Spectrum Collaboration)

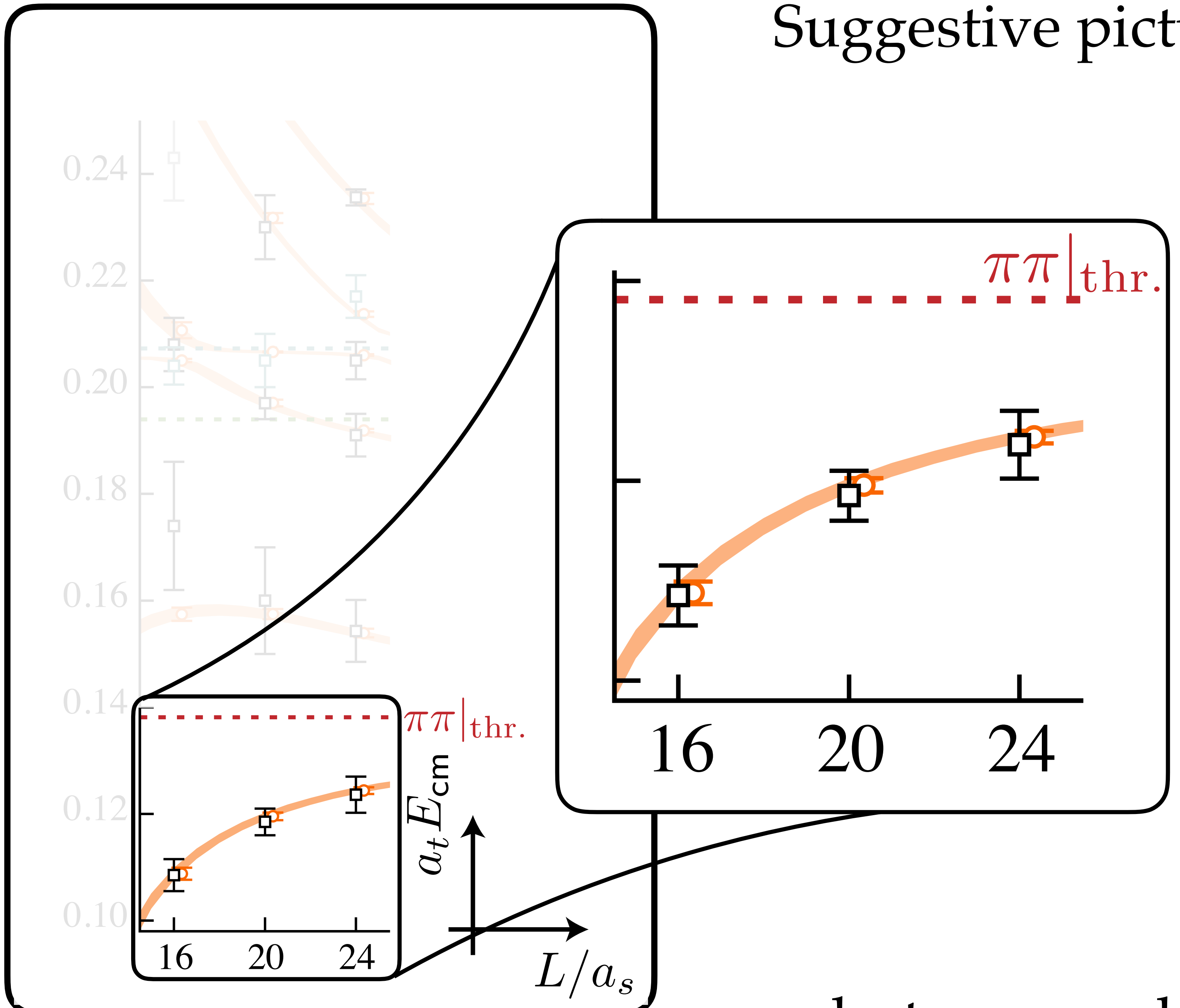
JLAI

Isoscalar $\pi\pi, K\overline{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson
(for the Hadron Spectrum Collaboration)
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606*
[2] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[3] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
[4] *School of Mathematics, Trinity College, Dublin 2, Ireland*

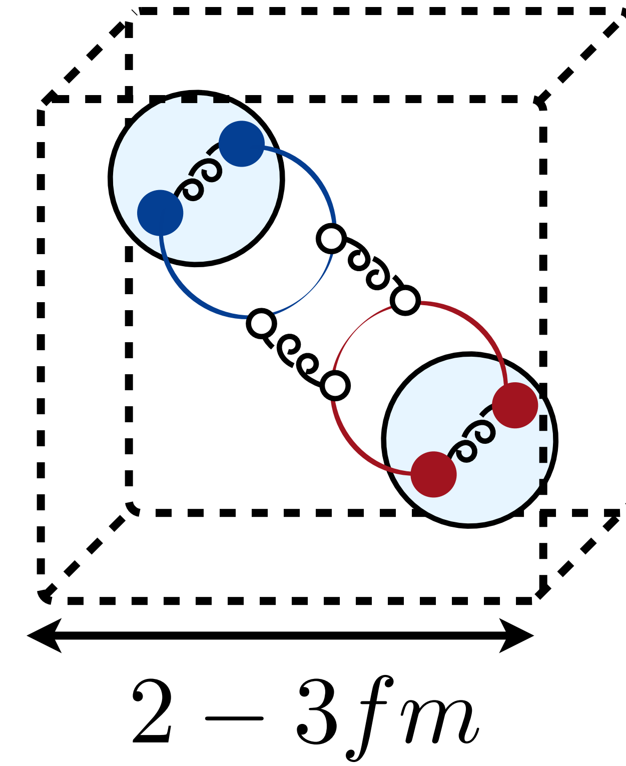
On the structure of states

Isoscalar 0^{++} channel:



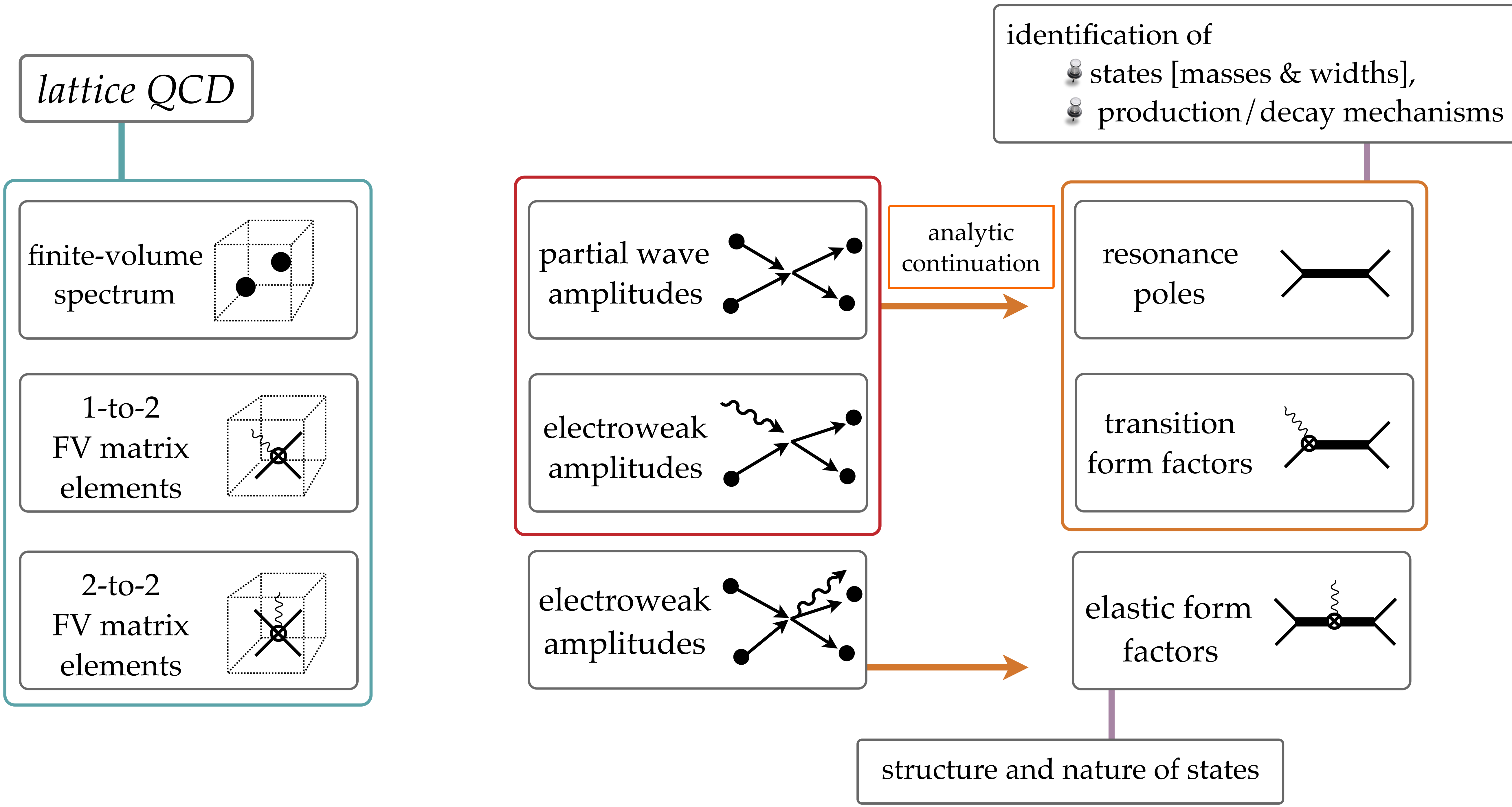
$m_\pi \sim 400$ MeV

Suggestive picture:

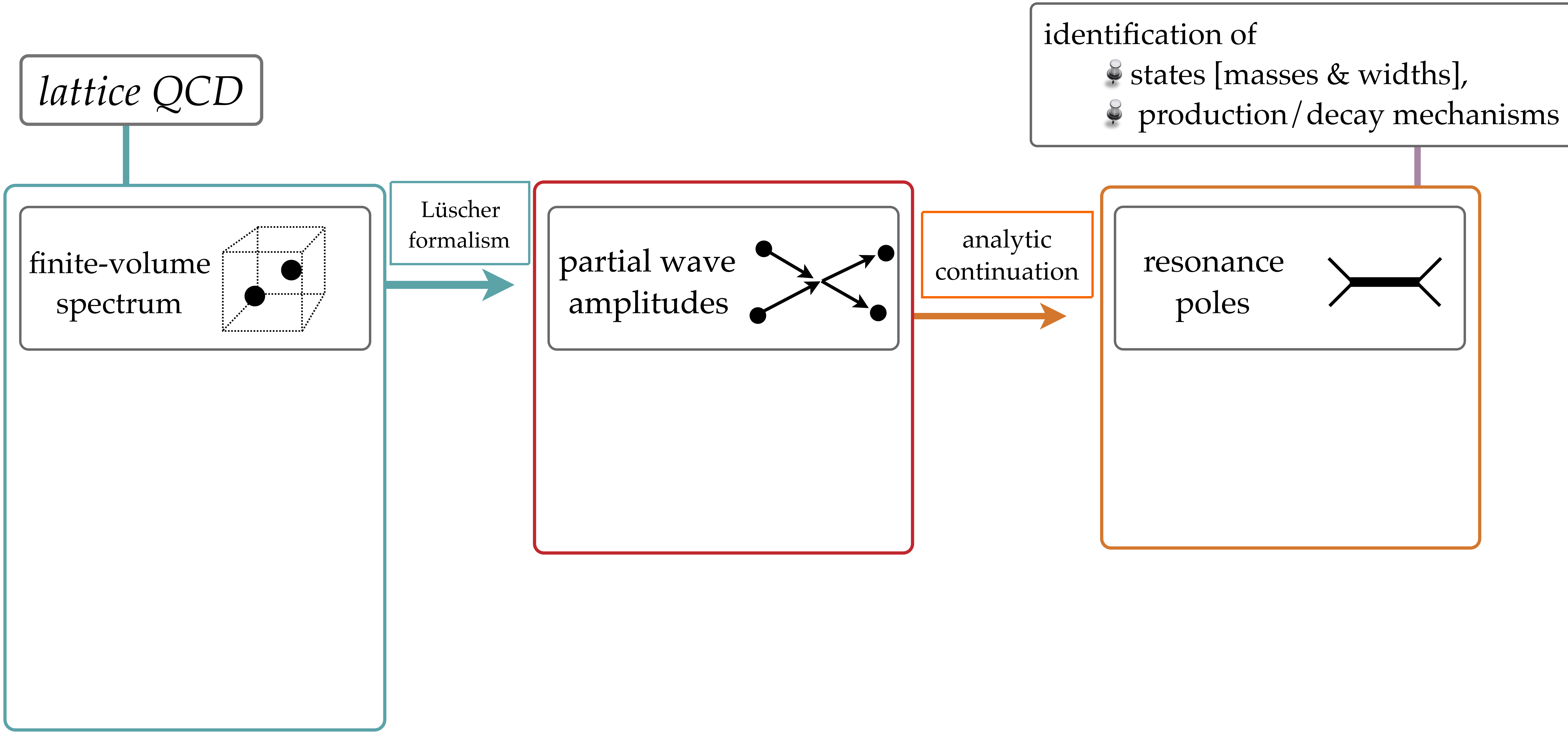


…but we can do much more…!

QCD spectroscopy



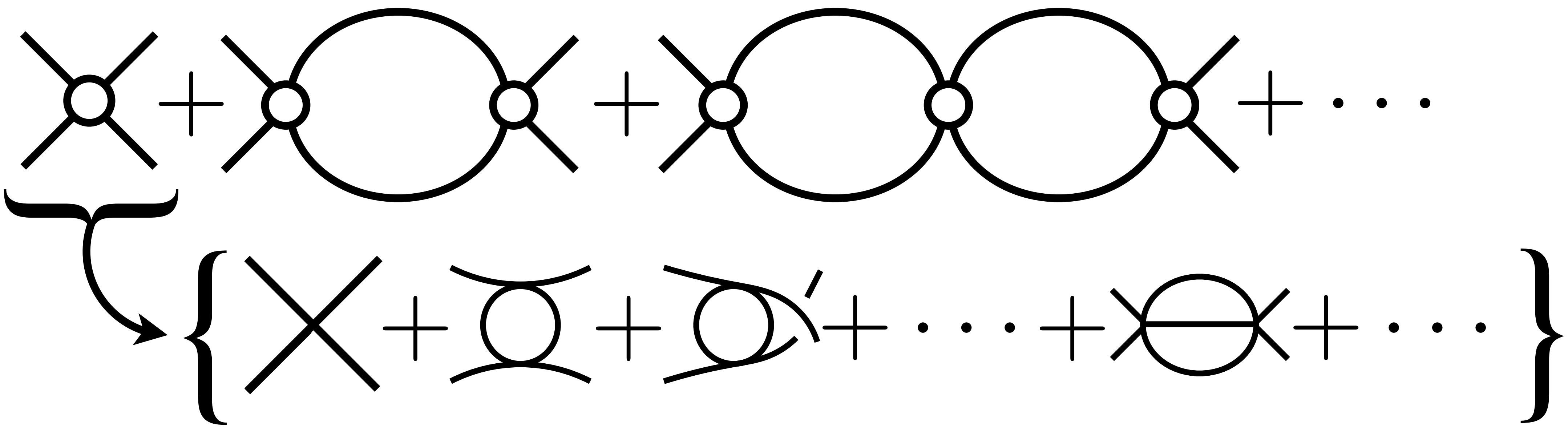
QCD spectroscopy



Two-body scattering

Unitarity using all orders perturbation theory:

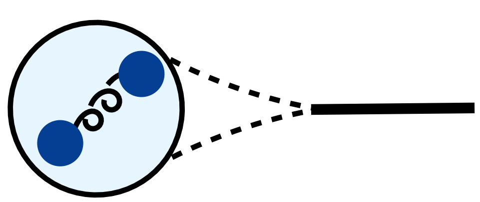
$i\mathcal{M} =$



non-perturbative kernel including all diagrams not shown…

"yep, the left hand cut is there"

IR limit of QCD, only interested in hadronic d.o.f.



Two-body scattering

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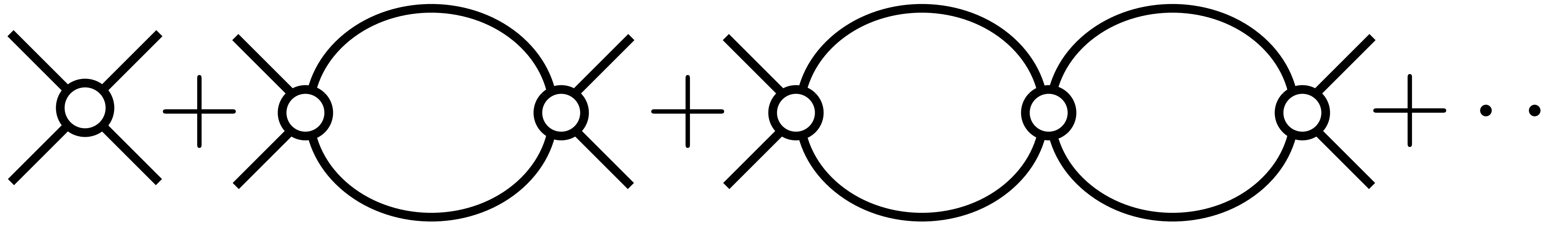
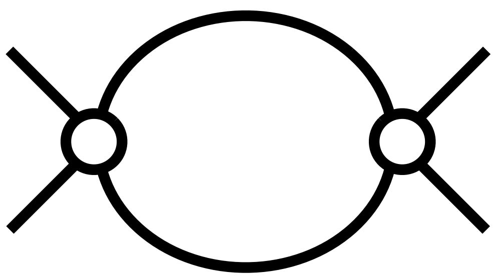
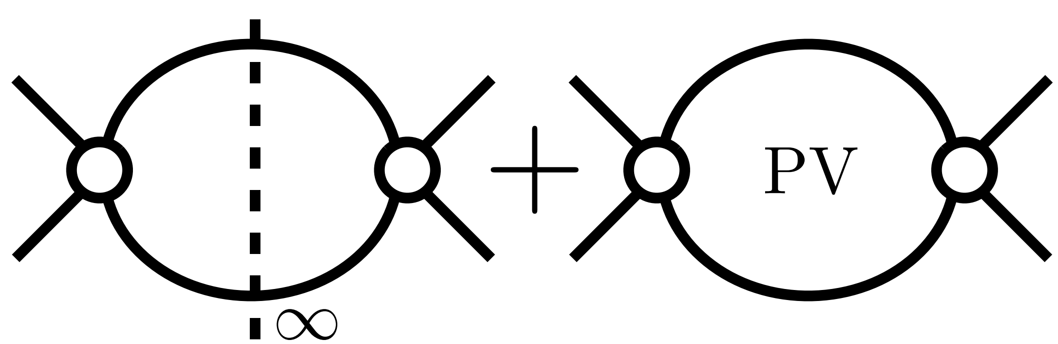
$i\mathcal{M} =$ 



$$= \int \frac{d^4k}{(2\pi)^4} [iB(k,P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P-k)^2 - m^2 + i\epsilon}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k,P)]^2}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} + \text{“smooth”}$$

$$= [iB_{on}]\rho\,[iB_{on}] + \text{“PV integral”}$$

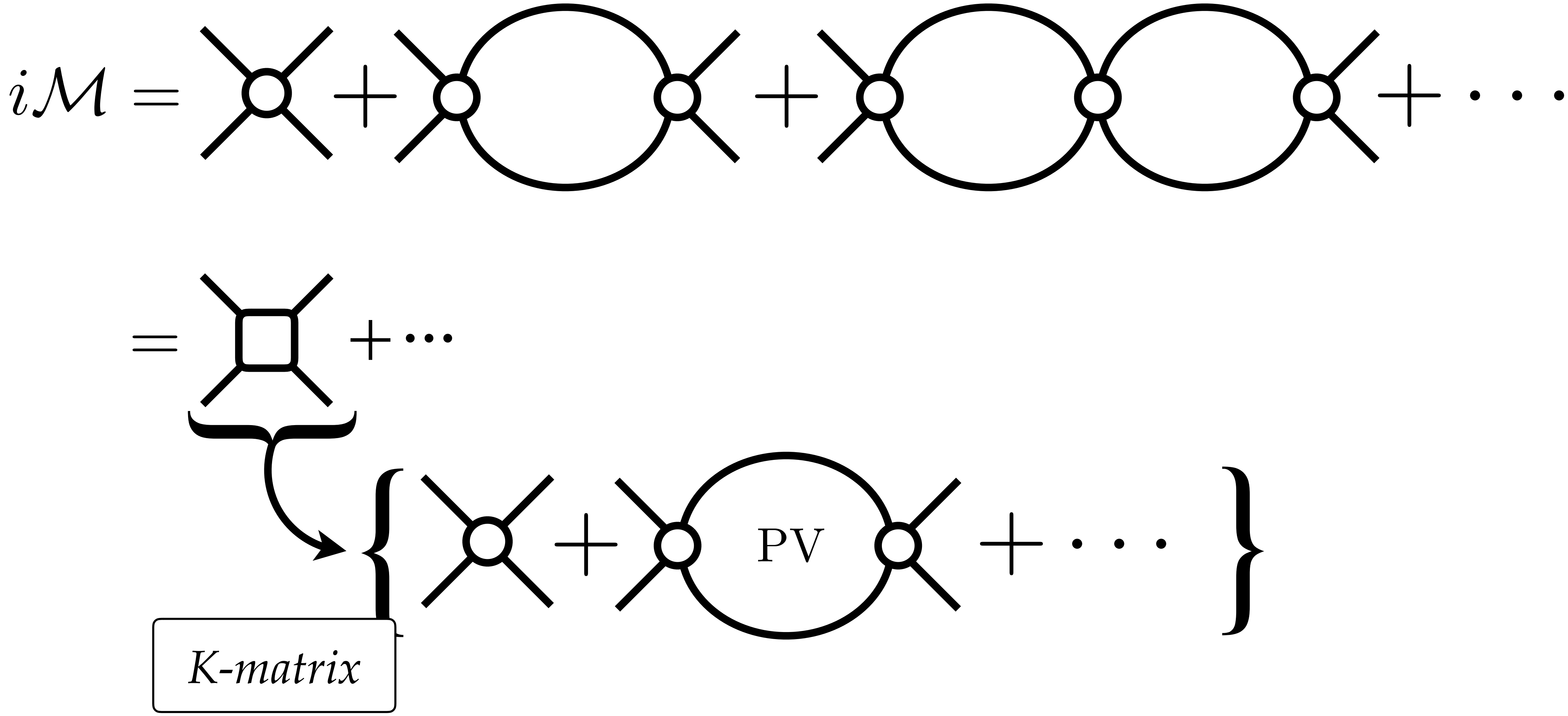
$=$ 

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-body scattering

Unitarity using all orders perturbation theory:



Two-body scattering

Unitarity using all orders perturbation theory:

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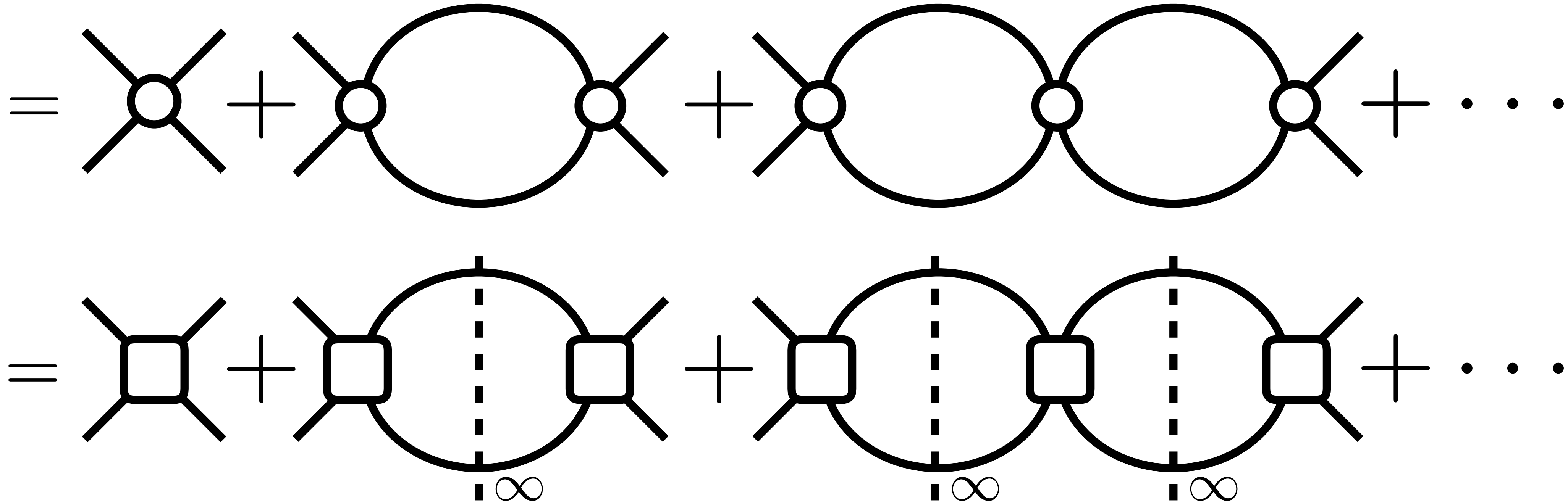
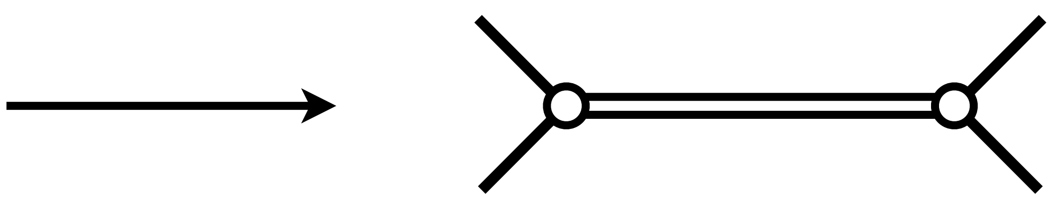
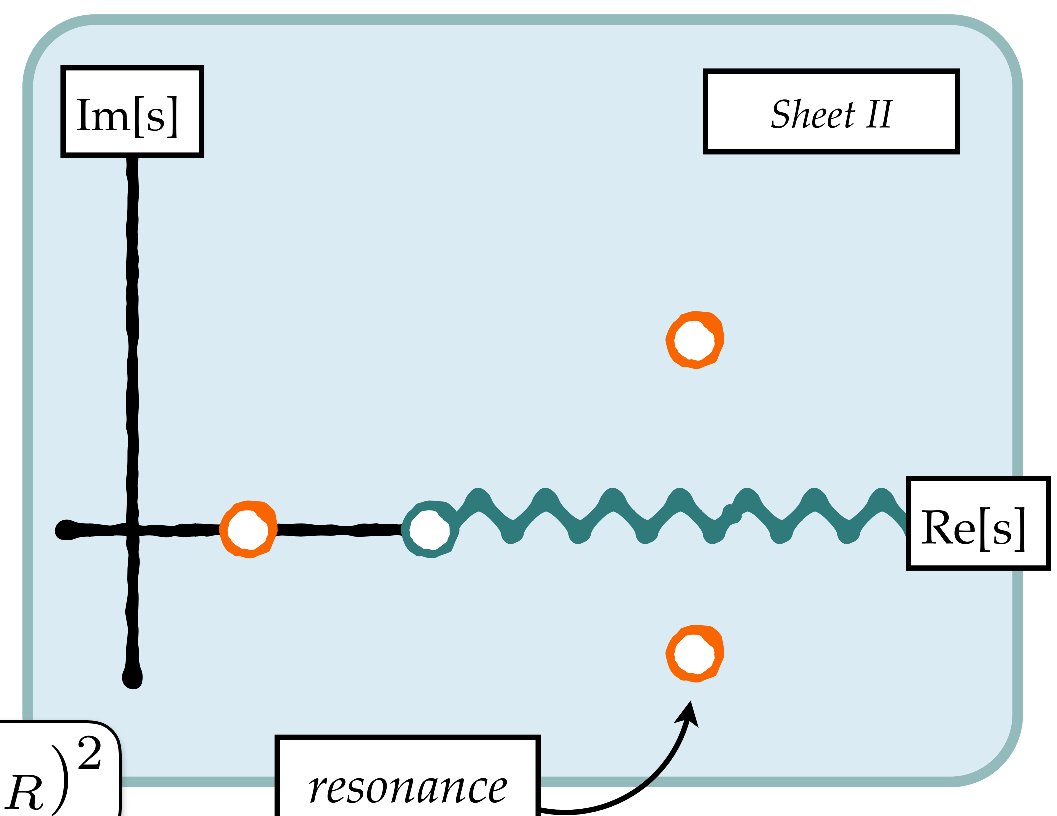


$$= i\mathcal{K}\,\frac{1}{1 - i\rho\mathcal{K}}$$



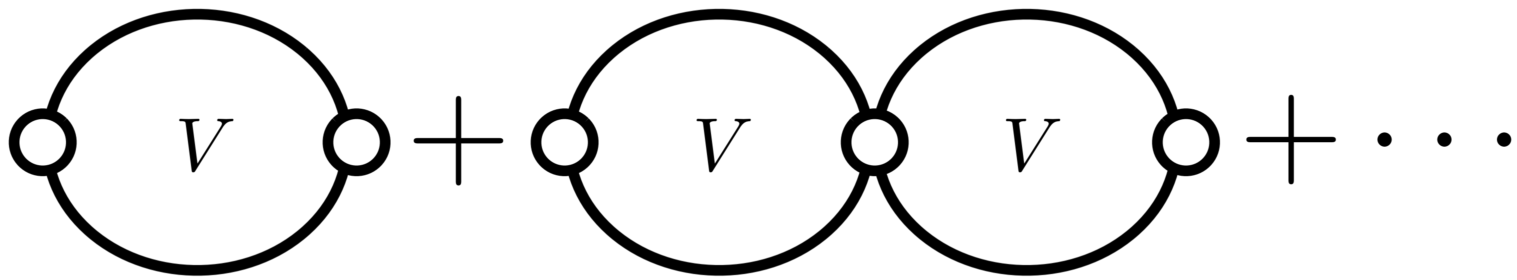


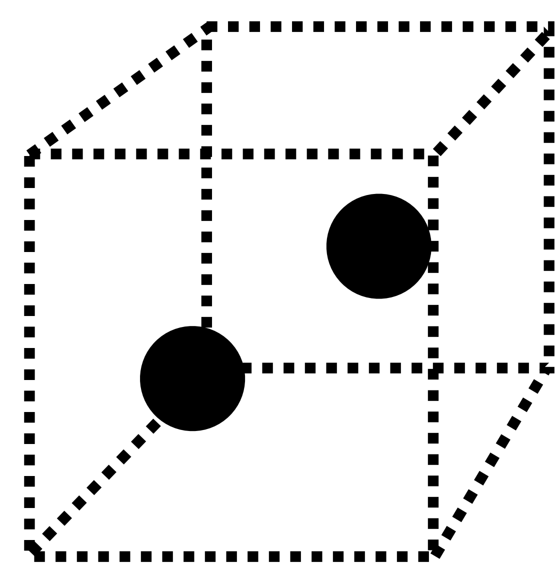
$$s_R = \left(E_R - \tfrac{i}{2}\Gamma_R\right)^2$$

Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

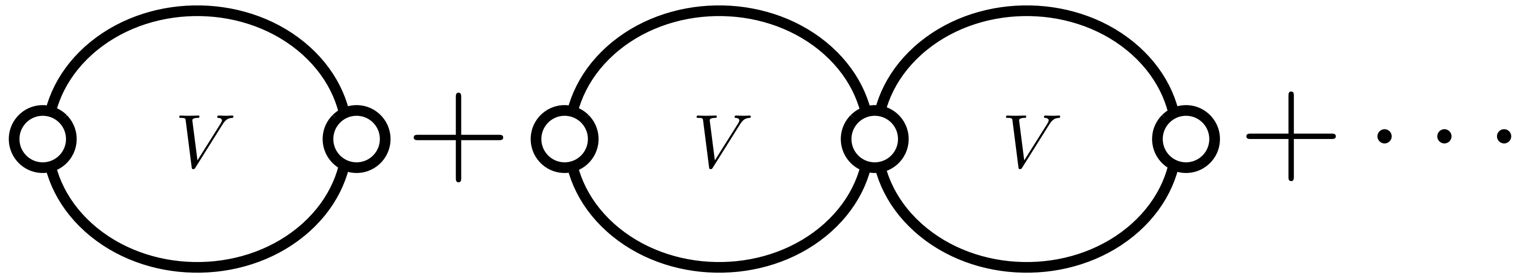
$$C_L^{2pt.}(P) =$$

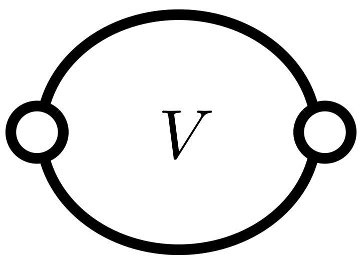




Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m):*

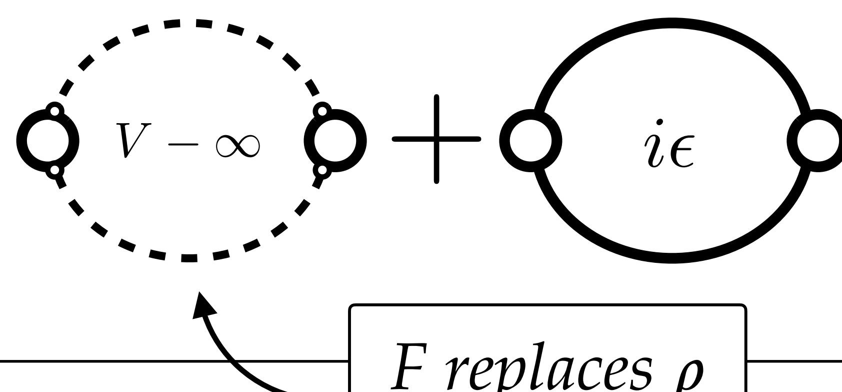
$C_L^{2pt.}(P) =$ 



$$= \frac{1}{L^3} \sum_{\mathbf{k}} \frac{iB^2}{(2\omega_k)^2} \frac{i}{E - 2\omega_k} + \text{“smooth”}$$

$$= (iB) \left(\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} \right) (iB) + i\epsilon \text{ integral}$$

$$\equiv [iB] \, iF \, [iB] + i\epsilon \text{ integral}$$

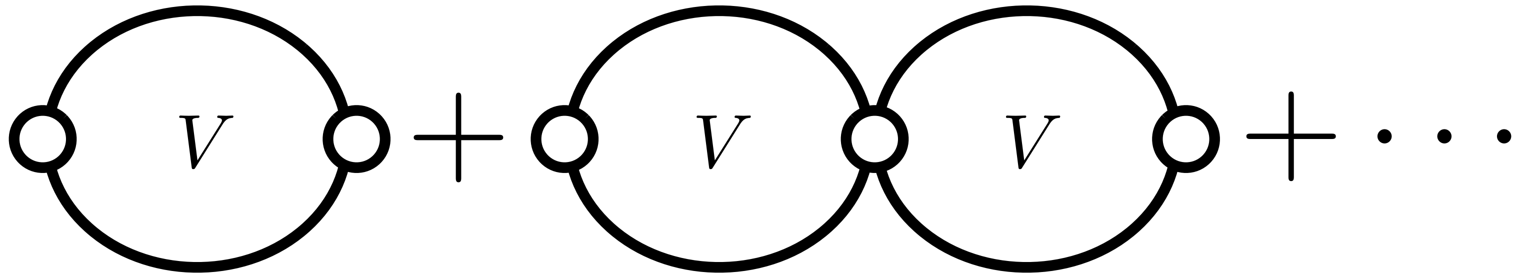
$=$ 

F replaces ρ

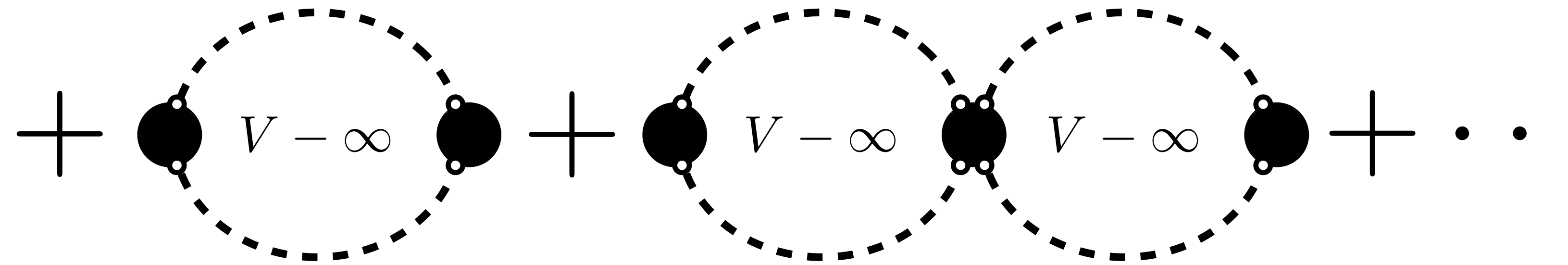
Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m*):

$$C_L^{2pt.}(P) =$$



$$= C_\infty(P) +$$

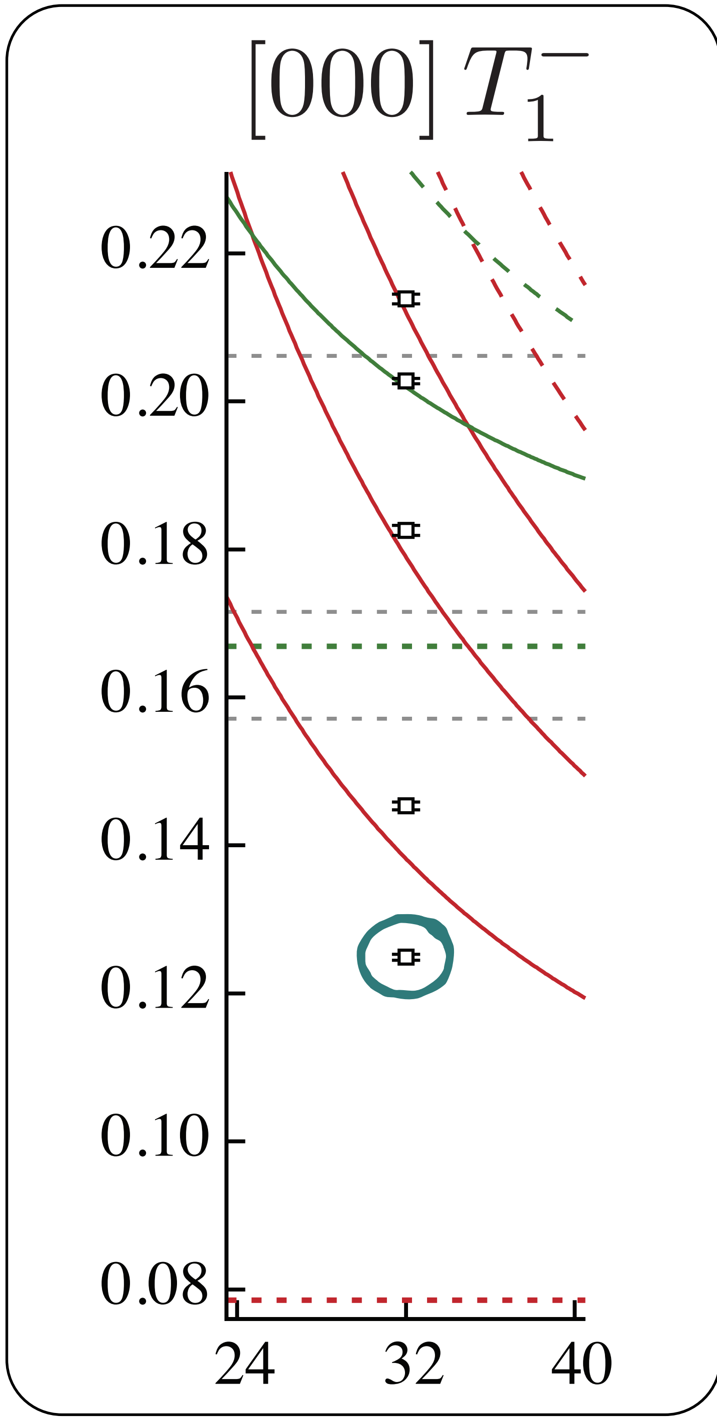


$$= \text{“smooth”} + A\frac{i}{F^{-1}+\mathcal{M}}B^\dagger$$

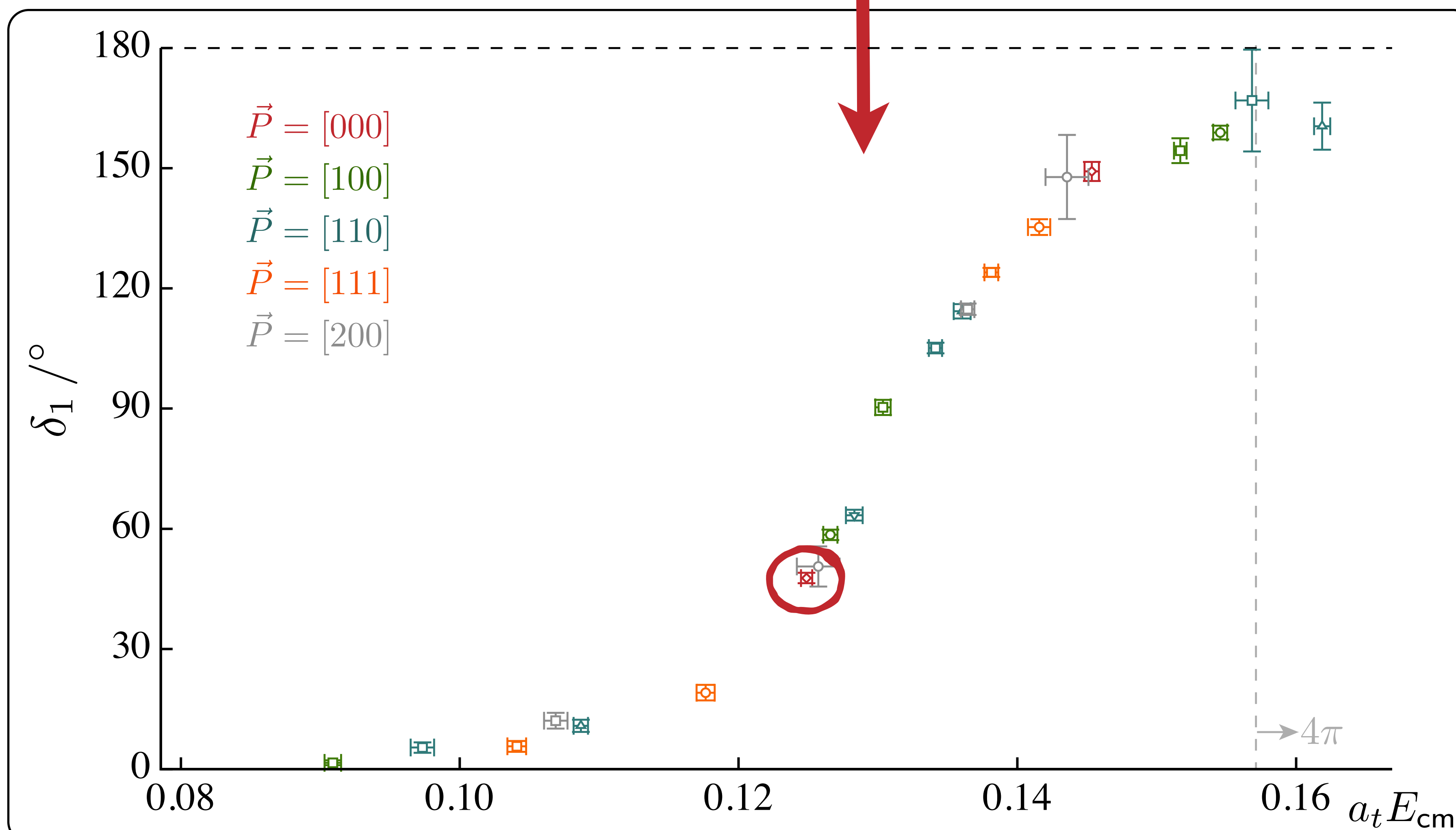
poles satisfy: $\det[F^{-1}(P,L)+\mathcal{M}(P)] = 0$

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

$\pi\pi$ Spectrum - (I=1 channel)



$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

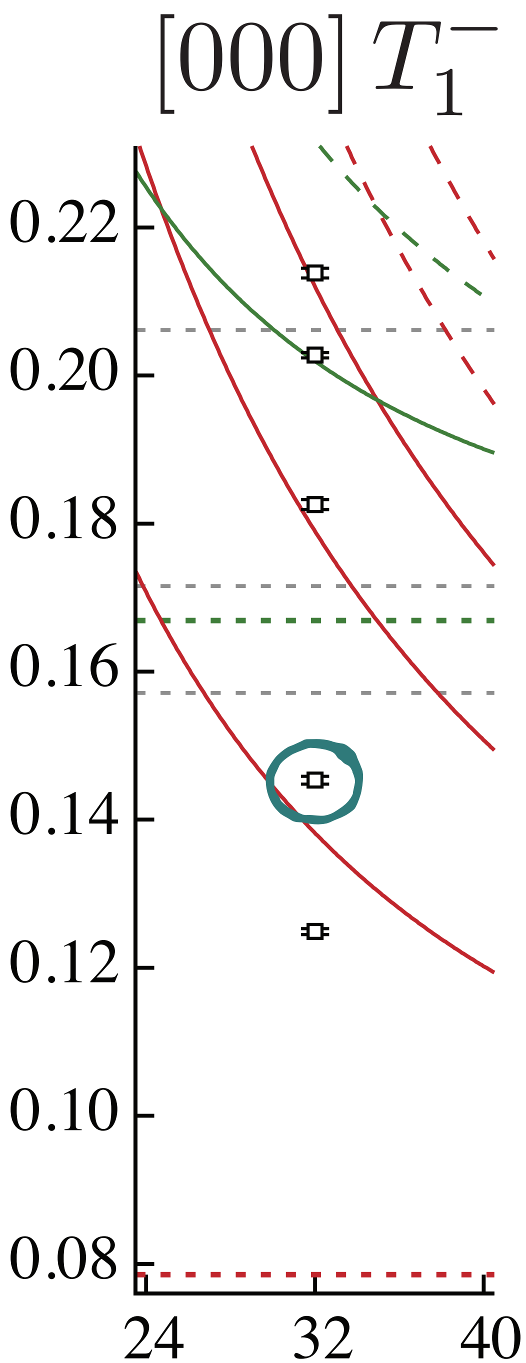


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

$m_\pi \sim 240$ MeV

Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ Spectrum - (I=1 channel)



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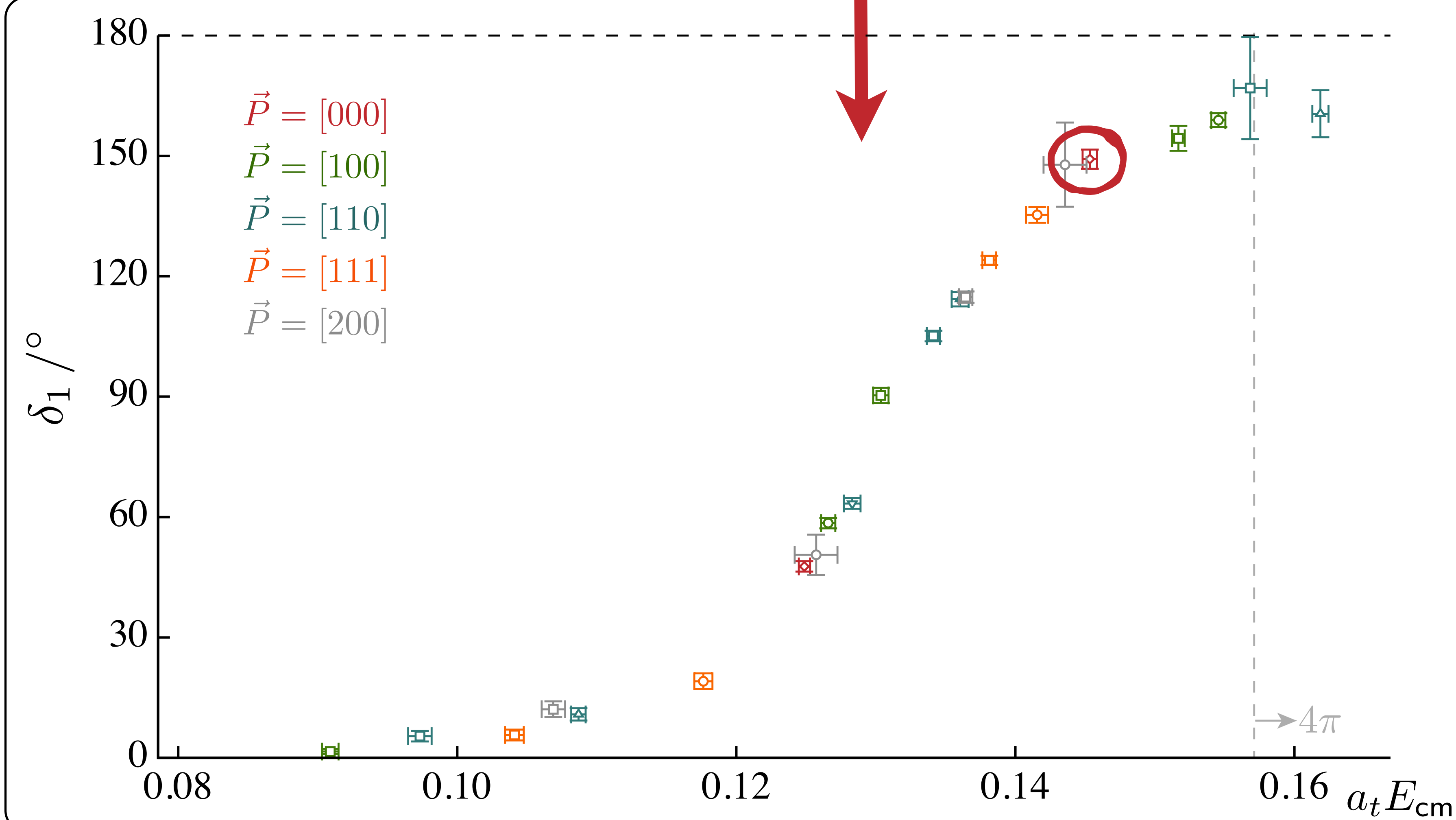

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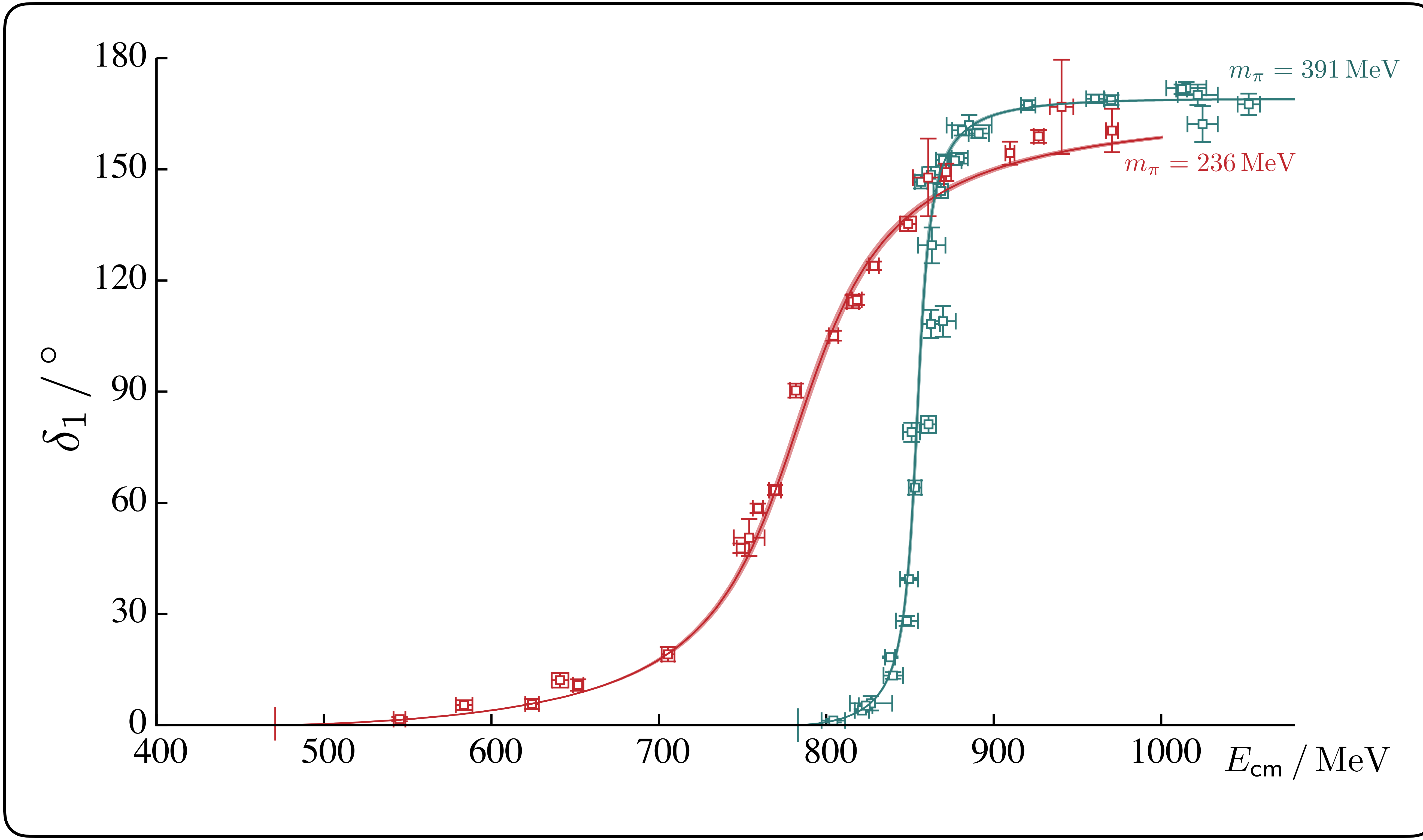
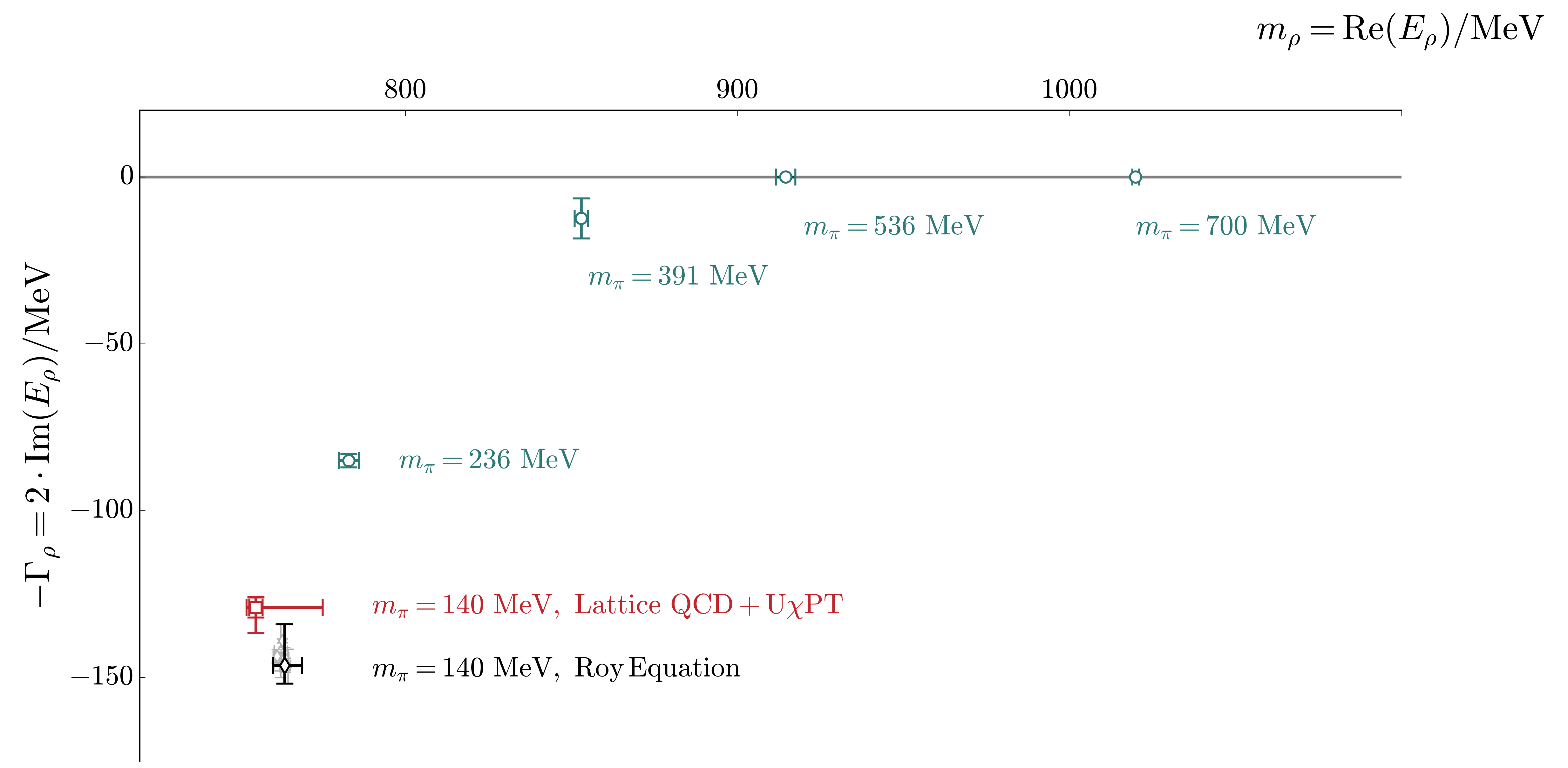


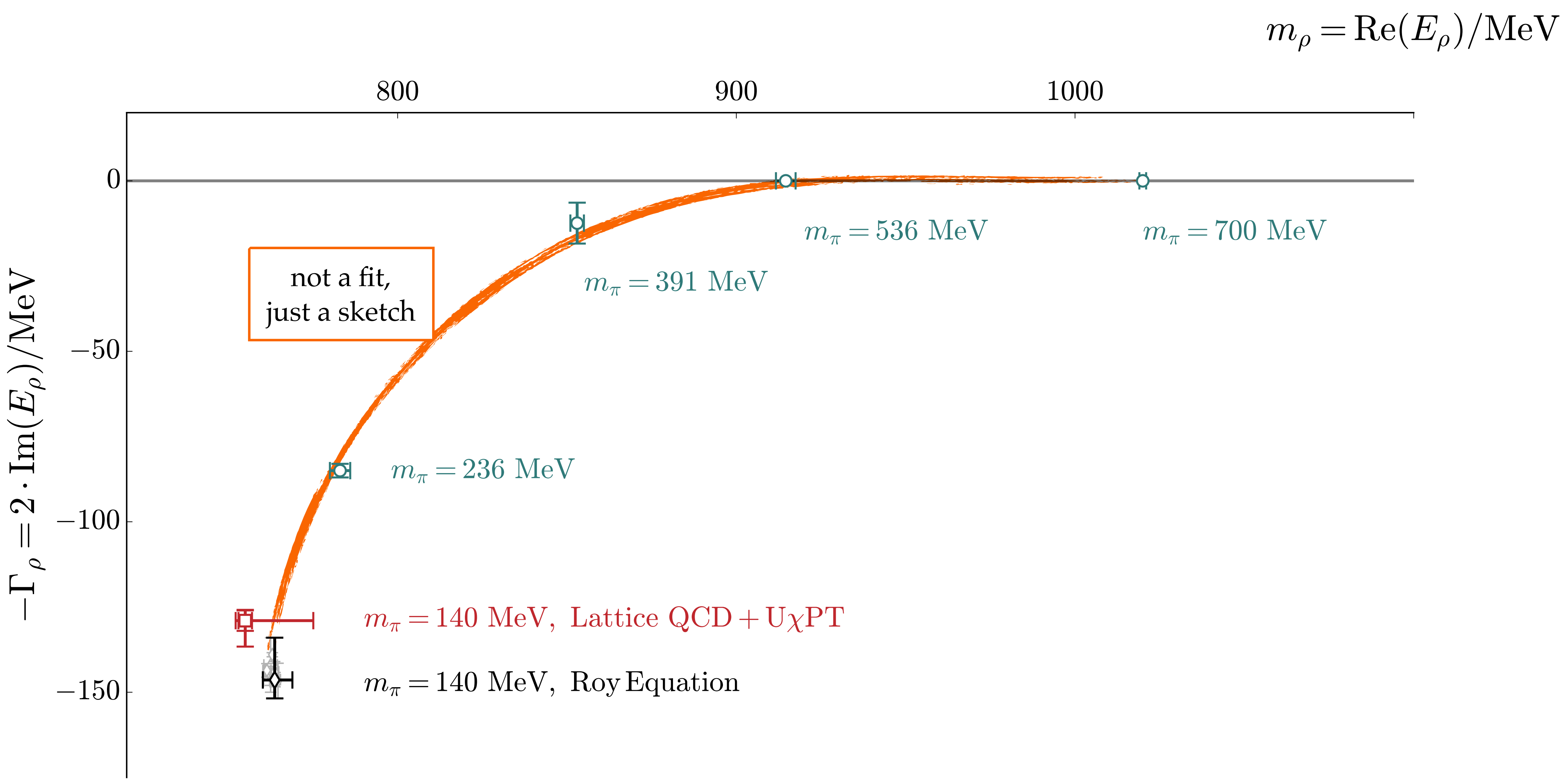
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

The ρ *vs* m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

The ρ *vs* m_π



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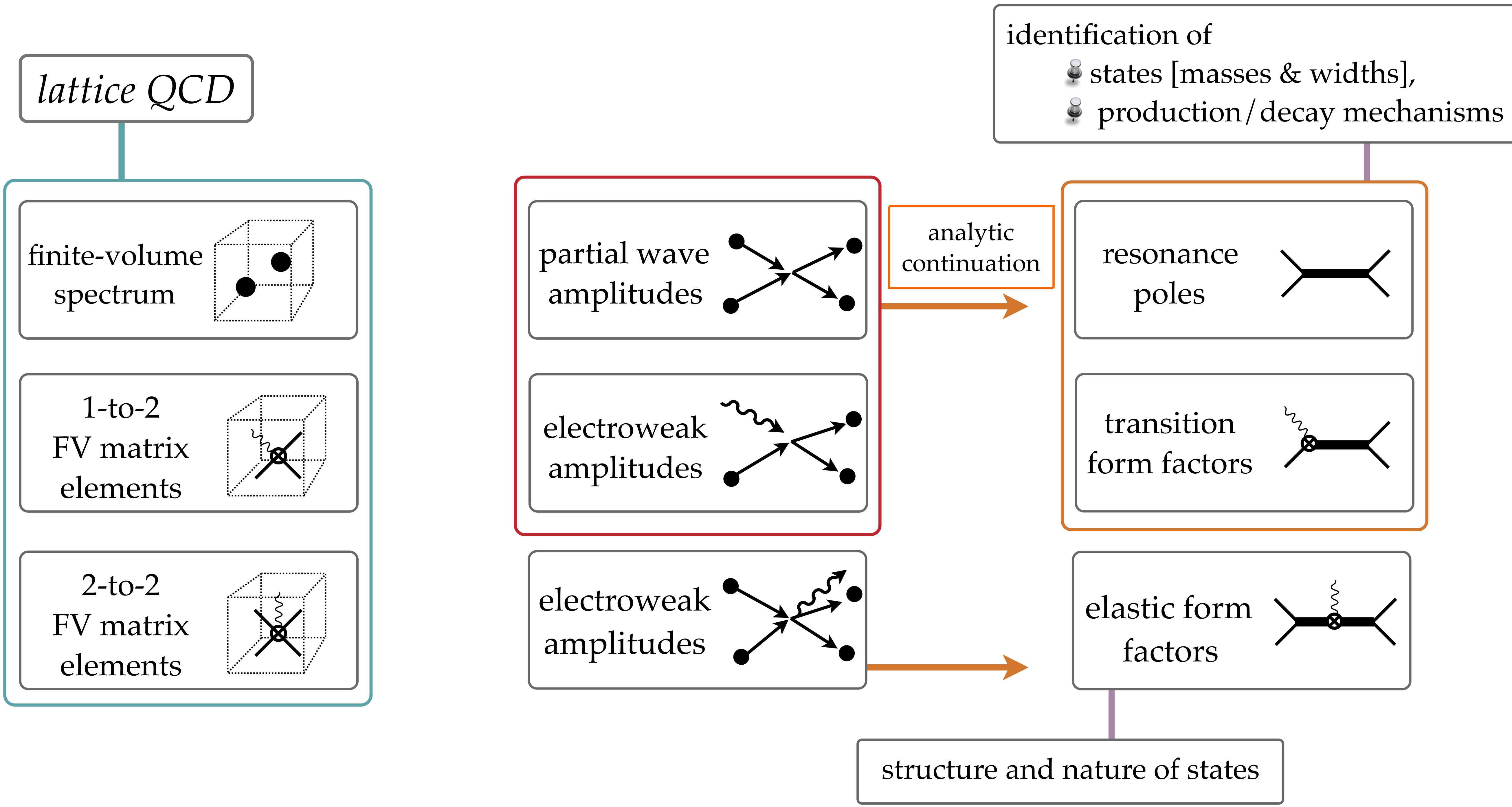
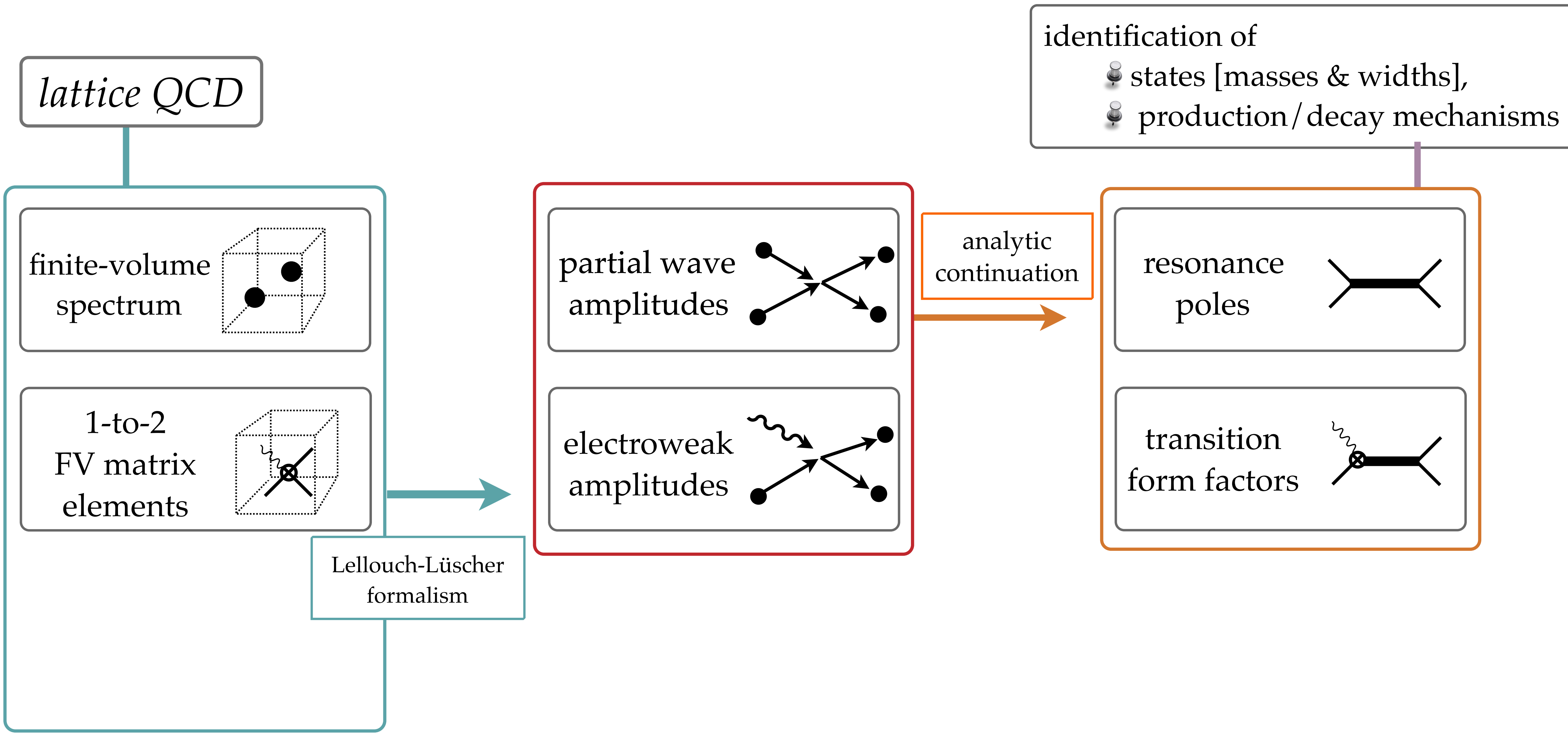
QCD spectroscopy



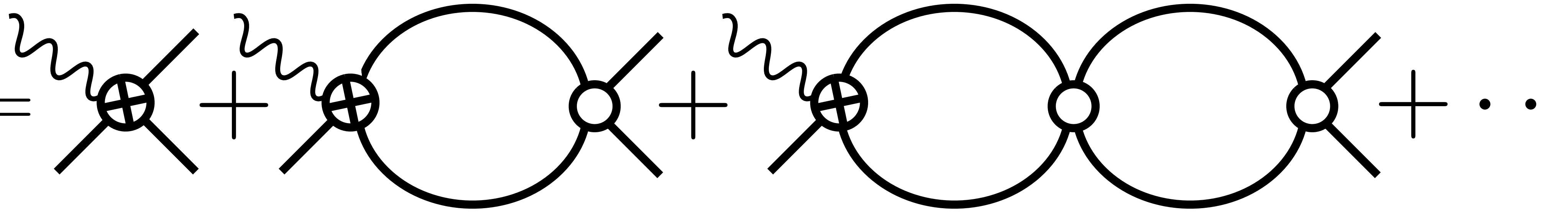
QCD spectroscopy



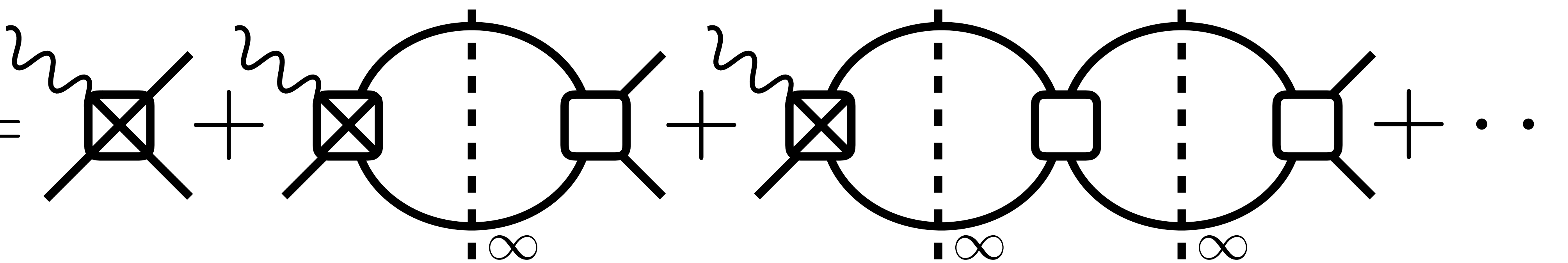
One-to-two transitions

Unitarity using all orders perturbation theory:

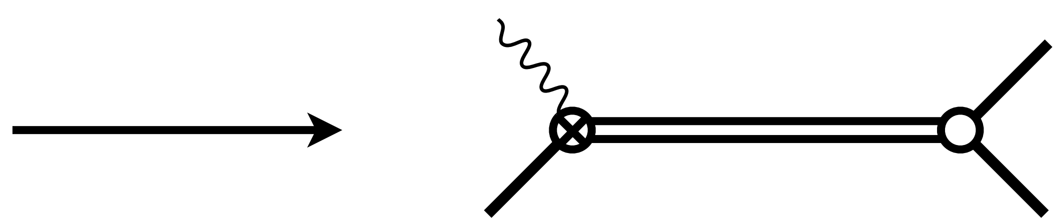
$i\mathcal{A} =$



$=$



$$= i\mathcal{H}\,\frac{1}{1 - i\rho\mathcal{K}}$$

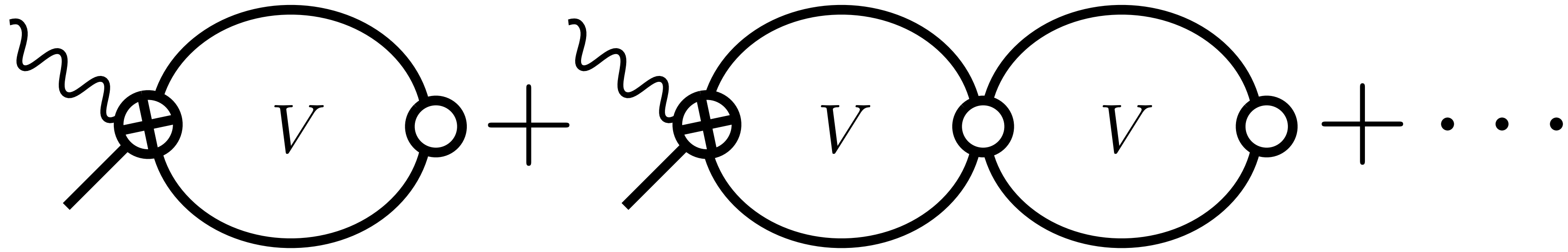


Watson's theorem

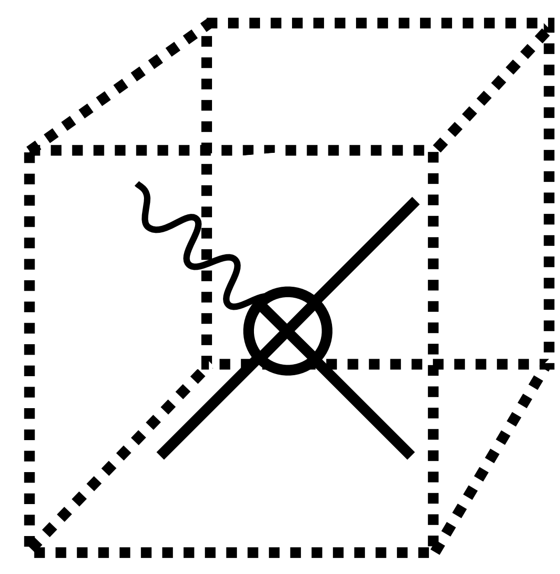
Same phase and analytic structure as the scattering amplitude

One-to-two in finite volume

Same as before…but with a current

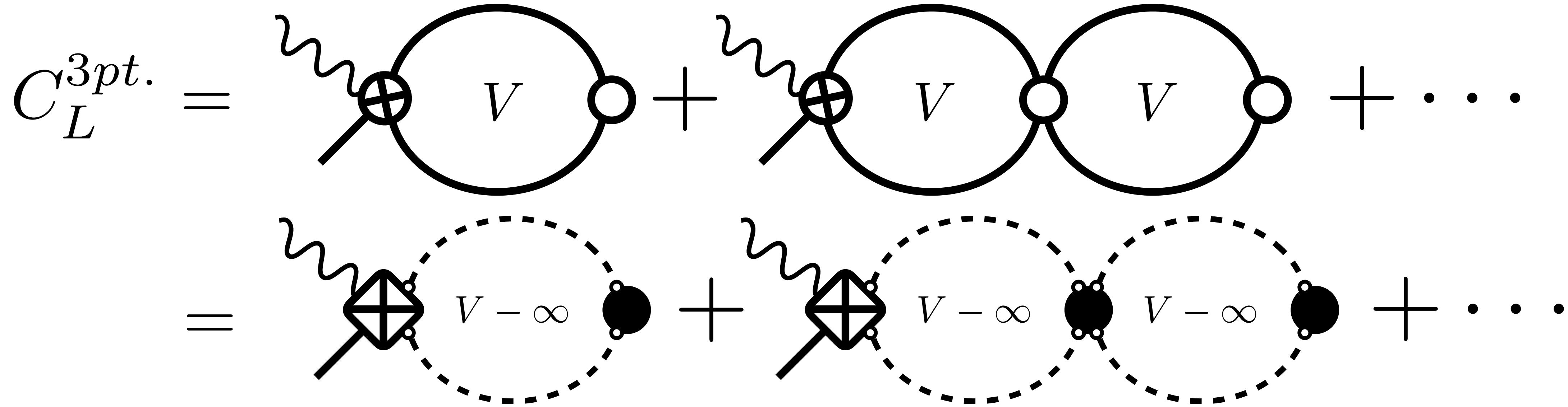
$C_L^{3pt.} =$ 

same topologies that we considered before…



One-to-two in finite volume

Same as before…but with a current



$$= \text{“smooth”} + \mathcal{A}\frac{i}{F^{-1}+\mathcal{M}}B^{\dagger}$$

- Lellouch & Lüscher (2000)
- Kim, Sachrajda, & Sharpe
- Christ, Kim & Yamazaki (2005)
- …
- Hansen & Sharpe (2012)
- RB, Hansen Walker-Loud (2014)
- RB & Hansen (2015)

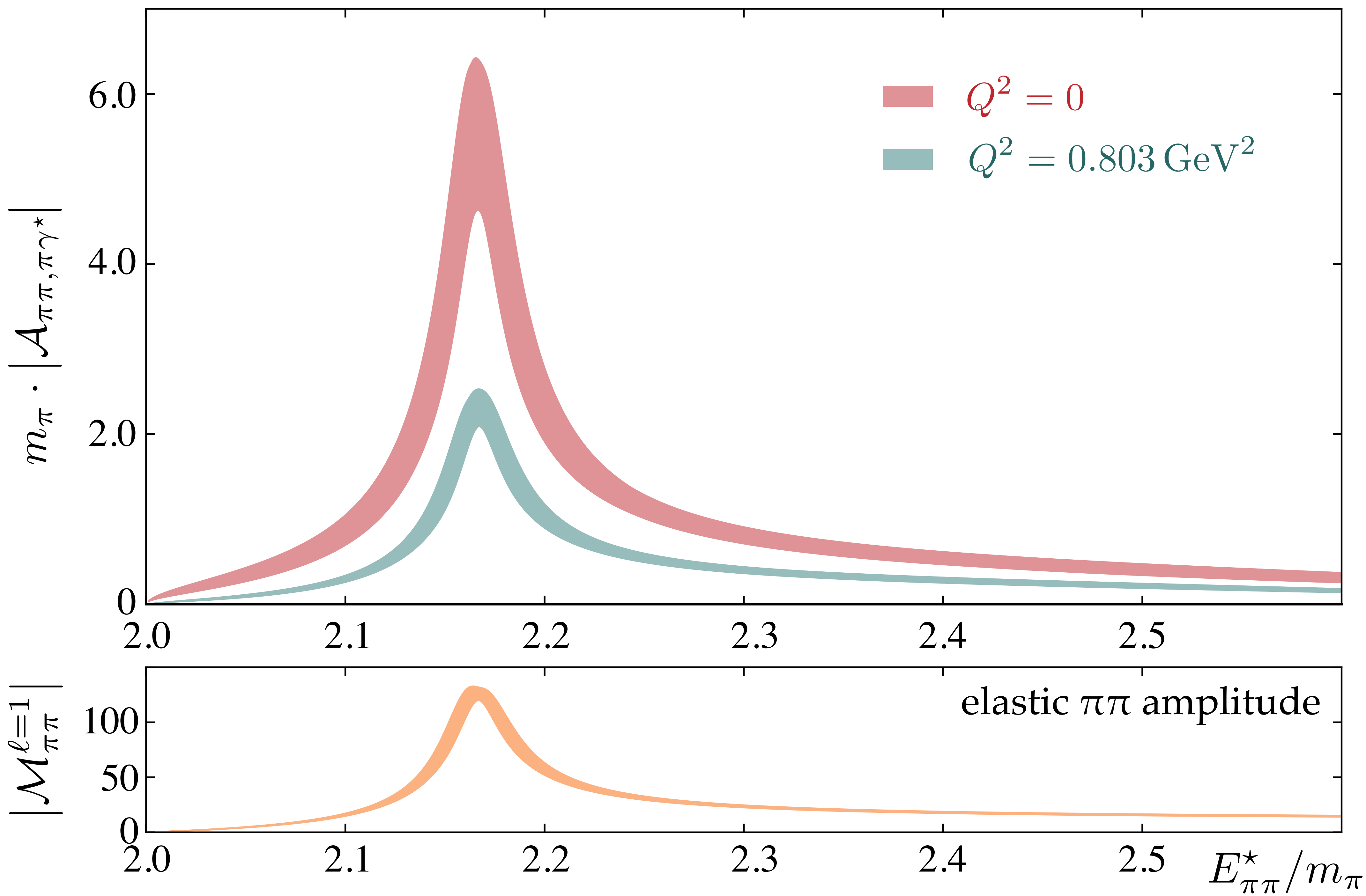
After lots of massaging…

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\mathcal{A}\,\mathcal{R}\,\mathcal{A}}$$

Lellouch-Lüscher matrix:

$$\mathcal{R}(E_n,\mathbf{P}) \equiv \lim_{E\to E_n}\left[\frac{(E-E_n)}{F^{-1}(P,L)+\mathcal{M}(P)}\right]$$

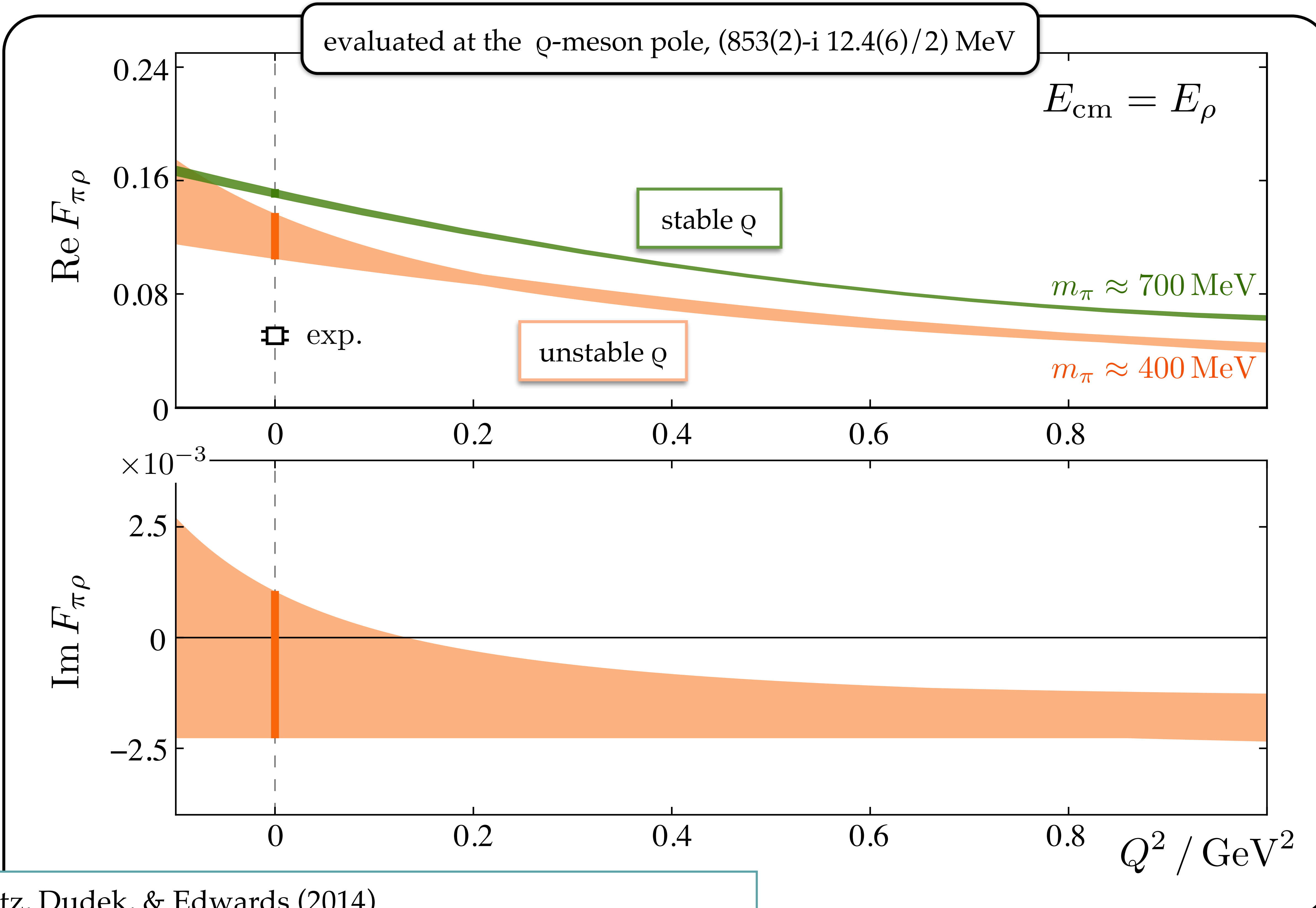
$\pi\gamma^*$-to-$\pi\pi$ amplitude



$m_\pi \sim 400$ MeV

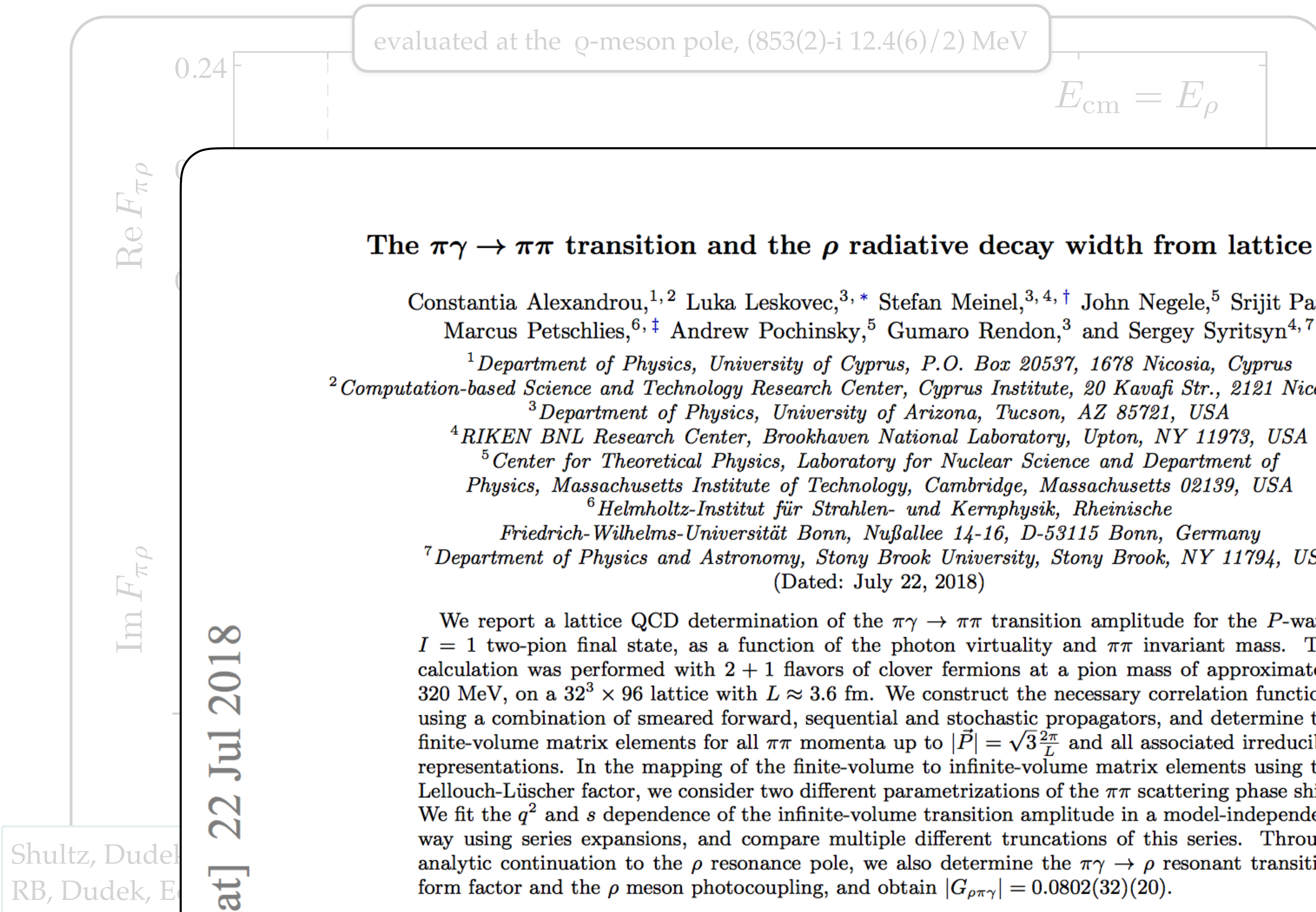
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

π-to-ρ form factor



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

π-to-ρ form factor



-lat] 22 Jul 2018

The $\pi\gamma \to \pi\pi$ transition and the ρ radiative decay width from lattice QCD

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Marcus Petschlies,[6,‡] Andrew Pochinsky,[5] Gumaro Rendon,[3] and Sergey Syritsyn[4,7]

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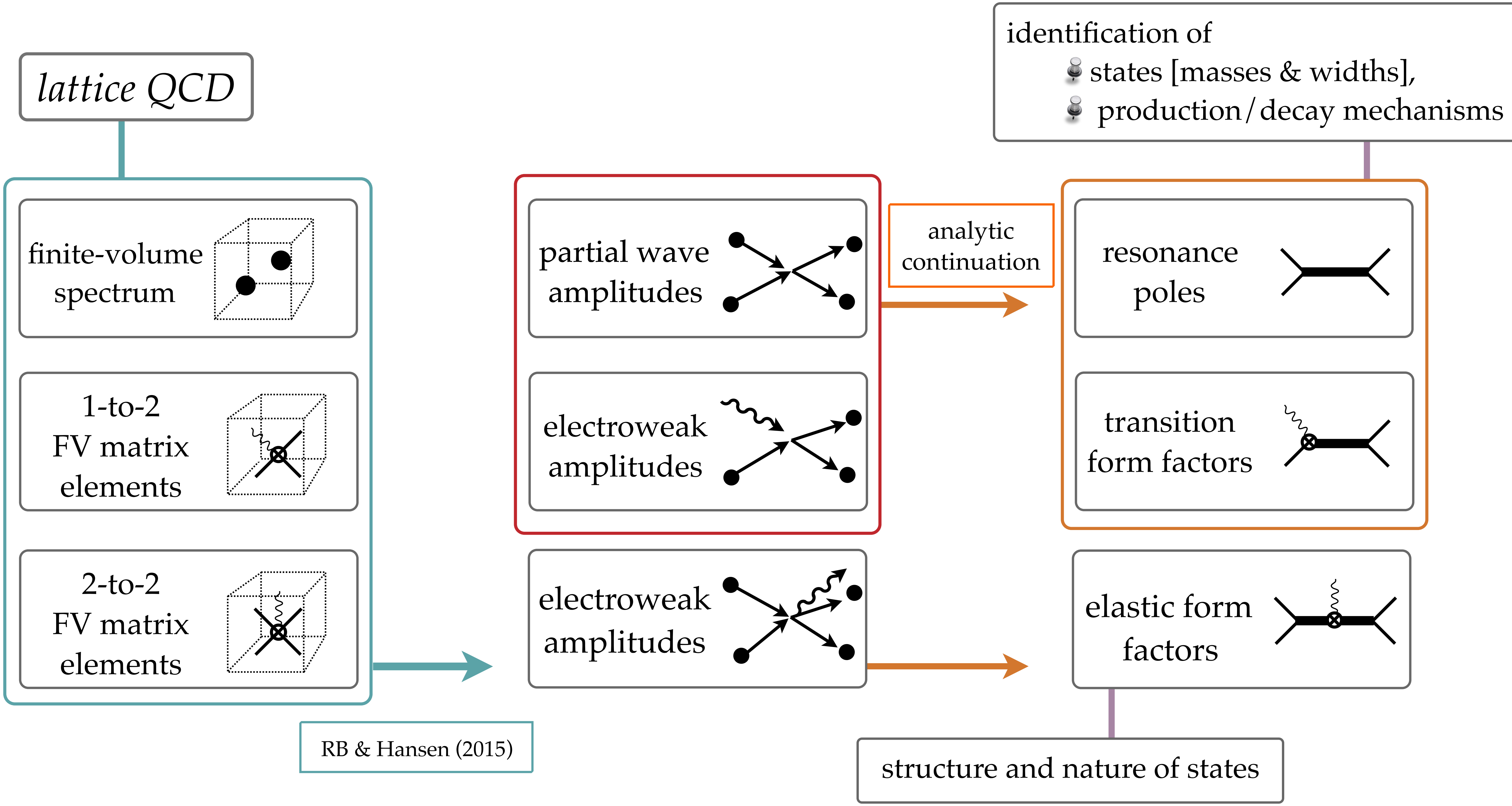
[7] *Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA*

(Dated: July 22, 2018)

We report a lattice QCD determination of the $\pi\gamma \to \pi\pi$ transition amplitude for the P-wave, $I = 1$ two-pion final state, as a function of the photon virtuality and $\pi\pi$ invariant mass. The calculation was performed with $2+1$ flavors of clover fermions at a pion mass of approximately 320 MeV, on a $32^3 \times 96$ lattice with $L \approx 3.6$ fm. We construct the necessary correlation functions using a combination of smeared forward, sequential and stochastic propagators, and determine the finite-volume matrix elements for all $\pi\pi$ momenta up to $|\vec{P}| = \sqrt{3}\frac{2\pi}{L}$ and all associated irreducible representations. In the mapping of the finite-volume to infinite-volume matrix elements using the Lellouch-Lüscher factor, we consider two different parametrizations of the $\pi\pi$ scattering phase shift. We fit the q^2 and s dependence of the infinite-volume transition amplitude in a model-independent way using series expansions, and compare multiple different truncations of this series. Through analytic continuation to the ρ resonance pole, we also determine the $\pi\gamma \to \rho$ resonant transition form factor and the ρ meson photocoupling, and obtain $|G_{\rho\pi\gamma}| = 0.0802(32)(20)$.

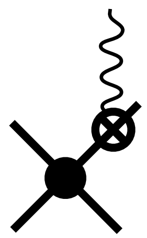
QCD spectroscopy



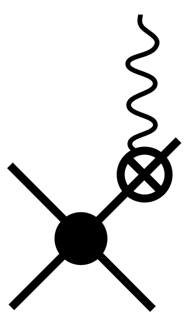
Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

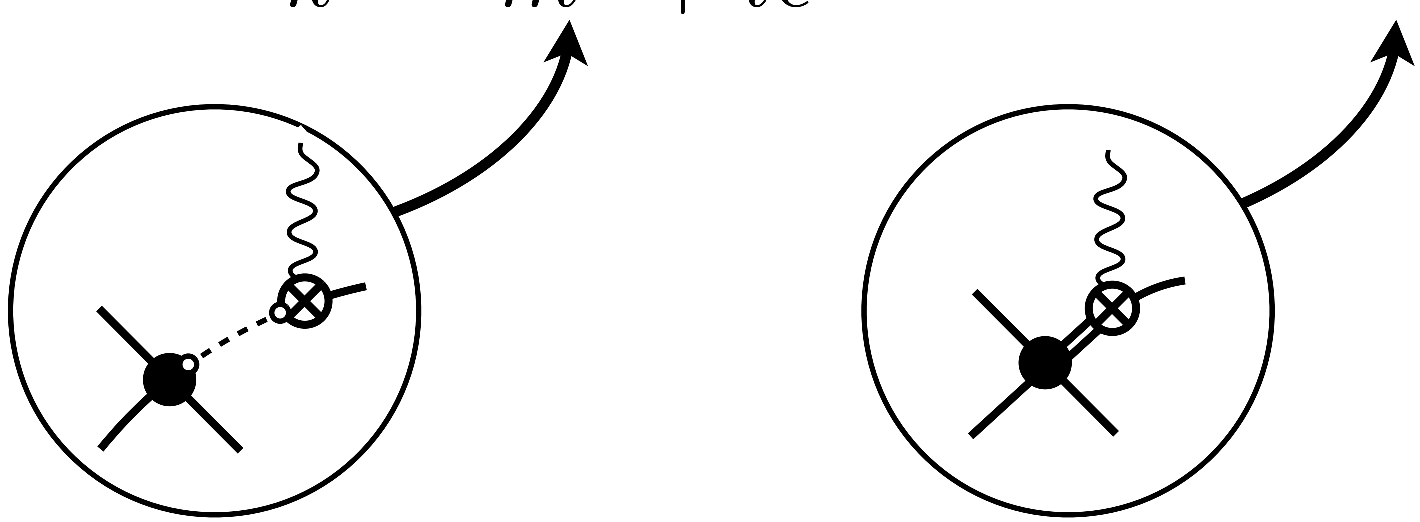
$$i\mathcal{W}_{\mu_1\cdots\mu_n} =$$



$$+ \cdots$$

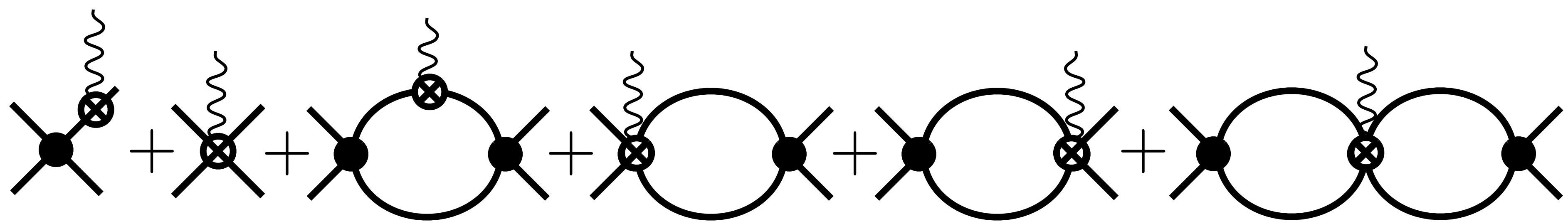


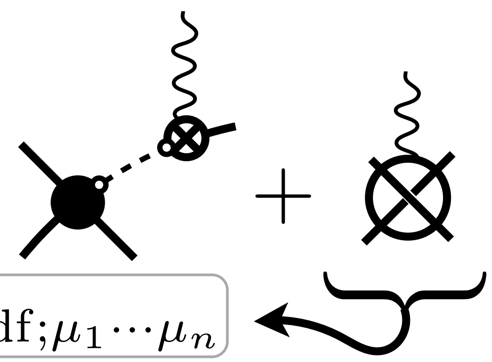
$$= i\mathcal{M}\frac{i}{k^2 - m^2 + i\epsilon}\, i\omega_{\mu_1\cdots\mu_n} + \text{“smooth”}$$



Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

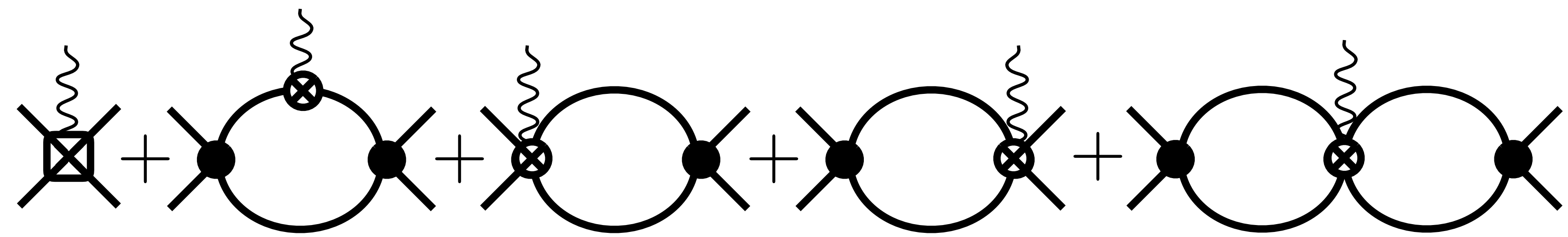
$i\mathcal{W}_{\mu_1\cdots\mu_n} =$ 

$=$ 

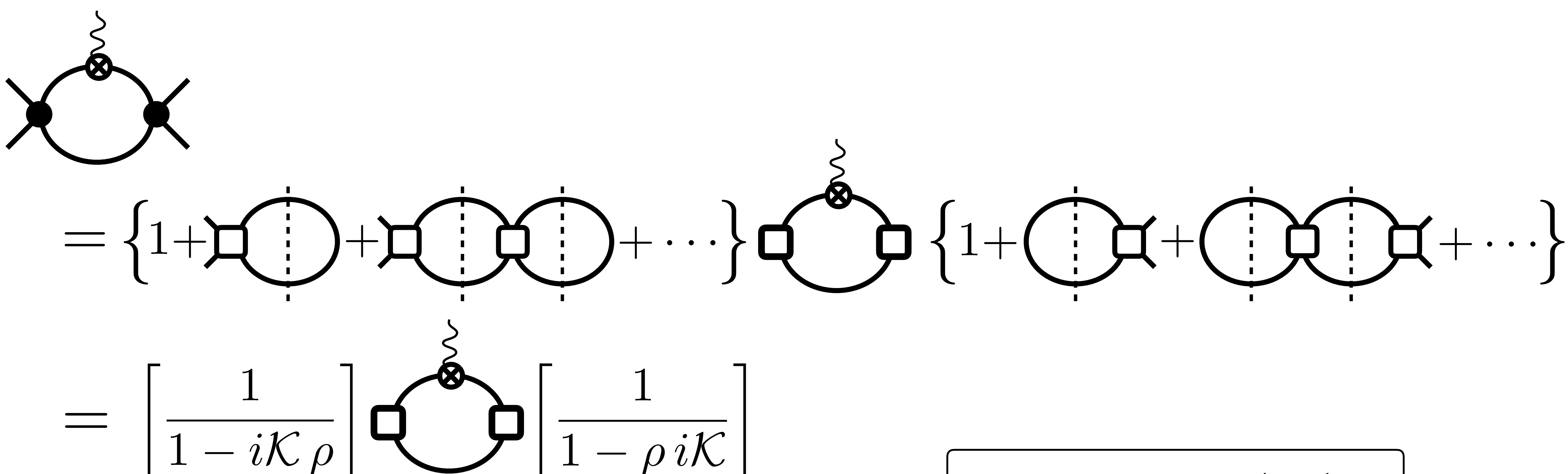
$i\mathcal{W}_{\mathrm{df};\mu_1\cdots\mu_n}$

Two-to-two scattering **with current** - (df amp.)

Divergence-free amplitude

$i\mathcal{W}_{\mathrm{df};\mu_1\cdots\mu_n} =$ 

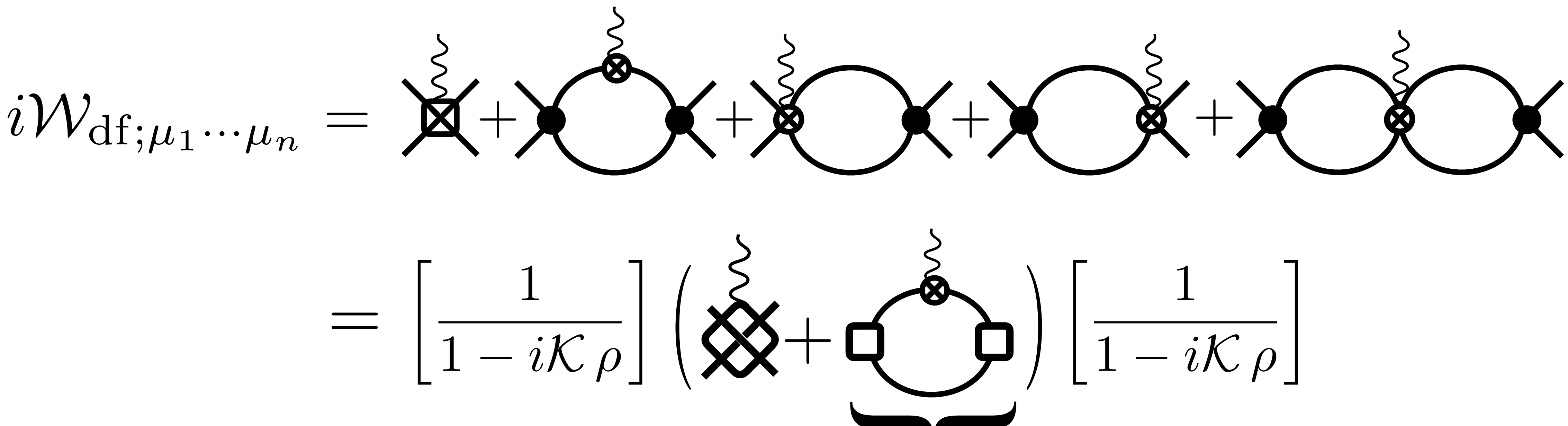
New class of diagrams:



same square-root (and possibly pole) singularities as two-body amplitudes

Two-to-two scattering **with current**

Divergence-free amplitude



Complex function…depending on the one-body form factors

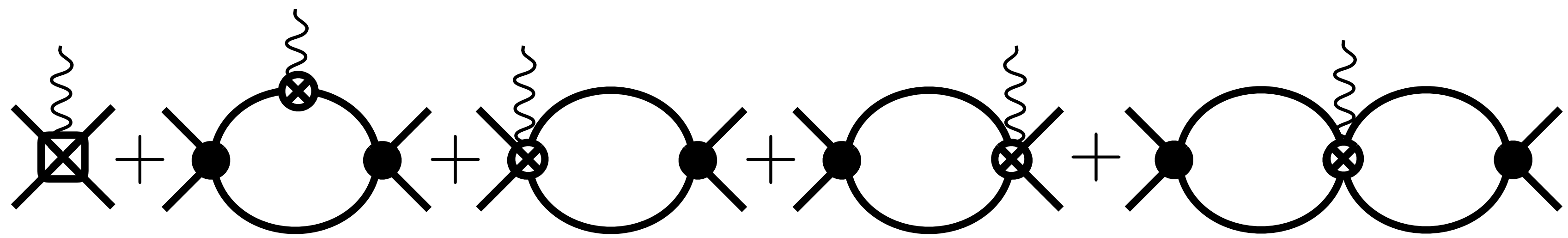
Naive Watson's theorem does not apply!

finite-volume quantities **must** be more complicated

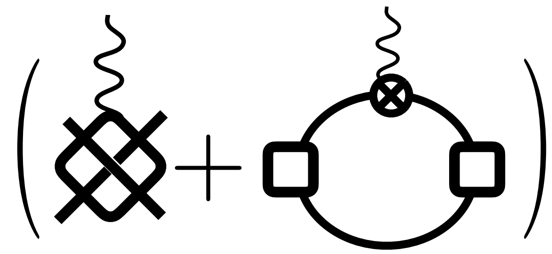
Two-to-two scattering **with current**

Divergence-free amplitude

$$i\mathcal{W}_{\mathrm{df};\mu_1\cdots\mu_n} =$$

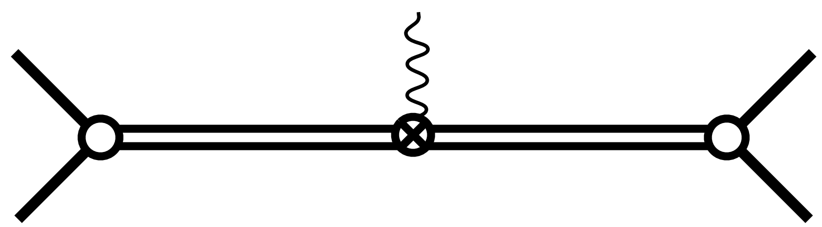


$$= \left[\frac{1}{1 - i\mathcal{K}\,\rho}\right]\Big($$

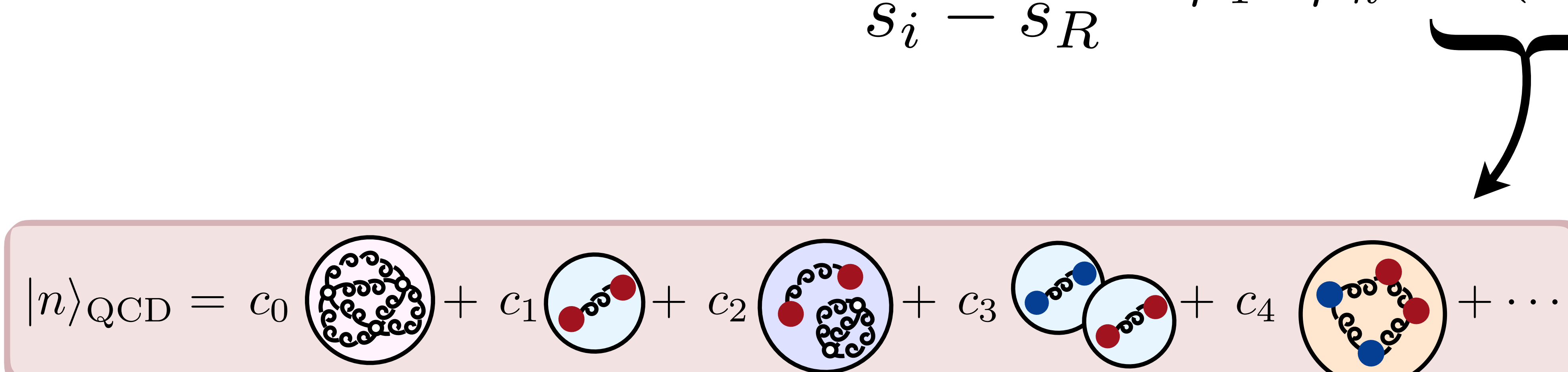


$$\Big)\left[\frac{1}{1 - i\mathcal{K}\,\rho}\right]$$

$$\xrightarrow{\lim s_i, s_f = s_R}$$



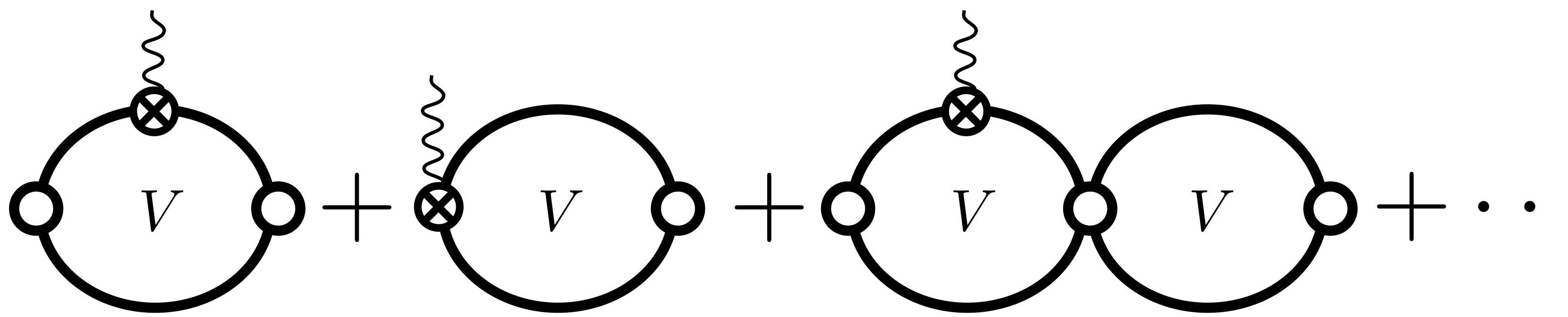
$$\sim \frac{g}{s_i - s_R}\, K_{\mu_1\ldots\mu_n}\, F_R(Q^2)\, \frac{g}{s_f - s_R}$$

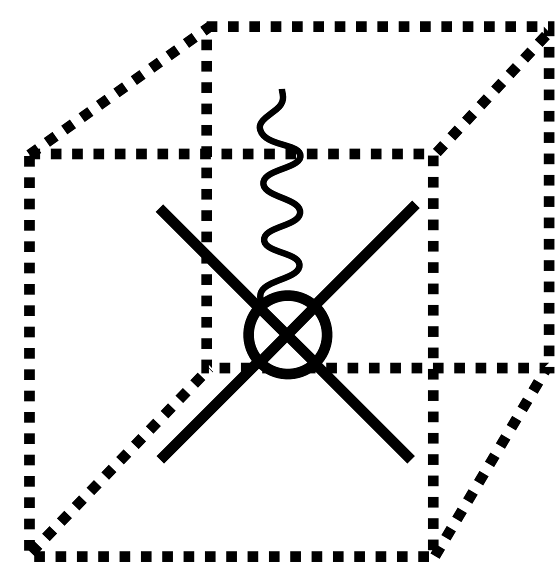


Two-particle in finite volume **with current**

Same as before…but with a current

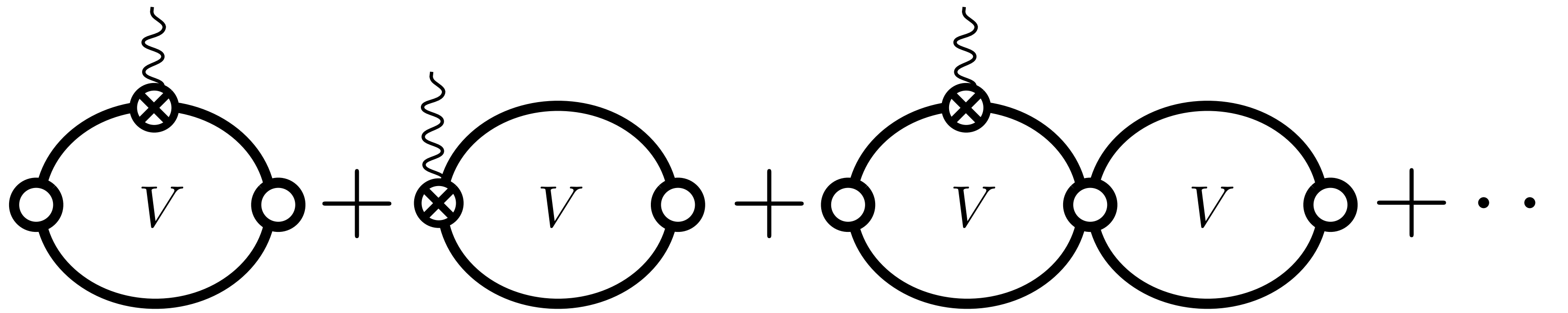
$$C_L^{3pt.} =$$



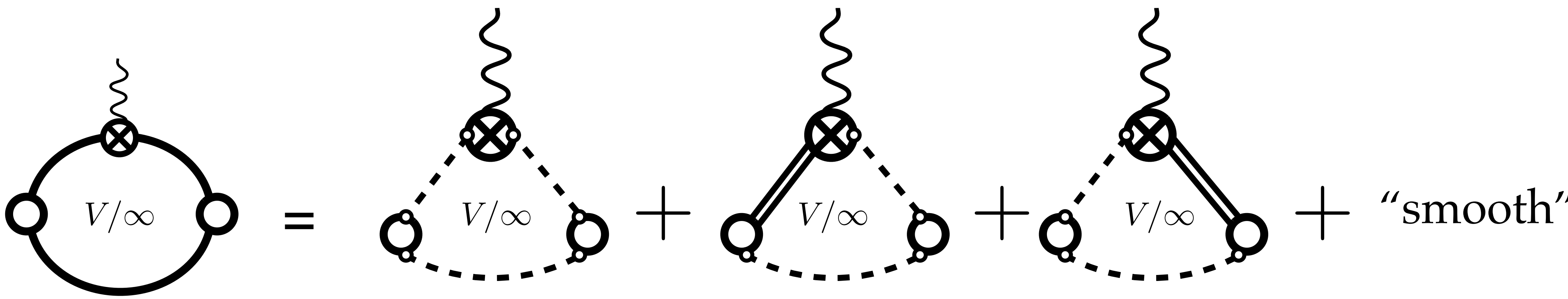


Two-particle in finite volume **with current**

Same as before…but with a current

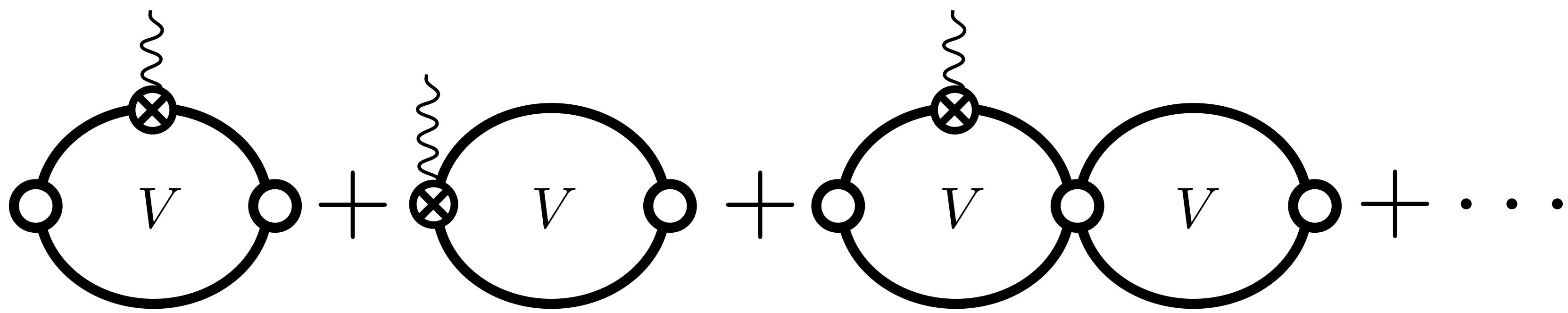
$C_L^{3pt.} =$ 

…everything is the same as before except for…

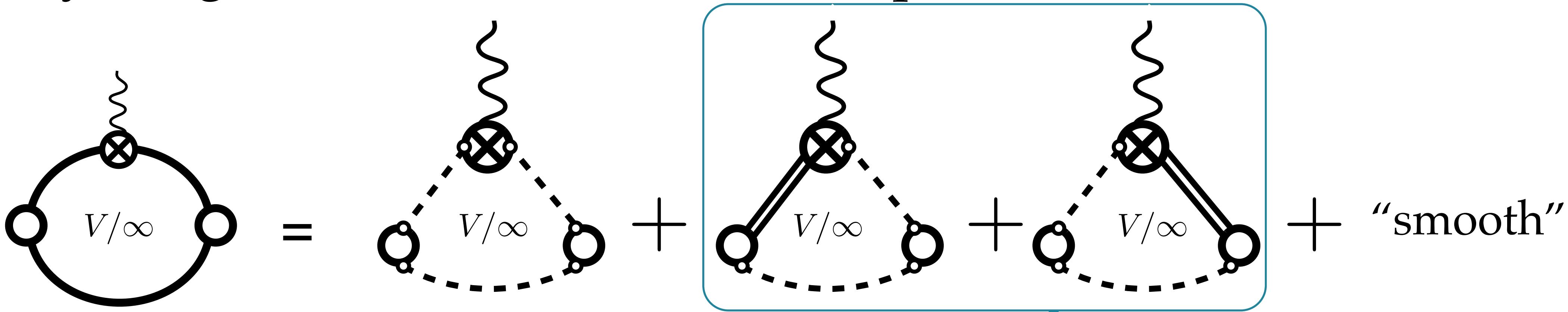


Two-particle in finite volume **with current**

Same as before…but with a current

$C_L^{3pt.} =$ 

…everything is the same as before except for…

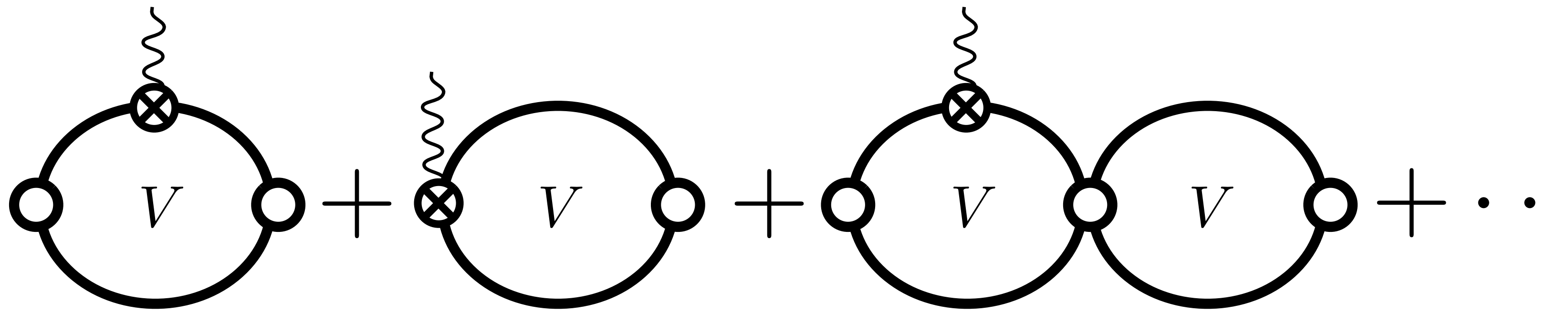


leads to the presence of F-functions…

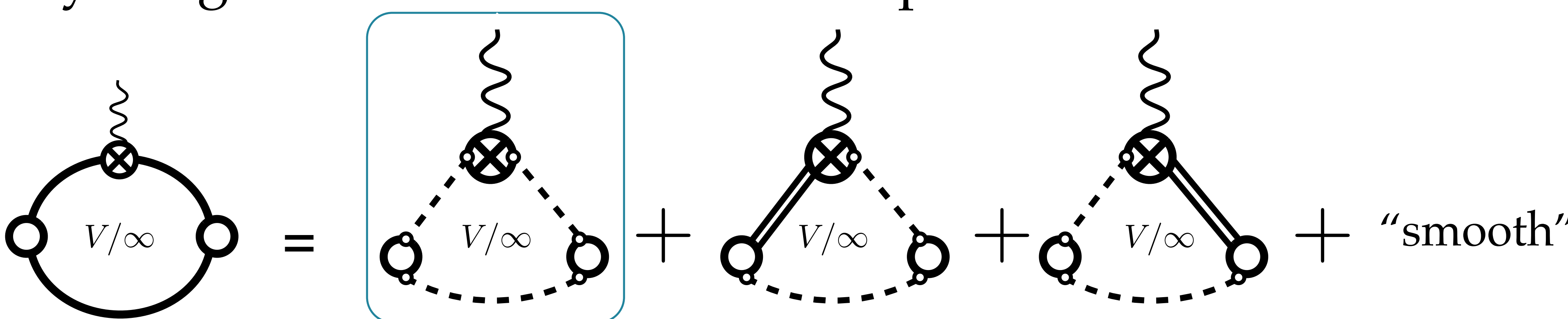
Two-particle in finite volume **with current**

Same as before…but with a current

$C_L^{3pt.} =$

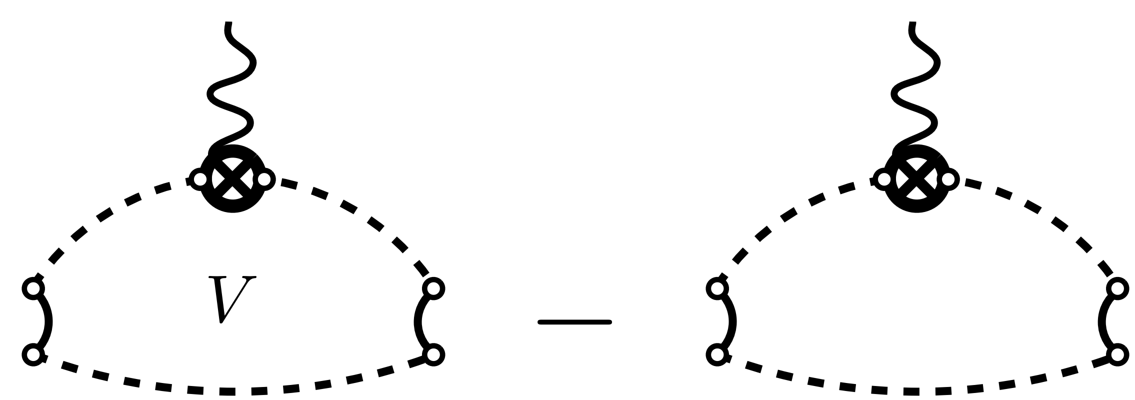


…everything is the same as before except for…



New finite-volume function:

$G_{\mu_1 \cdots \mu_n}(P_f, P_i, L) =$



Two-particle in finite volume **with current**

After lots of massaging…

$$\left|\langle 2|\mathcal{J}|2\rangle\right|_L^2 = \frac{1}{L^6}\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\right]$$

Building block #1) Lellouch-Luscher matrices:

$$\mathcal{R}(E_n,\mathbf{P}) \equiv \lim_{E\to E_n}\left[\frac{(E-E_n)}{F^{-1}(P,L)+\mathcal{M}(P)}\right]$$

"EASY"

-derivatives of amplitudes and F-function at the finite-volume spectra

Two-particle in finite volume **with current**

After lots of massaging…

$$\left|\langle 2|\mathcal{J}|2\rangle\right|^2_L = \frac{1}{L^6}\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\right]$$

Building block #2) stable particle form factor

$$\mathcal{W}_{L,\mathrm{df}} - \mathcal{W}_{\mathrm{df}}^{\mu_1\cdots\mu_n} \sim \sum_{n'}^{n} \mathcal{M}\,\left[G^{(j)}\,f^{(j)}(-q^2)\right]\,\mathcal{M}$$

“EASY”

- form factors of single-particle states

Two-particle in finite volume **with current**

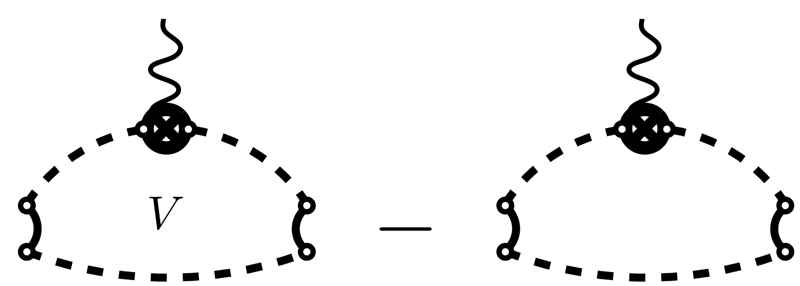
After lots of massaging…

$$\left|\langle 2|\mathcal{J}|2\rangle\right|^2_L = \frac{1}{L^6}\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\right]$$

Building block #3) G-function

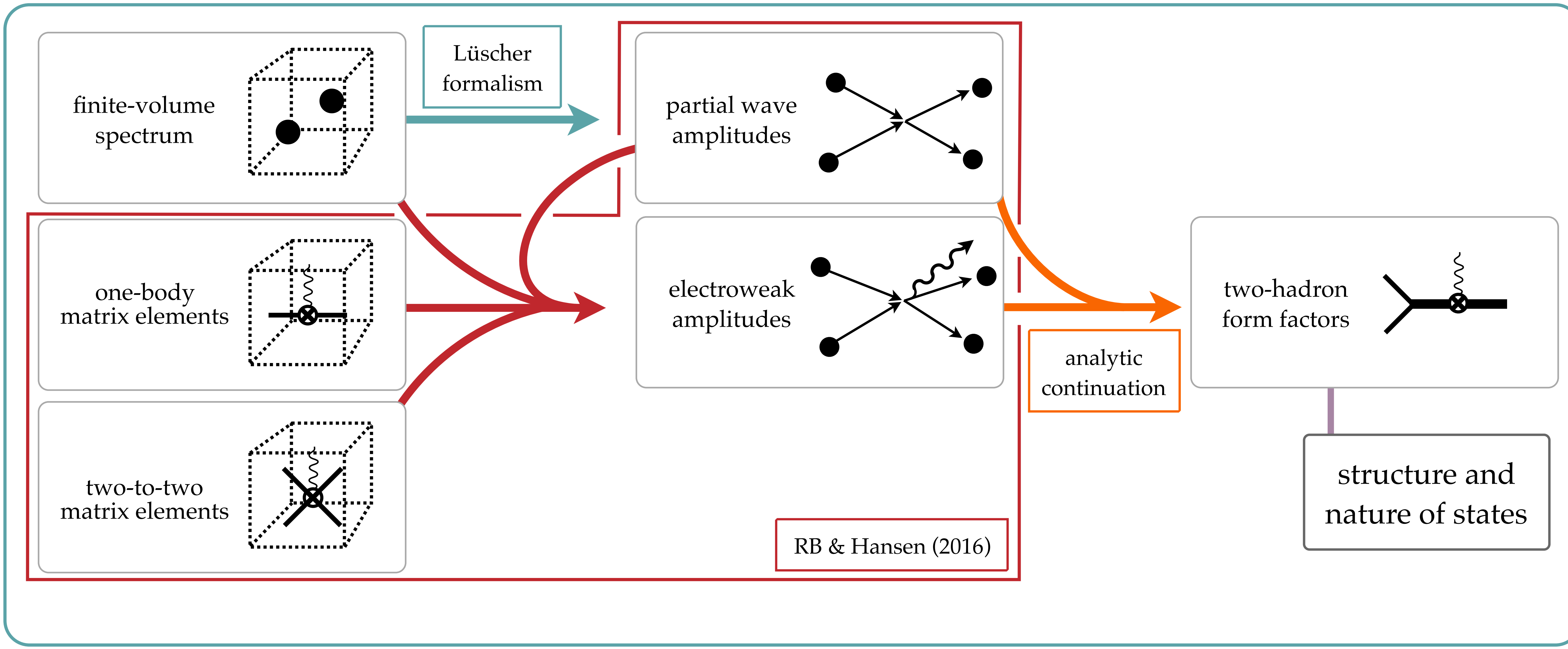
$$\mathcal{W}_{L,\mathrm{df}} - \mathcal{W}_{\mathrm{df}}^{\mu_1\cdots\mu_n} \sim \sum_{n'}^{n} \mathcal{M}\left[G^{(j)}\, f^{(j)}(-q^2)\right]\mathcal{M}$$

$G_{\mu_1\cdots\mu_n}(P_f, P_i, L) =$



- Integral is complicated
- if P_f=P_i:
  - poles coincide…we found an analytic solution for the integral
- otherwise:
  - write the integral in terms of two integrals:
    - 4D, singular, covariant, dim-reg integral [semi-analytical]
    - 3D, smooth, evaluate numerically…

Take-home message

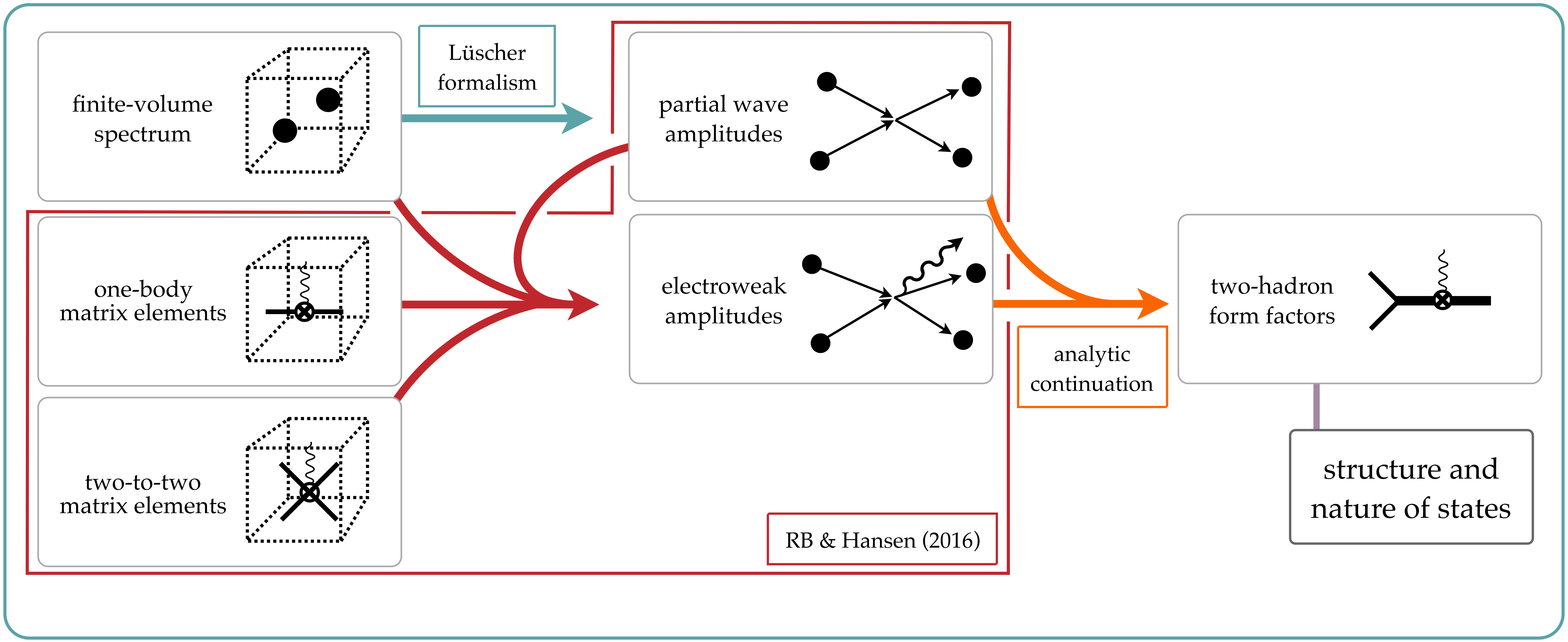


Multiple birds with one stone:

Access:

- transition electroweak amplitudes
- elastic electroweak amplitudes
- structural information composite states:
  - bound states
  - resonance
- remove all finite-volume systematics

Take-home message



Much harder than 0-to-2 and 1-to-2 transitions…but we have removed all major obstacles!