Quark masses from Latice QCD

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FLAG quark masses WG (Thomas Blum, Antonin Portelli)

Overview

Motivation

FLAG

Non-perturbative renormalization

New approaches to heavy quarks

Isospin breaking

Conclusions

Introduction

Before anything

- FLAG $\equiv$ Flavour Lattice Averaging Group
- Apologies: First time I give a review talk

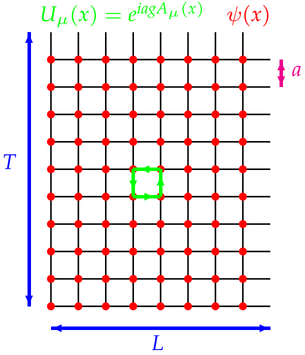
Points of the talk

- Not to review a bunch of numbers. Focus in a few points
- How does the lattice determine quark masses?
- Approaches to non-perturbative renormalization
- New approaches for heavy quarks
- Isospin breaking

Overview

Computing path integrals: Lattice field theory

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



- Discretize space-time in an hyper-cubic lattice (spacing a)
- Path integral $\longrightarrow$ multiple integral (one variable for each field at each point)
- Compute the integral numerically $\to$ Monte Carlo sampling.

$$\langle O \rangle = \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

Observable computed averaging over samples

Not a model of QCD

$$\lim_{a\to 0}; \qquad \lim_{L\to\infty}; \qquad \lim_{m\to m^{\text{phys}}}$$

Extrapolations always carry assumptions!

FLAG quality criteria

Each source of systematics is quantified

★ The parameter values and ranges used to generate the datasets allow for a satisfactory control of the systematic uncertainties;
○ The parameter values and ranges used to generate the datasets allow for a reasonable attempt at estimating systematic uncertainties, which however could be improved;
■ The parameter values and ranges used to generate the datasets are unlikely to allow for a reasonable control of systematic uncertainties.

Example: Continuum extrapolation $\lim_{a\to 0}$

★ at least 3 lattice spacings <u>and</u> at least 2 points below 0.1 fm <u>and</u> a range of lattice spacings satisfying $[a_{\mathrm{max}}/a_{\mathrm{min}}]^2 \geq 2$
○ at least 2 lattice spacings <u>and</u> at least 1 point below 0.1 fm <u>and</u> a range of lattice spacings satisfying $[a_{\mathrm{max}}/a_{\mathrm{min}}]^2 \geq 1.4$
■ otherwise

Quark masses: Fundamental parameters of the SM

How to determine quark masses?

- Confinement: Quark masses not directly accessibly from experiment.
- Renormalization: quark masses are **defined** by renormalization conditions.
- Different conditions $\Longrightarrow$ Different **schemes** $\Longrightarrow$ Different values for the quark masses.
- In order to compare, it is customary to quote $m_{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$ (different for heavy quarks c, b)

Conceptually three steps

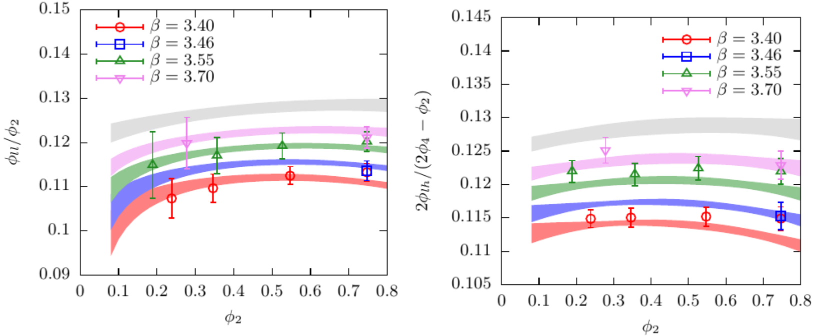
- Tune lattice bare parameters (am_q, g_0) to reproduce physical results
  - $N_f = 2 + 1$ simulations we need 3 physical inputs
  - f_π to set the scale
  - Fix M_π / f_π , M_K / f_π , . . . to its physical values to determine m_q / f_π
- Renormalize lattice bare quark mass

$$am_q \to a\bar{m}_q(\mu)$$

- Convert from our chosen scheme to $\overline{\mathrm{MS}}$ and remove lattice spacing a

$$a\bar{m}_q(\mu) \to m_{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$$

Lattice bare masses [J. Koponen@lat'18]



- Use technical scale $\sqrt{8t_0} = 0.415(4)(2)$ fm (determined from $f_\pi + f_k/2$).
- $\phi_2 = 8t_0 M_\pi^2$
- $\phi_{ll} = 8t_0 m_{ud}$
- $\phi_{ll}/\phi_2 = m_{ud}^2/M_\pi^2$

Overview

How to define Quark masses? Use Ward identities

One possibility: use PCAC relation

$$\partial_\mu (A_R)_\mu = 2 m_R P_R$$

with

$$\begin{aligned} (A)_\mu &= \bar{\psi}\gamma_\mu\gamma_5\psi(x)\,, \\ P &= \bar{\psi}\gamma_5\psi(x)\,, \end{aligned}$$

One can renormalize the operators $(A)_\mu, P$ with factors $Z_A(g_0^2), Z_P(g_0^2,\mu)$ to obtain

$$m_R(\mu) = \frac{Z_A(g_0^2)}{Z_P(g_0^2,\mu)} m \longrightarrow m_{\overline{\mathrm{MS}}}(\mu)\left[1 + \mathcal{O}(g^2)\right] .$$

- $Z_A(g_0^2), Z_P(g_0^2,\mu)$ can be determined in PT or non-perturbatively!

RI-(s)MOM non-perturbative renormalization

- Renormalization condition formulated in terms of Green functions with external momenta $p^2 \sim \mu^2$.
- Renormalization in infinite volume and $m = 0$
- 4-loop matching with $\overline{\text{MS}}$ known

Caveats

- Needs gauge fixing (Gribov ambiguities)
- Window problem:

$$\Lambda_{QCD} < \mu < 1/a$$

  (ameliorated with some “step-scaling procedure” [BMW ’10, RBC ’10], but still limited range of energies $\mu \sim 3 - 4\,\text{GeV}$
- Need dedicated simulations to take all $m \to 0$ (BMW, ETMC)

Some values from RI-MOM

Many of the most precise results use RI-MOM

- **[BMW '10]** ($N_f = 2+1$)

$$m_{ud}(2\,\text{GeV}) = 3.469(47)(48)\,\text{MeV}; \qquad m_s(2\,\text{GeV}) = 95.5(1.1)(1.5)\,\text{MeV}$$

- **[RBC '14]** ($N_f = 2+1$)

$$m_{ud}(2\,\text{GeV}) = 3.31(4)(4)\,\text{MeV}; \qquad m_s(2\,\text{GeV}) = 90.3(0.9)(1.0)\,\text{MeV}\,.$$

- **[ETMC '14]** ($N_f = 2+1+1$)

$$m_{ud}(2\,\text{GeV}) = 3.70(13)(11)\,\text{MeV}; \qquad m_s(2\,\text{GeV}) = 99.6(4.3)\,\text{MeV}\,.$$

Finite volume renormalization schemes (ALPHA, PACS-CS)

- Renormalization condition imposed in a finite volume.
- Gauge invariant
- Massless renormalization schemes, but simulations at $m_q = 0$ possible (1/L is the IR regulator).
- Solve the running non-perturbatively

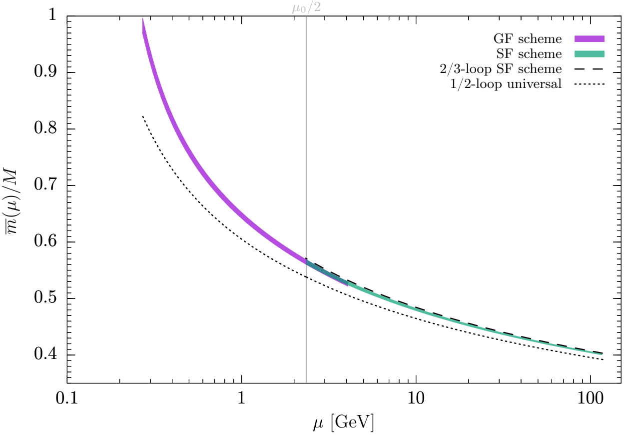
$$\begin{aligned} \mu \frac{\mathrm{d}}{\mathrm{d}\mu} g(\mu) &= \beta(g(\mu)) \,, \\ \mu \frac{\mathrm{d}}{\mathrm{d}\mu} m_i(\mu) &= \tau(g(\mu))\, m_i(\mu) \,, \qquad i = 1, \ldots, N_f \,. \end{aligned}$$

- Matching with PT at $\mu \sim 100\,\mathrm{GeV}$

Caveats

- Precision
- 2-loop matching with $\overline{\mathrm{MS}}$ known. (But matching with PT at $\mu \sim 100\,\mathrm{GeV}$)

Non-perturbative running at all scales [I. Campos et. al. '18]



Preliminary results [J. Koponen@Lat '18]

$$m_{ud} = 3.50(8)\,\text{MeV}; \qquad m_s = 94.1(1.5)\,\text{MeV}$$

Overview

New approaches to heavy quarks [Bazavov et. al. '18]

Remember three fundamental steps

1. Get lattice bare quark masses (am)
2. Renormalize them to some convenient scheme ($am \to \bar{m}$)
3. Convert to $\overline{\text{MS}}$ at $\mu = 2$ GeV

Avoiding renormalization on the lattice

- Determine an RGI quantity in terms of am
- Match to continuum PT

Use Heavy-Light meson mass as a function of the heavy quark mass

$$M_{hl} = m_h + \bar{\Lambda} + \frac{\mu_\pi - \mu_G(m_h)}{2m_h} + \mathcal{O}(1/m_h^2)\,.$$

- $\bar{\Lambda}$ Binding energy
- $\mu_\pi/2m_h$ Kinetic energy
- $\mu_G(m_h)$ Hyperfine energy
- m_h Pole mass of the heavy quark

Problem: Pole mass has terrible PT expansion → MRS scheme

$$m_h \sim \bar{m}_{\overline{\mathrm{MS}}} \left(1 + \sum_{k=0}^{\infty} r_n \alpha^{n+1}(\bar{m}_{\overline{\mathrm{MS}}}) \right)$$

with $r_n = (2b_0)^n \Gamma(n + 1 + b_1/(2b_0^2))$

- Remove the leading divergence in the asymptotic series (i.e renormalon substraction): minimal renormalon substraction scheme **[N. Bambrila et. al. '17]**

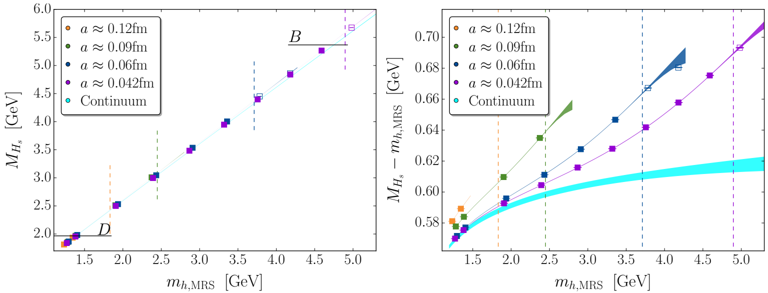
$$M_{hl} = m_{\mathrm{MRS}} + \bar{\Lambda}_{\mathrm{MRS}} + \frac{\mu_\pi - \mu_G(m_{\mathrm{MRS}})}{2m_{\mathrm{MRS}}} + \mathcal{O}(1/m^2_{\mathrm{MRS}})\,.$$

  with better PT properties

$$m_{\mathrm{MRS}} \sim \bar{m}_{\overline{\mathrm{MS}}} \left(1 + \sum_{k=0}^{\infty} [r_n - R_n] \alpha^{n+1}(\bar{m}_{\overline{\mathrm{MS}}}) \right)$$

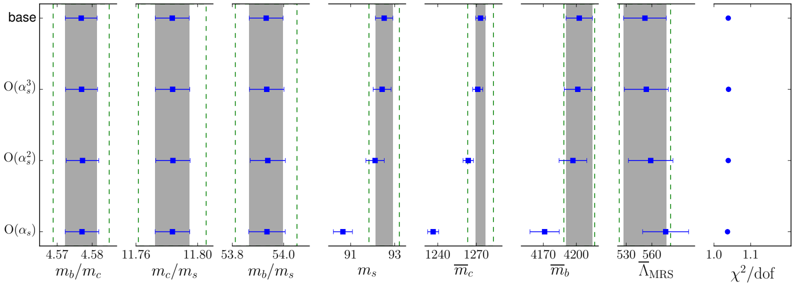
- Determine $am \to m_{\mathrm{MRS}}$ and fit meson mass M_{hl} as a function of m_{MRS}

Results



- Cut data with $am > 0.9$
- Cutoff effects significant at m_b

Results



PT truncation error under control

$$\begin{aligned} m_c(2\,\text{GeV}) &= 1090(5)_{\text{stat}}(2)_{\text{syst}}(6)_{\alpha_s}(1)_{f_\pi,PDG}\,\text{MeV}, \\ m_b(2\,\text{GeV}) &= 4990(17)_{\text{stat}}(2)_{\text{syst}}(29)_{\alpha_s}(1)_{f_\pi,PDG}\,\text{MeV}. \end{aligned}$$

Overview

SM at low energies: QCD+QED

From a practical point of view

- QED corrections can be computed to leading order (RM123-method).
- Probably enough for most(?) practical situations
- Some points to address in FLAG: FV effects are **not** exponentially suppressed (Quality criteria?)

But from a conceptual point of view many open questions

- How to simulate QCD+QED?
- NP renormalization/running?
- Self publicity: Local formulation of QCD+QED in finite volume **[B. Lucini et. al. '16]**
  - Gauge invariant description of electrically charged states **[M. Hansen et. al. '18]**

Overview

Conclusions

- Lattice QCD is the natural tool to connect well measured hadronic quantities with fundamental parameters
- Precision in many quantities $\sim 1\%$.
- Some quantities do not need more precision (ej. m_s). Others are needed with as much precision as possible (ej. m_b).
- Still many challenges
  - non-perturbative renormalization of QCD+QED
  - Isospin breaking effects in heavy quark determinations
  - Renormalization **and** running in QCD+QED
- FLAG can provides a quick overview
- Many (impressive) works not covered. Apologies.