

Status of QCD at nonzero temperature (and density)

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Introduction

QCD phase diagram

Lattice gauge theory

Selected results

Chiral symmetry restoration

Curvature of the crossover line

The equation of state at $O(\mu_B^6)$

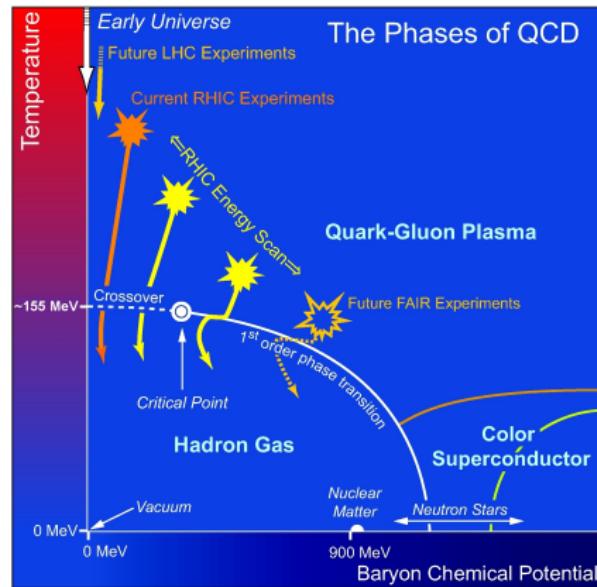
Constraints on the critical point

Equation of state at high temperature

Screening properties at high temperature

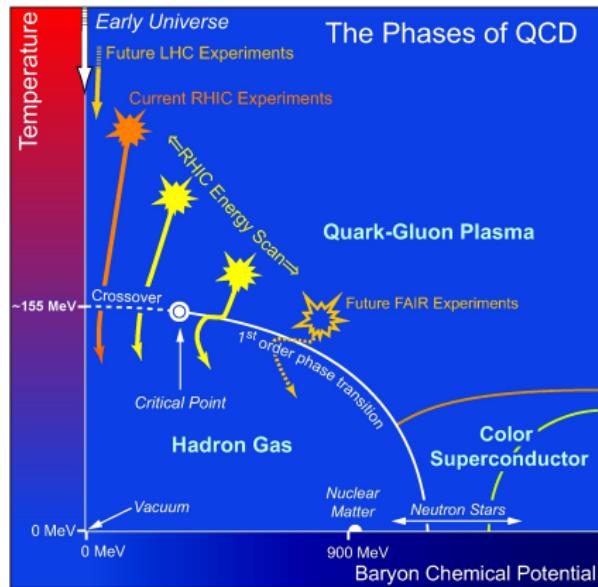
Conclusion

(Conjectured) QCD phase diagram



¹Collins, Perry (1975), Cabibbo, Parisi (1975)

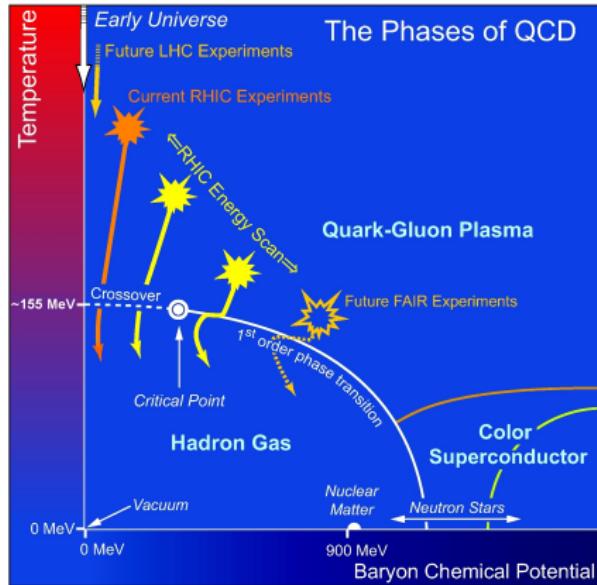
(Conjectured) QCD phase diagram



- ▶ Study response of the system to change of external parameters, i.e. temperature and baryon density, asymptotic freedom suggests a weakly interacting phase¹

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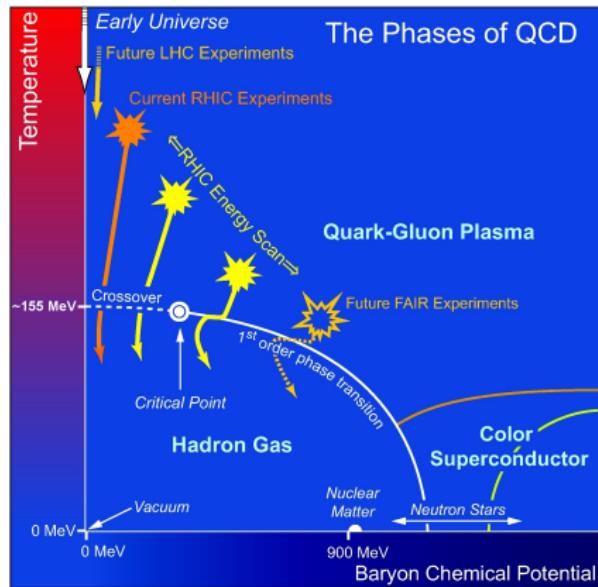
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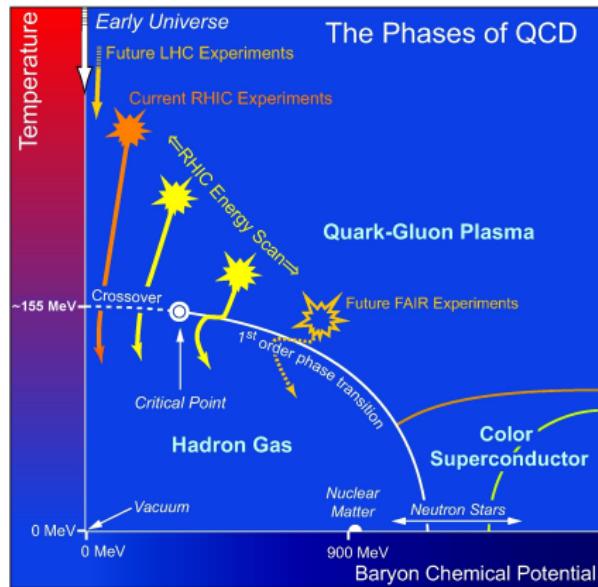
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- ▶ First-principle calculations are possible at $\mu_B/T = 0$, expansions/extrapolations at small μ_B/T

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Lattice gauge theory

- ▶ Start with the path integral quantization, Euclidean signature:

$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi] \mathcal{D}[\bar{\psi}] \mathcal{D}[A] \mathcal{O} \exp(-\mathcal{S}_E(T, V, \vec{\mu})), \\ \mathcal{Z}(T, V, \vec{\mu}) &= \int \mathcal{D}[\psi] \mathcal{D}[\bar{\psi}] \mathcal{D}[A] \exp(-\mathcal{S}_E(T, V, \vec{\mu})),\end{aligned}$$

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- ▶ The lattice spacing a acts as a UV cutoff, $p_{max} \sim \pi/a$
- ▶ The integrals can be evaluated with **importance sampling** methods

Taylor expansion in μ/T

- ▶ The chemical potentials for conserved charges B, Q, S :

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q,$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q,$$

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- ▶ The pressure can be expanded in Taylor series

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

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- ▶ The generalized susceptibilities are evaluated at vanishing chemical potential

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \left. \frac{\partial P(T, \hat{\mu}) / T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\hat{\mu}=0}, \quad \hat{\mu} \equiv \frac{\mu}{T}$$

Constrained series expansions

- ▶ The number densities can also be represented with Taylor expansions:

$$\frac{n_X}{T^3} = \frac{\partial P/T^4}{\partial \hat{\mu}_X}, \quad X = B, Q, S$$

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- ▶ These constraints can be fulfilled by

$$\begin{aligned}\hat{\mu}_Q(T, \mu_B) &= q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3 + q_5(T)\hat{\mu}_B^5 + \dots, \\ \hat{\mu}_S(T, \mu_B) &= s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3 + s_5(T)\hat{\mu}_B^5 + \dots\end{aligned}$$

Selected results

Chiral symmetry restoration

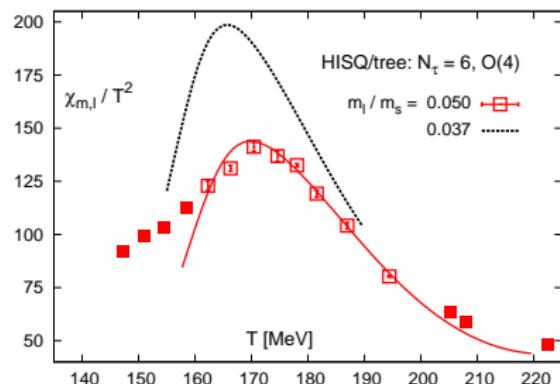
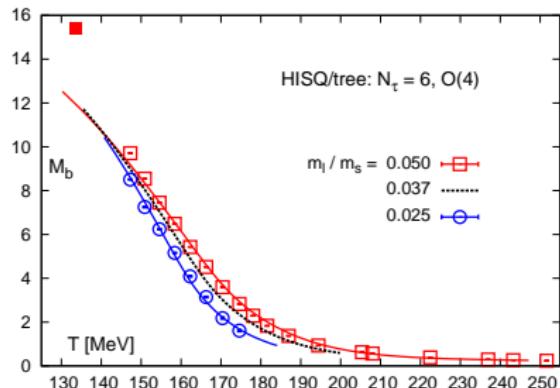
- ▶ Chiral condensate and susceptibility

$$\langle \bar{\psi} \psi \rangle_f = \frac{T}{V} \frac{\partial \ln \mathcal{Z}}{\partial m_f}, \quad \chi(T) = \frac{\partial \langle \bar{\psi} \psi \rangle_f}{\partial m_f}$$

Chiral symmetry restoration

- Chiral condensate and susceptibility

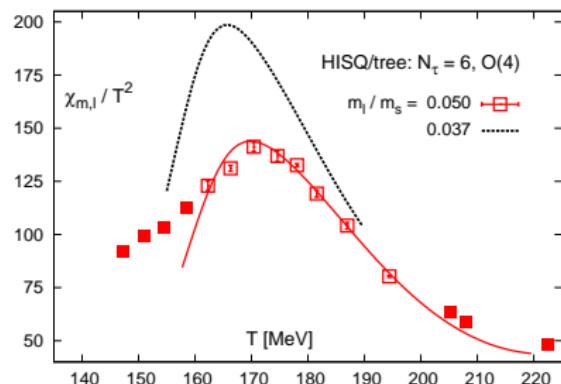
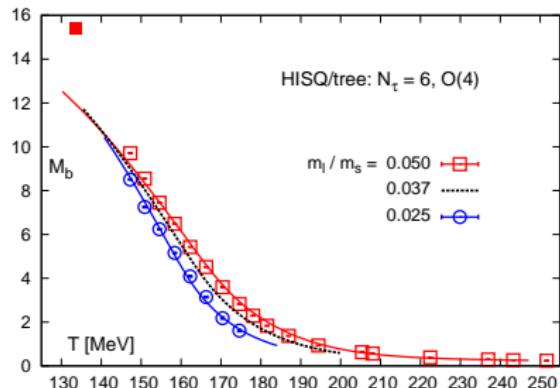
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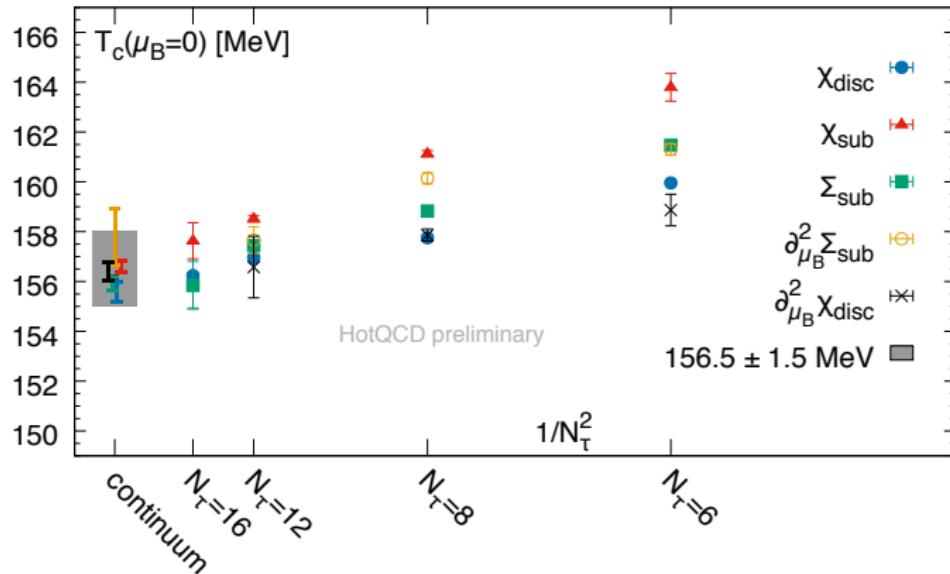
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The chiral crossover temperature at $\mu_B = 0$ (Borsanyi et al. [BW] (2010), Bazavov et al. [HotQCD] (2012))

$$T_c = 154 \pm 9 \text{ MeV}$$

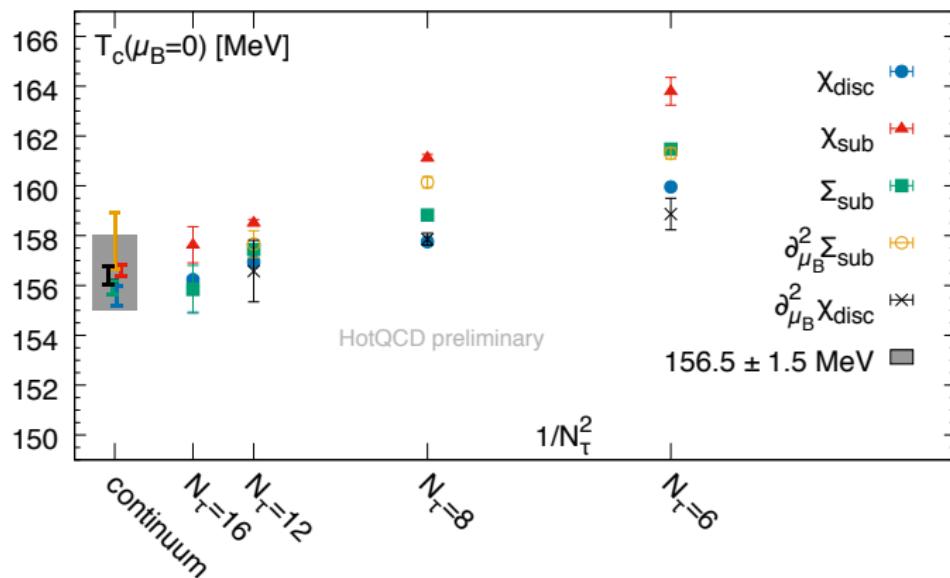
Chiral symmetry restoration (update)²



The chiral crossover temperature at $\mu_B = 0$ (HotQCD, preliminary)
 $T_c = 156.5 \pm 1.5$ MeV

²Figure from the talk at Quark Matter 2018 by P. Steinbrecher

Chiral symmetry restoration (update)²



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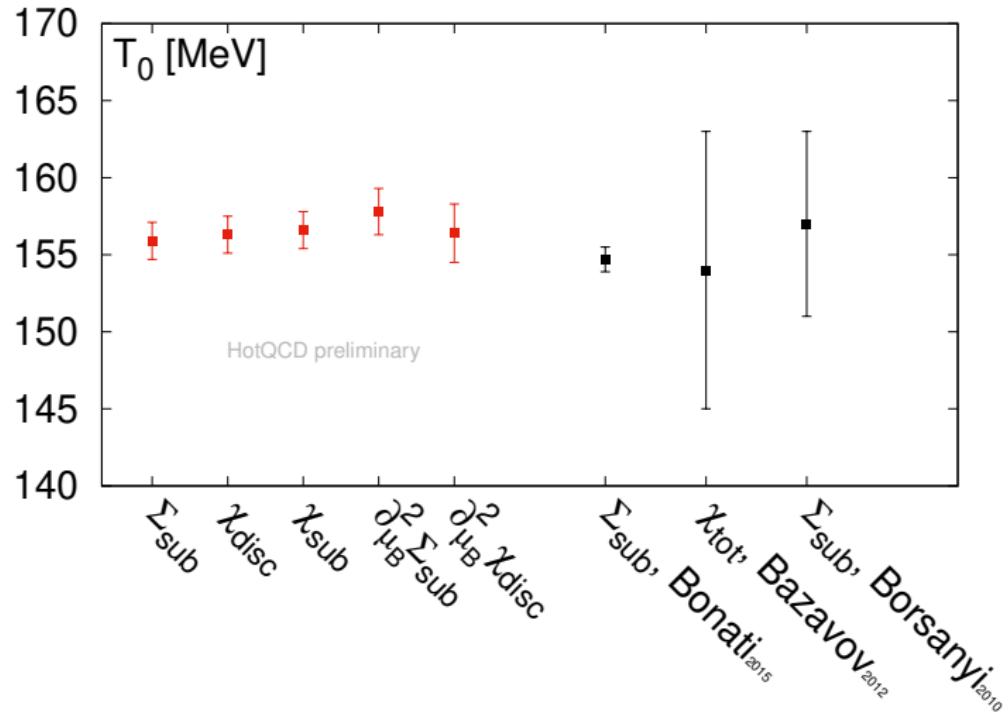
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See talk by C. Schmidt, Wed 14:00

²Figure from the talk at Quark Matter 2018 by P. Steinbrecher

Chiral symmetry restoration (update)³

- ▶ Comparison with earlier results

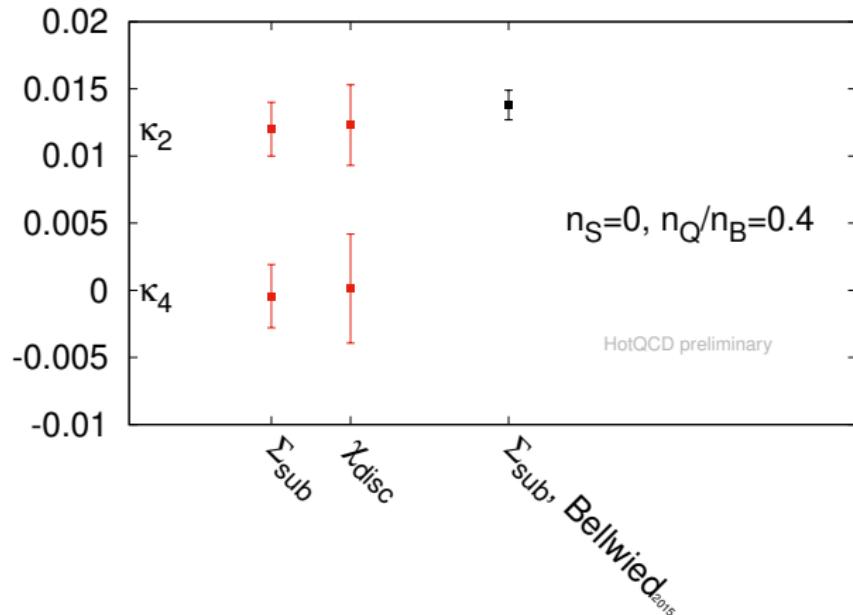


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Curvature of the chiral crossover line

- ▶ Change in the chiral crossover temperature with μ_B

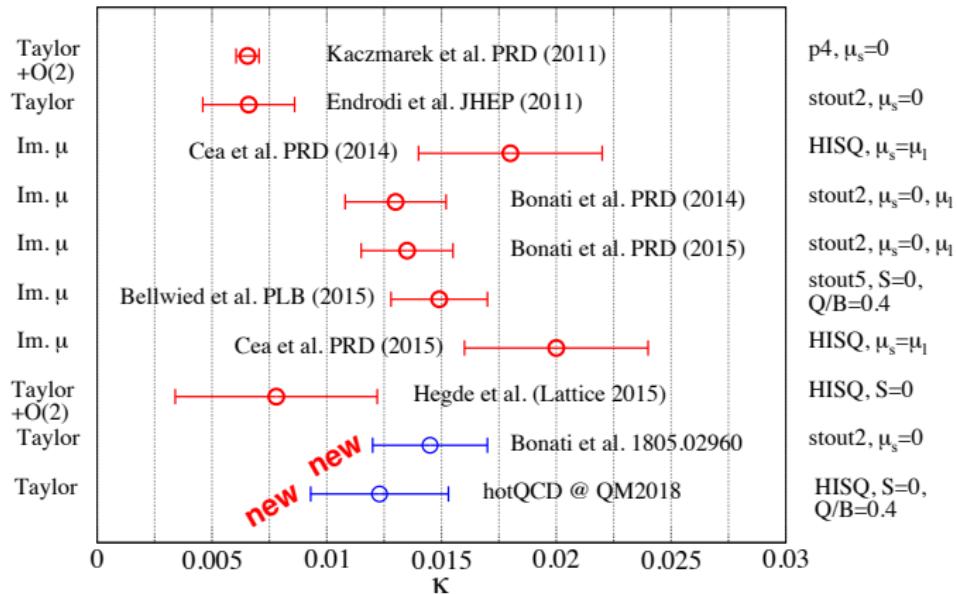
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(0)} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c(0)} \right)^4 + O(\mu_B^6)$$



Curvature of the chiral crossover line⁴

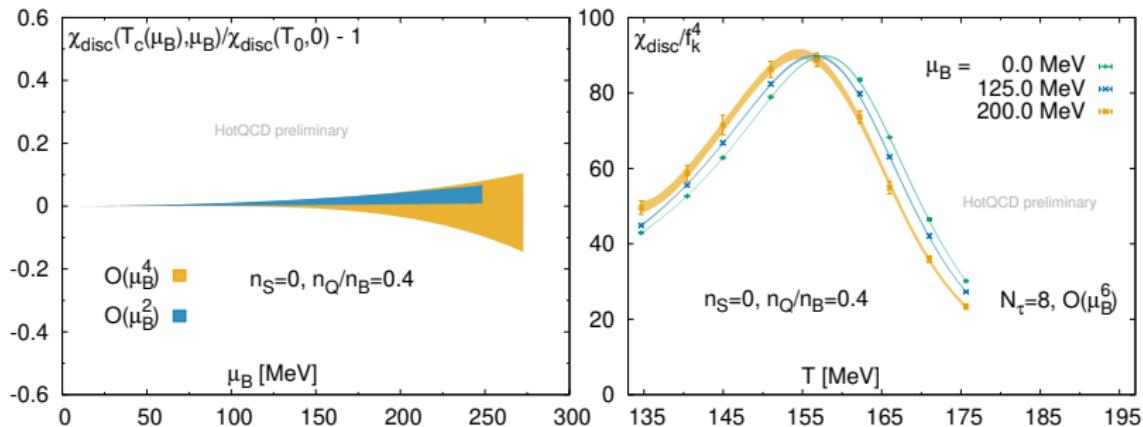
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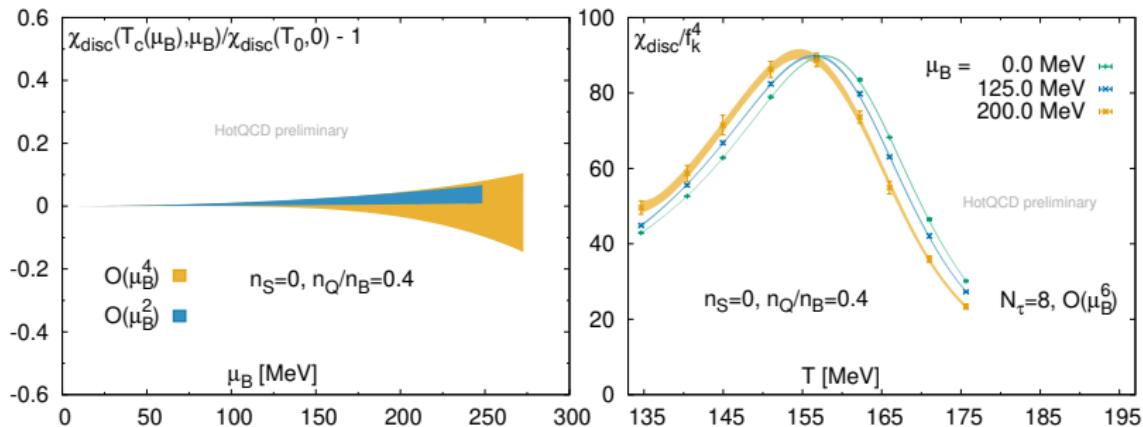


⁴Figure from the talk at Quark Matter 2018 by M. D'Elia

Chiral crossover at $\mu_B > 0$

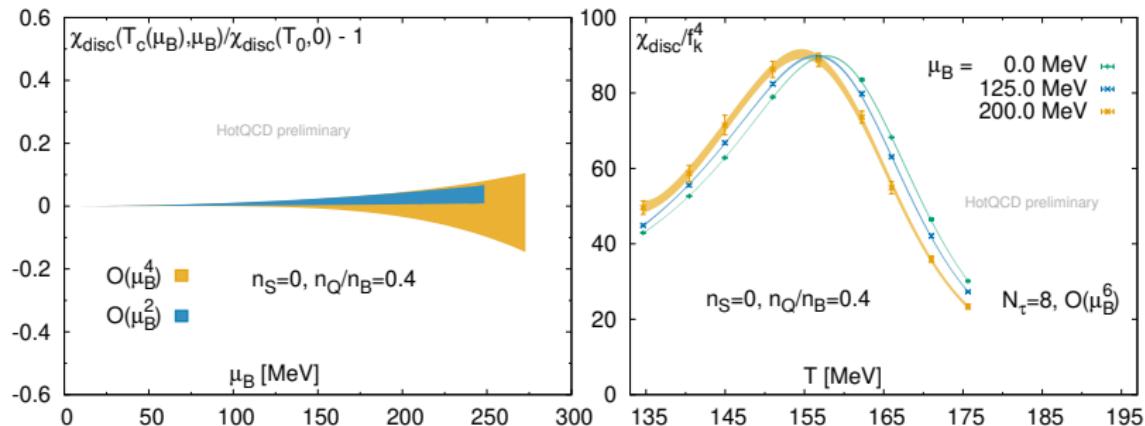


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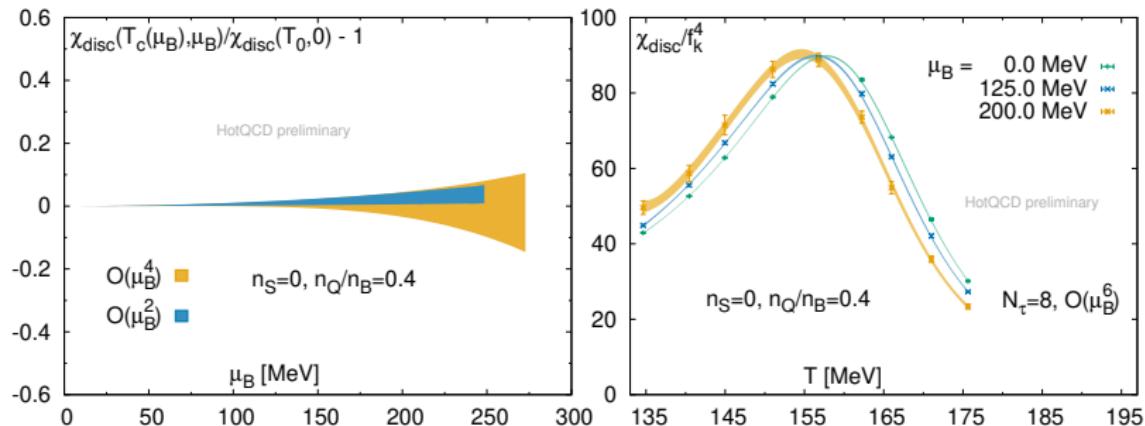
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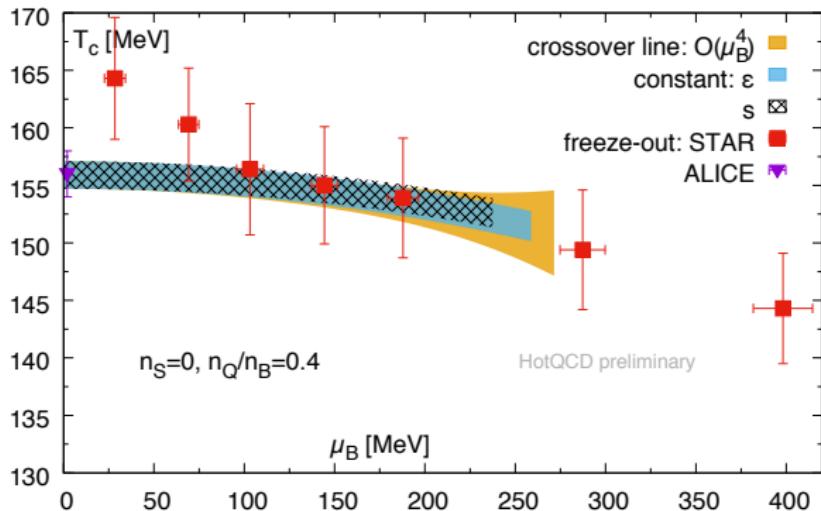
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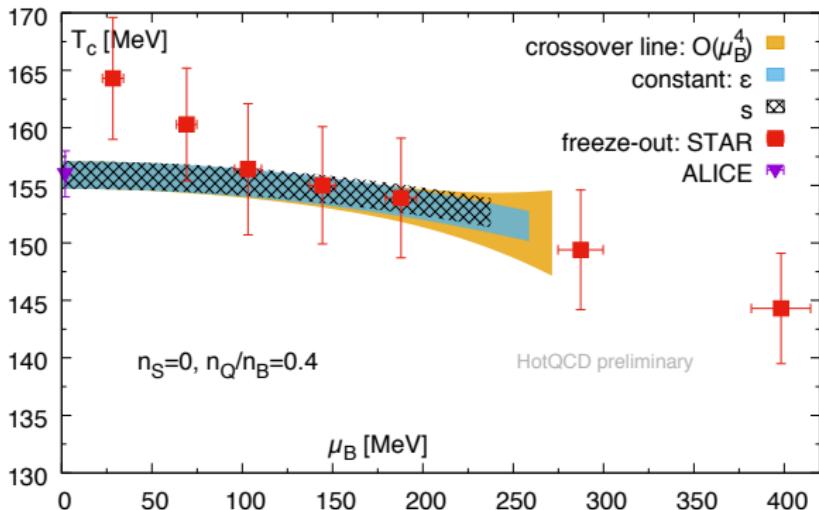


- ▶ The magnitude of the chiral susceptibility shows almost no change with increasing $\mu_B > 0$
- ▶ No indication that the crossover is getting stronger
- ▶ Similar conclusion from the baryon number fluctuations along the crossover line

Freeze-out line

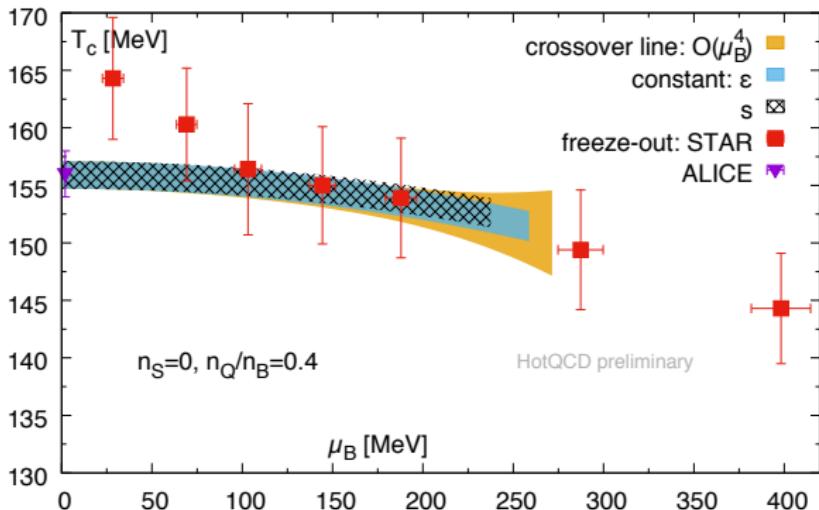


Freeze-out line



- ▶ The chiral crossover line coincides with the freeze-out line (data from ALICE:1408.6403, STAR:1701.07065)

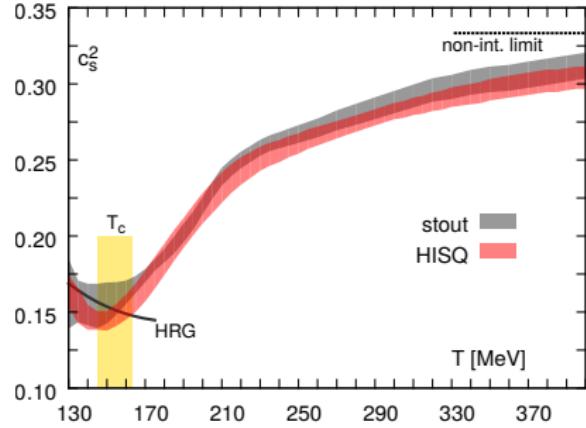
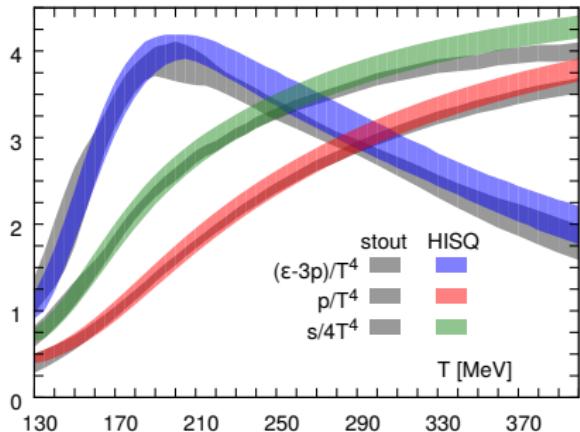
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- ▶ The chiral crossover line coincides with the freeze-out line (data from ALICE:1408.6403, STAR:1701.07065)
- ▶ The energy and entropy densities are constant along the chiral crossover line

The equation of state at $O(\mu_B^6)$

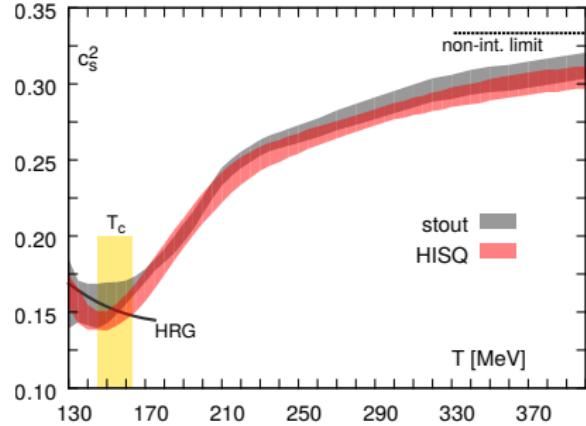
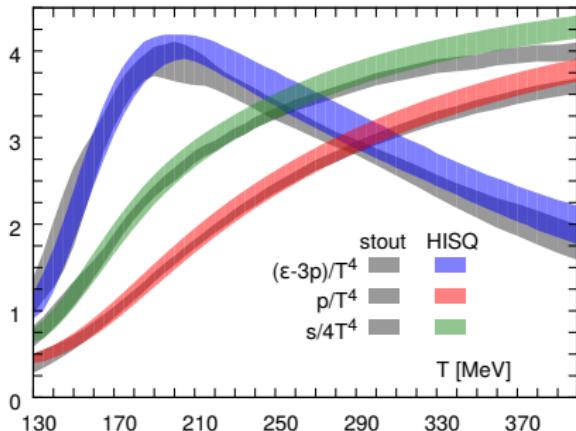
- The equation of state at $\mu_B = 0^5$



⁵Borsanyi et al. [WB] (2014), Bazavov et al. [HotQCD] (2014)

The equation of state at $O(\mu_B^6)$

- The equation of state at $\mu_B = 0$ ⁵

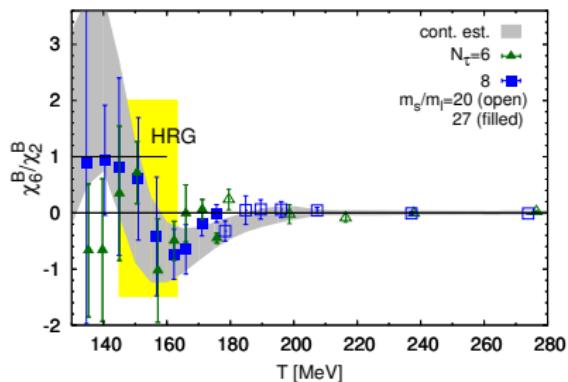
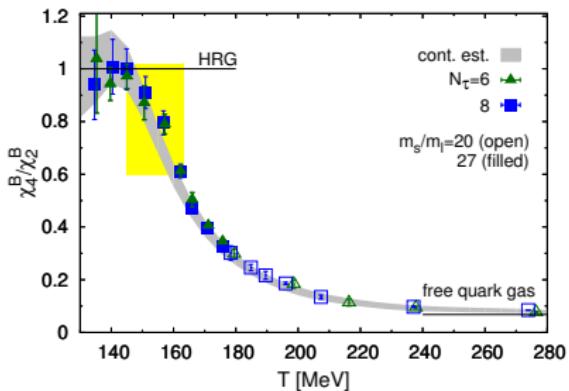
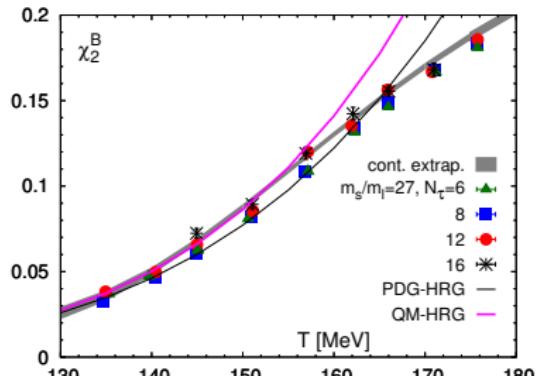
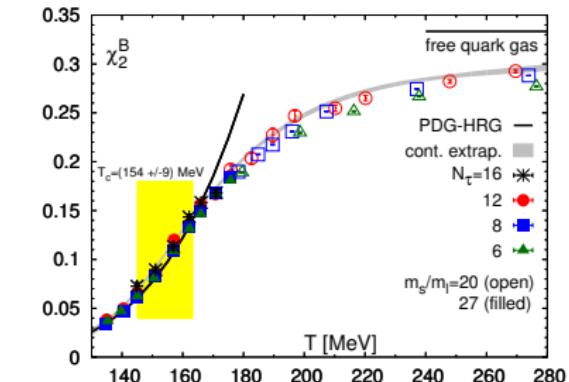


- Additional contribution at $\mu_B > 0$, $\mu_Q = \mu_S = 0$:

$$\frac{\Delta P}{T^4} = \frac{1}{2} \chi_2^B(T) \hat{\mu}_B^2 \left(1 + \frac{1}{12} \frac{\chi_4^B(T)}{\chi_2^B(T)} \hat{\mu}_B^2 + \frac{1}{360} \frac{\chi_6^B(T)}{\chi_2^B(T)} \hat{\mu}_B^4 + \dots \right)$$

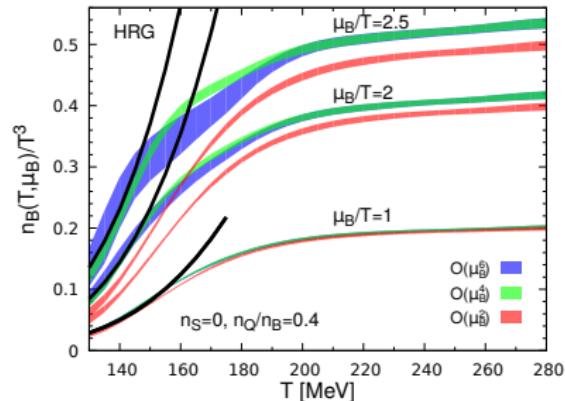
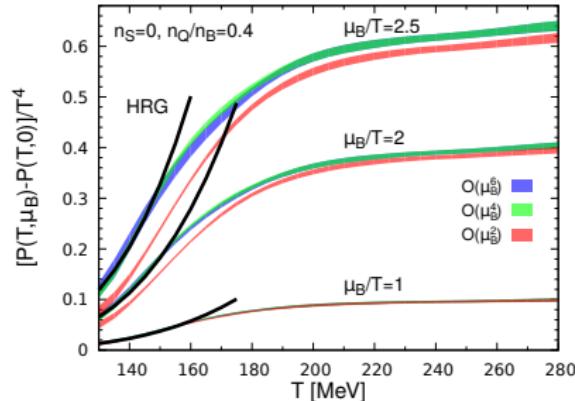
⁵Borsanyi et al. [WB] (2014), Bazavov et al. [HotQCD] (2014)

The equation of state at $O(\mu_B^6)^6$



⁶HotQCD (2017); up to $O(\mu_B^8)$ – BW 1805.04445

The equation of state at $O(\mu_B^6)$



- ▶ The contribution to the pressure due to finite chemical potential (left) and the baryon number density (right) for strangeness neutral systems:

$$n_S = 0, \quad \frac{n_Q}{n_B} = 0.4$$

Constraints on the critical point

- For $\mu_Q = \mu_S = 0$ the net baryon-number susceptibility is

$$\chi_2^B(T, \mu_B) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} \chi_{2n+2}^B \hat{\mu}_B^{2n}$$

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$$r_{2n}^\chi \equiv \sqrt{\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}}$$

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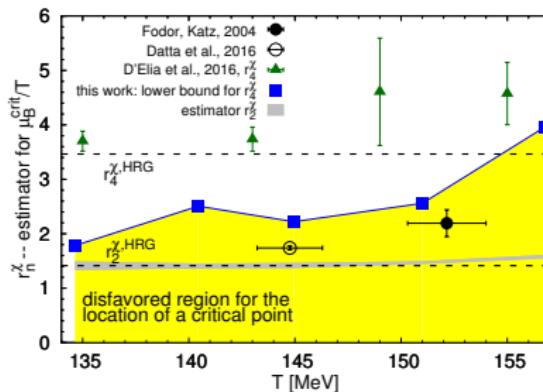
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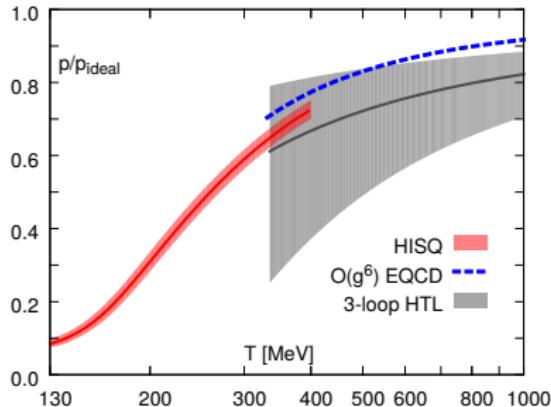
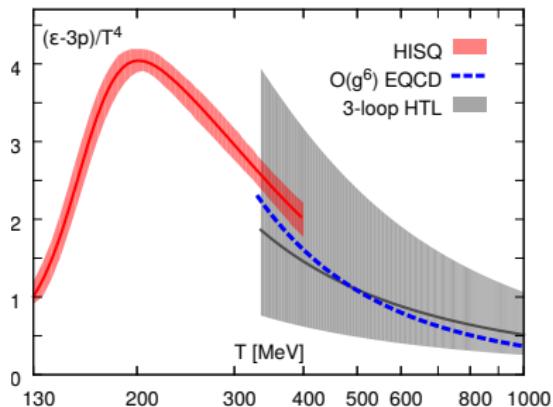
- The radius of convergence

$$r_{2n}^{\chi} \equiv \sqrt{\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}}$$

- We observe $\chi_6^B / \chi_4^B < 3$ for $135 < T < 155$ MeV $\Rightarrow r_4^{\chi} \geq 2$



Equation of state at high temperature

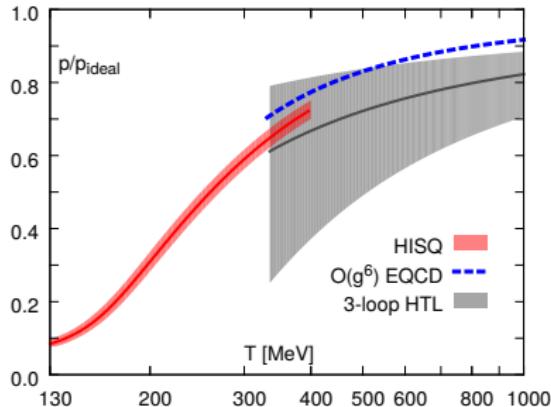
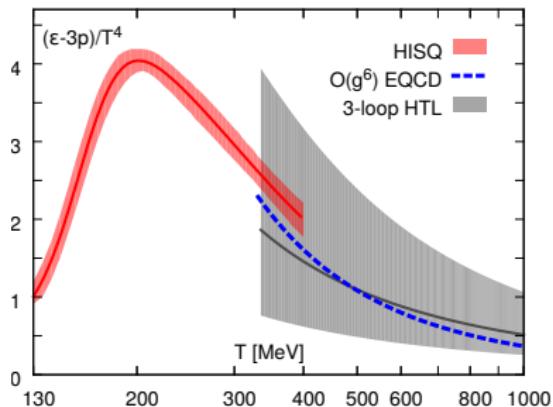


- ▶ The trace anomaly (left) and pressure (right) compared with (HTL)⁷ and Electrostatic QCD (EQCD)⁸ calculations
- ▶ The black line is the HTL calculation with the renormalization scale $\mu = 2\pi T$

⁷Haque et al. (2014)

⁸Laine and Schroder (2006)

Equation of state at high temperature

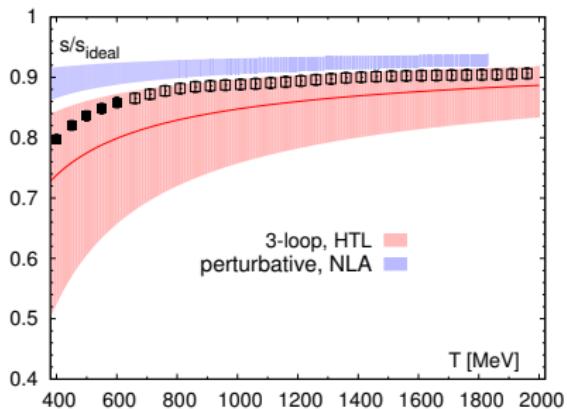
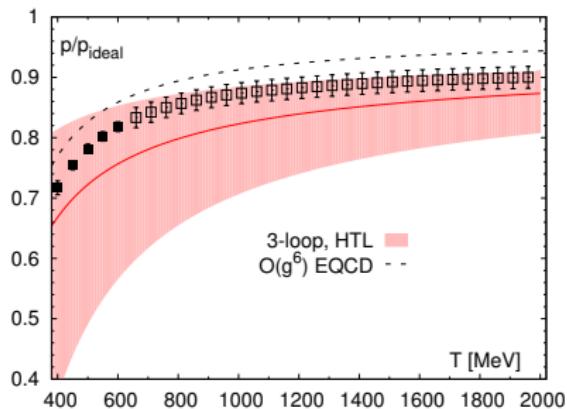


- ▶ The trace anomaly (left) and pressure (right) compared with (HTL)⁷ and Electrostatic QCD (EQCD)⁸ calculations
- ▶ The black line is the HTL calculation with the renormalization scale $\mu = 2\pi T$
- ▶ Extension of the 2+1 flavor equation of state to higher temperatures – **see talk by J. Weber, Wed 14:30**

⁷Haque et al. (2014)

⁸Laine and Schroder (2006)

Equation of state at high temperature



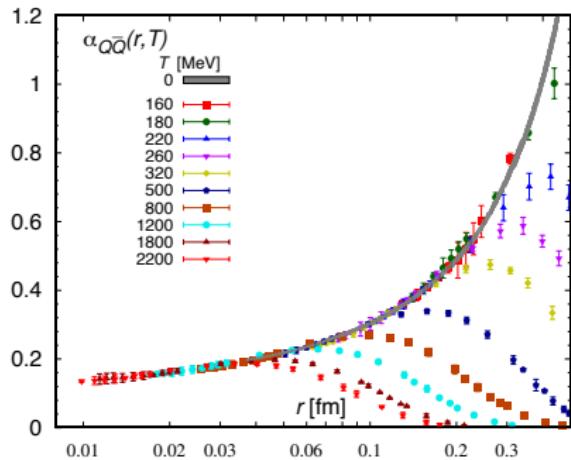
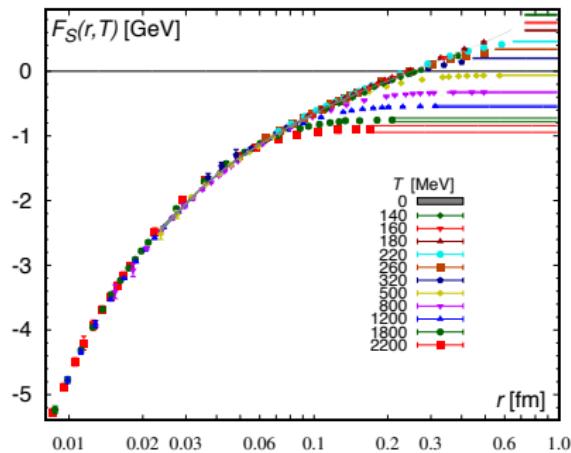
- ▶ Left: Comparison of the pressure obtained on the lattice with the HTL⁹ and EQCD¹⁰ results
- ▶ Right: Comparison of the entropy density obtained on the lattice with the HTL and NLA¹¹ results

⁹Haque et al. (2014)

¹⁰Laine and Schroder (2006)

¹¹Rebhan (2003)

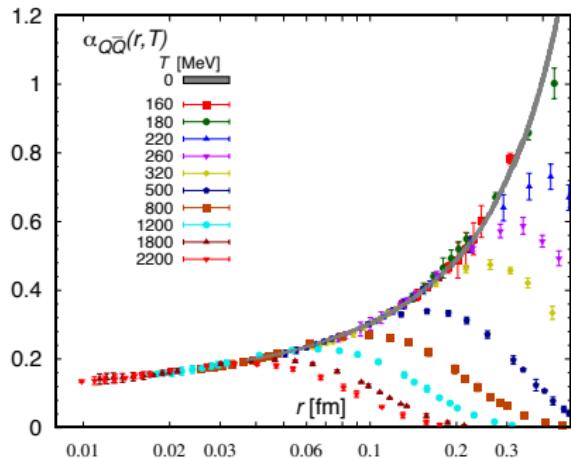
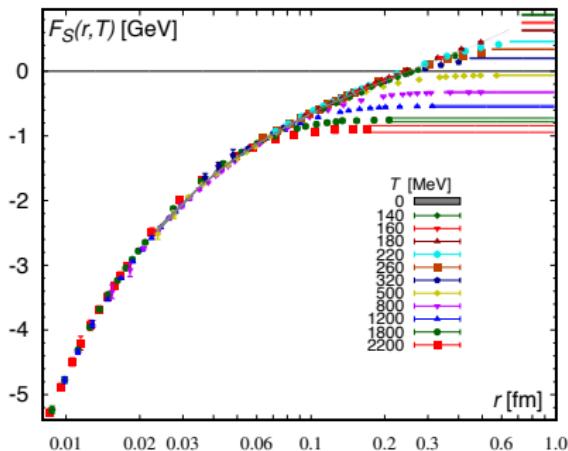
Screening properties at high temperature



- ▶ The singlet free energy (left) and the effective coupling (right) at temperatures up to 2.2 GeV¹²

¹²TUMQCD, 1804.10600

Screening properties at high temperature



- ▶ The singlet free energy (left) and the effective coupling (right) at temperatures up to 2.2 GeV¹²
- ▶ Comparison with weak-coupling calculations shows three distinct regimes: for $rT \lesssim 0.3$ medium effects are small, consistent with pNRQCD; for $0.3 \lesssim rT \lesssim 0.6$ screening effects are described by perturbative EQCD; for $rT > 0.6$ non-perturbative chromo-magnetic effects become important

¹²TUMQCD, 1804.10600

Conclusion

- ▶ Lattice QCD calculations are now in the regime of the physical light quark masses and continuum limit is possible for many observables
- ▶ The most studied region of the QCD phase diagram is at $\mu_B = 0$
- ▶ The region of small μ/T can be explored with expansions in μ/T or by analytic continuation from imaginary μ
- ▶ (Preliminary) updates on the chiral crossover temperature
- ▶ Generalized susceptibilities are now calculated up to 8th order in μ_B
- ▶ The equation of state is now known up to the 6th order in μ_B
- ▶ Recent lattice calculations strongly disfavor QCD critical point in the region of $\mu_B < 2T$ in the temperature range $135 < T < 155$ MeV
- ▶ At $\mu_B = 0$ the 2+1 flavor QCD equation of state has been calculated up to $T = 2$ GeV