Lattice determination of α_s

Alberto Ramos <alberto.ramos@maths.tcd.ie>

Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

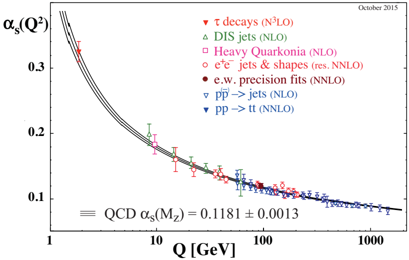
ALPHA Collaboration

Based on: M. Dalla Brida, P. Fritzsch, T. Korzec, A. R., S. Sint, R.Sommer,
Phys.Rev.Lett. 117 (2016) no.18; Phys.Rev.D (2017) no.95; Eur.Phys.J. C78 (2018) no.5

M. Bruno, M. Dalla Brida, P. Fritzsch, T. Korzec, A. R.,
S. Schaefer, S. Sint, H. Simma, R.Sommer,
Phys.Rev.Lett. 119 (2017) no.10, 102001

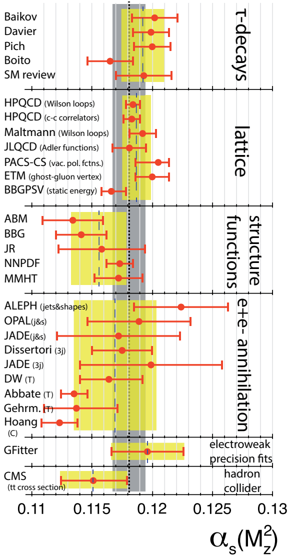
Overview

DETERMINATIONS OF $\alpha_{\overline{\mathrm{MS}}}(m_Z)$ [PDG '16]



- Low energy determinations are more precise (!!?)...

- ... But PT more accurate at high energies.



Determining the fundamental parameters of the SM.

A theoretical problem in strongly coupled QFT

$$S_{\rm QCD}[A_\mu, \psi, \overline{\psi}] = \int d^4x \, \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_f \overline{\psi_f} \gamma_\mu (\partial_\mu + m_q + \imath g A_\mu) \psi_f$$

- Fundamental parameters ($m_{\overline{\rm MS}}, \alpha_{\overline{\rm MS}}$) naturally defined at high energies (EW scale).
- Well measured and clean QCD quantities naturally defined at low energies ($M_p, M_\pi, \ldots$).
- In principle one-to-one correspondence, but…
- …**Common approach** to compute observables in a QFT: perturbation theory **only works in the weakly coupled regime!!**

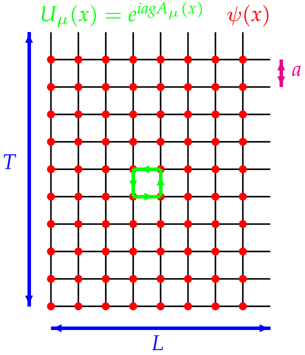
Fundamental parameters ($m_{\overline{\rm MS}}, \alpha_{\overline{\rm MS}}$) from hadronic input $M_p, M_\pi, \ldots$

- Lattice QCD
- Finite size scaling

Overview

COMPUTING PATH INTEGRALS: LATTICE FIELD THEORY

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



- Discretize space-time in an hyper-cubic lattice (spacing a)
- Path integral $\longrightarrow$ multiple integral (one variable for each field at each point)
- Compute the integral numerically $\rightarrow$ Monte Carlo sampling.

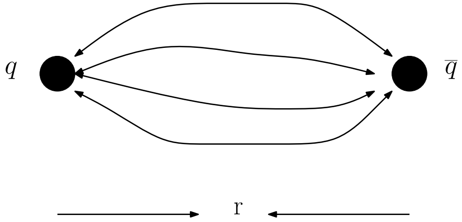
$$\langle O \rangle = \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

Observable computed averaging over samples

- This works both in the perturbative and non-perturbative regimes!

$$S_G[U] = \frac{\beta}{6} \sum_{p \in \text{Plaquettes}} Tr(1 - U_p - U_p^+) \xrightarrow[a \to 0]{} -\frac{1}{2} \int d^4x \, \text{Tr}(F_{\mu\nu} F_{\mu\nu})$$

The strength of YM



- Take $O(\mu) = \frac{3r^2}{4} F(r)\Big|_{\mu=1/r}$
- This defines the "potential scheme". Non-perturbative coupling definition.

$$\alpha_{qq}(\mu) = \frac{3r^2}{4} F(r)\Big|_{\mu=1/r}$$

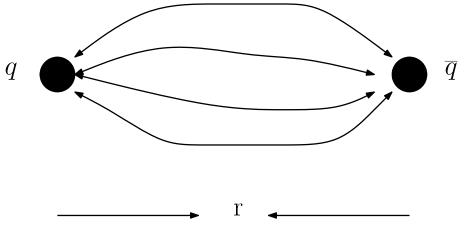
- **If** $\alpha(\mu)$ is small (small r), perturbation theory tells

$$\alpha_{qq}(\mu) = \alpha_{\overline{\text{MS}}}(\mu) + c_1 \alpha^2_{\overline{\text{MS}}}(\mu) + \dots$$

Physical (non-perturbative) couplings

"Any" observable can be used for a non perturbative definition of the strong coupling, but...
We need to evaluate $O(\mu)$ non-peturbatively $\Longrightarrow$ Lattice

The strength of YM



Viable strategy?

- Determine experimental quantity in Lattice units (e.g. aM_p)
- Determine short distance quantity ($F(r/a) \equiv \bar{q}q$ force at distance $rM_p = 1/100$)
- Continuum limit, …, and compare with PT $\Longrightarrow \alpha_{\overline{\rm MS}}(100\, M_p^{\rm exp})$

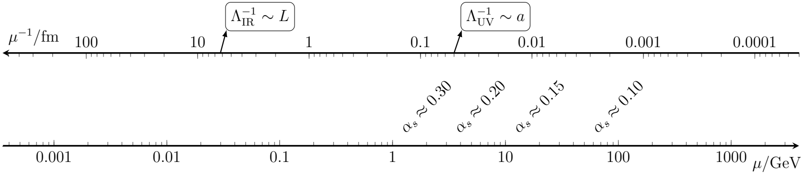
- ****If**** $\alpha(\mu)$ is small (small r), perturbation theory tells

$$\alpha_{qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

Physical (non-perturbative) couplings

"Any" observable can be used for a non perturbative definition of the strong coupling,

Lattice QCD typical scales



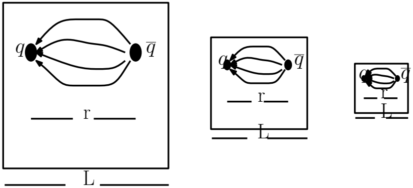
CLS ensembles ($N_f = 3$ QCD) **[Bruno et al. '15]**

Lattice sp. a	UV cutoff a^{-1}	L^{-1}	M_π	M_K
0.086 fm	2.3 GeV	35 – 70 MeV	130 – 420 MeV	420 – 480 MeV
0.064 fm	3.1 GeV	50 – 64 MeV	200 – 420 MeV	420 – 480 MeV
0.05 fm	3.9 GeV	60 MeV	260 – 420 MeV	420 – 470 MeV
0.04 fm	4.97 GeV	75 MeV	420 MeV	420 MeV

- Pushing $1/a \longrightarrow 100\,\text{GeV} \Longrightarrow \times 6400000$ in CPU cost (scaling $\propto (L/a)^6$).
- Reducing α **exponentially** difficult problem!

Overview

The soution: Finite size scaling [Lüscher, Weisz, Wolff '91]



Finite volume renormalization schemes: fix $\mu L =$ constant

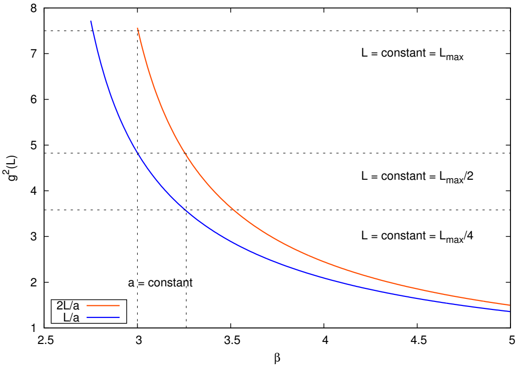
- Coupling $\alpha(\mu)$ depends on no other scale but L (Notation: $\alpha(L), \alpha(1/L)$).
- Small $L \Longrightarrow$ small $\alpha(L)$
- $a \ll 1/\mu$ easily achieved: $L/a \sim 10 - 40$
- Step scaling function: How much changes the coupling when we change the renormalization scale:

$$\sigma(u) = g^2(\mu/2)\Big|_{g^2(\mu)=u}$$

  achieved by simple changing $L/a \to 2L/a$!
- $1/L$ is a IR cutoff $\Rightarrow$ simulate directly $m_q = 0$
- We need dedicated simulations of the **femto-universe**

The soution: Finite size scaling [Lüscher, Weisz, Wolff '91]

$$\beta \Longleftrightarrow a; \qquad g^2(L) \Longleftrightarrow L \Longleftrightarrow \mu$$



Step scaling function

$$\Sigma(u, a/L) = g^2(2L)\Big|_{g^2(L)=u}$$

Continuum limit

$$\sigma(u) = \lim_{a/L \to 0} \Sigma(u, a/L)$$

Simulate several pair of lattices

Details of the work

- Schrödinger Functional (Dirichlet) boundary conditions **[Lüscher et al. '92; S. Sint '94]**.
- Three massless NP-$\mathcal{O}(a)$ improved Wilson fermions (CLS **[Bulava, Schaefer '13]**).
- Schrödinger Functional (SF) coupling **[Lüscher et al. '92]**

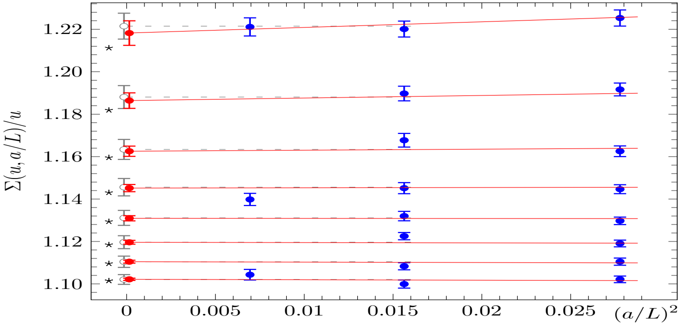
$$100\,\mathrm{GeV} > \mu > 2\,\mathrm{GeV}$$

- Gradient Flow (GF) coupling in the SF **[P. Fritzsch, A. Ramos. '13]**

$$4\,\mathrm{GeV} > \mu > 0.2\,\mathrm{GeV}$$

- Non-perturbative matching at $\mu_0 \sim 2\,\mathrm{GeV}$
- Excellent tunning to zero mass ($Lm < 0.0005$)
- More than 150 simulations (!!) in lattices with $L/a = 6 - 32$.

Precise determination of $\sigma_{\rm SF}(u)$ ($\sim 100 - 2$ GeV)



- Accurate determination of $\sigma_{\rm GF}(u)$ with $L/a = 6, 8, 12 \to 12, 16, 24$
- At $u = 2.0201, 1.7943, 1.6173, 1.4783, 1.3629, 1.2657, 1.1845, 1.1122$
- Investigated a 1-parameter family of renormalization schemes
- Detailed study on how "good" is perturbation theory
- Challenging to keep precision at lower energies

Gradient flow coupling [P. Fritzsch, A. Ramos '13]

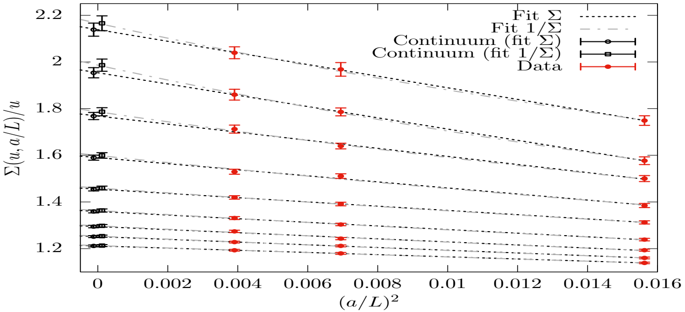
L/a	6	8	10	12	16
β	5.2638	5.4689	5.6190	5.7580	5.9631
$\kappa_{\rm sea}$	0.135985	0.136700	0.136785	0.136623	0.136422
$N_{\rm meas}$	12160	8320	8192	8280	8460
$\bar{g}^2_{\rm SF}(L_1)$	4.423(75)	4.473(83)	4.49(10)	4.501(91)	4.40(10)
$\bar{g}^2_{\rm GF}(\mu)$ ($c = 0.3$)	4.8178(46)	4.7278(46)	4.6269(47)	4.5176(47)	4.4410(53)
$\bar{g}^2_{\rm GF}(\mu)$ ($c = 0.4$)	6.0090(86)	5.6985(86)	5.5976(97)	5.4837(97)	5.410(12)
$\bar{g}^2_{\rm GF}(\mu)$ ($c = 0.5$)	7.106(14)	6.817(15)	6.761(19)	6.658(19)	6.602(24)

Advantages of GF coupling definition

- $\mathcal{O}(10^3)$ less expensive at $g^2 \sim 4$ (1 CPU day $\rightarrow$ some CPU years).
- Finite variance when $a \rightarrow 0$ (i.e. $\mathcal{V} \sim \langle E^2(t)\rangle - \langle E(t)\rangle^2$).
- Statistical precision independent of coupling value $\delta g^2/g^2 \sim$ constant.
- Smaller $c \Longrightarrow$ Larger cutoff effects, more precision. ($c \in [0.3, 0.5]$)

Ideal for matching with hadronic regime of QCD

Precise determination of $\sigma_{\rm GF}(u)$ ($\sim 4 - 0.2$ GeV)



- Accurate determination of $\sigma_{\rm GF}(u)$ with $L/a = 8, 12, 16 \rightarrow 16, 24, 32$
- At $u = 6.5489, 5.8673, 5.3013, 4.4901, 3.8643, 3.2029, 2.7359, 2.3900, 2.1257$
- Cutoff effects larger than in the $\alpha_{\rm SF}$ scheme
- Detailed investigation in a EFT approach **[A.R., S. Sint Eur. Phys. J., 2016, C76, 15]**
- +20 pages discussion in **[Phys.Rev.D (2017) no.95, 014507]**
- +4 variations to fit cutoff effects

$\sigma(u) = g^2(2L)|_{g^2(L)=u} \Longrightarrow \beta(g) \Longrightarrow$ Whatever!

- Once $\sigma(u)$ is known, we can determine $\beta(g) = \mu \frac{\partial g(\mu)}{\partial \mu}$

$$\log 2 = -\int_{g(\mu)}^{g(\mu/2)} \frac{dx}{\beta(x)} = -\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{dx}{\beta(x)}$$

- …also the Λ parameter

$$\frac{\Lambda}{\mu_{\rm ref}} = \left[b_0 g^2(\mu_{\rm ref})\right]^{\frac{-b_1}{2b_0^2}} e^{-\frac{1}{2b_0 g^2(\mu_{\rm ref})}} \exp\left\{-\int_0^{g(\mu_{\rm ref})} dx \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0 x}\right]\right\}$$

With ssf define $g_0 = g(\mu_{\rm ref})$ and $g_k = g(2^k \mu_{\rm ref}) = \sqrt{\sigma^{-1}(g_{k-1}^2)}$. Now

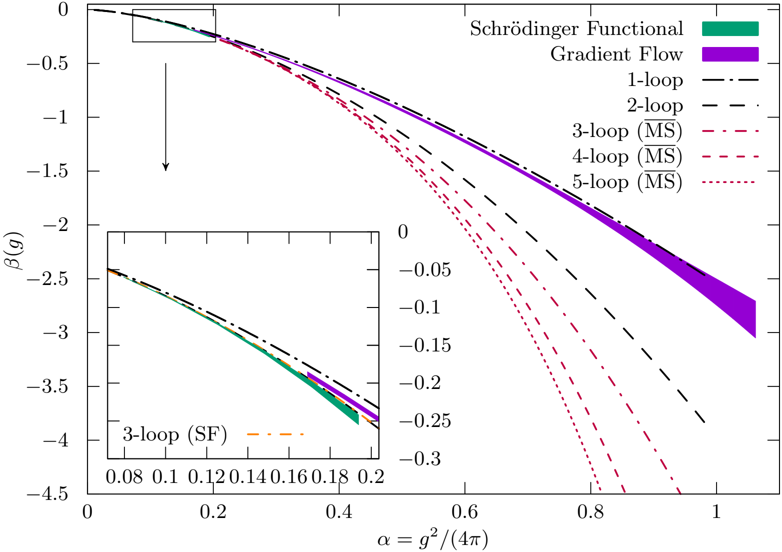
$$\int_0^{g(\mu_{\rm ref})} = \int_{g_1}^{g_0} + \int_{g_1}^{g_0} + \cdots + \int_0^{g_N} dx \left[\frac{1}{\beta_{\rm PT}(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0 x}\right]$$

- …and any ratio of scales

$$\log \frac{\mu_1}{\mu_2} = -\int_{g(\mu_1)}^{g(\mu_2)} \frac{dx}{\beta(x)}$$

A precise determination of $\alpha_s(m_Z)$ from three-flavor QCD

[M. Dalla Brida, P. Fritzsch, T. Korzec, A. Ramos, S. Sint, R.Sommer, M. Bruno, S. Schaefer, H. Simma]



A precise determination of $\alpha_s(m_Z)$ from three-flavor QCD

[M. Dalla Brida, P. Fritzsch, T. Korzec, A. Ramos, S. Sint, R.Sommer, M. Bruno, S. Schaefer, H. Simma]

$$\Lambda^{(3)}_{\overline{\rm MS}} = \frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\Lambda^{(3)}_{\rm SF}} \times \frac{\Lambda^{(3)}_{\rm SF}}{\mu_0} \times \frac{\mu_0}{\mu_{\rm had}} \times \frac{\mu_{\rm had}}{F_k + \frac{1}{2}F_\pi} \times (F_k + \frac{1}{2}F_\pi)^{\rm exp} = 0.341(12)\,\text{GeV}$$

- Analitically known
- High energy running (SF scheme): 57%
- Low energy running (GF scheme): 30%
- Hadronic matching (CLS): 13%

Results

- Use perturbation theory

$$\Lambda^{(3)}_{\overline{\rm MS}} = 0.341(12)\,\text{GeV} \to \Lambda^{(4)}_{\overline{\rm MS}} = 0.298(12)(3)\,\text{GeV} \to \Lambda^{(5)}_{\overline{\rm MS}} = 0.215(10)(3)\,\text{GeV}\,.$$

- Finally, the strong coupling

$$\alpha_s(M_Z) = 0.11852(84)\,, \quad [0.7\%]\,.$$

Overview

Conclusions

- **Lattice QCD** and **Finite size scaling** provide a sound theoretical approach to many multi-scale problems
  - Determinning quark masses
  - HQET
  - Renormalization of 4-quark operators
- Schemes based on the **gradient flow** allow to reach a new level of precision.
- **High energy** determination of $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ from hadronic input
  - Non-perturbative running from 200 MeV to 100 GeV
  - PT only used at $\mu > 100$ GeV
  - Clean hadronic low energy quantity as input
- Use of PT to cross thresholds must be studied in detail

Perspectives

- If any of us see the discovery of new physics, this will be most likely the result of precision searches.
- One of our "best" hopes (LHC), is a "dirty" hadronic machine: knowing α_s to 0.5% is really important.
- I am convinced that now we have the tools that will make this computation possible.

Overview

Continuum limit of flow quantities

Accuracy of PT

Solving the flow equation on the lattice

The continuum equation

$$\frac{dB_\mu(x,t)}{dt} = D_\nu G_{\nu\mu}(x,t); \quad \left(\sim -g_0^2 \frac{\delta S_{\rm YM}[B]}{\delta B_\mu}\right)$$

How do the links $V_\mu(x,t) = \exp[B_\mu(t,x)]$ change with the t?

$$a^2 \frac{d}{dt} V_\mu(x,t) = -g_0^2 \frac{\delta S^{\rm latt}[V]}{\delta V_\mu(x,t)} V_\mu(x,t)$$

- Is this the best option?
- Which lattice action $S^{\rm latt}$?

The Zeuthen flow

$$a^2 \frac{d}{dt} V_\mu(x,t) = -g_0^2 \left(1 + \frac{a^2}{12} D_\mu D_\mu^*\right) \frac{\delta S^{\rm LW}[V]}{\delta V_\mu(x,t)} V_\mu(x,t)$$

This equation is **the result of a computation**.

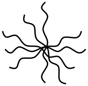
Lattice people hate discovering "new physics"

$S =$ Standard model + Quantum Gravity

At low energies ($\ll M_{\rm pl}$)
$\langle O\rangle = \langle O\rangle_{SM} + \mathcal{O}(1/M_{\rm pl})$

We simulate

$S = \sum_{x,\mu\neq\nu} \mathrm{Tr}(1 - U_\mu(x)U_\nu(x+\mu)\cdots)$



Multi gluon interactions:
6,8,10,12,... gluon vertices.
at energy scales $1/a$

UNIVERSALITY
(Symmetries, dimensions, ...)

We obtain QCD

$S = -\frac{1}{2}\int \mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu}) + \ldots$

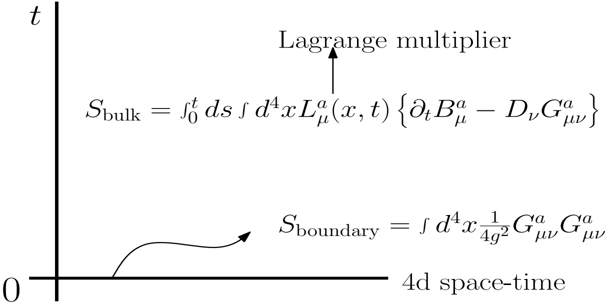
At low energies ($\ll 1/a$)
$\langle O\rangle_{\rm latt} = \langle O\rangle_{\rm QCD} + \mathcal{O}(a^2)$

Symanzik improvement program

Fine tune (i.e. cook) a lattice action $S^{\rm latt}$ such that the efective theory at energy scales much smaller than the cutoff looks as close as possible to the continuum.

5d Local formulation [Lüscher, Weisz '11]

We can see the theory as a 5d local field theory [Zinn-Justin '86, Zinn-Justin, Zwanziger '88]



$$S_{\rm Total} = S_{\rm bulk} + S_{\rm boundary}$$

Theory finite to all orders of perturbation theory

- Power counting
- Theory has BRS invariance
- No loops on the bulk ⇒ No extra counterterms ⇒ No operator mixing for $t > 0$.

The Symanzik effective action for the Gradient flow

Action composed of bulk part and boundary part

$$S_{\mathrm{bndry}} = -\frac{1}{2g_0^2}\int d^4x\,\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_i \alpha_i \int d^4x\, O_i^{\mathrm{d}=6}(x)$$

$$S_{\mathrm{bulk}} = -2\int_0^\infty dt \int d^4x\,\mathrm{Tr}\left\{L_\mu(x,t)[\partial_t B_\mu(x,t) - D_\mu G_{\mu\nu}]\right\} + \sum_i \int_0^\infty dt \int d^4x\, O_i^{\mathrm{d}=8}(x,t)$$

Possible bulk counterterms

- Remember: No loops in the bulk ⇒ No new counterterms are generated.
- Classical improvement in the bulk is equivalent to **non-perturbative** improvement.

CLASSICAL EXPANSION OF THE FLOW EQUATION

$$\left(a^2\partial_t V_\mu\right) V_\mu^{-1} = a^3\partial_t B_\mu + \frac{1}{2}a^4 D_\mu\partial_t B_\mu + \frac{1}{6}a^5 D_\mu^2\partial_t B_\mu + \mathcal{O}(a^6)$$

$$-\partial_{x,\mu}\left[g_0^2 S_{\mathrm{lat}}(V)\right] = \sum_{\nu=0}^{3}\Big\{a^3 D_\nu G_{\nu\mu} + \frac{1}{2}a^4 D_\mu D_\nu G_{\nu\mu} + \frac{1}{12}a^5\Big[(1+12(c_1-c_2))\left(2D_\nu D_\mu^2 + D_\nu^3\right) - 12(c_1-c_2)D_\mu^2 D_\nu + 12c_2\sum_{\rho=0}^{3}\left(3D_\rho^2 D_\nu - 4D_\rho D_\nu D_\rho + 2D_\nu D_\rho^2\right)\Big]G_{\nu\mu}\Big\} + \mathcal{O}(a^6)$$

Some conclusions

- Correct continuum flow equation

$$\partial_t B_\mu = D_\nu G_{\nu\mu}$$

- $\mathcal{O}(a)$ corrections cancel.
- No value of c_1, c_2 for which the $\mathcal{O}(a^2)$ corrections cancel!

CLASSICAL EXPANSION OF THE FLOW EQUATION

$$\left(a^2\partial_t V_\mu\right) V_\mu^{-1} = a^3\partial_t B_\mu + \frac{1}{2}a^4 D_\mu\partial_t B_\mu + \frac{1}{6}a^5 D_\mu^2\partial_t B_\mu + \mathcal{O}(a^6)$$

$$-\partial_{x,\mu}\left[g_0^2 S_{\text{lat}}(V)\right] = \sum_{\nu=0}^{3}\Bigg\{a^3 D_\nu G_{\nu\mu} + \frac{1}{2}a^4 D_\mu D_\nu G_{\nu\mu}$$

$$+ \frac{1}{12}a^5\Big[(1+12(c_1-c_2))\left(2D_\nu D_\mu^2 + D_\nu^3\right) - 12(c_1-c_2)D_\mu^2 D_\nu$$

$$+ 12c_2\sum_{\rho=0}^{3}\left(3D_\rho^2 D_\nu - 4D_\rho D_\nu D_\rho + 2D_\nu D_\rho^2\right)\Big] G_{\nu\mu}\Bigg\} + \mathcal{O}(a^6)$$

the Symanzik/LW flow ($c_1 = -1/12$, $c_2 = 0$), is "almost" $\mathcal{O}(a^2)$ improved

$$\partial_t B_\mu = \sum_{\nu=0}^{3}\left\{D_\nu G_{\nu\mu}(x,t) - \frac{1}{12}a^2 D_\mu^2 D_\nu G_{\nu\mu} + \mathcal{O}(a^3)\right\}$$

The Zeuthen flow

$$\left(a^2\partial_t V_\mu(x,t)\right) V_\mu(x,t)^{-1} = -g_0^2\left(1 + \frac{1}{12}a^2 D_\mu^* D_\mu\right)\partial_{x,\mu}\left[g_0^2 S_{\text{LW}}(V)\right]$$

$$aD_\mu F(x) = V_\mu(x,t)F(x+a\hat{\mu})V_\mu(x,t)^\dagger - F(x), \ldots$$

Continuum limit of flow quantities

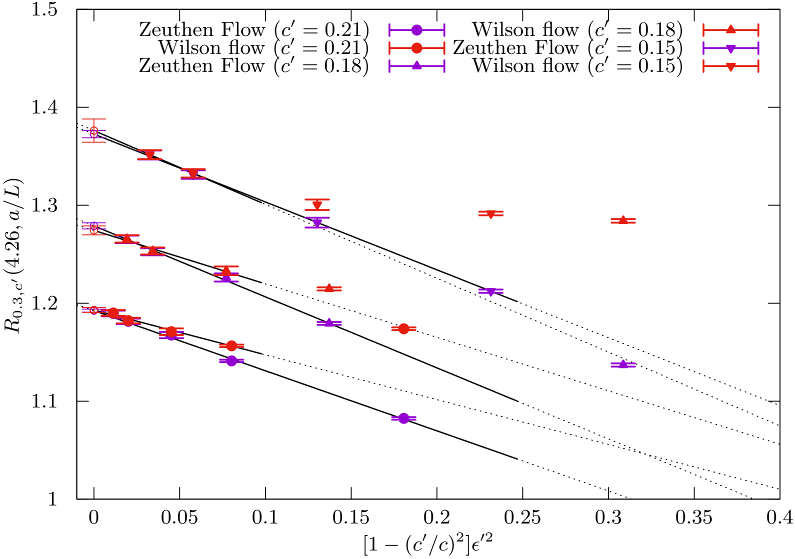
Getting some insight in the scalig of the $\Sigma(u, a/L)$

$$R_{c,c'}(u, a/L, s) = \left.\frac{g_c^2(L)}{g_{c'}^2(sL)}\right|_{g_c^2(L)=u} = R_{c,c'}(u, 0, s)\left\{1 + A_{c,c'}(u)[\epsilon^2 - \epsilon'^2] + \ldots\right\},$$

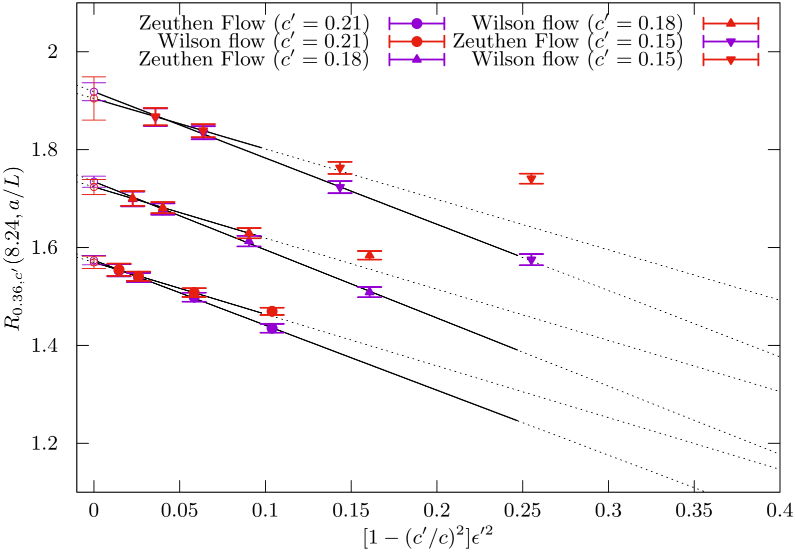
with $\epsilon = a/(cL)$ and $\epsilon' = a/(c'sL)$.

- $R_{c,c'}(u, a/L, s)$ is mainly a function of sc'.
- Step scaling function $\Longrightarrow R_{c,c}(u, a/L, 2) = u/\Sigma(u, a/L)$.
- Instead study $R_{c,c'}(u, a/L, 1) \Longrightarrow$ we can use $L/a = 8, 12, 16, 24, 32$

Scaling of the ratios $R_{c,c'}(u, a/L, 1)$



Scaling of the ratios $R_{c,c'}(u, a/L, 1)$



SCALING OF THE RATIOS $R_{c,c'}(u, a/L, 1)$

Four sources of cutoff effects in flow quantities [A. Ramos, S. Sint '15]

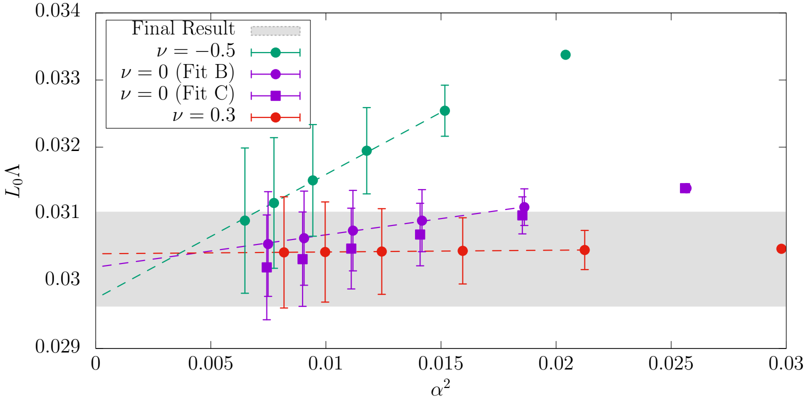
- Quantum effects at $t = 0$. Very complicated dependence on g_0^2
  - Choice of action.
  - Choice of initial condition in the flow equation (i.e. $V_\mu(0, x) = U_\mu(x)$)
- *Integrating the flow equation* Zeuthen flow
- *Evaluating an observable* Classically improved discretization.

Conclusions: Still lot to understand!

- In our data:
  - Wilson Flow: Breaking of scaling at $(a/cL)^2 = 0.15$
  - Zeuthen Flow: Breaking of scaling at $(a/cL)^2 = 0.3$
  - We use $L/a = 8, c = 0.3 \Longrightarrow (a/cL)^2 = 0.17$
- Zeuthen flow **not** cooked for this!
- $\mathcal{O}(a^2)$ effects still significant!
- Main suspect: The initial condition of the flow equation.

Overview

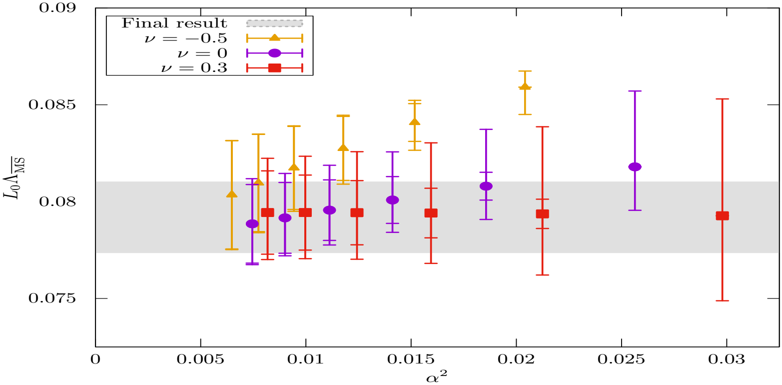
Accuracy of PT: Test in different schemes



Not all schemes are goot!

- We use different schemes ν
- And different energy scales (x-axis, α)
- All schemes have good PT properties
  - No leading renormalon contributions
  - β-function know up to NNLO with good apparent convergence

Accuracy of PT: Test in different schemes



Not all schemes are goot!

- We use different schemes ν
- And different energy scales (x-axis, α)
- All schemes have good PT properties
  - No leading renormalon contributions
  - β-function know up to NNLO with good apparent convergence