α_s from high-energy collider data

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Strong coupling constant (1992)

	$\alpha_s(\mathrm{M}_\mathrm{Z}^2)$
R_τ	$0.117\ {}^{+\ 0.010}_{-\ 0.016}$
DIS	0.112 ± 0.007
Υ Decays	0.110 ± 0.010
$R_{e^+e^-}(s < 62$ GeV$)$	0.140 ± 0.020
$p\bar{p} \to W + jets$	0.121 ± 0.024
$\Gamma(Z \to \mathrm{hadrons})/\Gamma(Z \to l\bar{l})$	0.132 ± 0.012
Jets at LEP	0.122 ± 0.009
Average	0.118 ± 0.007

G. Altarelli (1992)
in *QCD - 20 Years Later,*
CERN-TH-6623-92

Essential facts

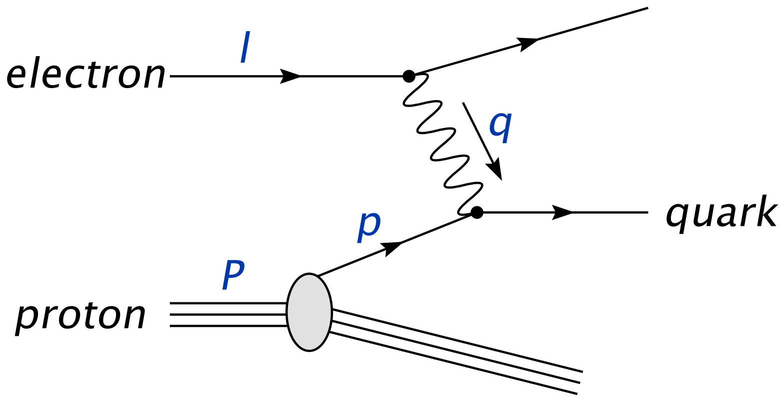
- World average 1992 $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to $\sim 0.0050 - 0.0040$ (theory scale uncertainty)

Strong coupling constant from DIS (2018)

BBG	$0.1134\ ^{+\,0.0019}_{-\,0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
JR08	0.1128 ± 0.0010	dynamical approach	Jimenez-Delgado, Reya '08
	0.1162 ± 0.0006	including NLO jets	
ABKM09	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
	0.1129 ± 0.0014	HQ: BSMN	
MSTW	0.1171 ± 0.0014		Martin, Stirling, Thorne, Watt '09
Thorne	0.1136	[DIS+DY, HT*] (2013)	Thorne '13
$\mathrm{ABM11}_J$	$0.1134 \ldots 0.1149\ \pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. '11
NN21	0.1173 ± 0.0007	(+ heavy nucl.)	NNPDF '11
ABM12	0.1133 ± 0.0011		Alekhin, Blümlein, S.M. '13
	0.1132 ± 0.0011	(without jets)	
CT10	0.1140	(without jets)	Gao et al. '13
CT14	$0.1150\ ^{+\,0.0060}_{-\,0.0040}$	$\Delta\chi^2 > 1$ (+ heavy nucl.)	Dulat et al. '15
JR14	0.1136 ± 0.0004	dynamical approach	Jimenez-Delgado, Reya '14
	0.1162 ± 0.0006	standard approach	
MMHT	0.1172 ± 0.0013	(+ heavy nucl.)	Martin, Motylinski, Harland-Lang, Thorne '15
ABMP16	0.1147 ± 0.0008		Alekhin, Blümlein, S.M., Placakyte '17
NN31	0.1185 ± 0.0012	including NLO jets	NNPDF '18

- Measurements at NNLO (last ~ 10 years) from DIS data
- Large spread of fitted values at NNLO: $\alpha_s(M_Z) = 0.1128 \ldots 0.1185$
- [yellow box] taken for 2017 PDG average: $\alpha_s(M_Z) = 0.1156 \pm 0.0021$

Theory description of deep-inelastic scattering



Kinematic variables

- momentum transfer $Q^2 = -q^2$
- Bjorken variable $x = Q^2/(2p \cdot q)$

- Structure functions (up to order $\mathcal{O}(1/Q^2)$)

$$F_a\left(x, Q^2\right) = \sum_i \left[C_{a,i}\left(\alpha_s(\mu^2), \mu^2/Q^2\right) \otimes PDF\left(\mu^2\right)\right](x)$$

- Perturbative expansion of coefficient functions up to N^{3}LO

$$C_{a,i} = \alpha_s^n \left(c_{a,i}^{(0)} + \alpha_s c_{a,i}^{(1)} + \alpha_s^2 c_{a,i}^{(2)} + \alpha_s^3 c_{a,i}^{(3)} + \dots\right)$$

- Application to DIS data requires careful consideration of kinematic region in Q^2 and x
  - invariant mass of the hadronic system $W^2 = M_P^2 + Q^2(1-x)/x$
  - cuts $W^2 \geq 12.5\ \mathrm{GeV}^2$ and $Q^2 \geq 2.5 \div 10\ \mathrm{GeV}^2$

- Additional corrections for $F_a\left(x,Q^2\right)$ necessary dependent on cuts
  - higher twist and target mass corrections

Higher twist

- Operator product expansion predicts inifinte tower of $(1/Q^2)^n$ of power corrections (higher twist terms)
- Physical interpretation as multi-parton correlations
- Higher twist terms modify structure functions (up to order $\mathcal{O}(1/Q^4)$)

$$F_i^{\rm ht}(x,Q^2) \quad = \quad F_i^{\rm TMC}(x,Q^2) + \frac{H_i^{\tau=4}(x)}{Q^2}\,, \qquad\qquad i \;=\; 2,T$$

Target mass corrections

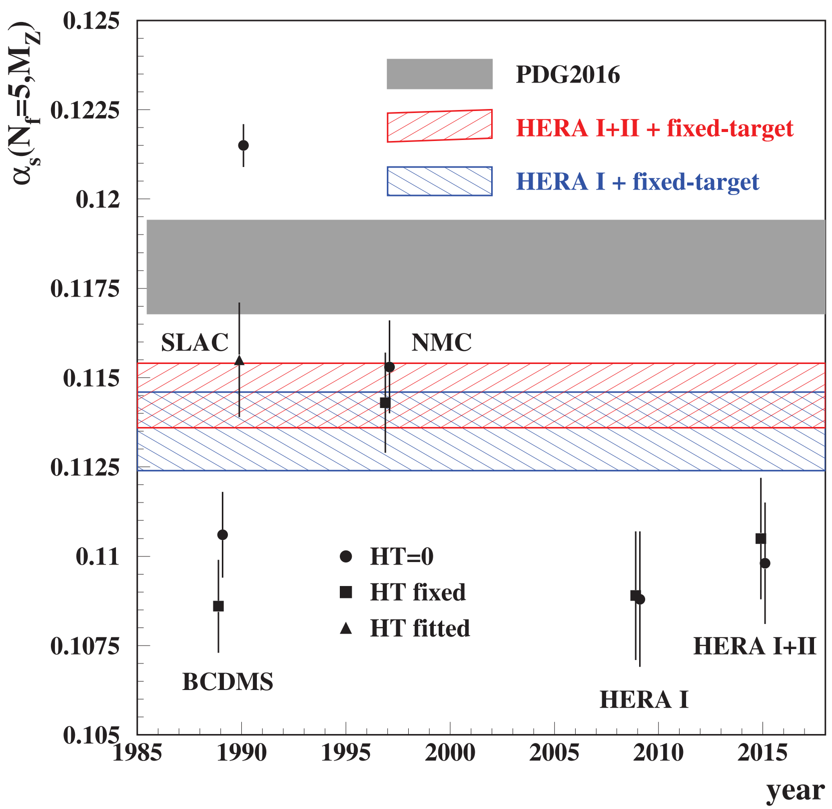
- Finite nucleon mass leads to target mass corrections up to $\mathcal{O}(M_N^2/Q^2)$

$$F_2^{\rm TMC}(x,Q^2) \;=\; \frac{x^2}{\xi^2\gamma^3}\,F_2(\xi,Q^2) + 6\,\frac{x^3 M_N^2}{Q^2\gamma^4}\int\limits_{\xi}^{1}\frac{d\xi'}{\xi'^2}\,F_2(\xi',Q^2)$$

  - kinematic variable $\xi = 2x/(1+\gamma)$
  - Nachtmann variable $\gamma = (1+4x^2 M_N^2/Q^2)^{1/2}$

Impact of higher twist on α_s determinations

- Correlation of errors among different data DIS sets
- Target mass corrections (powers of nucleon mass M_N^2/Q^2)
- Variants with no higher twist give larger α_s values Alekhin, Blümlein, S.M. '17



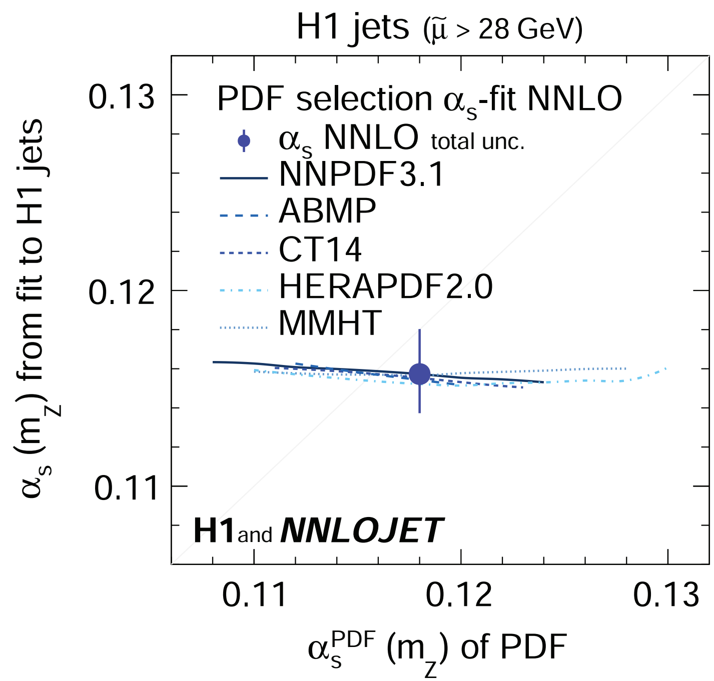
- Theoretical uncertainty of α_s at NNLO from DIS data $\gtrsim \mathcal{O}(1\ldots2)\%$

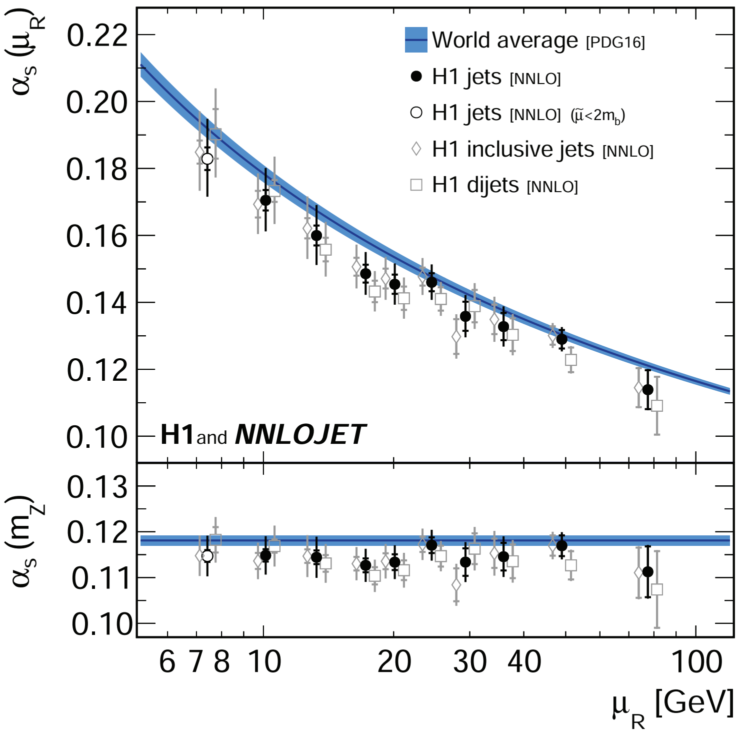
DIS jet cross section measurements

- Determination of $\alpha_s(M_Z)$ at NNLO from DIS jets data Britzger (H1 coll.) '17

  $$\alpha_s(M_Z) = 0.1157 \pm (0.0020)_{\rm exp}\,(0.0029)_{\rm th}$$

  - kinematic cut $\tilde{\mu} = \sqrt{Q^2_{\rm av} + p^2_{T,\,\rm av}}$
  - theory error with scale variation and PDF uncertainties derived from NNPDF3.1





- Central values at NNLO systematically below average of PDG
- Alternative Britzger (H1 coll.) '17: combined fit (inclusive DIS and jet data) with determination of parton distributions

  $$\alpha_s(M_Z) = 0.1142 \pm (0.0028)_{\rm tot}$$

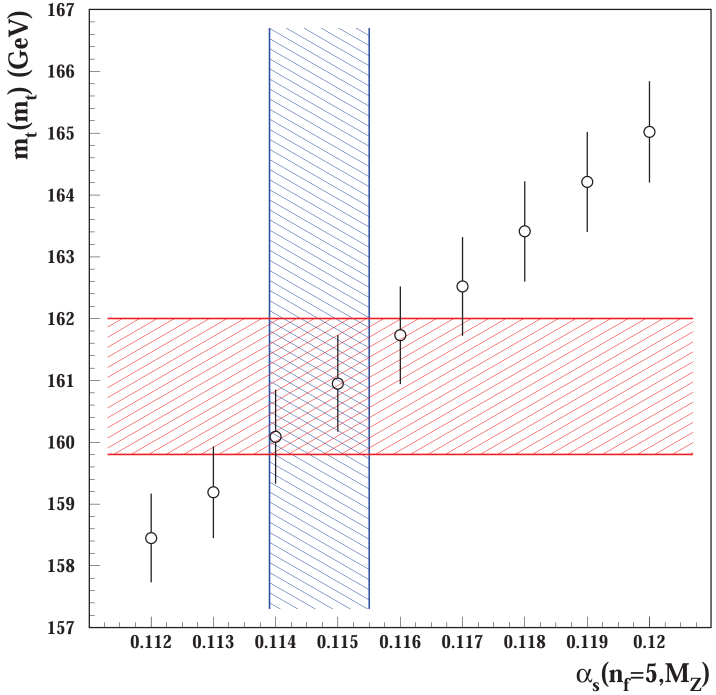
Top-quark pair production

- Cross section for $t\bar{t}$-production with parametric dependence

$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to t\bar{t}} \left(\alpha_s(\mu^2), Q^2, \mu^2, m_t^2\right)$$

  - correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and m_t

- Fits of data for top-quark pair production with fixed values of m_t and $\alpha_s(M_Z)$ carry significant bias
  - illustration of correlation of m_t and $\alpha_s(M_Z)$ in ABMP16 fit



- Joint determination of $\alpha_s(M_Z)$ and m_t from inclusive cross section for top-quark pair production CMS coll. '14

$$\alpha_s(M_Z) = 0.1151 \pm (0.0033)_{\rm tot}$$

α_s *in parton distributions*

PDF sets	$\alpha_s(M_Z)$	method of determination
ABMP16 Alekhin, Blümlein, S.M., Placakyte '17	0.1147 ± 0.0008	fit at NNLO
CJ15 Accardi, Brady, Melnitchouk et al. '16	0.118 ± 0.002	fit at NLO
CT14 Dulat et al. '15	0.118	assumed at NNLO
HERAPDF2.0 H1+Zeus Coll.	$0.1183\ ^{+0.0040}_{-0.0034}$	fit at NLO
JR14 Jimenez-Delgado, Reya '14	0.1136 ± 0.0004	dynamical fit at NNLO
	0.1162 ± 0.0006	standard fit at NNLO
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	0.118	assumed at NNLO
	0.1172 ± 0.0013	best fit at NNLO
NNPDF3.1 Ball et al. '17	0.118	assumed at NNLO
Ball et al. '18	0.1185 ± 0.0012	best fit at NNLO
PDF4LHC15 Butterworth et al. '15	0.118	assumed at NLO
	0.118	assumed at NNLO

- Values of $\alpha_s(M_Z)$ often assumed and not fitted (no correlations)
- Large spread of fitted values at NNLO: $\alpha_s(M_Z) = 0.1136 \ldots 0.1185$
- PDF4LHC: order independent recommendation
  - use $\alpha_s(M_Z) = 0.118$ at NLO and NNLO

Higgs cross section dependence on α_s

- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_s)$ for $m_H = 125.0$ GeV at $\sqrt{s} = 13$ TeV with $\mu_R, \mu_F = m_H$ and nominal α_s

PDF sets	$\sigma(H)^{\mathrm{NNLO}}$ [pb] nominal $\alpha_s(M_Z)$
ABMP16 Alekhin, Blümlein, S.M., Placakyte '17	40.20 ± 0.63
CJ15 Accardi, Brady, Melnitchouk et al. '16	$42.45 \,^{+\,1.73}_{-\,1.12}$
CT14 Dulat et al. '15	$42.33 \,^{+\,1.43}_{-\,1.68}$
HERAPDF2.0 H1+Zeus Coll.	$42.62 \,^{+\,0.35}_{-\,0.43}$
JR14 (dyn) Jimenez-Delgado, Reya '14	38.01 ± 0.34
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	$42.36 \,^{+\,0.56}_{-\,0.78}$
NNPDF3.1 Ball et al. '17	42.98 ± 0.40
PDF4LHC15 Butterworth et al. '15	42.42 ± 0.78

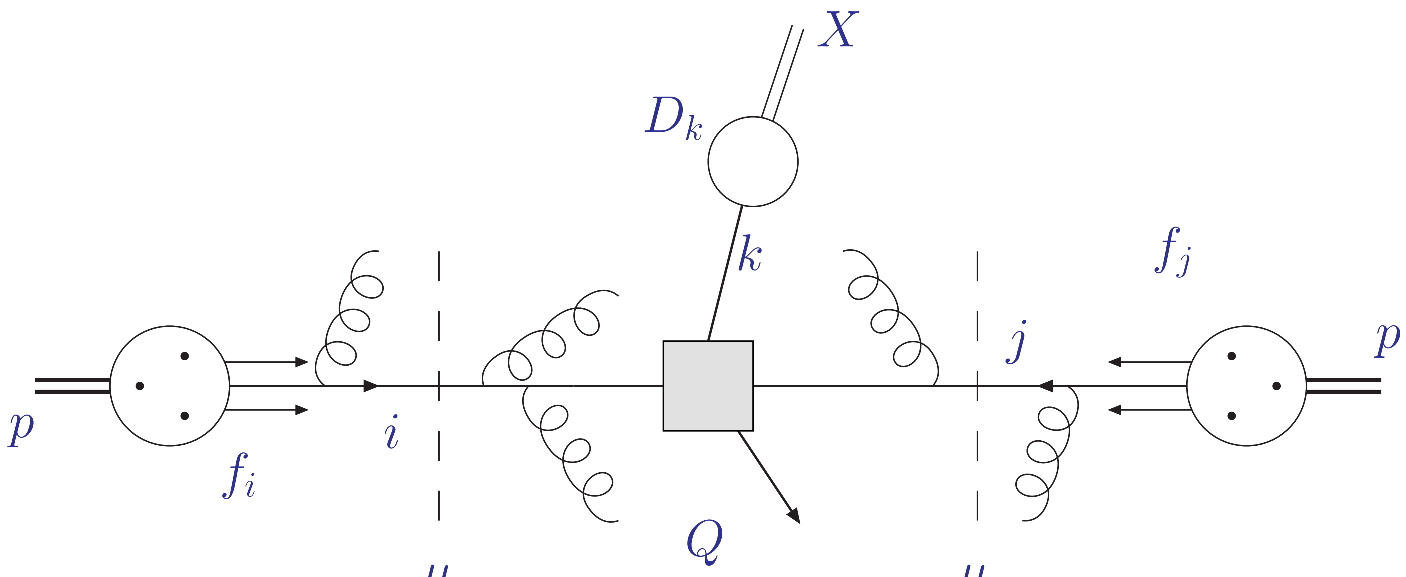
- Large spread for predictions from different PDFs $\sigma(H) = 38.0 \ldots 43.0$ pb
- PDF and α_s differences between sets amount to up to 12%
  - significantly larger than residual theory uncertainty due to N^{3}LO QCD and NLO electroweak corrections

Summary

- Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders lower than world average
  - $\alpha_s(M_Z) = 0.118$ at NNLO not preferred by data
  - details of kinematic cuts, treatment of higher twist, target mass corrections are essential
- Determinations of $\alpha_s(M_Z)$ from hadron collider data (e.g., top-quark pair production) need to account for correlations
  - correlations among PDFs, strong coupling constant $\alpha_s(M_Z)$ and quark masses are significant and may not be neglected
- Value of $\alpha_s(M_Z)$ is a fundamental parameter of QCD and motivates high precision measurements

Back-up

QCD factorization

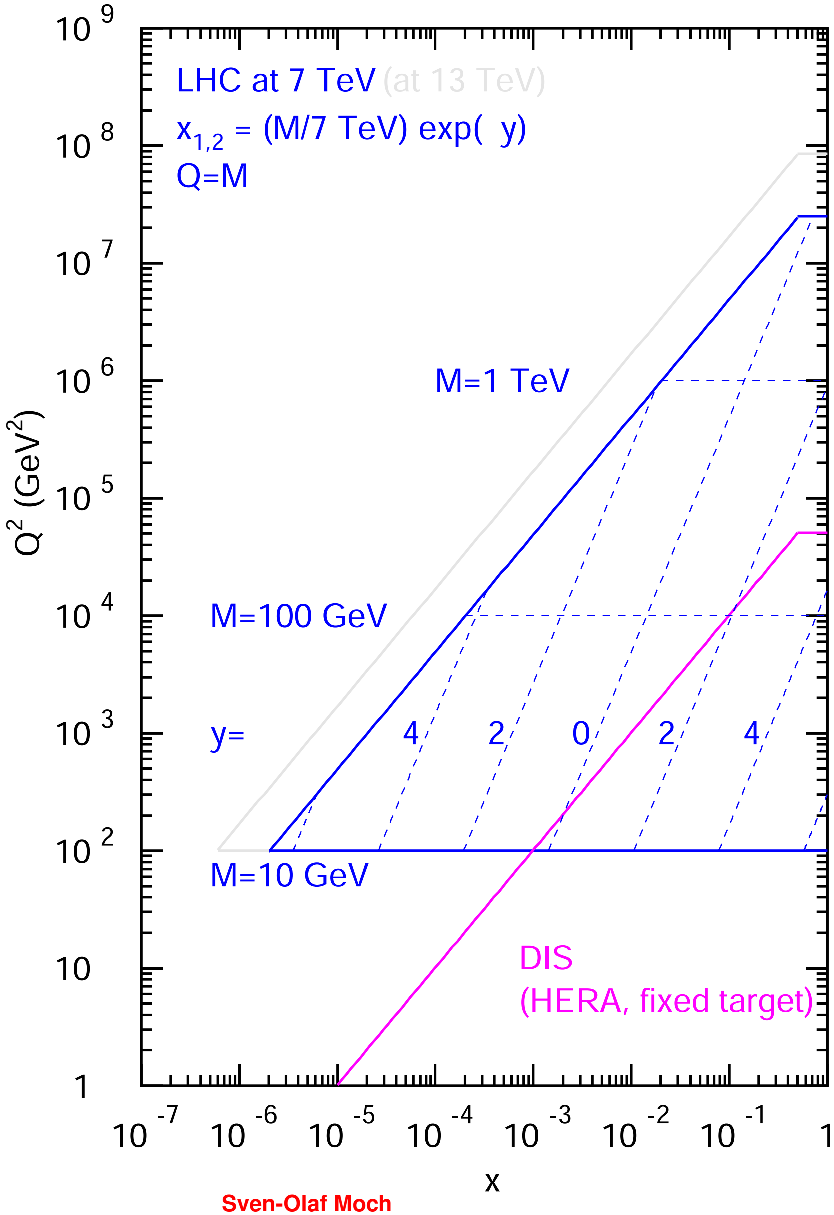


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage

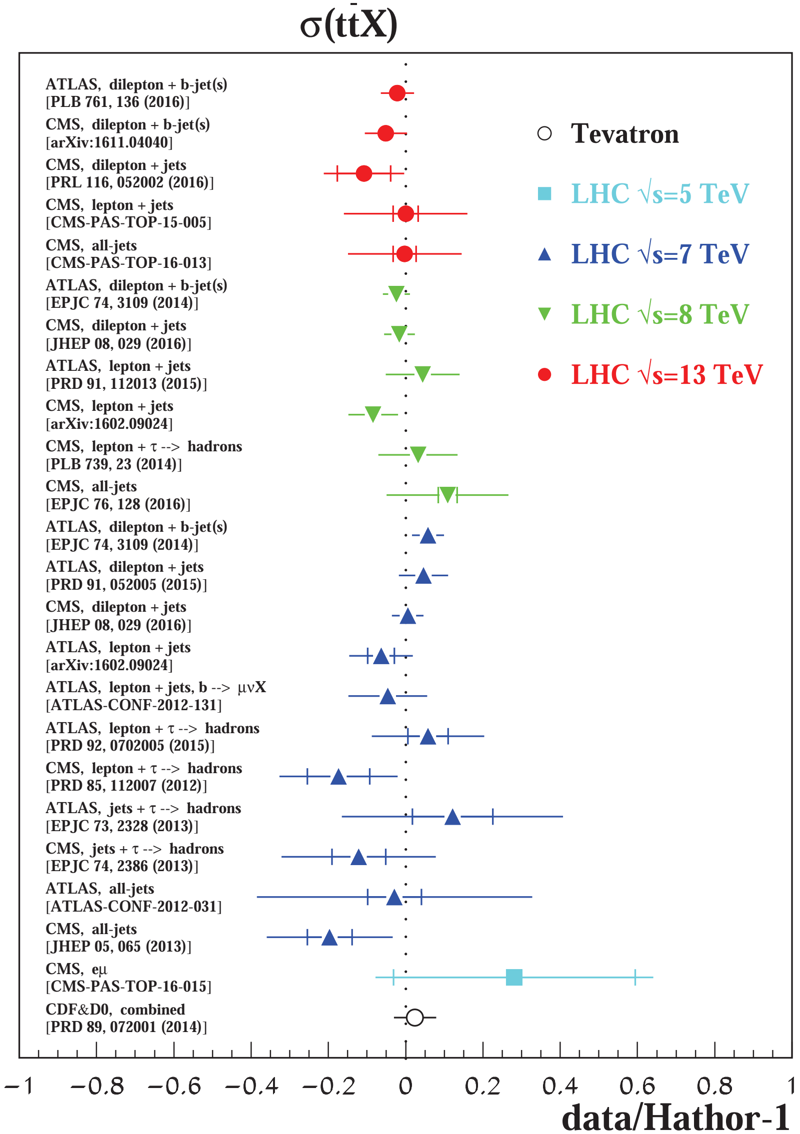


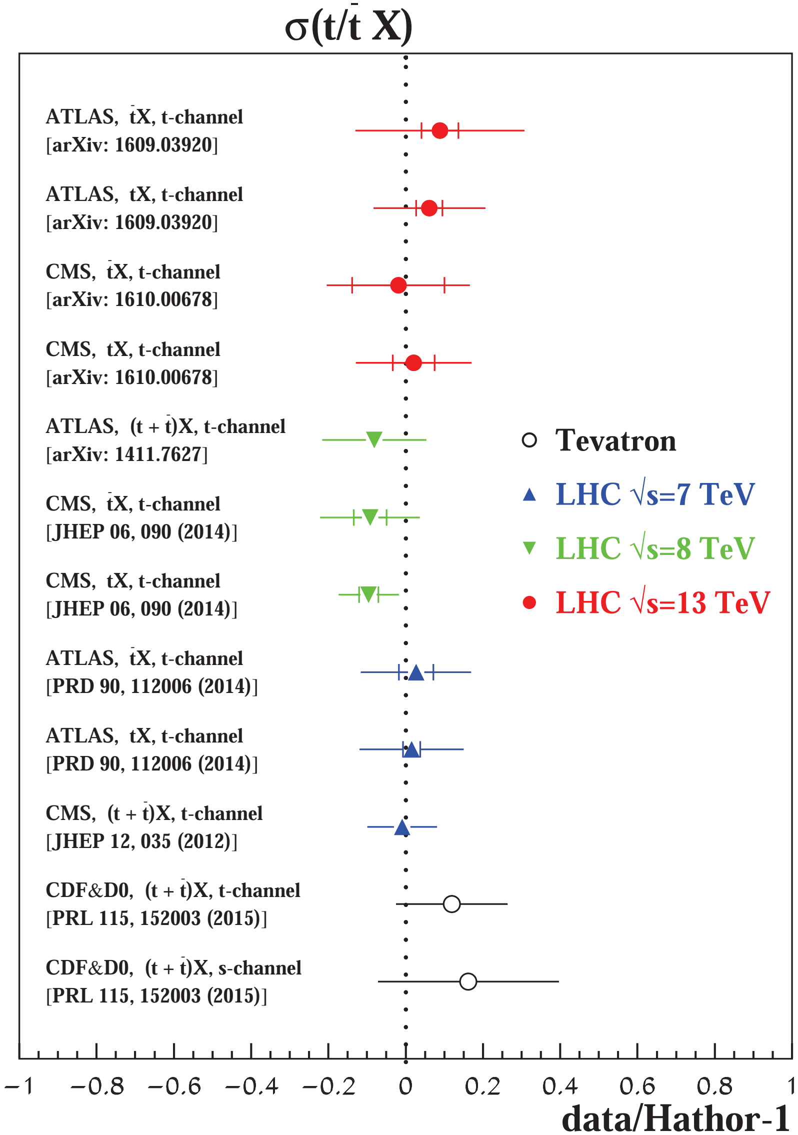
- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S}\mathrm{e}^{\pm y}$
  - forward rapidities sensitive to small-x
- Cross section depends on convolution of parton distributions
  - small-x part of f_i and large-x PDFs f_j

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots\right]$$

Data on top-quark cross sections

- Pulls for $t\bar{t}$- and single-t inclusive cross sections in ABMP16





Fit quality

- Goodness-of-fit estimator χ^2 for extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values in ABMP16
  - χ^2 of global fit with $NDP = 2834$
  - data on top-quark production with $NDP = 36$ D0, ATLAS, CMS, LHCb