Strong Coupling Determinations from e^+e^- Event-shapes

(using SCET, but not only)

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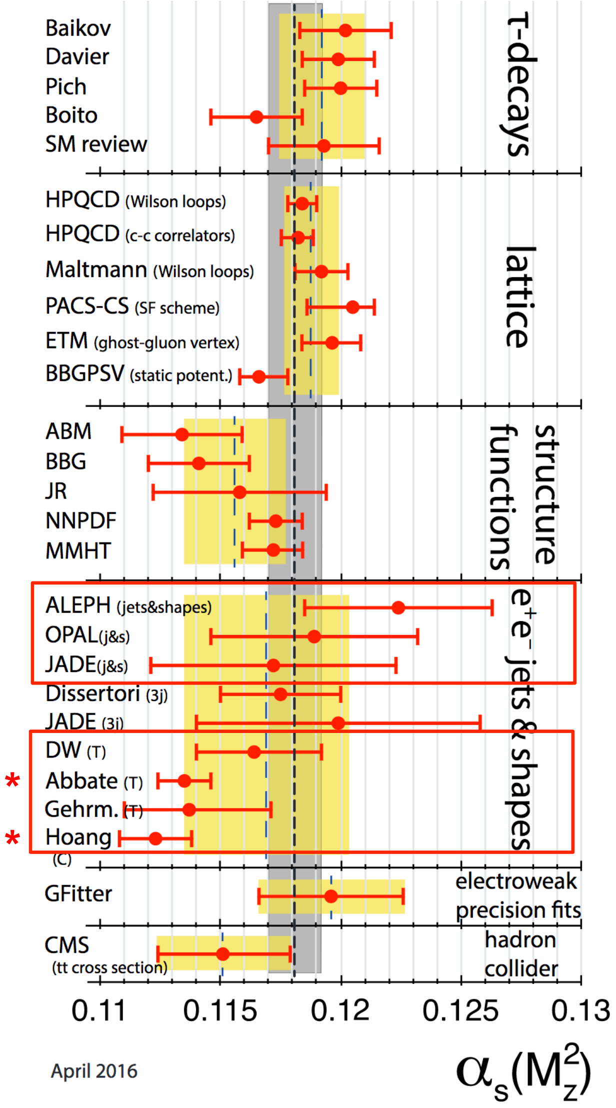
Motivation for Strong Coupling Determinations



Overall aim:

- Improved measurements with smaller errors **(Uncertainties)**
- Tests of our overall understanding of QCD **(Concistency)**

Current situation: → inconsistent results

- How to deal with very precise determinations that seem inconsistent ?

This talk:

- Theory issues for event-shapes
- Anatomy of SCET description – other approaches
- Previous results: Thrust, C
- New: Heavy jet mass
- Consistency of results on event-shapes
- Outlook

Event-Shapes

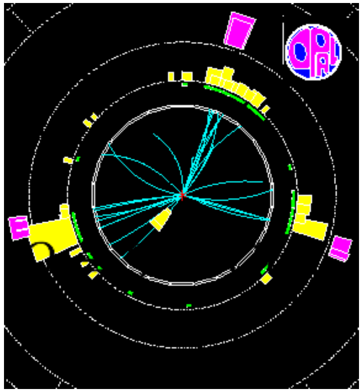
→ Classic method for determining $\alpha_s(M_z)$

Single-variable distributions (“2-jet-likeness”)

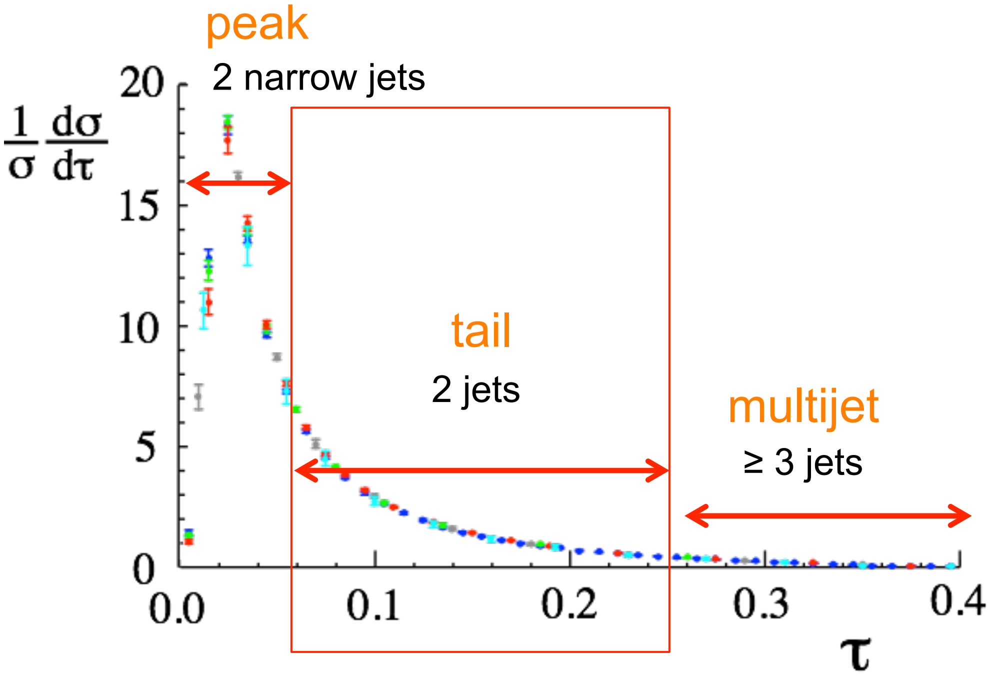
e.g. Thrust $$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

C-parameter $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

Heavy jet mass $$\rho = \frac{\max(M_1, M_2)}{Q^2}$$



OPAL 3 jet event



peak

2 narrow jets

tail

2 jets

multijet

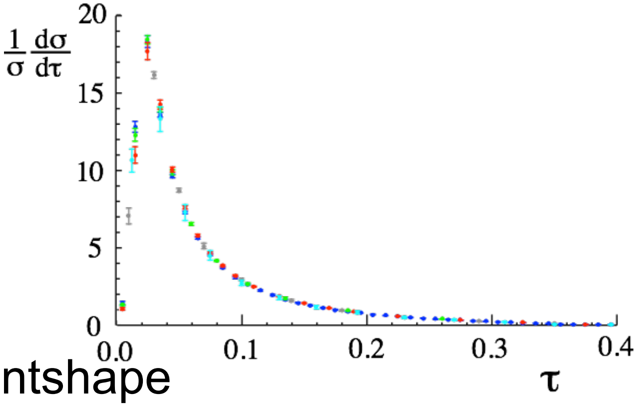
≥ 3 jets

Distribution for τ>0 exists only due to QCD radiation!

Experimental Data

Experimental Data:

Experiment:	Values of Q :
ALEPH	[91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0]
DELPHI	[45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0]
OPAL	[91.0, 133.0, 177.0, 197.0]
L3	[41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2]
SLD	[91.2]
TASSO	[14.0, 22.0, 35.0, 44.0]
JADE	[35.0, 44.0]
AMY	[55.2]



Lots of data available: ~ 800 bins <u>for each</u> eventshape
~ 500 in the tail region useful for analyses

Structure of Perturbative Corrections

Cross section anatomy (e.g. thrust)

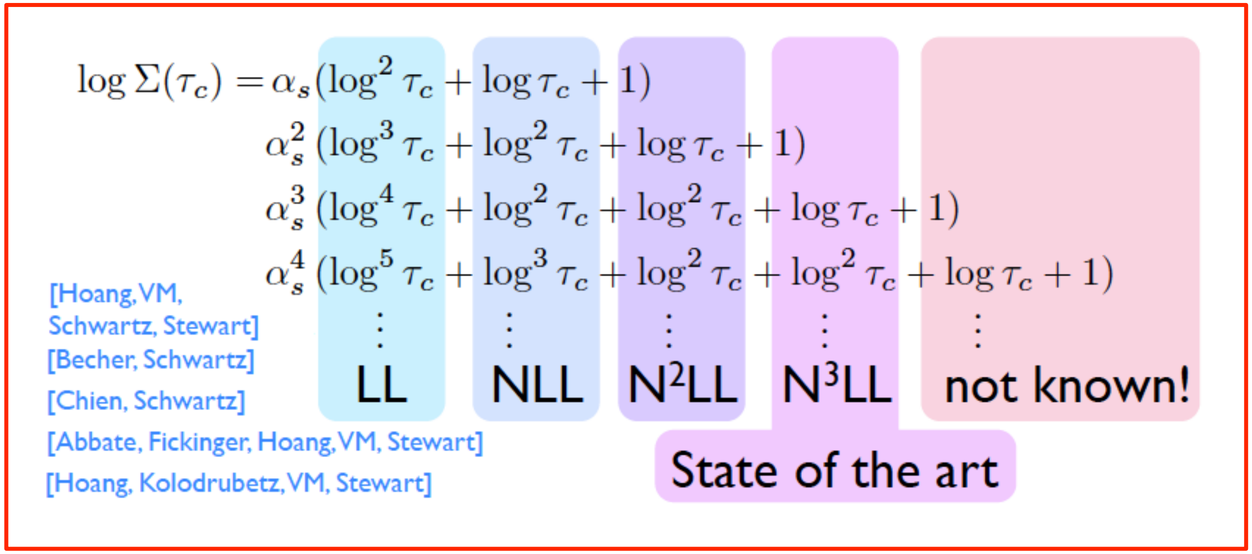
$$\frac{1}{\sigma_{\rm tot}^{\rm Born}}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{C_F\alpha_s}{\pi}\left[\left(\frac{\pi^2}{6}-\frac{1}{2}\right)\delta(\tau) + \frac{-3+9\tau+3\tau^2-9\tau^3}{2\tau(1-\tau)} - \frac{2-3\tau+3\tau^2}{(1-\tau)}\left(\frac{\ln(\frac{\tau}{1-2\tau})}{\tau}\right)_+\right]$$

$$= \boxed{\delta(\tau) + \frac{C_F\alpha_s}{\pi}\left[\left(\frac{\pi^2}{6}-\frac{1}{2}\right)\delta(\tau) - \frac{3}{2}\left(\frac{1}{\tau}\right)_+ - 2\left(\frac{\ln(\tau)}{\tau}\right)_+\right.} + \boxed{\{\text{non-sing. terms}\}\Big]}$$

+ Non-perturbative Corrections

$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} \mathrm{d}\tau \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

Singular terms



$$\begin{aligned}\log\Sigma(\tau_c) = {} & \alpha_s(\log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^2\,(\log^3\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^3\,(\log^4\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \alpha_s^4\,(\log^5\tau_c + \log^3\tau_c + \log^2\tau_c + \log^2\tau_c + \log\tau_c + 1)\\ & \quad\vdots\end{aligned}$$

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

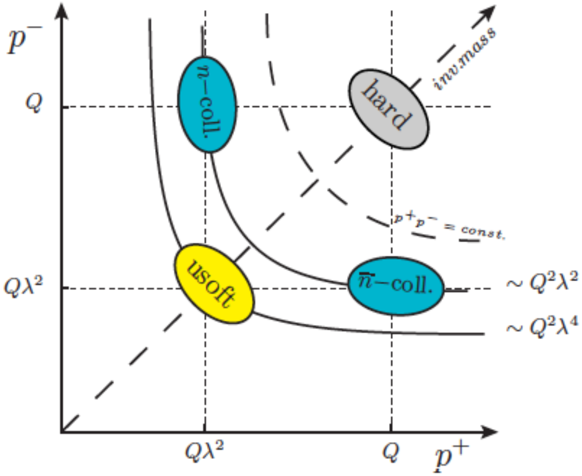
Log resummations essential!

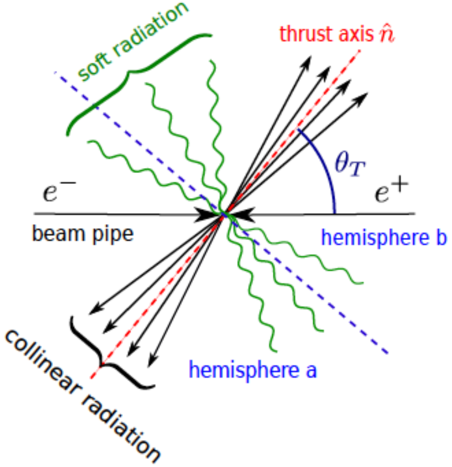
Anatomy of Event Shapes

Singular Cross section (e.g. C-parameter)

$$\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part} \sim \sigma_0 H(Q,\mu_Q)U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J\Big(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\Big)\, J_\tau(Q\ell',\mu_J)\, S_C(\ell-\Delta,\mu_S)$$

- Soft and collinear radiation related to widely separated quantum modes
- Approaches: (in principle equivalent results − but not all used to same precision)
  - pQCD resummation (coh. branching) — Catani, Trentadue, Turnock, Webber; etal.
  - QCD factorization — Korchemsky, Sterman; etal.
  - Effective field theory (SCET) — Fleming, Mantry, Stewart, AHH Schwartz





Anatomy of Event-Shapes (SCET)

Matrix element and hard matching terms (fixed-order)

$$\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part} \sim \sigma_0 H(Q,\mu_Q)\,U_H(Q,\mu_Q,\mu_s) \int d\ell\, d\ell'\, U_J\left(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\right) J_\tau(Q\ell',\mu_J)\, S_C(\ell-\Delta,\mu_S)$$

Hard function

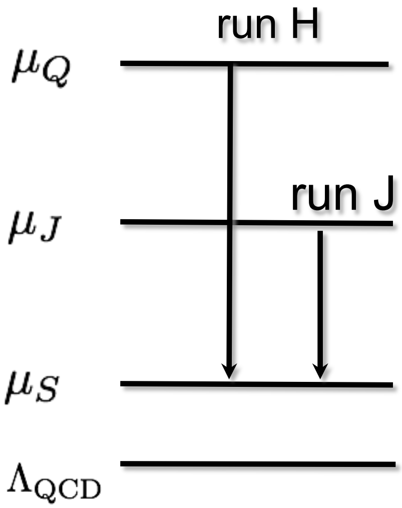
Jet function

Soft function

order-by-order renormalon subtraction

Fixed-order functions

(matching coefficients and matrix elements)



Anatomy of Event-Shapes (SCET)

Summation of large logarithms

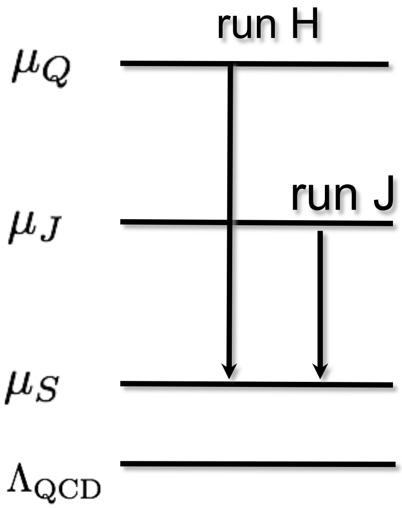
$$\left(\frac{d\sigma}{dC}\right)^{\text{sing}}_{\text{part}} \sim \sigma_0 H(Q,\mu_Q)\,U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J\left(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\right) J_\tau(Q\ell',\mu_J)\, S_C(\ell-\Delta,\mu_S)$$

2-jet production current

$$\mu\frac{d}{d\mu}H_Q(Q,\mu) = \gamma_{H_Q}(Q,\mu)\, H_Q(Q,\mu)$$

$$\gamma_{H_Q}(Q,\mu) = \Gamma_{H_Q}[\alpha_s]\ln\left(\frac{\mu^2}{Q^2}\right) + \gamma_{H_Q}[\alpha_s]$$

Renormalization group equations for matrix elements



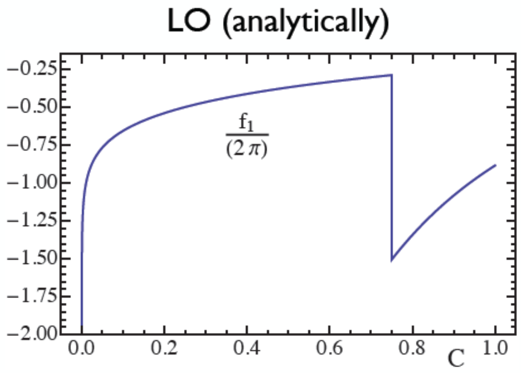
Jet function evolution

$$\mu\frac{d}{d\mu}J(y,\mu) = \gamma_J(y,\mu)\, J(y,\mu) = \left[2\Gamma^{\text{cusp}}(\alpha_s)\ln(iy\mu^2 e^{\gamma_E}) + \gamma_J(\alpha_s)\right] J(y,\mu)$$

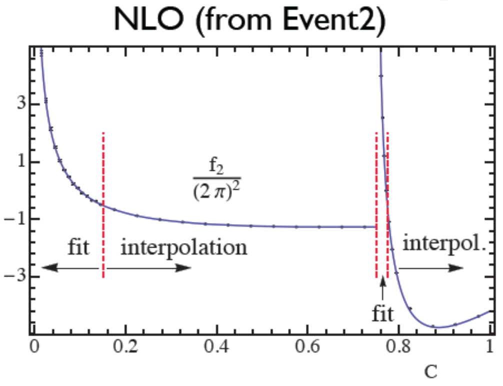
Anatomy of SCET Prediction

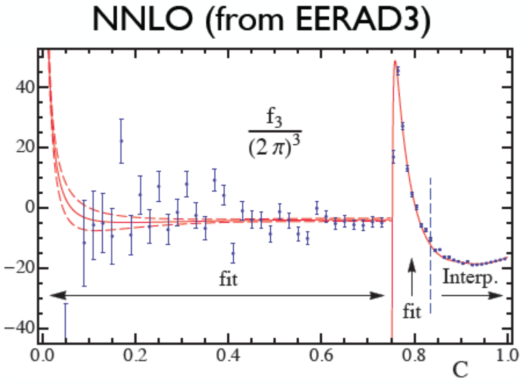
Combination for hadron level prediction

$$\left(\frac{d\sigma}{dC}\right) = \int d\ell \left[\left(\frac{d\sigma}{dC}\right)^{\rm sing}_{\rm part}\left(C-\frac{\ell}{Q}\right) + \left(\frac{d\sigma}{dC}\right)^{\rm nonsing}_{\rm part}\left(C-\frac{\ell}{Q}\right)\right] S^{\rm mod}(\ell,\Delta(R))$$

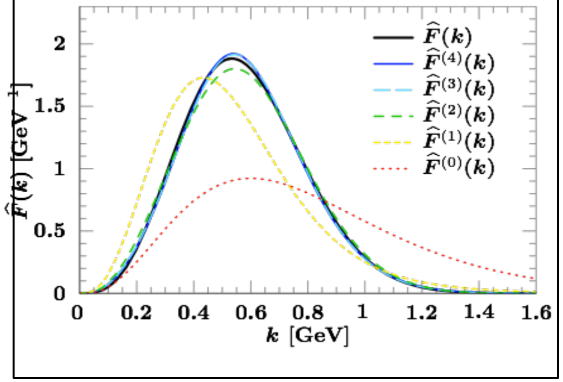

LO (analytically)

Fixed-order minus terms already resummed


NLO (from Event2)


NNLO (from EERAD3)

Soft matrix element model function (renormalon subtrated)

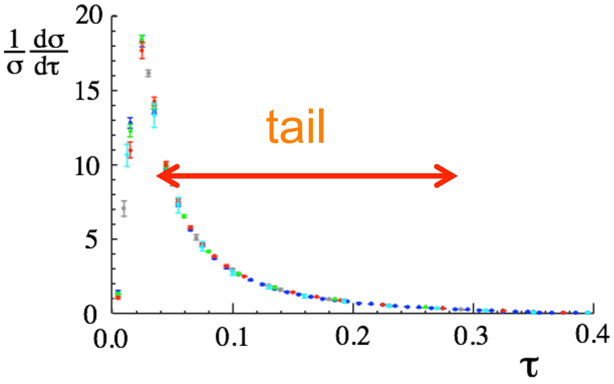


Anatomy of SCET Prediction

Field theory expansion for non perturbative power correction in the tail:

- For C $\gg \Lambda_{QCD}/Q$, in the tail region, the soft model function can be expanded in an OPE.

$$\left(\frac{d\sigma}{dC}\right)^{\text{tail}} \approx \frac{d\hat{\sigma}}{dC} - \frac{\Omega_1^C}{Q}\,\frac{d^2\hat{\sigma}}{dC^2} \approx \frac{d\hat{\sigma}}{dC}\left(C - \frac{\Omega_1^C}{Q}\right)$$



Only two fit parameters: α_s and Ω_1^C

Analogous for thrust.

- Universality of power-corrections: $\Omega_1^C = \frac{3\pi}{2}\,\Omega_1^\tau = 4.2\,\Omega_1^\tau$

Lee, Sterman

- Hadron mass effects break the relation, but very small effect in the relation between thrust and C-parameter.

Large breaking effects possible for HJM.

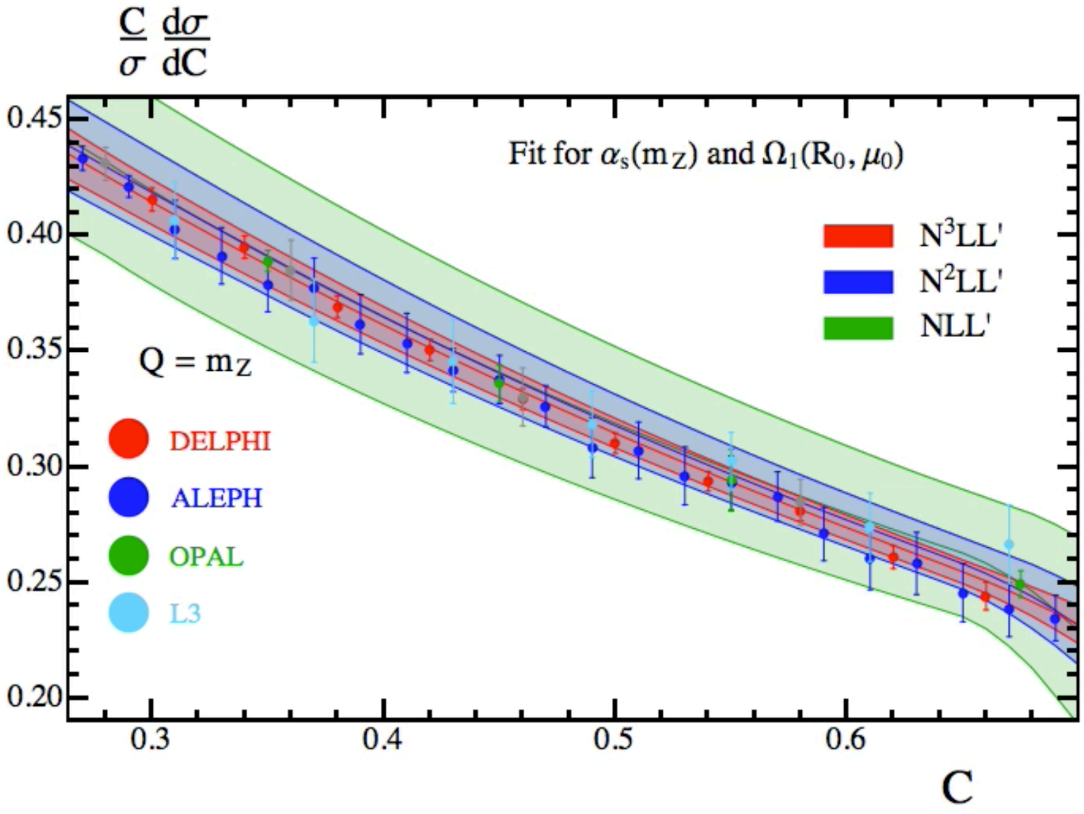
Mateu, Stewart, Thaler

Salam, Wicke

Strong Coupling Determination

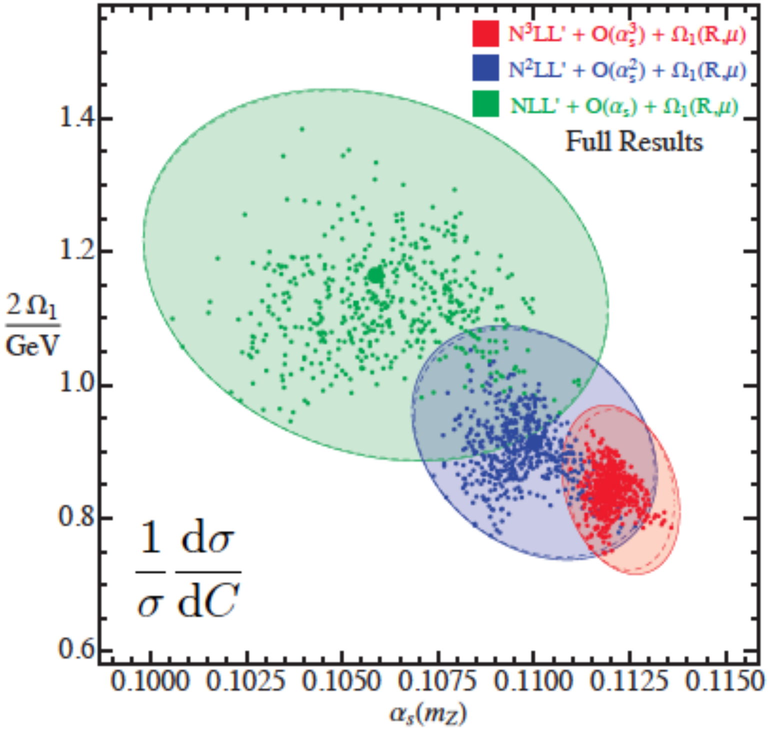
Convergence (using Random Scan scale variation)

→ Excellent convergence when order of description is increased.
(Picture for best fit)

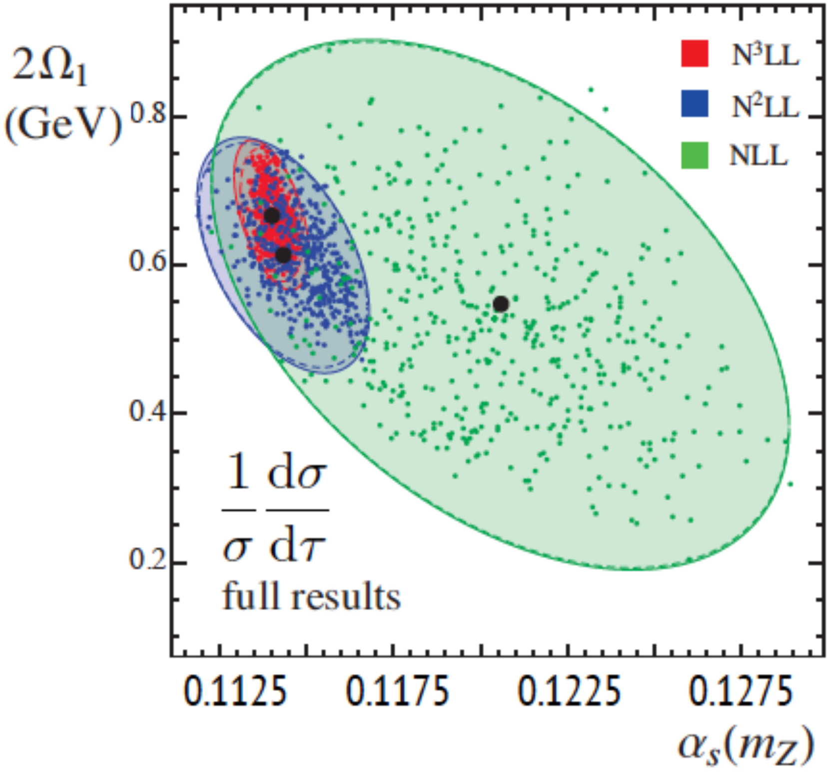


Strong Coupling Determination

C-parameter Tail Global Fit


[Hoang, Kolodrubetz,VM Stewart]

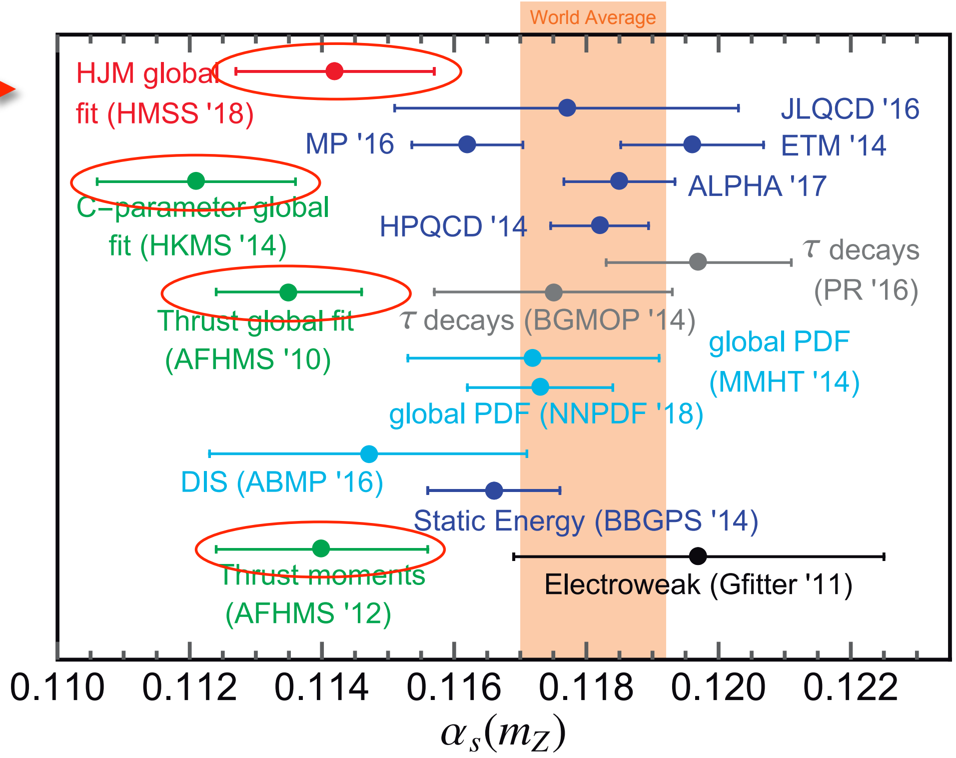
Thrust Tail Global Fit


[Abbate, Fickinger, Hoang,VM Stewart]

- Different behavior of fits with increase order
- Very good agreement at N^{3}LL + O(α_s^3) with renormalon subtraction.

Status of α_s Determinations using SCET

NEW (preliminary) →



Thrust distribution (2010):

$$\alpha_s(M_Z) = 0.1135 \pm 0.0010$$

Thrust 1st moment (2012):

$$\alpha_s(M_Z) = 0.1140 \pm 0.0016$$

C-parameter distribution (2015):

$$\alpha_s(M_Z) = 0.1123 \pm 0.0015$$

HJM distribution (2018):

$$\alpha_s(M_Z) = 0.1142 \pm 0.0015$$

← NEW (Preliminary)

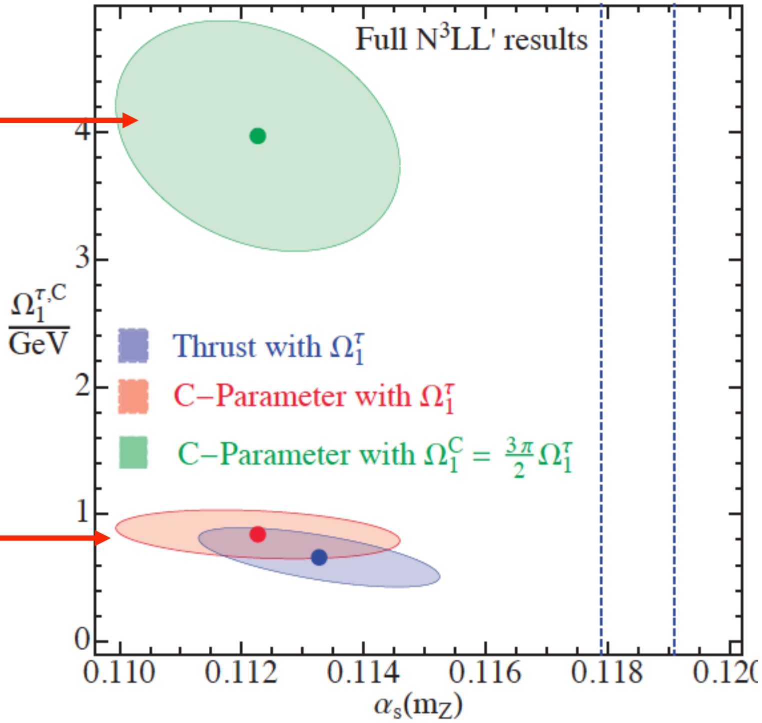
[2010] Abbate, Fickinger, Mateu, Stewart, AHH; PRD 83 (2011) 074021

[2012] Abbate, Fickinger, Mateu, Stewart, AHH; PRD 86 (2012) 094002

[2015] Kolodrubetz, Mateu, Stewart, AHH; PRD 91 (2015) 9, 094018

[2018] Mateu, Schwartz, Stewart, AHH; to appear

Consistency of Thrust and C-parameter Analyses

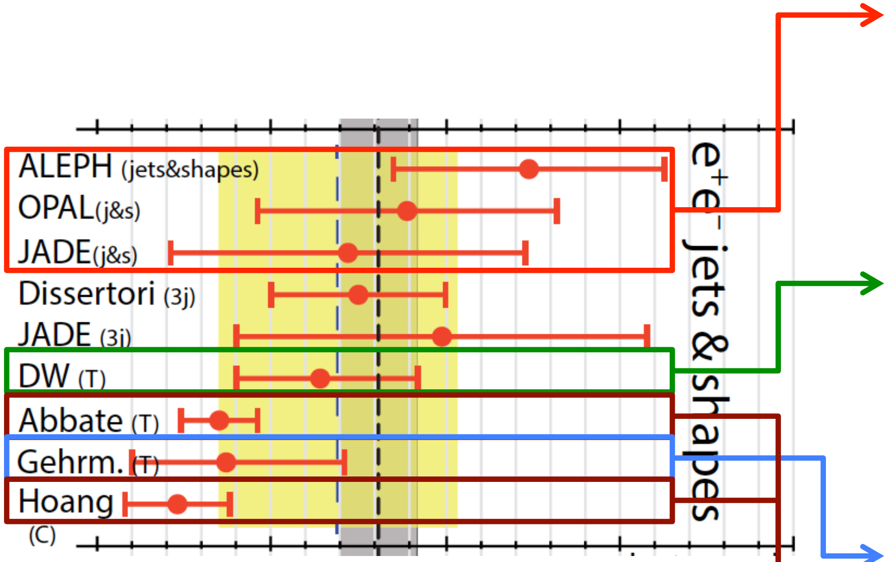


C-parameter norm

Thrust norm

- Consistency between fitted size of Ω_1 between C-parameter and Thrust
- Predicted universality relation confirmed from experimental data.

Eventshape Analyses in PDG Average



- NNLO, O(α_S^3) fixed-order results
- NLL resummation (coherent branching formalism)
- MC hadronization corrections
- Single variable α_S fit
- Global fit

- NLO, O(α_S^2) fixed-order results
- NLL resummation (coherent branching formalism)
- Dispersive hadronization model
- Simultaneous fit: α_S, α_0
- Thrust

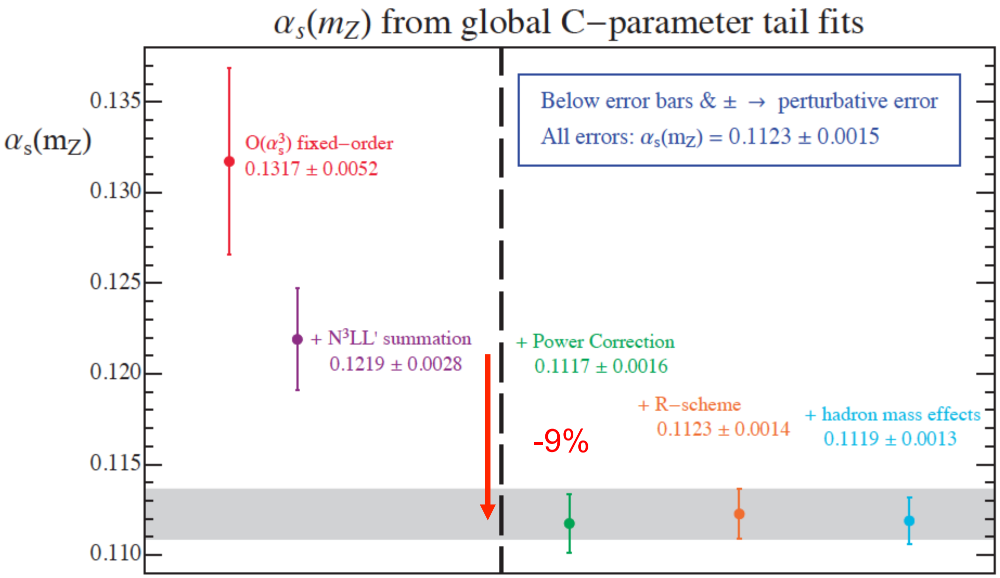
- NNLO, O(α_S^3) fixed-order results
- N^{2}LL resummation (coherent branching formalism)
- Dispersive hadronization model
- Simultaneous fit: α_S, α_0
- Thrust

- NNLO, O(α_S^3) fixed-order results
- N^{3}LL resummation (SCET)
- Shape function hadronization approach
- Simultaneous fit: α_S, Ω_1
- Thrust, C-parameter

Spread of results partly due to different perturbative orders.

Using MC estimate of non-pert. corrections leads to higher fitted α_S values.

Order and Size of Non-Perturbartive Effects



- Large dependence on perturbative precision.
- Finite non-perturbative effects drive strong coupling small.
- Simultaneous fits lead to substantially better fits.
- Consistency of results from SCET with fits by other groups
- Non-perturbative corrections: obtained from MC $\ll$ obtained from simultaneous fits
- MC hadronization problematic because precision of parton shower is NLL only and because of different IR regularization (shower cut).

Conclusions

- Event-shapes are a high-precision tool to extract the strong coupling.
- SCET allows for high-precision calculations at NNLO+N^{3}LL, but same results can be obtained in other approaches as well.
- Strong coupling comes out low from event shapes at highest order for simultaneous fits of α_S and hadronization corrections.
- Hadronization corrections from MC are much smaller, but likely incorrect because MC is less precise and uses IR cutoff regulator: Thrust, C-parameter, Heavy Jet Mass (new)
- “Low α_S problem” persists for all event shapes analyzed up to now at NNLO+N^{3}LL

Possible future tasks:

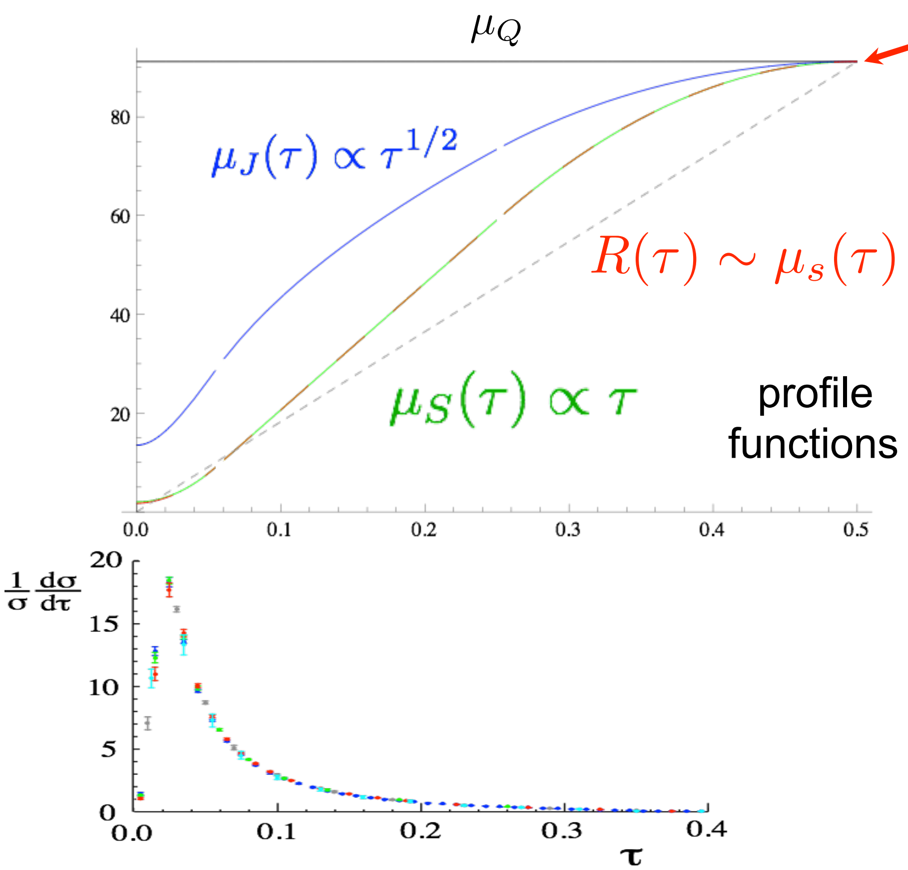
- NNLO+N^{3}LL hadron level descriptions for more event shapes
- Groomed event-shapes: e.g. soft-dropped thrust Baron, Marzani, Theeuwes
- P_T-dependent event shapes (SCET II)
- Personal comment: The methods of ALL α_S-determinations should be scrutinized critically regardless of whether they resulted in low or “world-average” values.

Backup Slides

Anatomy of SCET Prediction

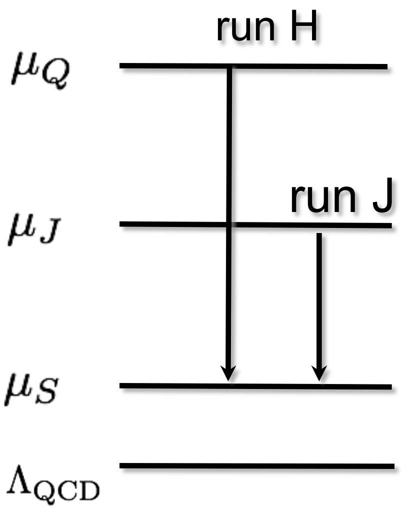
Summation of large logarithms

$$\left(\frac{d\sigma}{dC}\right)^{\text{sing}}_{\text{part}} \sim \sigma_0 H(Q,\mu_Q)U_H(Q,\mu_Q,\mu_s)\int d\ell d\ell'\, U_J\left(\frac{QC}{6}-\ell-\ell',\mu_J,\mu_S\right) J_\tau(Q\ell',\mu_J)\, S_C(\ell-\Delta,\mu_S)$$

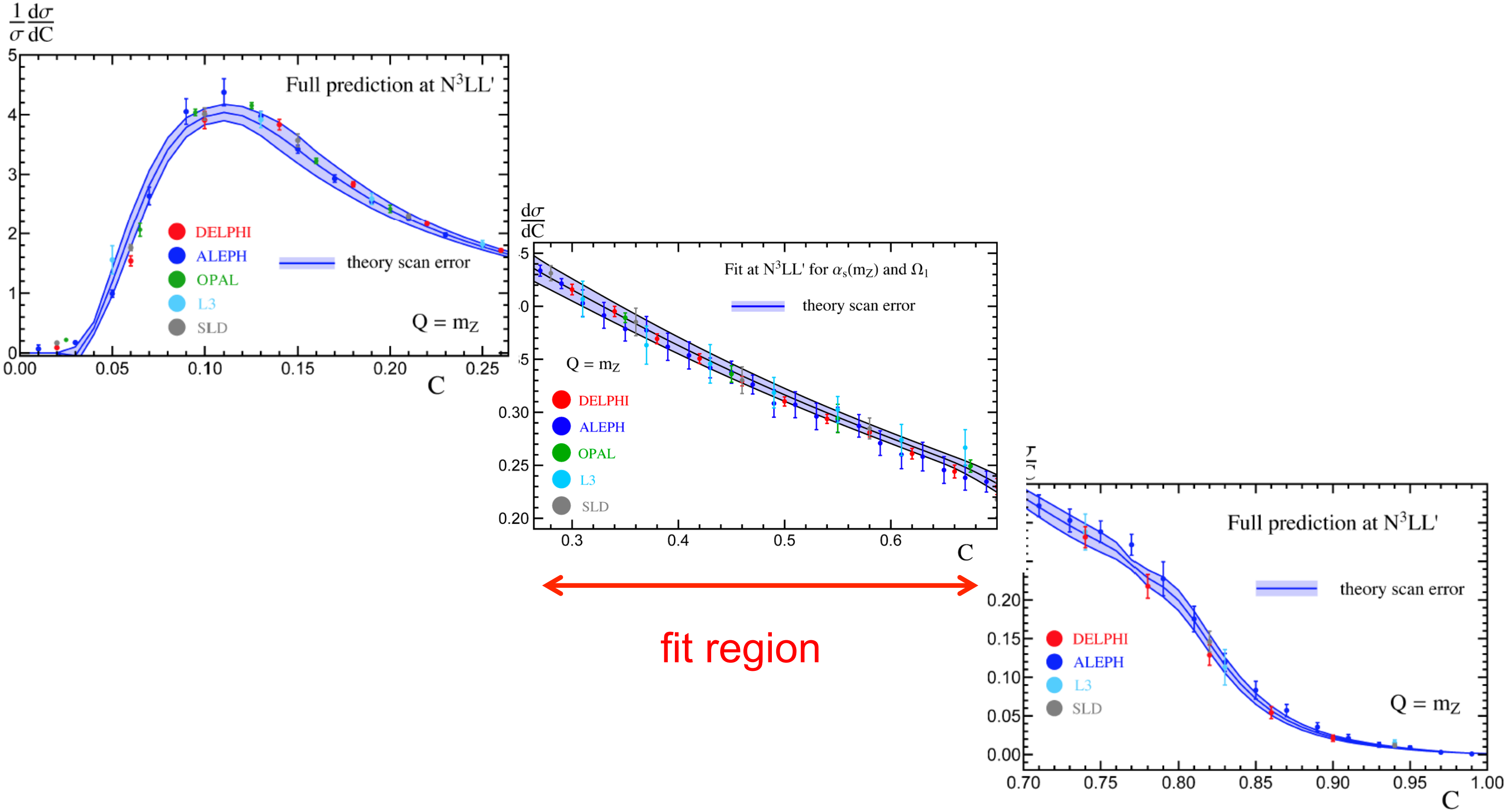


profile functions

scales become equal for multijet region



Overall description of data (C-parameter)

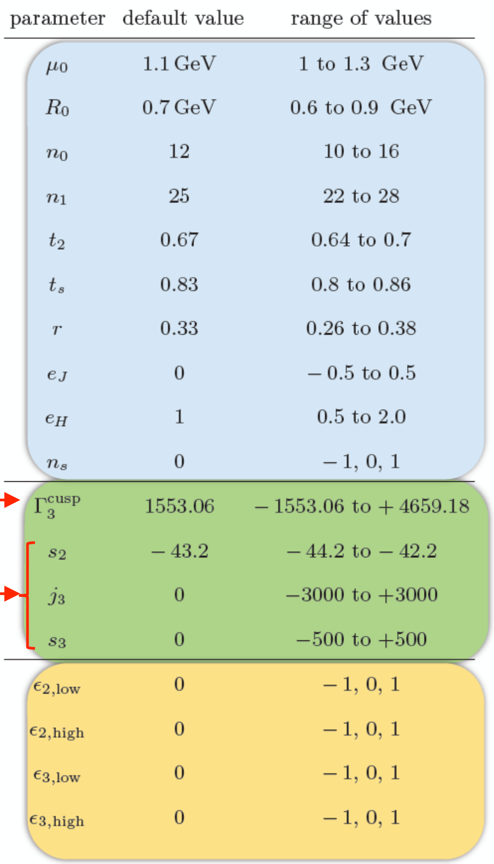


- Very good agreement with data for the entire spectrum, even outside fit region
- Demonstrates ability of or approach to cover the whole spectrum.

Scale Variation – Profile Functions

Scale and uncertainty parameter variations: **"Random Scan"**

- Pick 500 random points and fit for each choice separately (numerically costly!).
- More conservative than error band method OR qudratic sum of individual variations.

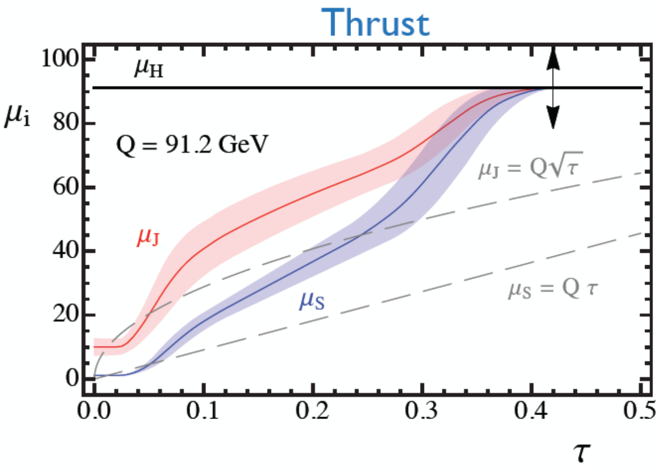


	parameter	default value	range of values
scale variation	μ_0	1.1 GeV	1 to 1.3 GeV
	R_0	0.7 GeV	0.6 to 0.9 GeV
	n_0	12	10 to 16
	n_1	25	22 to 28
	t_2	0.67	0.64 to 0.7
	t_s	0.83	0.8 to 0.86
	r	0.33	0.26 to 0.38
	e_J	0	-0.5 to 0.5
	e_H	1	0.5 to 2.0
	n_s	0	$-1, 0, 1$
unknowns	$\Gamma_3^{\rm cusp}$	1553.06	-1553.06 to $+4659.18$
	s_2	-43.2	-44.2 to -42.2
	j_3	0	-3000 to $+3000$
	s_3	0	-500 to $+500$
non-singular	$\epsilon_{2,\rm low}$	0	$-1, 0, 1$
	$\epsilon_{2,\rm high}$	0	$-1, 0, 1$
	$\epsilon_{3,\rm low}$	0	$-1, 0, 1$
	$\epsilon_{3,\rm high}$	0	$-1, 0, 1$

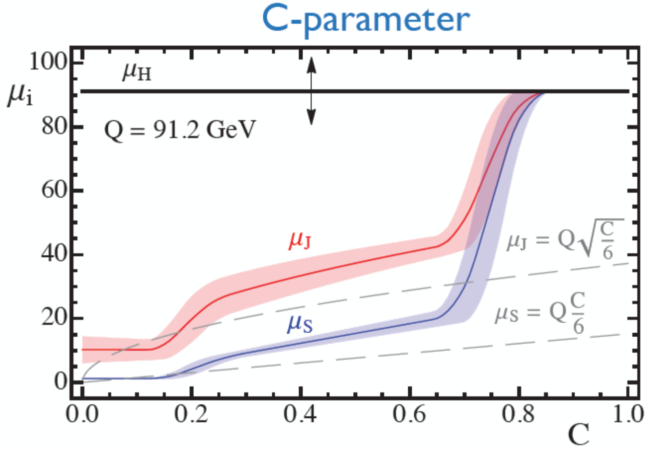
4-loop cusp anomalous dimension →

non-log $O(\alpha_s^3)$ corrections →

Thrust



C-parameter

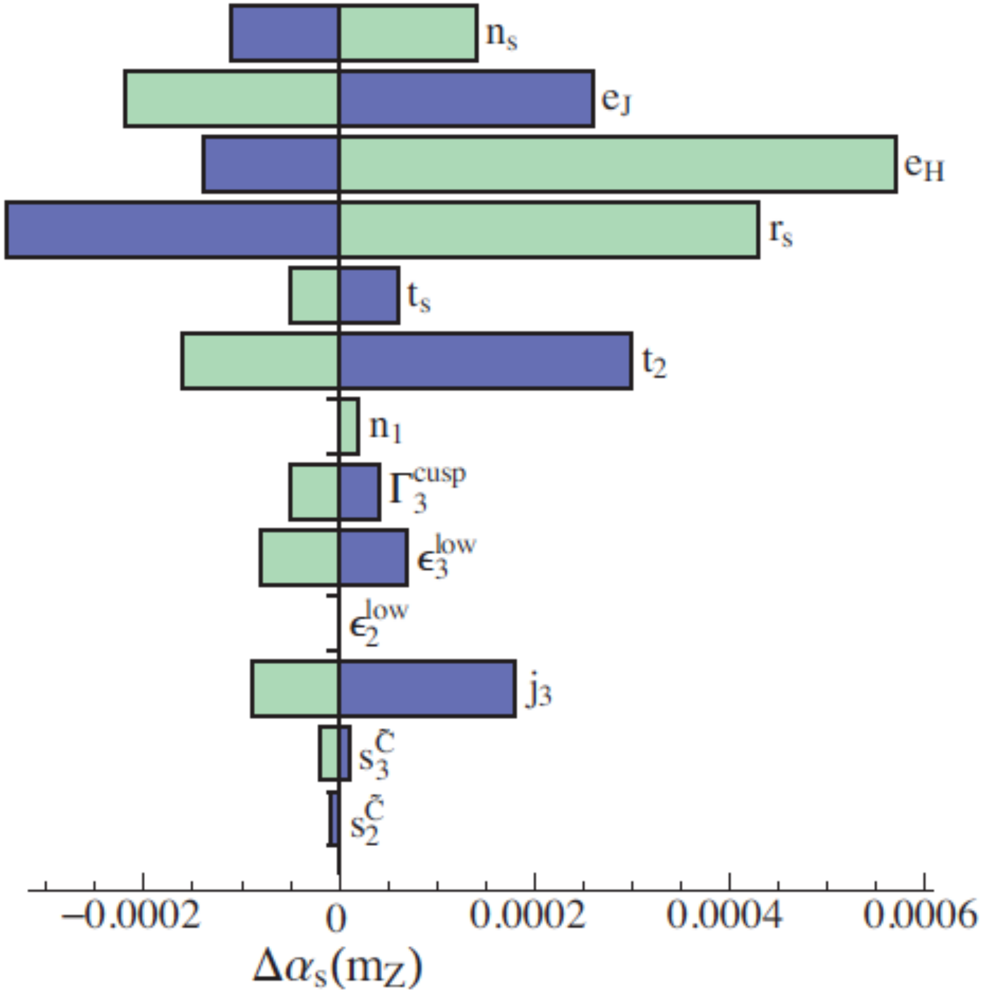


Theory Error Budget (C-parameter)

Scale and uncertainty parameter variations:

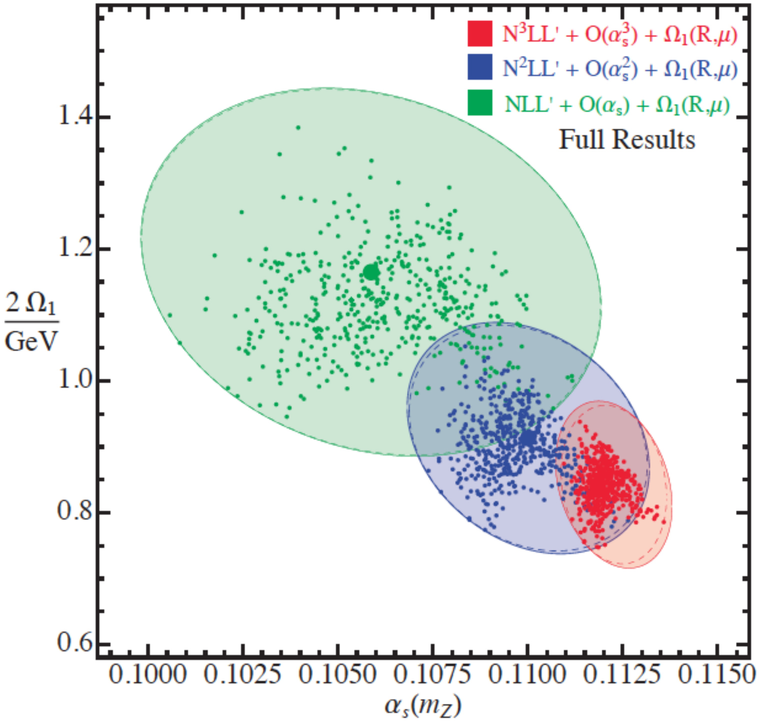
Renormalization
scale
parameters

Unknowns and
numerical
uncertainties

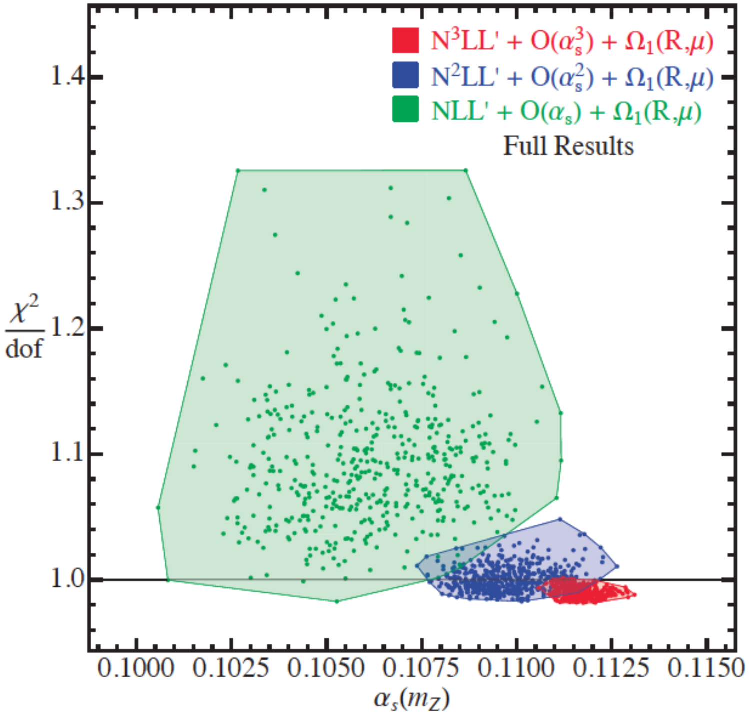


Quality of fits with order (C-parameter)

$\alpha_s - \Omega_1$ plane



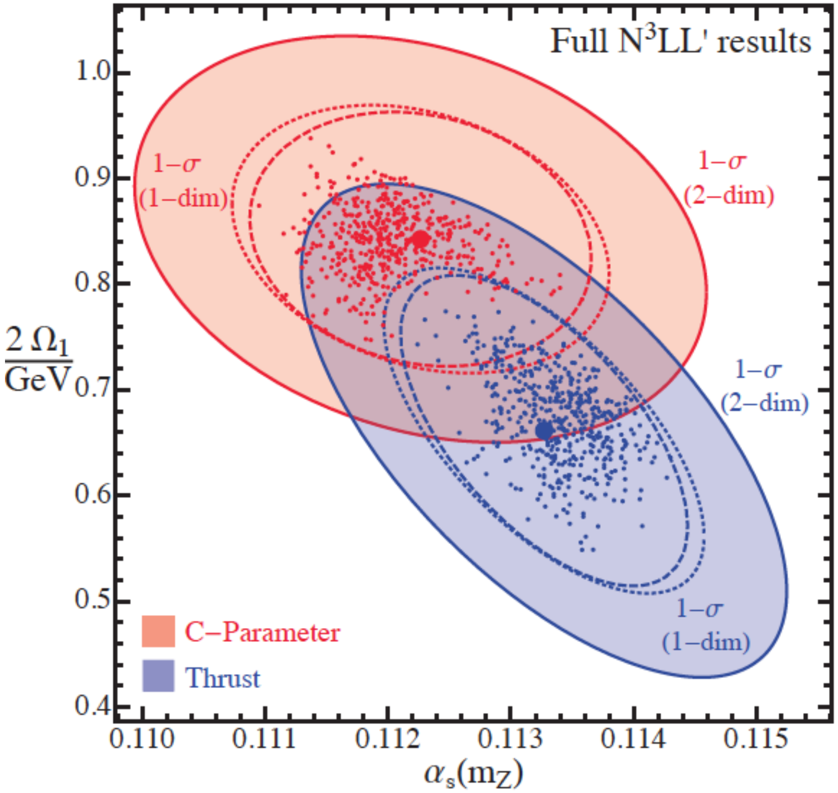
$\alpha_s - \chi^2$ plane



- → Good convergence of the fit when approaching higher order.
- → Improved quality of theoretical description with increasing order.

Strong Coupling Determination

C-parameter versus Thrust Tail Global Fit

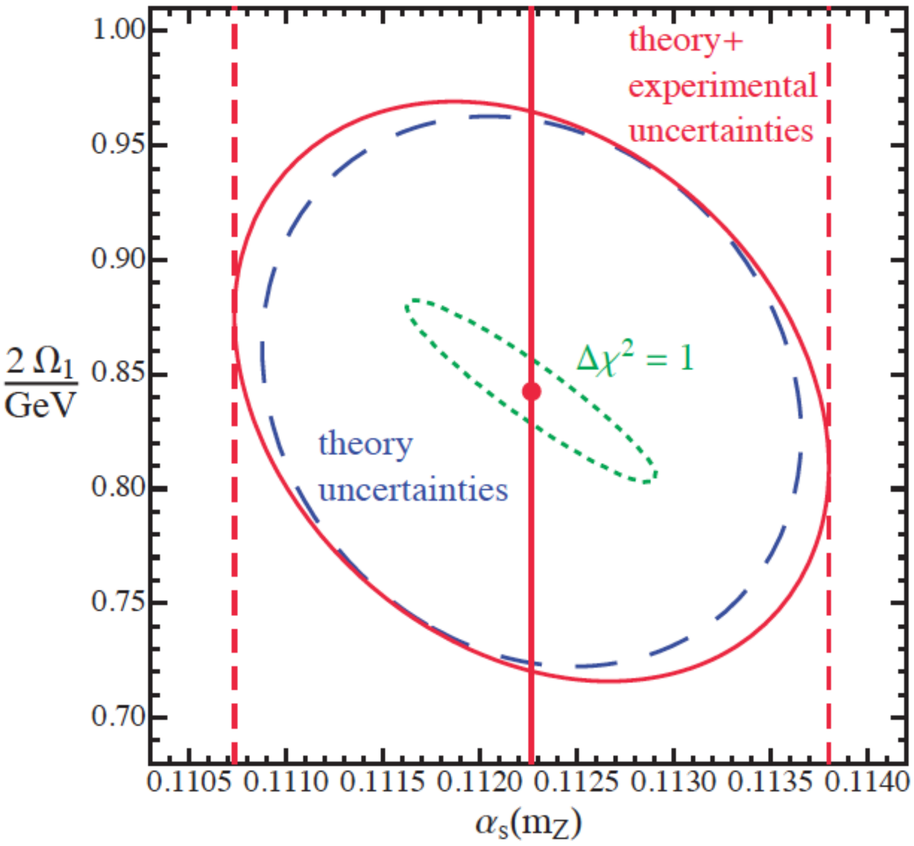


→ Very good agreement at N^{3}LL + O(α_s^3) with renormalon subtraction.

Strong Coupling Determination

Theoretical vs. Experimental vs. Hadronization Uncertainties:

C-parameter Tail Global Fit

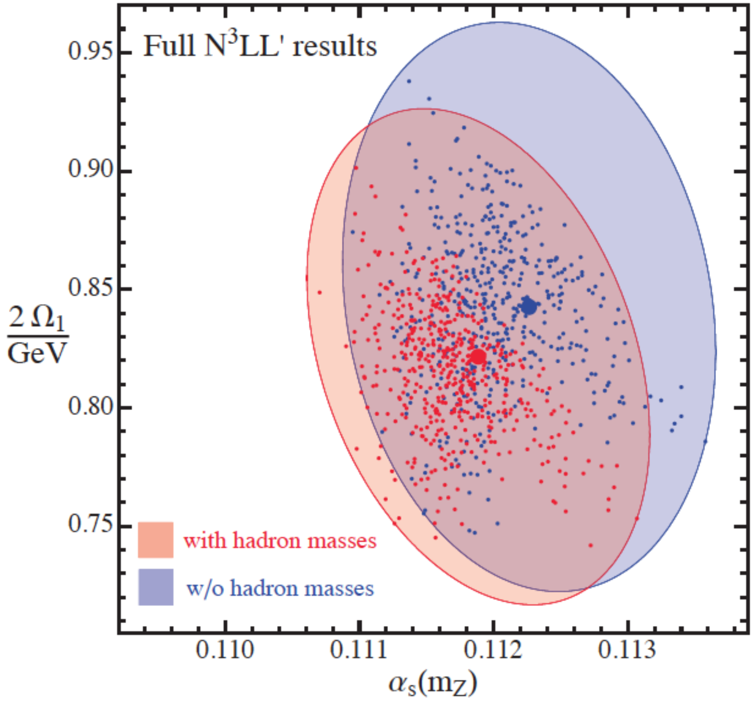


$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\text{exp}}$$
$$\pm 0.0007_{\text{hadr}}$$
$$\pm 0.0014_{\text{pert}}$$

- Perturbative errors dominate
- Experimental errors smallest
- Similar pattern for other eventshape analyses.

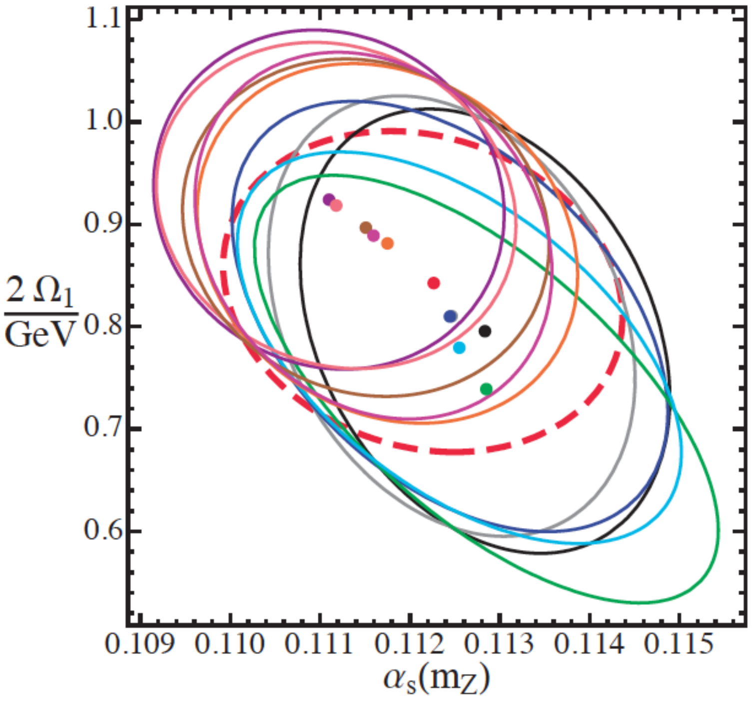
Cross Checks (C-parameter)

Hadron mass effects



- Hadron mass effects modify the way how the soft function enters the theory prediction.
- Effect is very small and

Data set dependence



- Dependence on the upper and lower boundary of fit intervals.
- Dependence compatible with theory uncertainty. (NOT ADDITIONAL ERROR!)