Calculating light-cone correlations on lattice using large-momentum effective theory

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XIIIth Quark Confinement and Hadron Spectrum

Aug. 3, 2018, Maynooth University, Ireland

Outline

- Introduction to Electron-Ion Colliders
- Problems with light-like correlations and momentum renormalization group
- Large-momentum effective field theory and lattice QCD
- Examples
- Conclusions

Introduction to Electron-Ion Collider

REACHING FOR THE HORIZON

The Site of the Wright Brothers' First Airplane Flight

The 2015 LONG RANGE PLAN for NUCLEAR SCIENCE

RECOMMENDATION III

Gluons, the carriers of the strong force, bind the quarks together inside nucleons and nuclei and generate nearly all of the visible mass in the universe. Despite their importance, fundamental questions remain about the role of gluons in nucleons and nuclei. These questions can only be answered with a powerful new electron ion collider (EIC), providing unprecedented precision and versatility. The realization of this instrument is enabled by recent advances in accelerator technology.

We recommend a high-energy high-luminosity polarized EIC as the highest priority for new facility construction following the completion of FRIB.

NAS report has been released!

Statement by Brookhaven Lab, Jefferson Lab, and the Electron-Ion Collider Users Community on National Academy of Sciences Electron-Ion Collider Report

July 24, 2018

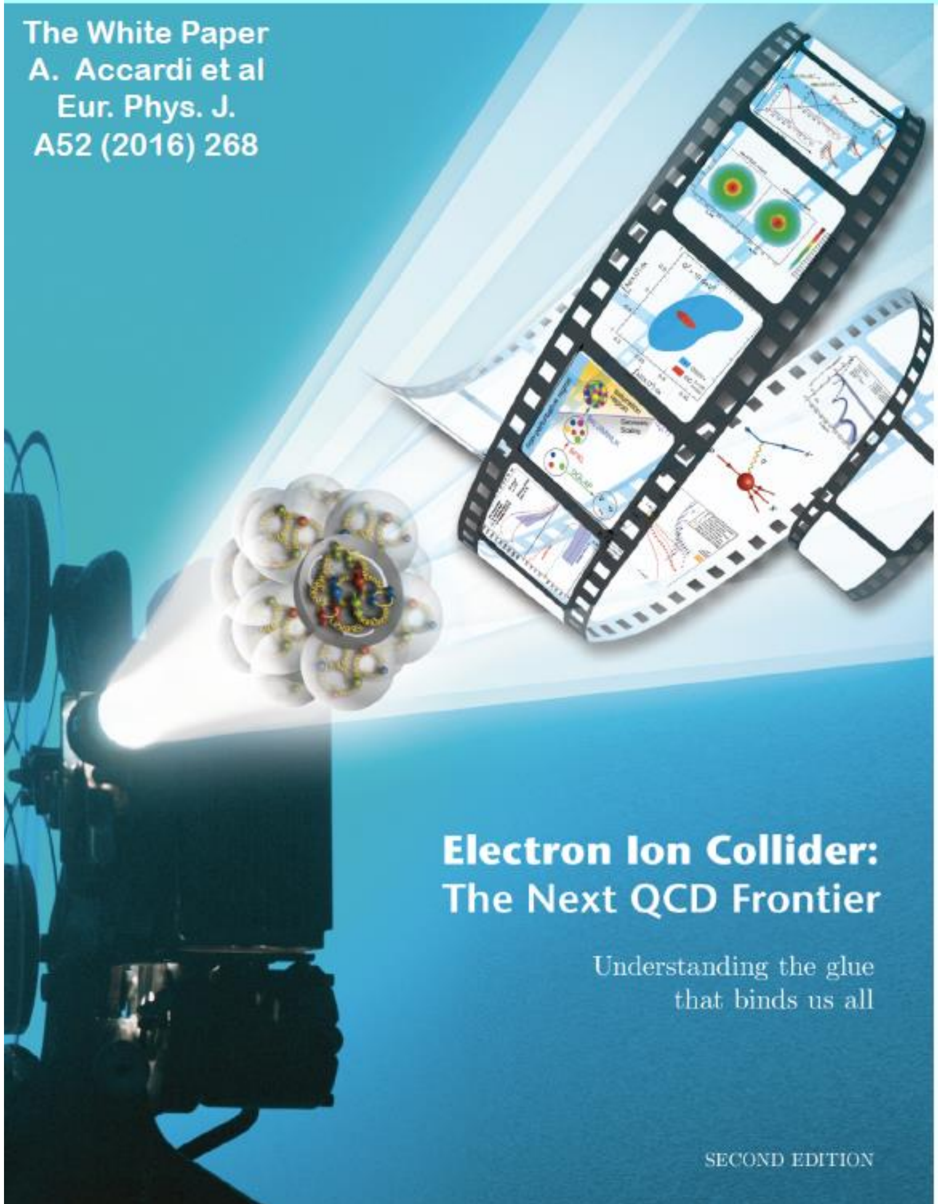
On July 24, 2018, a National Academy of Sciences (NAS) committee issued a report of its findings and conclusions related to the science case for a future U.S.-based Electron-Ion Collider (EIC) and the opportunities it would offer the worldwide nuclear physics community.

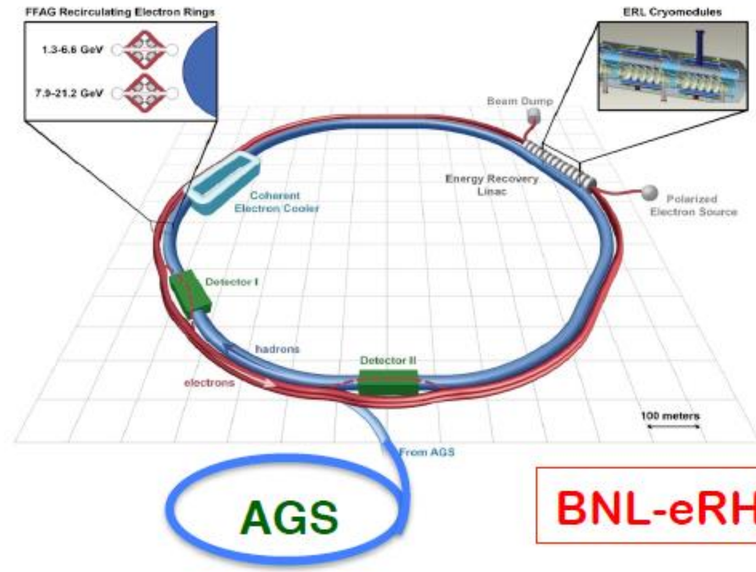
The committee's report—commissioned by the U.S. Department of Energy (DOE)—comes after 14 months of deliberation and meetings held across the U.S. to gather input from the nuclear science community. The report's conclusions include the following:

- The committee concludes that the science questions regarding the building blocks of matter are compelling and that an EIC is essential to answering these questions.
- The answers to these fundamental questions about the nature of the atoms will also have implications for particle physics and astrophysics and possibly other fields.
- Because an EIC will require significant advances and innovations in accelerator technologies, the impact of constructing an EIC will affect all accelerator-based sciences.
- In summary, the committee concludes that an EIC is timely and has the support of the nuclear science community. The science that it will achieve is unique and world leading and will ensure global U.S. leadership in nuclear science as well as in the accelerator science and technology of colliders.

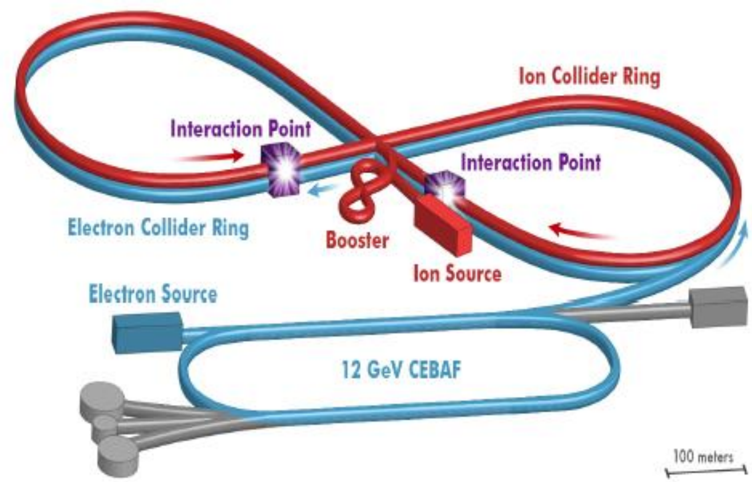
US EIC – two options of realization

The White Paper
A. Accardi et al
Eur. Phys. J.
A52 (2016) 268





BNL-eRHIC



JLab-JLEIC

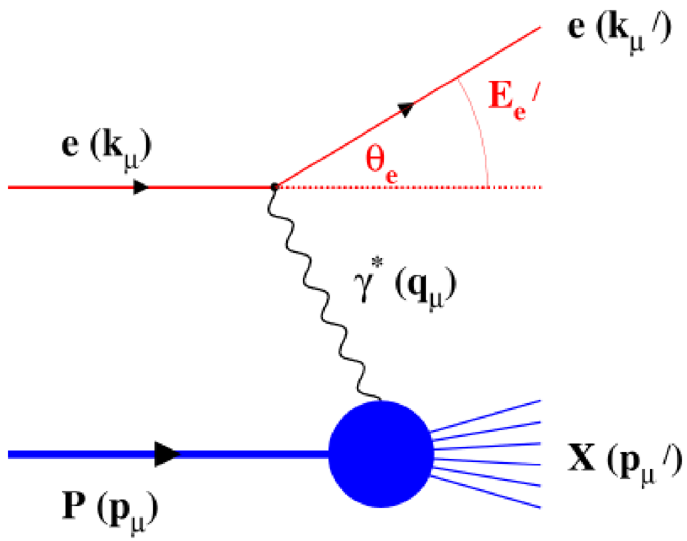
EIC: the World Wide Interest

	HERA@DESY	LHeC@CERN	eRHIC@BNL	JLEIC@JLab	HIAF@CAS	ENC@GSI
E_{CM} (GeV)	320	800-1300	45-175	12-140	12 → 65	14
proton x_{min}	1×10^{-5}	5×10^{-7}	3×10^{-5}	5×10^{-5}	$7 \times 10^{-3} \rightarrow 3 \times 10^{-4}$	5×10^{-3}
ion	p	p to Pb	p to U	p to Pb	p to U	p to ~ ^{40}Ca
polarization	-	-	p, ^{3}He	p, d, ^{3}He (^{6}Li)	p, d, ^{3}He	p,d
L [$cm^{-2} s^{-1}$]	2×10^{31}	10^{33}	$10^{33\text{-}34}$	$10^{33\text{-}34}$	$10^{32\text{-}33} \rightarrow 10^{35}$	10^{32}
IP	2	1	2+	2+	1	1
Year	1992-2007	2022 (?)	2022	Post-12 GeV	2019 → 2030	upgrade to FAIR

The past

Possible future

❑ Lepton-hadron facility:



Q^2 → Measure of resolution

y → Measure of inelasticity

x → Measure of momentum fraction of the struck quark in a proton

$Q^2 = S\,x\,y$

Inclusive events: e+p/A → e'+X

Detect only the scattered lepton in the detector

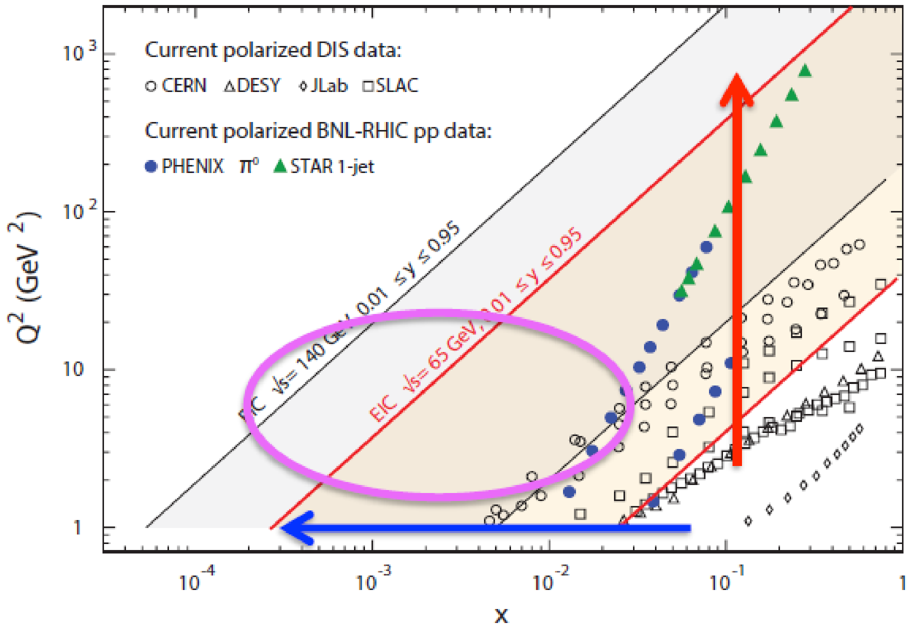
Semi-Inclusive events: e+p/A → e'+h(π,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

Exclusive events: e+p/A → e'+ p'/A'+ h(π,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

US EIC – Kinematic reach & properties

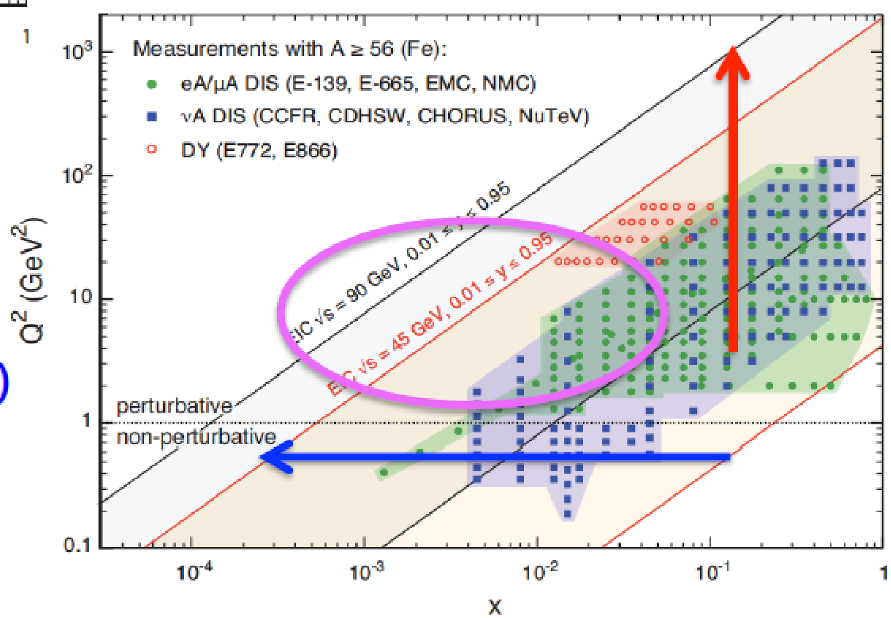


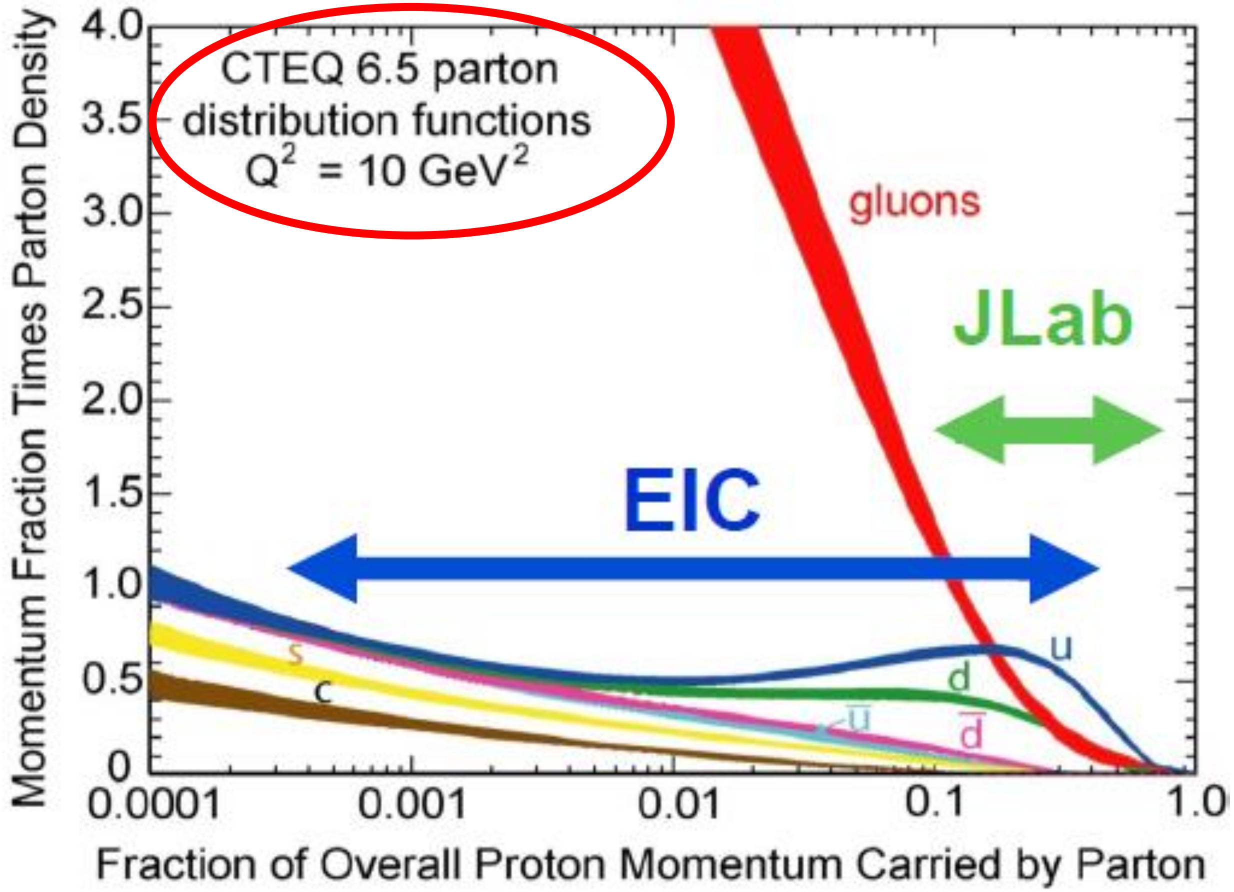
For e-N collisions at the EIC:

- ✓ Polarized beams: e, p, d/^{3}He
- ✓ Variable center of mass energy
- ✓ Wide Q^2 range → evolution
- ✓ Wide x range → spanning from valence to low-x physics
- ✓ 100-1K times of HERA Luminosity

For e-A collisions at the EIC:

- ✓ Wide range in nuclei
- ✓ Variable center of mass energy
- ✓ Wide Q^2 range (evolution)
- ✓ Wide x region (high gluon densities)

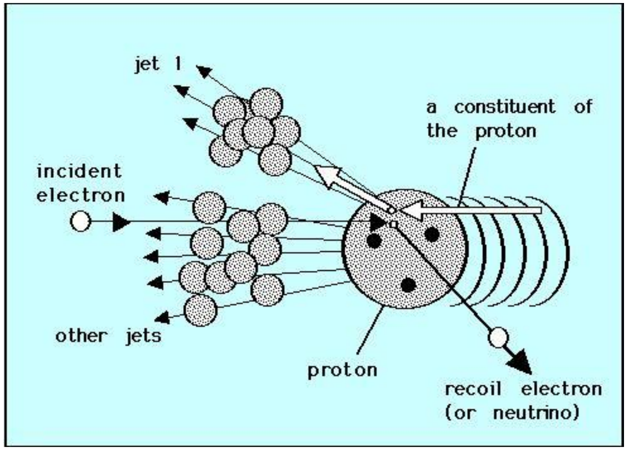




EIC explores the "sea" and the "glue", the "valence" with a huge level arm

Factorization theorems

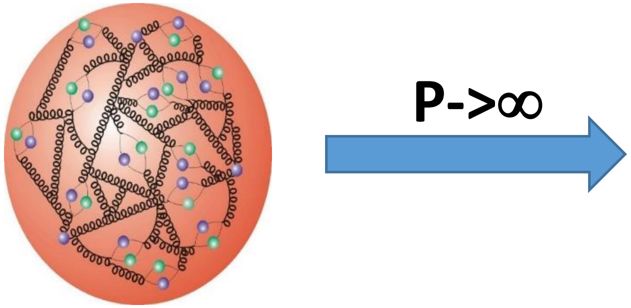
- Structure (long distance) and probe (short) physics are separated through perturbative analysis-> factorization theorems.



- Experimental cross sections can be written as products of calculable perturbative parts and non-perturbative matrix elements!

Parton content of composite systems!

- In relativistic theories, wave functions are frame-dependent.
- DIS probe is hard and relativistic. According to Feynman, physics simplifies considerably in the infinite momentum frame (IMF)



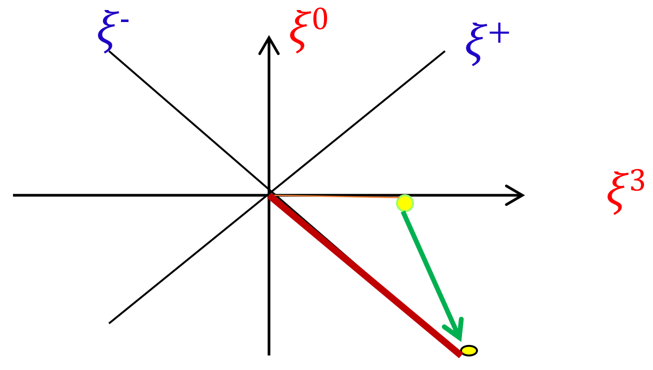
- The nucleons and nuclei are studied in IMF as Feynman partons

Light-cone correlations

- The IMF hadron state can be obtained by an infinite boost,

  $|p\rangle = U(\Lambda(p))\ |p=0\rangle$, Λ is related to the boost K_i

- The infinite boost will make any correlation functions becoming light-cone correlations



Parton physics of nucleon and nuclei, from EIC

- Light-cone wave function, $\psi_n(x_i, k_{\perp i})$

$$\left|\psi_p(P^+, \vec{P}_\perp)\right\rangle = \sum_n \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \left|n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\right\rangle$$

- Distributions amplitudes, $\psi_n(x_i)$
- Parton distributions, $f(x)$
- Transverse momentum dependent (TMD) parton distributions, $f(x, k_\perp)$
- Generalized parton distributions (GPD), $F(x,\xi,r_\perp)$
- Wigner distributions, $W(x, k_\perp, r_\perp)$

TMD's and GPD's

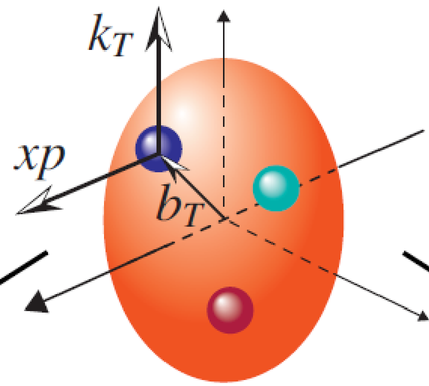
- **3D boosted partonic structure:**

Momentum Space

TMDs

Confined motion

$\int d^2 b_T$



$\int d^2 k_T$

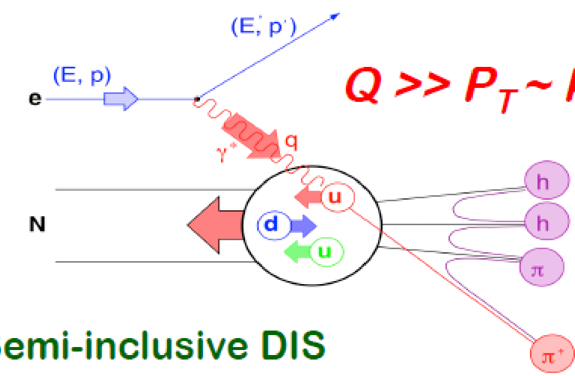
Coordinate Space

GPDs

Spatial distribution

$f(x, k_T)$ Two-scales observables $f(x, b_T)$

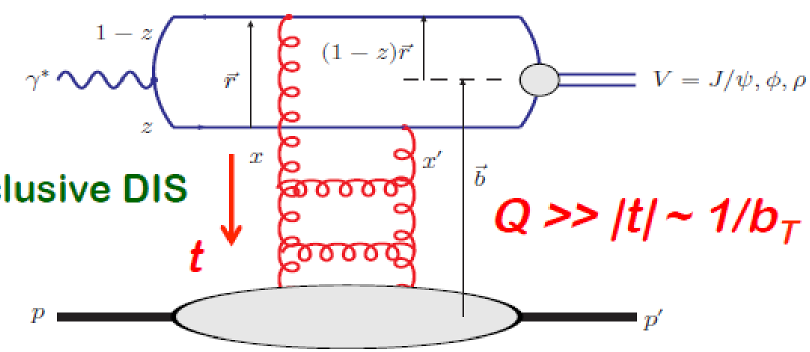
3D momentum space images



$Q >> P_T \sim k_T$

Semi-inclusive DIS

2+1D coordinate space images



Exclusive DIS

t

$Q >> |t| \sim 1/b_T$

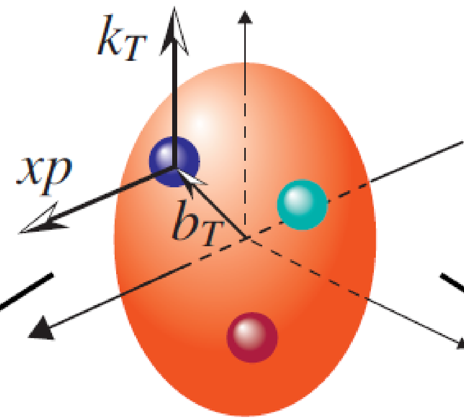
❑ 3D boosted partonic structure:

Momentum Space

TMDs

Coordinate Space

GPDs



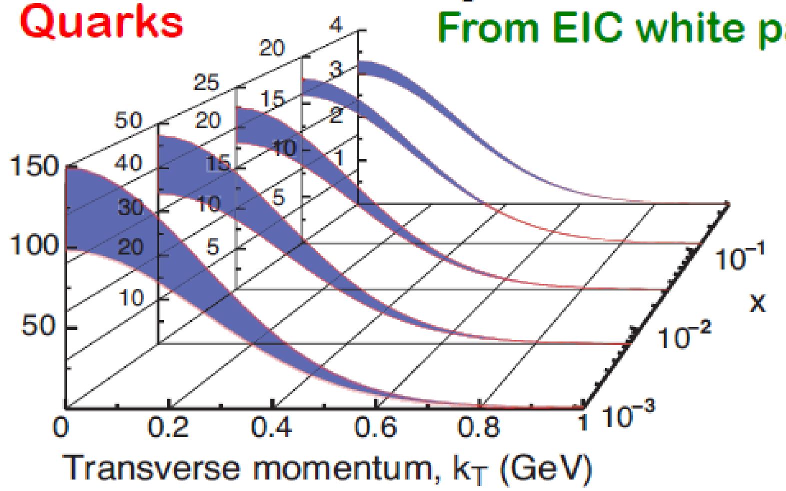
$\int d^2 b_T$ $\qquad$ $\int d^2 k_T$

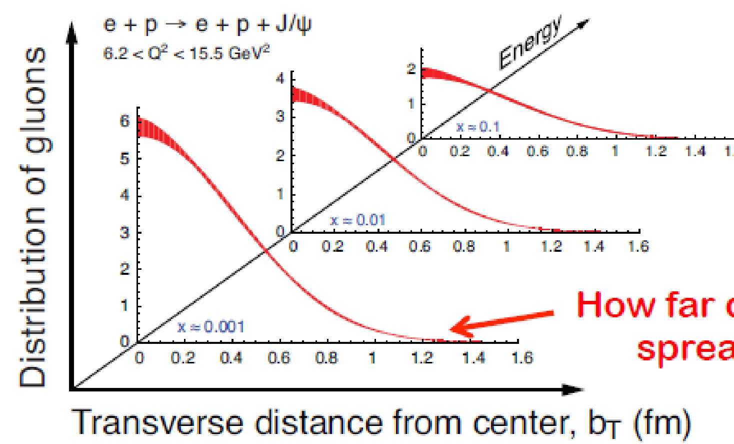
$f(x,k_T)$ $\qquad$ $f(x,b_T)$

Quarks

From EIC white paper: arXiv:1212.1701

Gluons





How far does it spread?

Problems with light-like correlations and momentum renormalization

Origin of the problem: sign

- When naively formulated, time-dependent (dynamical) correlations have sign problem: NP problem
- The same problem exists in condensed matter systems (Hubbard model, High-Tc)
- There is no general algorithm for a classical computer to solve NP problem!
  - A quantum computer? (M. Savage and others)
  - Converting NP into P problem.

Wilson and light-cone quantization

- Wilson realized in the end of 1980's that his lattice QCD approach cannot calculate PDFs, and therefore he led an new effort to solve QCD using

light-cone quantization (LCQ)

- Despite many years of efforts, light-cone quantization has not yielded a systematic approach to calculating non-perturbative parton physics directly from QCD

Momentum renormalization group

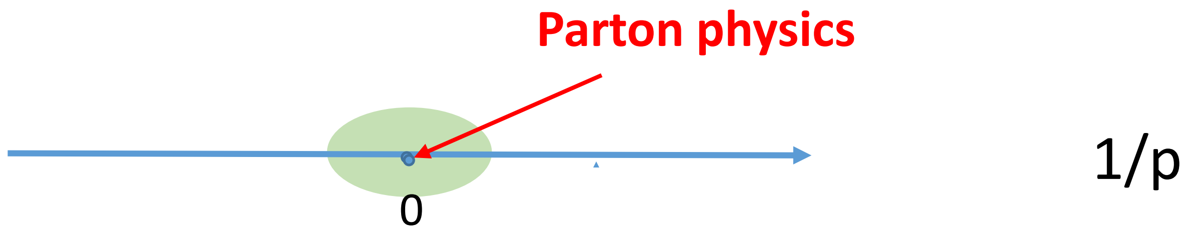
- Taking infinite-momentum (center-of-mass motion) limit in field theory is not a trivial process.
- One can derive a RG equation for the momentum-dependence of an composite operator at large momentum

$$dO(p,a)/dlnp \ = \gamma_o(p,a)O(p,a)$$

- The RG equation has a fixed point at P=∞, $\gamma_o = 0$
- Thus the parton physics corresponds to frame-dependent physical obervables at the fixed point of the frame transformations.

Partons, and large but not infinite momentum

- P=∞ is a fixed point， near this QCD is an approximately scale-invariant theory



- Momentum renormalization group relates physical system in the attraction basin of P= ∞
- P=∞ parton physics can be obtained from physics at large but finite P. (Hopefully around 2-5 GeV！)

Large-momentum effective theory

Large-momentum effective theory

- It has been realized in 2013 that the large momentum observables in lattice calculations provide an approach to calculating all parton physics

Large momentum effective theory, or LaMET

- **X. Ji, Phys. Rev. Lett. 110, 262002 (2013) arXiv:1305.1539 [hep-ph].**
- **X. Ji, Sci. China Phys. Mech. Astron. 57, 1407 (2014), arXiv:1404.6680 [hep-ph].**

Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

A large-momentum expansion

- Composite Euclidean operators will have external momentum P dependence
- One can expand lattice observable at large momentum,

$$O(P,a) = \mathrm{c}_0\left(\frac{\mu}{P}, aP\right) o(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

fixed-point physics is $o(\mu)$.

C_0 can be calculated in perturbation theory, summed with renormalization group equation.

General recipe

- The recipe:
  - Deconstructing (unboosting) the light-cone operator to get an Euclidean operator
    $A^+ \to A^3 \ or \ A^0$
  - Lattice computation of the Euclidean matrix elements with large momentum
  - Effective field theory interpretation

Universality class

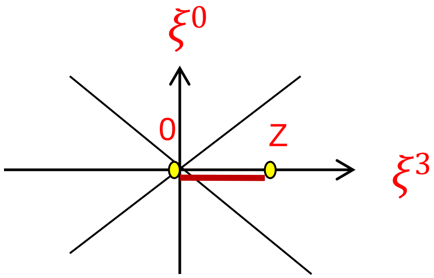
- Just like the same PDF can be extracted from different hard scattering processes, the same light-cone physics can be extracted from different lattice operators (0 and 3 components are similar).
- All operators that yield the same light-cone physics form a universality class.
- Universality class allows one exploring different operator O so that a result at finite P can be as close to that at large P as possible.

Hatta, Ji, and Zhao, Phys.Rev. D89 (2014) no.8, 085030

Examples

A Euclidean quasi-distribution

- Consider space correlation in a large momentum P in the z-direction.



$$\tilde{q}(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P | \overline{\psi}(z) \gamma^z \times \exp\left(-ig \int_0^z dz' A^z(z')\right) \psi(0) | P \rangle$$

- Quark fields separated along the z-direction
- The gauge-link along the z-direction
- The matrix element depends on the momentum P.
- Calculable in lattice QCD

X. JI, Phys. Rev. Lett. 110, 112001 (2013)

Factorization or matching at one-loop

- Since the IR behavior of the quasi-distribution is the same as the LC one, one can write down easily a factorization to one-loop order.

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

$$Z(x,\mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P^z) + \ldots$$

where the matching factor is perturbative

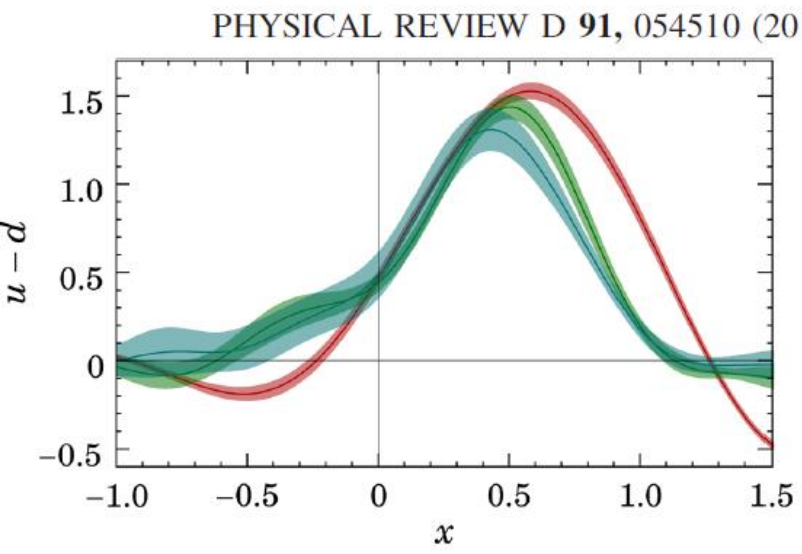
Xiong et al, Phys.Rev. D90 (2014) no.1, 014051

Factorization to all orders, Y. Ma and J. W. Qiu, 2014

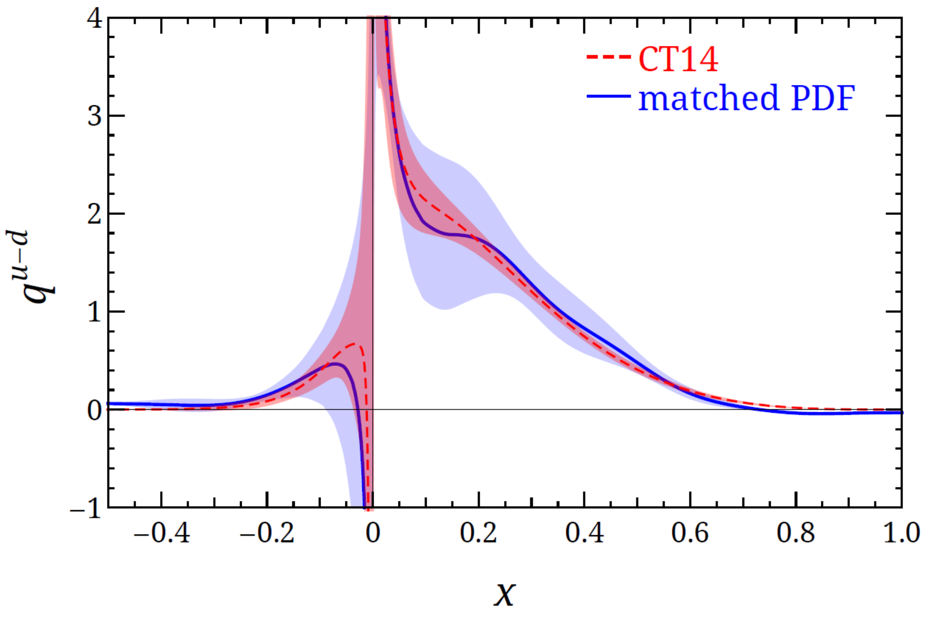
Quark distributions

- For isospin non-singlet distributions, calculations are easier on lattice.
- A number of lattice groups:
  - LP3 collaboration (MSU,MIT, NTU, U Conn, Regensburg, TD Lee Inst., ITP, Beijing)
  - ETMC (C. Alexandrou (U. Cyprus) , M. Constantinou (Temple U.), K.Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al)
  - Jefferson lab theory group, US
  - BNL theory group , US
  - Regensburg U., Germany
  - …

LP3 collaboration



H.W.Lin et al, PRD, 2005



To be published

ETMC recent result

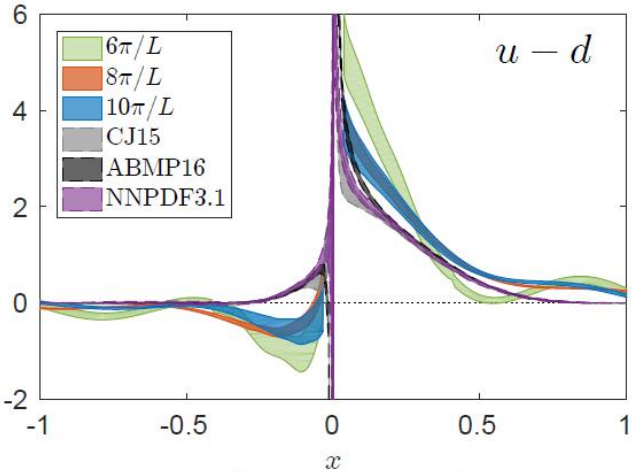


FIG. 4: Comparison of unpolarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), and ABMP16 [39] (NNLO), NNPDF [40] (NNLO) and CJ15 [38] (NLO) phenomenological curves.

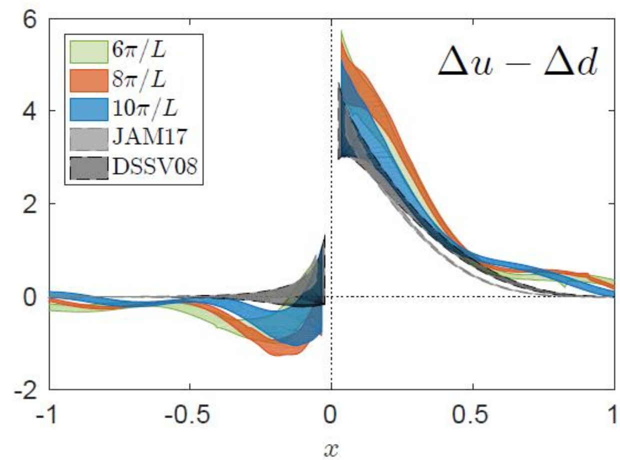


FIG. 5: Comparison of polarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), DSSV08 [41] and JAM17 NLO phenomenological data [42].

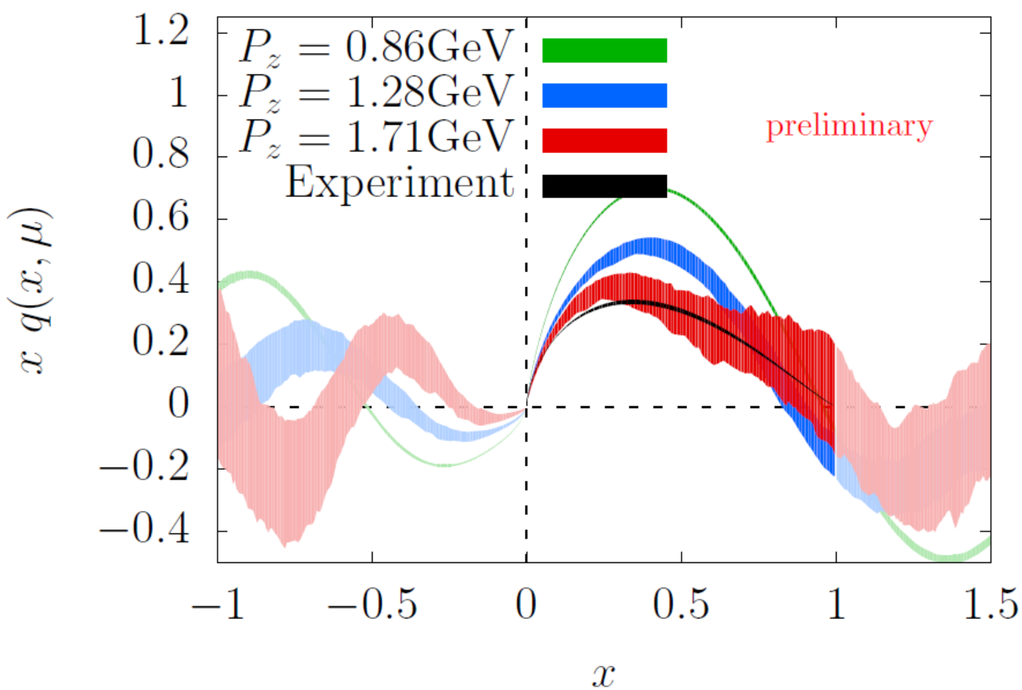
arXiv:1803.02685

Pion PDF from BNL&SBU

P. Petreczky
Parallel Session
talk on Tuesday

Pion PDF and comparison to extraction from experiments

$p_z^R = 1.28$ GeV, $p_T^R = 1.48$ GeV $\mu = 3.2$ GeV



comparison to pion PDF from data on πA DY and neutron electro-production at HERA: Barry et al (JAM), 2018

Other related approaches

- There are other proposals for PFDs other than quasi-PDF in the literature since the 2013 work

  A.Radyushkin, J. W. Qiu et al, …

- However, they are similar in the sense that they fall into the same universality class of the large-momentum effective theories (LaMET).
- Moreover, the so-called Ioffe-time or pseudo distribution are entirely equivalent. (Izubuchi et al, arXiv:1801.03917), see talk by S. Zafeiropoulos in parallel session

References

- **Restoration of rotational symmetry to calculate higher moments**

  Z. Davoudi and M. Savage, 2012.

- **OPE of flavor-changing current-current correlation**

  D. Lin and W. Detmold, 2006.

- **OPE of the Compton amplitude** A. J. Chambers et al. (QCDSF), 2017.
- **Direct computation of the physical hadronic tensor**

  K.F. Liu (et al.), 1994, 1999, 1998, 2000, 2017.

- **Smeared Quasi-PDF with Gradient flow** C. Monahan and K. Orginos, 2017.
- **Pseudo-PDF (alternative to quasi-PDF)** A. Radyushkin, 2017; K. Orginos et al., 2017.
- **Lattice cross section** Y.-Q. Ma and J. Qiu, 2014, 2017.
- **Factorization of Euclidean correlations in coordinate space**

  V. M. Braun and D. Mueller, 2008;G. S. Bali, V. M. Braun, A. Schaefer, et al., 2017.

Work on TMD & GPD

- GPD calculations are straightforward, no new theoretical issues.
- However, TMD's are more complicated
  - F. Yuan et al,
  - MIT group
  - Others…
- Distribution amplitudes
  - Talk by G. Bali in parallel session
  - LP3

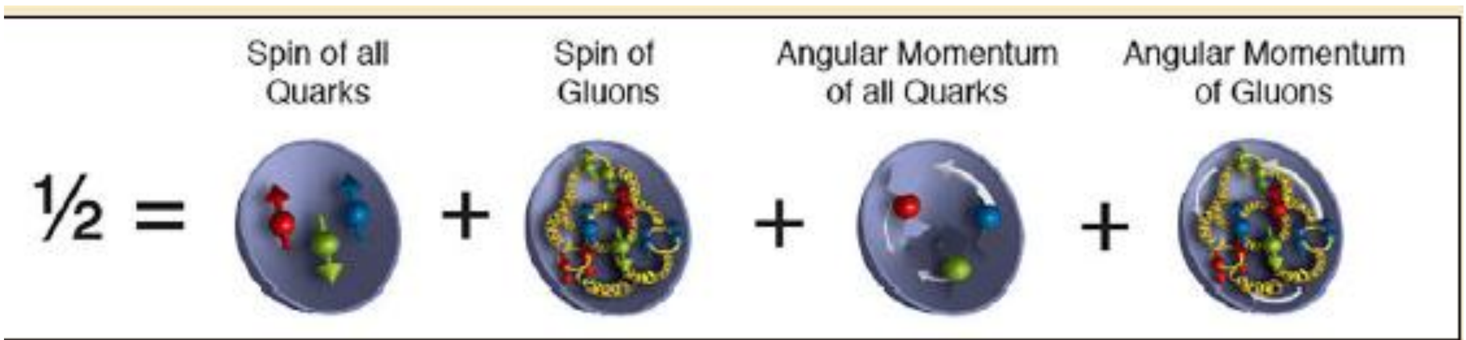
Proton spin crisis

- In the simple quark model, the proton is made of 3 quarks.
- The spin of the proton is entirely from the spin of unpaired quark.
- In 1987, EMC at CERN published a paper showing that only a small fraction of the proton spin comes from quark spin.
- Where is the proton spin coming from ?

"Spin crisis"

Spin structure of the proton

- An important topic for EIC



- Gluon contribution shall be important
- Polarized RHIC program was pushed forward, resulting important result on gluon polarization

QCD factorization

- In QCD factorization, one can show that the gluon polarization is a matrix element of non-local light-cone correlations.

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-) \times \mathcal{L}^{ab}(\xi^-, 0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle,$$

- No one knows how to calculate this for nearly 30 years!

LaMET calculations

- In LaMET theory, one can start with the local operator $\vec{E} \times \vec{A}$, in a physical gauge in the sense that the gauge condition shall allow transverse polarized gluons:
  - Coulomb gauge $\nabla \cdot E = 0$
  - Axial guage A_z=0
  - Temporal gauge A_0 =0
- Their matrix elements in the large momentum limit all go to ΔG.

Ji, Zhang, Zhao, Phys. Rev. Lett.,111,112002(2013)

First calculation (Yang et al, PRL (2017))

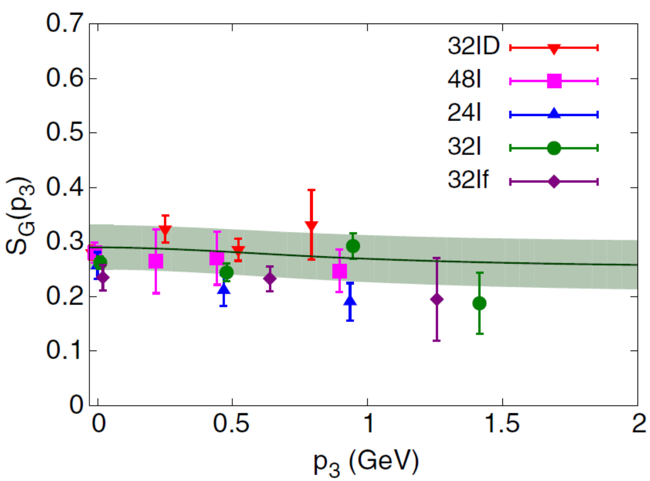
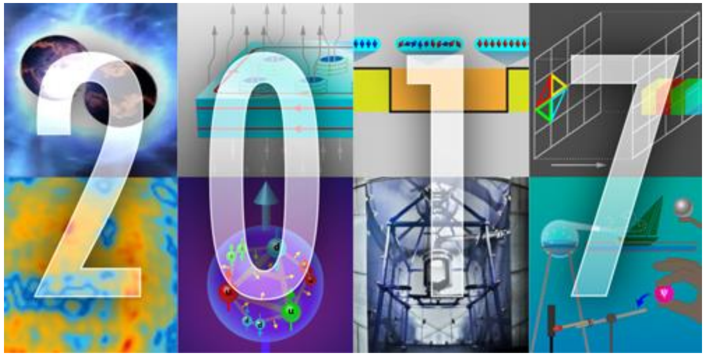


FIG. 4. The results extrapolated to the physical pion mass as a function of the absolute value of $\vec{p} = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\text{MS}}$ at $\mu^2 = 10\ \text{GeV}^2$. The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit [with the empirical form in Eq. (11)] of the results.



Gluons Provide Half of the Proton's Spin
The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: [Spinning Gluons in the Proton](#)). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from

Conclusions

- The main goal of EIC is to explore the parton physics of nucleon and nuclei. This presents a great opportunity for lattice community
- It appears that the only efficient way we know how to calculate partons on lattice is LaMET, it is important to explore this new emerging area in in the next decade.
- Dedicated lattice efforts needed: configurations, large momentum states, higher-twist analysis,…