Heavy Quark Mass Definition, Minimal Renormalon Subtracted Mass (MRS) and Precise Quark Mass Extraction

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on behalf of the

Fermilab Lattice, MILC, and TUMQCD Collaborations

Based on the papers:

MRS
mass:Relations between Heavy-light Meson and Quark Masses
N. Brambilla,^{1,2} J. Komijani,^{1,2,•} A.S. Kronfeld,^{2,3} and A. Vairo¹arXiv:1712.04983 (TUMQCD Collaboration)Phys.Rev. D97 (2018) no.3, 034503

quarks mass determination:

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Up-, down-, strange-, charm-, and bottom-quark masses from four-flavor lattice QCD

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t arXiv:1802.04248

- Introduction and review of different methods
- Extraction of quark masses from heavy-light meson masses
- •Renormalons in pole mass: MRS mass definition
- •HISQ ensembles with (2+1+1) flavors of dynamical quarks
- •EFT fit to lattice QCD data
- Determinations of heavy-quark masses and HQET matrix elements
- Light quark mass results
- •Comparison and conclusions

Introduction

Six of the fundamental parameters of the Standard Model are quark masses

- they cannot be measured directly (confined inside hadrons)
- must be extracted indirectly from physical observables
- For observable particles such as electrons
 - the position of the pole in the propagator is the definition of its mass
 - the pole mass is the rest mass of an isolated particle
- The masses of quarks can be defined as theoretical parameters
 - renormalized, e.g., in the $\overline{\text{MS}}$ scheme at a given scale μ
- Precise values of quark masses are needed for precise calculations in SM/BSM
- In lattice QCD simulations, the bare quark masses can be tuned to obtain physical observables
- The resulting bare masses must be renormalized, but multiloop lattice-QCD calculations are difficult (⇒ limited accuracy)

 Methods that require only nonperturbative lattice-QCD calculations and continuum perturbative calculations yield better accuracy:

Nonperturbative calculation of quark mass renormalization constant

Quark masses are calculated in an intermediate scheme (variants of RI-MOM), and then converted to the $\overline{\text{MS}}$ scheme. Employed by BMW, ETM, RBC/UKQCD, χ QCD, HPQCD, \cdots

Heavy-quark correlator moments

By comparing moments calculated on lattice and QCD perturbation theory. Employed by HPQCD, JLQCD, hotQCD, ···

Extraction based on dependence of meson masses on quark masses

A new method developed by Fermilab/MILC/TUMQCD collaborations to extract heavy quark masses from heavy-light meson masses (based on HQET):

guark pole mass

meson mass

quark <mark>MS</mark> mass

Finite volume step scaling

Employed by the Alpha Collaboration

Extraction of quark masses from heavy-light meson masses

HQET description of a HL meson mass in terms of its heavy quark mass

$$M_H = m_h + \bar{\Lambda} + rac{\mu_\pi^2 - \mu_G^2(m_h)}{2m_h} + \mathcal{O}(1/m_h^2)$$

- $\bar{\Lambda}$: energy of light quarks and gluons inside the system
- $\mu_{\pi}^2/2m_h$: kinetic energy of the heavy quark inside the system
- $\mu_G^2(m_h)/2m_h$: hyperfine energy due to heavy quark's spin (can be estimated from B^* -B splitting $\Rightarrow \mu_G^2(m_b) \approx 0.35 \,\text{GeV}^2$)
- *m_h* is the pole mass of the heavy quark

(The pole mass can be calculated at each order in PT, but it suffers from renormalon divergence)

- For the heavy quark mass, we use the minimal renormalon subtracted (MRS) scheme [PRD97, 034503 (2018)]
 - removes the leading infrared renormalon from the pole mass
 - has an asymptotic expansion identical to the perturbative pole mass (does not spoil the HQET power counting)
 - is a gauge- and scale-independent scheme; it does not introduce any factorization scale (unlike, e.g., the RS or kinetic scheme)

Minimal renormalon subtracted mass

The pole mass can be calculated at each order in perturbation theory

$$m_{\text{pole}} = \overline{m} \left(1 + \sum_{n=0}^{N} r_n \alpha_s^{n+1}(\overline{m}) + \mathcal{O}(\alpha_s^{N+2}) \right)$$

- \overline{m} is the $\overline{\text{MS}}$ mass at scale $\mu = \overline{m}$
- The series diverges because $r_n \propto (2\beta_0)^n \Gamma(n+b+1)$ as $n \to \infty$
- The divergent expression can be interpreted using the Borel transform

involves an integral of form
$$\int_0^\infty dz \, \frac{e^{-z/(2\beta_0\alpha_s)}}{(1-z)^{1+b}}$$
 with $b = \beta_1/(2\beta_0^2)$

The idea in the MRS scheme is to divide the integral as

$$\int_{0}^{1} dz \; \frac{e^{-z/(2\beta_{0}\alpha_{s})}}{(1-z)^{1+b}} \quad \rightarrow \quad \mathcal{J}_{\mathrm{MRS}}(\mu)$$
$$\int_{1}^{\infty} dz \; \frac{e^{-z/(2\beta_{0}\alpha_{s})}}{(1-z)^{1+b}} \quad \rightarrow \quad \delta m \propto (-1)^{b} \Lambda_{\mathrm{QCD}}$$

and subtract the ambiguous term δm from the pole mass

The MRS mass is defined as

$$m_{\text{MRS}} = m_{h,\text{pole}} - \delta m$$
$$= \overline{m} \left(1 + \sum_{n=0}^{\infty} \left[r_n - R_n \right] \alpha_s^{n+1}(\overline{m}) \right) + \mathcal{J}_{\text{MRS}}(\overline{m}) + \Delta m_{(c)}$$

• For a theory with $n_l = 3$ massless quarks, and $R_0 = 0.535$:

 $r_n - R_n = (-0.1106, -0.0340, 0.0966, 0.0162, \ldots)$

The smallness of $r_n - R_n$ reduces the truncation error in our work

With the MRS mass for heavy quarks, we proceed to map bare quark masses to the MRS mass Mapping bare quark masses to the MS and MRS masses

Introduce a "reference mass", and construct the identity (up to lattice artifacts)

$$m_{h,\text{MRS}} = m_{r,\overline{\text{MS}}}(\mu) \frac{\overline{m}_{h}}{m_{h,\overline{\text{MS}}}(\mu)} \frac{m_{h,\text{MRS}}}{\overline{m}_{h}} \frac{am_{h}}{am_{r}}$$

- 1) First factor: a fit parameter (we set $am_r = am_{p4s}$ and $\mu = 2$ GeV)
- Second factor: running factor governed by the mass anomalous dimension (the five-loop result is known [JHEP 1410 (2014) 076])
- Third factor:

$$m_{h,\text{MRS}} = \overline{m}_h \left(1 + \sum_{n=0}^3 \left[r_n - R_n \right] \alpha_s^{n+1}(\overline{m}_h) + \mathcal{O}(\alpha_s^5) \right) + \mathcal{J}_{\text{MRS}}(\overline{m}_h) + \Delta m_{(c)}$$

- Last factor: simulation inputs
- The 2nd and 3rd factors require the strong coupling constant; we use $\alpha_{\overline{MS}}(5 \text{ GeV}; n_f = 4) = 0.2128(25)$ [HPQCD, arXiv:1408.4169]

• Discretization errors should be incorporated as powers of $(am_h)^2$ and $(a\Lambda)^2$

MILC ensembles with (2+1+1)-flavors of dynamical quarks

Ensembles with physical mass for the strange quark:

$\approx a \text{ (fm)}$	m_l/m_s	size	L (fm)	$M_{\pi}L$	M_{π} (MeV)
0.15	1/5	$16^{3} \times 48$	2.38	3.8	314
0.15	1/10	$24^{3} \times 48$	3.67	4.0	214
0.15	1/27	$32^{3} \times 48$	4.83	3.2	130
0.12	1/5	$24^{3} \times 64$	3.00	4.5	299
0.12	1/10	$24^3 \times 64$	2.89	3.2	221
0.12	1/10	$32^{3} \times 64$	3.93	4.3	216
0.12	1/10	$40^3 \times 64$	4.95	5.4	214
0.12	1/27	$48^3 \times 64$	5.82	3.9	133
0.09	1/5	$32^{3} \times 96$	2.95	4.5	301
0.09	1/10	$48^3 \times 96$	4.33	4.7	215
0.09	1/27	$64^{3} \times 96$	5.62	3.7	130
0.06	1/5	$48^3 \times 144$	2.94	4.5	304
0.06	1/10	$64^{3} \times 144$	3.79	4.3	224
0.06	1/27	$96^{3} \times 192$	5.44	3.7	135
0.042	1/5	$64^{3} \times 192$	2.91	4.34	294
0.042	1/27	$144^{3} \times 288$	6.12	4.17	134
0.03	1/5	$96^{3} \times 288$	3.25	4.84	294

The fermion action is "highly improved staggered quark" (HISQ) action
 Physical-mass ensembles at most lattice spacings

Heavy-light mesons with HISQ action



- We have 24 Ensembles:
 - 6 lattice spacings
 - several sea masses
- We calculate masses of pseudoscalar mesons for various light and heavy quarks with masses:
 - light valence: $m_{ud} \lesssim m_v \lesssim m_s$
 - heavy valence: $m_{c} \lesssim m_{h} \lesssim m_{b}$
- We use only am_h < 0.9 to avoid large discretization errors

EFT description of heavy-light meson masses

We employ HQET and heavy-meson staggered ChPT to describe the dependence of meson masses on both heavy and light quark masses and incorporate taste-breaking lattice artifacts

• Include HMrPQAS χ PT and higher order HQET terms

 $M_H = m_{h,\rm MRS} + \overline{\Lambda}_{\rm MRS} + \frac{\mu_\pi^2 - \mu_G^2(m_h)}{2m_{h,\rm MRS}} + {\rm HMrPQAS}\chi {\rm PT} + {\rm higher \ order \ HQET}$

- $m_{h,\text{MRS}}$ is a function of am_h/am_{p4s} and $am_{p4s,\overline{\text{MS}}}(2 \text{ GeV})$
- The higher order terms are typically polynomials in dimensionless, "natural" expansion parameters:
 - Light-quark and gluon discretization: $(a\Lambda)^2$ with $\Lambda=600~{
 m MeV}$
 - Heavy-quark discretization: $(2am_h/\pi)^2$
 - Light valence and sea quark mass effects: $B_0 m_q/(4\pi^2 f_\pi^2)$
 - HQET: Λ/m_{h,MRS} with Λ = 600 MeV
- Our fit function has 77 parameters and 384 data points

A snapshot of the fit and data



Dashed lines: $am_h \approx 0.9$; open symbols: data points omitted from fit Vertical axis: heavy-strange meson masses Horizontal axis: the fit values for the RS mass projected to continuum (no lattice artifacts)

- The combined-correlated fit gives $\chi^2/d.o.f \approx 1$, p = 0.3
- After extrapolating to continuum, experimental masses of D_s and B_s with EM effects subtracted are used to determine the charm- and bottom-quark masses

Stability of results under variation in number of loops

- We use
 - four-loop relation between the pole and MS mass
 - five-loop results for the quark mass anomalous dimension
 - five-loop results for beta function
- The plot shows the dependence of our final results on number of loops;



In the fits labeled by $O(\alpha_s^n)$, we keep n subleading orders; the green dashed lines show the total errors.

We do not introduce any systematic error associated with truncation in PT

Results for the strange, charm and bottom quarks

• The strange quark masses in a theory with 4 active flavors:

 $m_{s,\overline{\text{MS}}}(2 \text{ GeV}) = 92.52(40)_{\text{stat}}(18)_{\text{syst}}(52)_{\alpha_s}(12)_{f_{\pi,\text{PDG}}} \text{ MeV}$

For quark mass ratios:

$$\begin{split} m_c/m_s &= 11.784(11)_{\rm stat}(17)_{\rm syst}(00)_{\alpha_s}(08)_{f_{\pi,\rm PDG}} \\ m_b/m_s &= 53.93(7)_{\rm stat}(8)_{\rm syst}(1)_{\alpha_s}(5)_{f_{\pi,\rm PDG}} \\ m_b/m_c &= 4.577(5)_{\rm stat}(7)_{\rm syst}(0)_{\alpha_s}(1)_{f_{\pi,\rm PDG}} \end{split}$$

For heavy quarks:

$$\overline{m}_{c} = 1273(4)_{\text{stat}}(1)_{\text{syst}}(10)_{\alpha_{s}}(1)_{f_{\pi,\text{PDG}}} \text{ MeV}$$
$$\overline{m}_{b}^{(n_{f}=5)} = 4197(12)_{\text{stat}}(1)_{\text{syst}}(8)_{\alpha_{s}}(1)_{f_{\pi,\text{PDG}}} \text{ MeV}$$

where $\overline{m}_h = m_{h,\overline{MS}}(m_{h,\overline{MS}})$.

- Uncertainties:
- "stat") Statistics and EFT fit
- "syst") Various systematic uncertainties in inputs: FV, EM, topological charge freezing, contamination from higher order states...
 - α_s) Uncertainty in the strong coupling constant

 $\alpha_{s,\overline{\text{MS}}}(5 \,\text{GeV}; n_f = 4) = 0.2128(25)$ [HPQCD, arXiv:1408.4169]

 $f_{\pi,\text{PDG}}$) Uncertainty in the PDG value of $f_{\pi^{\pm}} = 130.50(13)$ MeV, which is used for scale setting

For HQET parameters we have

$$\overline{\Lambda}_{\text{MRS}} = 552(25)_{\text{stat}}(6)_{\text{syst}}(16)_{\alpha_s}(2)_{f_{\pi,\text{PDG}}} \text{ MeV}$$
$$\mu_{\pi}^2 = 0.06(16)_{\text{stat}}(14)_{\text{syst}}(06)_{\alpha_s}(00)_{f_{\pi,\text{PDG}}} \text{ GeV}^2$$
$$\mu_{G}^2(m_b) = 0.38(01)_{\text{stat}}(01)_{\text{syst}}(00)_{\alpha_s}(00)_{f_{\pi,\text{PDG}}} \text{ GeV}^2$$

(Note that the prior value of $\mu_G^2(m_b)$ is set to $0.35(7) \,\text{GeV}^2$ [Gambino and Schwanda, arXiv:1307.4551])

Results for the up and down quark masses

• To calculate the light quark masses we combine our determination of $m_{s,\overline{\mathrm{MS}}}(2\mathrm{GeV})$ and separate determination of mass ratios m_s/m_l and m_d/m_u

$$\begin{array}{ll} m_{l,\overline{\text{MS}}}(2 \ \text{GeV}) &= 3.404(14)_{\text{stat}}(08)_{\text{syst}}(19)_{\alpha_s}(04)_{f_{\pi,\text{PDG}}} \ \text{MeV} \\ m_{u,\overline{\text{MS}}}(2 \ \text{GeV}) &= 2.118(17)_{\text{stat}}(32)_{\text{syst}}(12)_{\alpha_s}(03)_{f_{\pi,\text{PDG}}} \ \text{MeV} \\ m_{d,\overline{\text{MS}}}(2 \ \text{GeV}) &= 4.690(30)_{\text{stat}}(36)_{\text{syst}}(26)_{\alpha_s}(06)_{f_{\pi,\text{PDG}}} \ \text{MeV} \end{array}$$

 m_u and m_d values depend on separate calculation of EM effects on light-light mesons [MILC, arXiv:1807.05556]

Our result is shown as a magenta burst, with the gray band showing how it compares directly with the other lattice and nonlattice results; see [arXiv:1802.04248 [hep-lat]] for details.



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Recalling the three major methods used by lattice collaborations, we find very good agreement between different results.

Conclusion

- We developed a method based on HQET to extract quark masses from heavy-light meson masses
- We employed heavy-meson (staggered) ChPT to describe the dependence of heavy-light mesons on masses of light valence and sea quarks, and we performed a combined correlated, multidimensional fit to 384 data at multiple lattice spacings
- We presented results for up, down, strange, charm and bottom quark masses determined by Fermilab/MILC/TUMQCD collaborations
- Comparing these results and other lattice calculations, we find good agreement between quark masses obtained with different methods
- We obtain a new definition of the mass: the minimal renormalon subtracted (MRS) mass which is not scale dependent and is optimal for mass extraction
- We presented results for the HQET parameters in MRS scheme

BACKUP

Scale setting and calculating tuned quark masses

- Scale setting is done using f_{p4s} (the decay constant of a fiducial pseudoscalar meson with both valence masses equal to $m_{p4s} \equiv 0.4m_s$)
- The physical value of f_{p4s} is set from f_{π}
- This method yields a simultaneous determination of both the lattice spacing a and the quark mass am_{p4s} (and in turn m_s = 2.5m_{p4s})
- The values of f_{p4s} and quark mass ratio m_s/m_l are determined by analyzing light-light data from the same ensembles
 ⇒ Various systematic errors (such as FV, EM, continuum extrapolation, etc.) in estimate of f_{p4s} and tuned quark masses must be incorporated to our estimate of uncertainties

• $\mathcal{J}_{\mathrm{MRS}}(\mu)$ is defined as

$$\mathcal{J}_{\rm MRS}(\mu) = \frac{R_0}{2\beta_0} \mu e^{-1/[2\beta_0 \alpha_{\rm g}(\mu)]} \sum_{n=0}^{\infty} \frac{1}{n!(n-b)} \left(\frac{1}{2\beta_0 \alpha_{\rm g}(\mu)}\right)^n$$

where $b = \beta_1/(2\beta_0^2)$, R_0 is the overall normalization of the leading renormalon in the pole mass, and $\alpha_g(\mu)$ is the coupling constant in the scheme with

$$\beta\left(\alpha_{\rm g}(\mu)\right) = -\frac{\beta_0 \alpha_{\rm g}^2(\mu)}{1 - (\beta_1/\beta_0)\alpha_{\rm g}(\mu)}$$

For the relations between the RS and MRS schemes:

$$m_{\rm RS}(\nu_f) = m_{\rm MRS} - \mathcal{J}_{\rm MRS}(\nu_f)$$
$$\overline{\Lambda}_{\rm RS}(\nu_f) = \overline{\Lambda}_{\rm MRS} + \mathcal{J}_{\rm MRS}(\nu_f)$$

Discussion on smallness of truncation error

In the MRS scheme, we use

$$m_{h,\text{MRS}} = \overline{m}_h \left(1 + \sum_{n=0}^{\infty} \left[r_n - R_n \right] \alpha_s^{n+1}(\overline{m}_h) \right) + \mathcal{J}_{\text{MRS}}(\overline{m}_h) + \Delta m_{(c)}$$

- $J_{\rm MRS}(\overline{m}_h)$ has a convergent expression in powers of $1/\alpha_s(\overline{m}_h)$
- Coefficients are small: r_n − R_n = (-0.1106, -0.0340, 0.0966, 0.0162) for n = (0, 1, 2, 3), three active flavors, and R₀ = 0.535.
 ⇒ the errors from truncating perturbative QCD relations are negligible
- This is not necessarily the case when one uses other schemes
- Using the RS scheme $~[{\rm hep-ph}/0105008]$, which introduces a factorization scale $\nu \ll \overline{m}_h$ as

$$m_{h,\mathrm{RS}}(\nu) = \overline{m}_h \left(1 + \sum_{n=0}^{\infty} c_n(\nu, \overline{m}_h, \mu) \; \alpha_s^{n+1}(\mu) \right) + \Delta m_{(c)}$$

we then have $c_n(1\text{GeV}, 4.2\text{GeV}, 4.2\text{GeV}) = (0.30, 0.52, 1.1, 2.2, \cdots)$ $c_n(1\text{GeV}, 4.2\text{GeV}, 3\text{GeV}) = (0.30, 0.38, 0.59, 0.68, \cdots)$ the truncation error is expected to be of size $2.20\alpha_s^4(4.2\text{GeV}) \times \overline{m}_h \approx 20$ MeV and $0.68\alpha_s^4(3\text{GeV}) \times \overline{m}_h \approx 10$ MeV In order to incorporate heavy quark discretization errors, in our fit function:

$$m_{h,\mathrm{MRS}} \rightarrow m_{h,\mathrm{MRS}} \times \left(1 + lpha_{\overline{\mathrm{MS}}}(2 \ \mathrm{GeV}) \sum_{n=1}^{4} k_n x_h^n \right) \quad \mathrm{with} \ x_h = (2am_h/\pi)^2$$

The prior values of the k_n are set to 0 ± 1, and the posterior values of k_n from our base fit:

$$k_n = (0.19, 0.07, -0.12, -0.46)$$
 for $n = (1, 2, 3, 4)$

When we include one more term:

$$k_n = (0.19, 0.06, -0.12, -0.37, -0.19)$$
 for $n = 1, 2, 3, 4, 5$

with extremely small change in our final results