Soft-Collinear Effective Theory and Jets in QCD

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Quark Confinement and the Hadron Spectrum
Maynooth University, Ireland
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Outline

- Introduction to Jets, SCET, and Factorization
- Precision QCD: fixed order $\mathcal{O}(\alpha_s^k)$ and resummation $\alpha_s^k \sum_j (\alpha_s \ln^2)^j$
- Exclusive and Inclusive Jets: Jet Vetoes and Jet Mass
- Jet Substructure
- Power Corrections to Collinear and Soft limits
- Conclude

Cover a few examples here, for many more see SCET 2018:

Outline

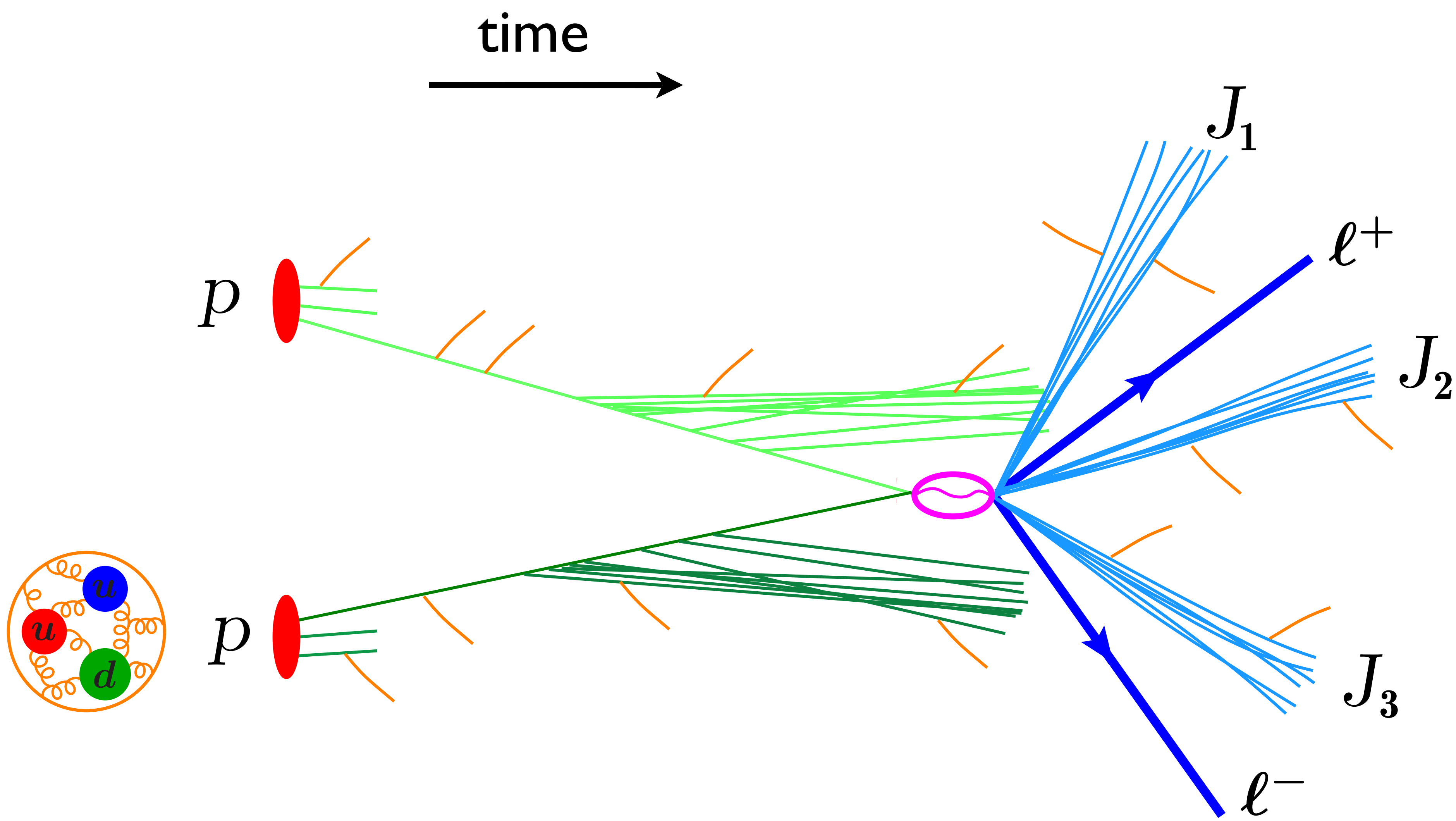
- Introduction to Jets, SCET, and Factorization
- Precision QCD: fixed c $\ln^2)^j$
- Exclusive and Inclusive J
- Jet Substructure
- Power Corrections to C
- Conclude

Jets are useful for:

- measuring parameters: $\alpha_s, m_t, \ldots$
- measuring non.pert. distributions:
  PDFs, hadronization, …
- studying QCD dynamics:
  convergence of pert. QCD,
  collinear & soft limits (jet dynamics),
  jet constituents, fragmentation,
  power corrections, …
- key ingredient in new physics searches

Cover a few examples here, for many more see SCET 2018:

Exclusive Jet Production with a Hard Interaction:



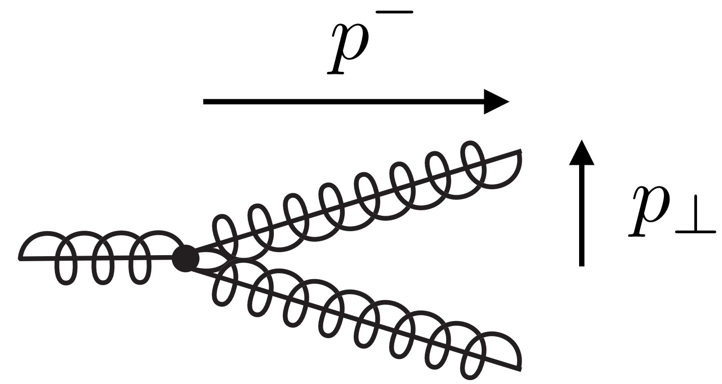
Relevant Momentum Regions:

- Collinear Splittings

$p^z > 0 \longrightarrow \hat{n} = \hat{z}$

$p^- = p^0 + p^z$

$p^+ = p^0 - p^z$

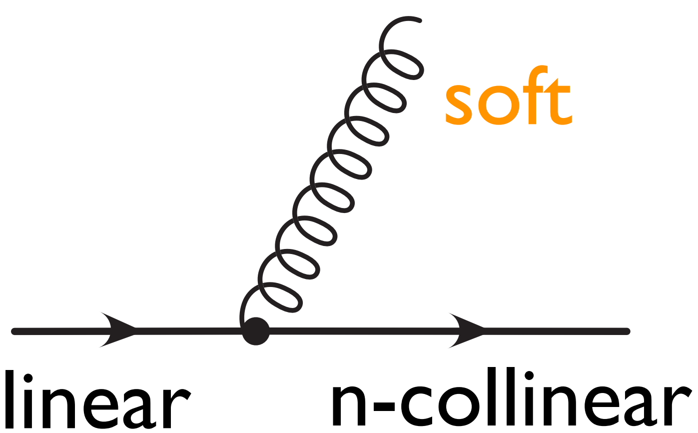


"n-collinear"

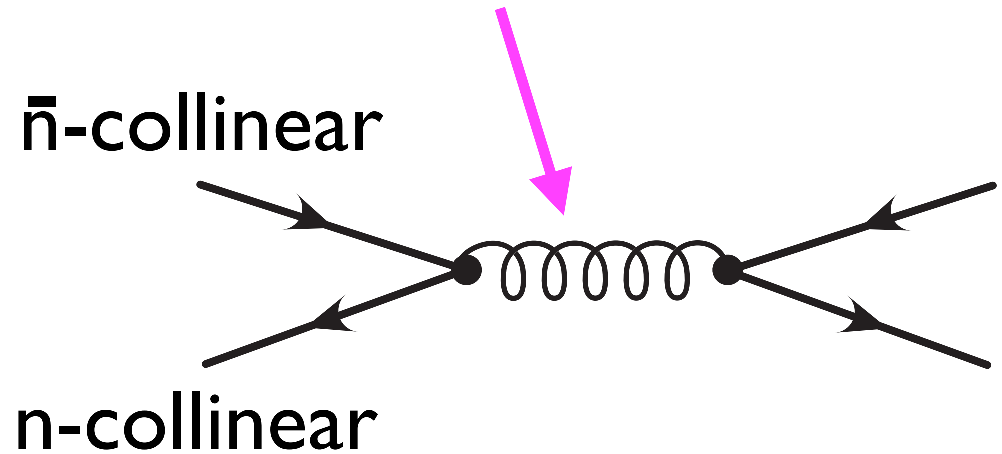
$p^- \gg p_\perp \gg p^+$

onshell: $p^+ p^- = \vec{p}_\perp^{\,2}$

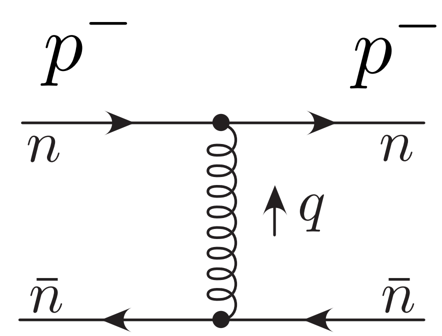
- Soft Emission



- Hard Propagators (short dist.)



- Glauber Exchange

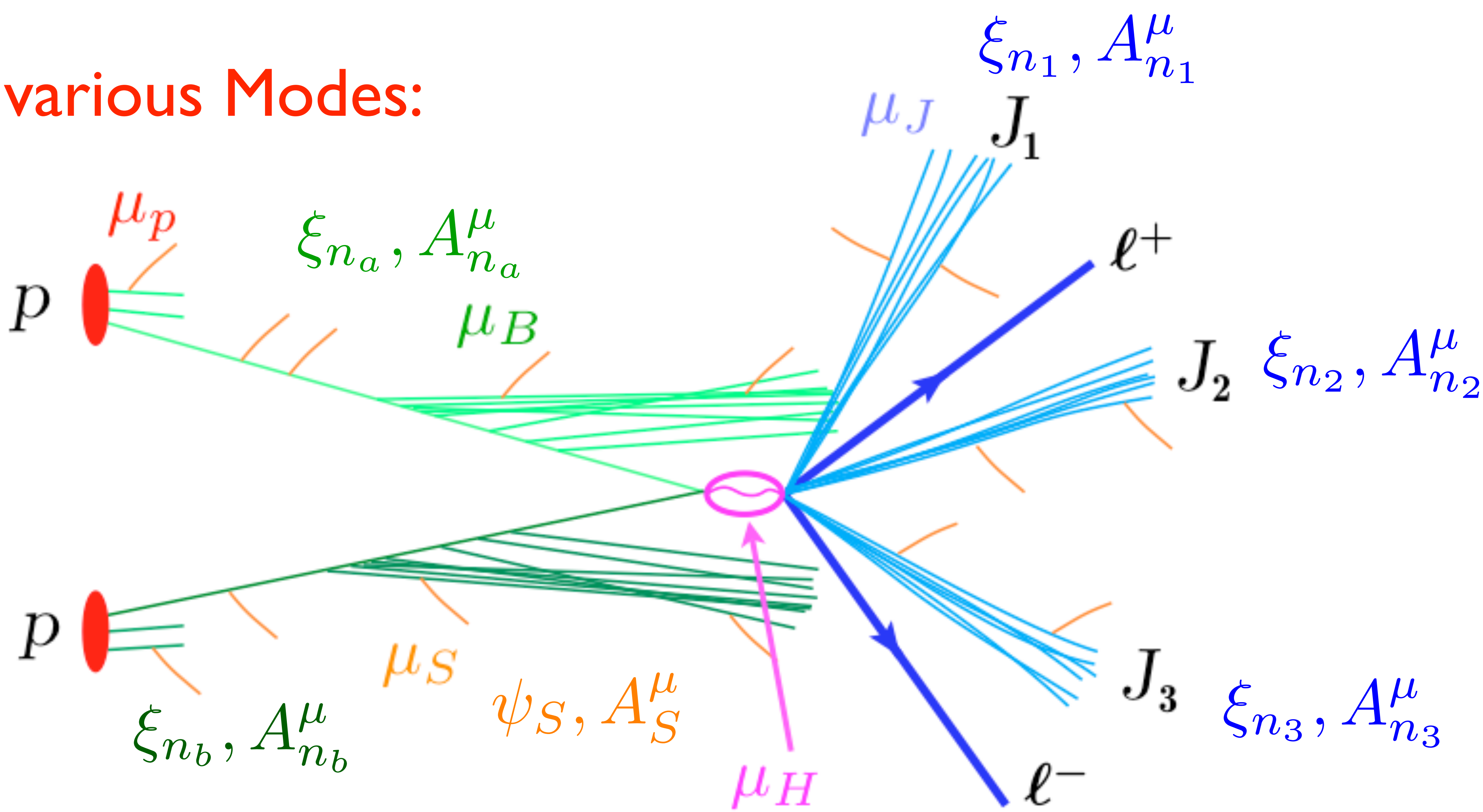


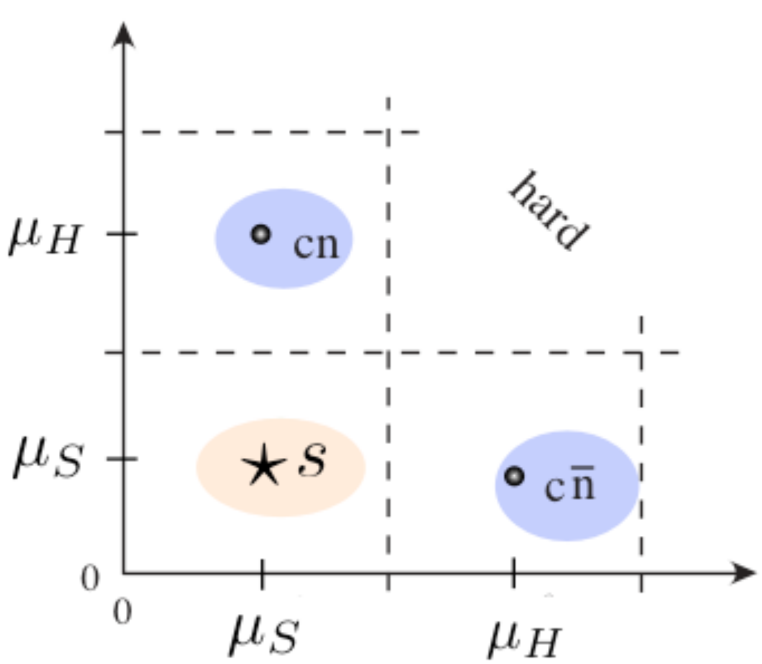
forward scattering

factorization violating

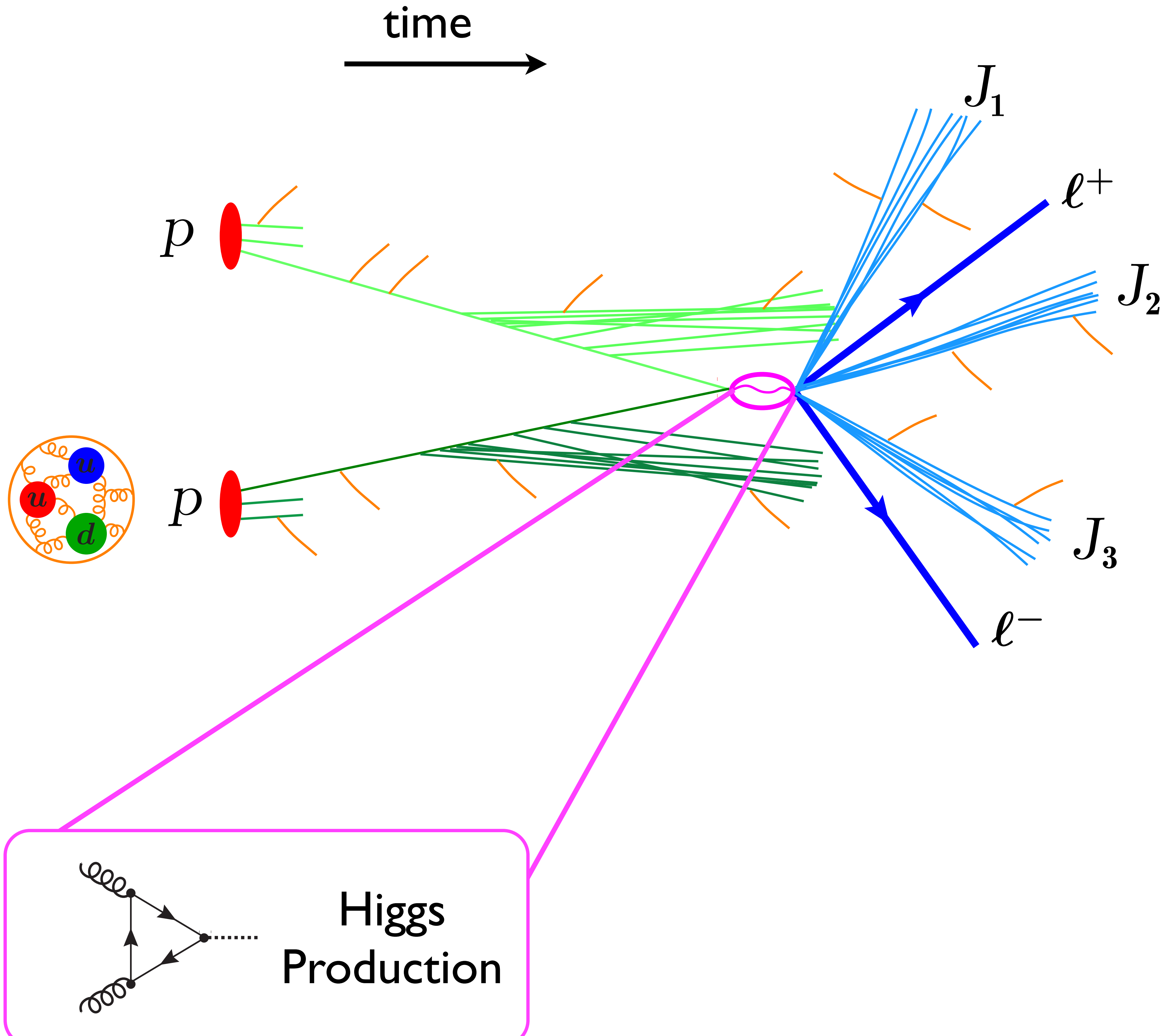
EFT for collider physics = Soft Collinear Effective Theory

Fields for various Modes:

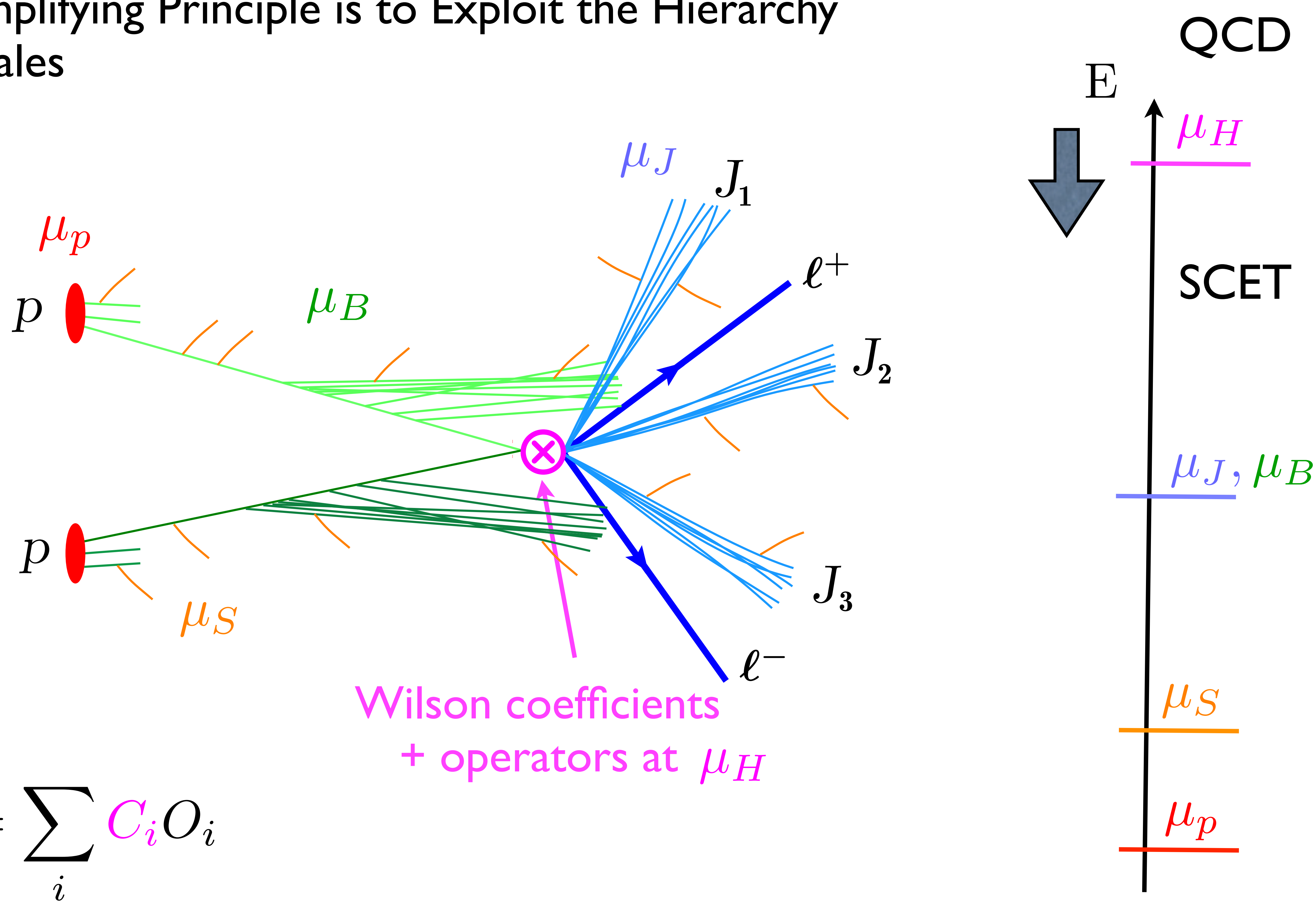




- dominant contributions from isolated regions of momentum space
- use subtractions rather than sharp boundaries to preserve symmetry


time
J_1
ℓ^+
J_2
p
p
u
u
d
J_3
ℓ^-
Higgs
Production

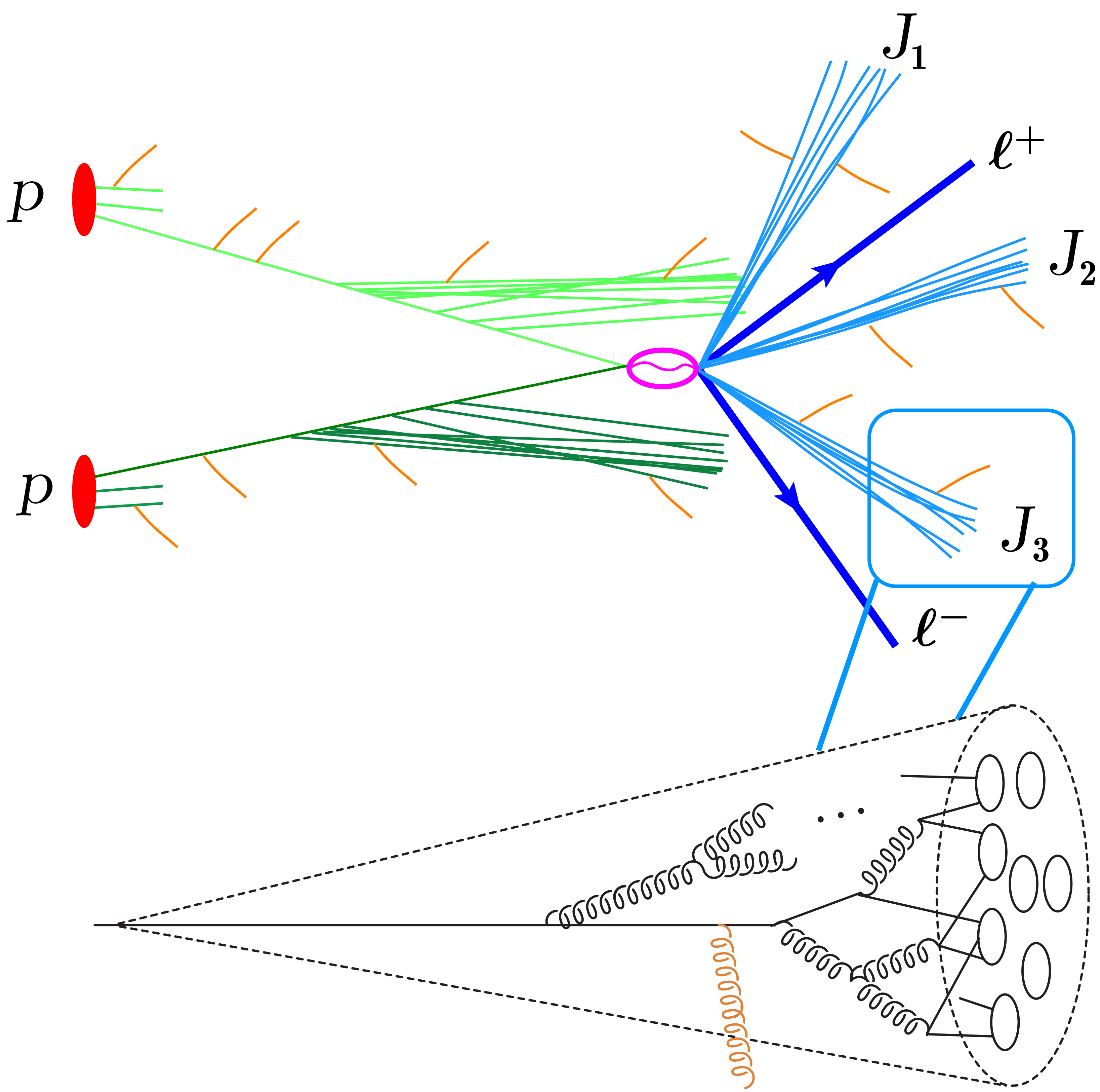
Key Simplifying Principle is to Exploit the Hierarchy of Scales



$$\mathcal{L} = \sum_i C_i O_i$$

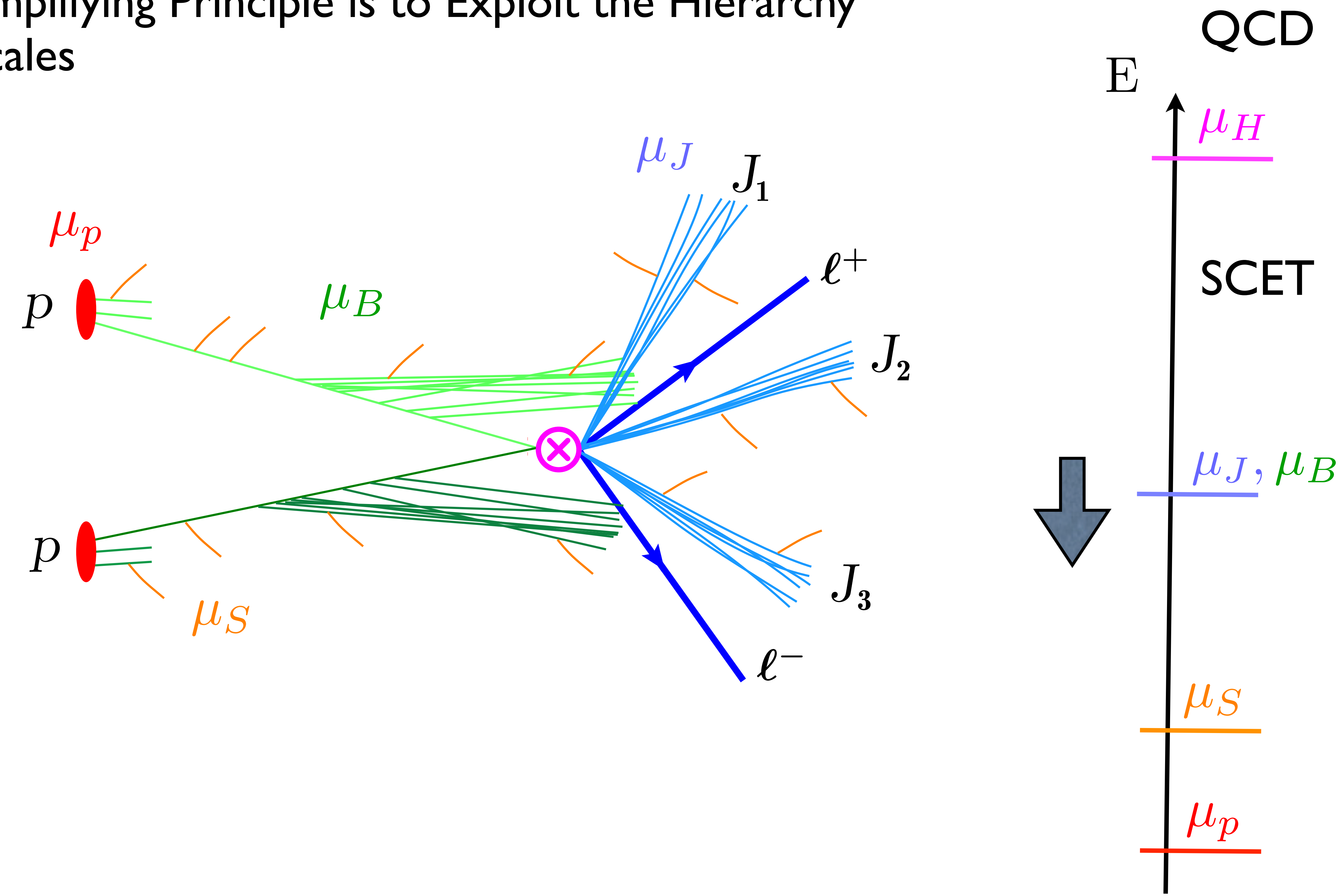
$$d\sigma = \int (\text{phase space}) \left| \sum_i C_i \langle O_i \rangle \right|^2 = \sum_j H_j \otimes (\text{longer distance dynamics})_j$$

Exclusive Jet Production with a Hard Interaction:

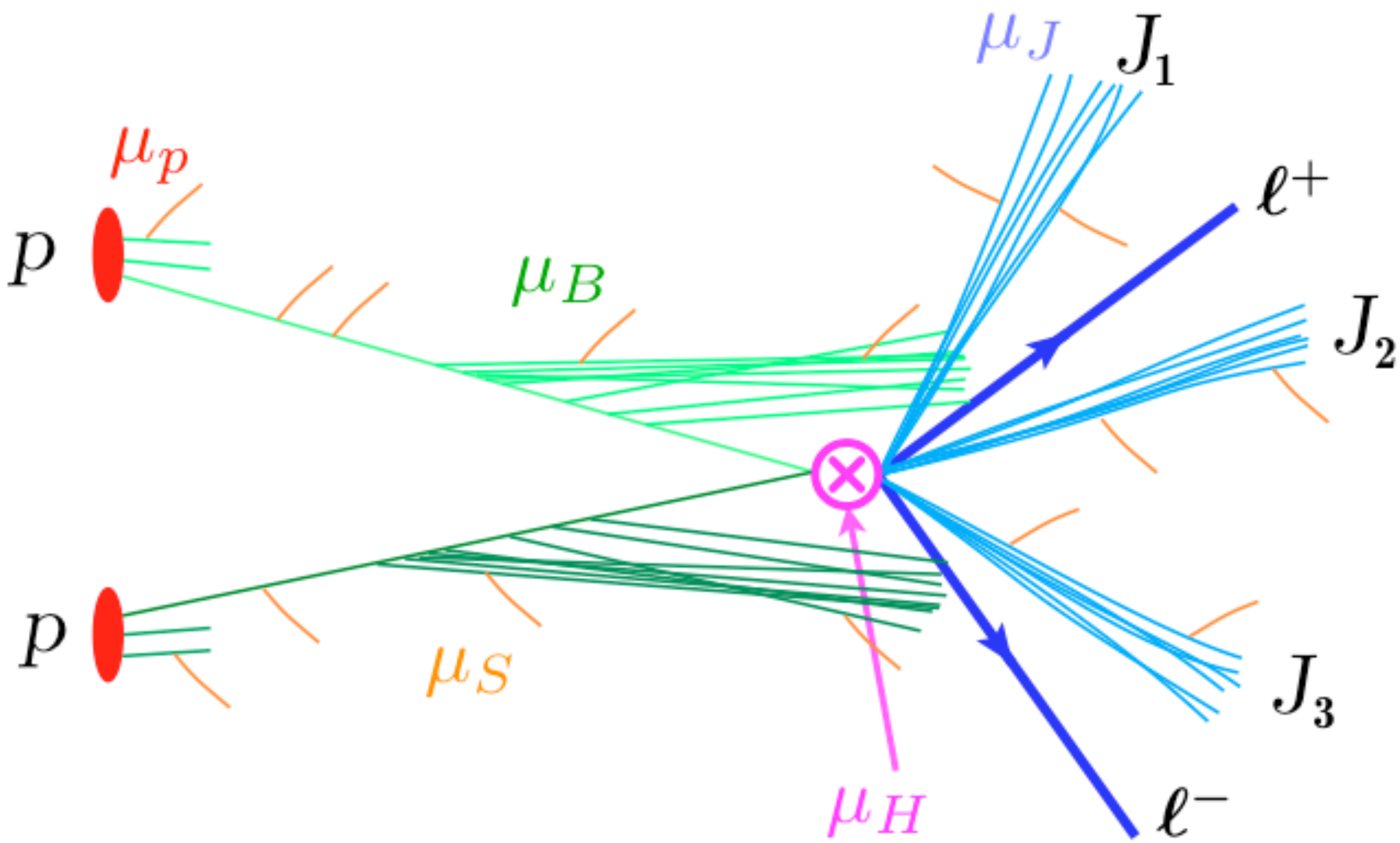


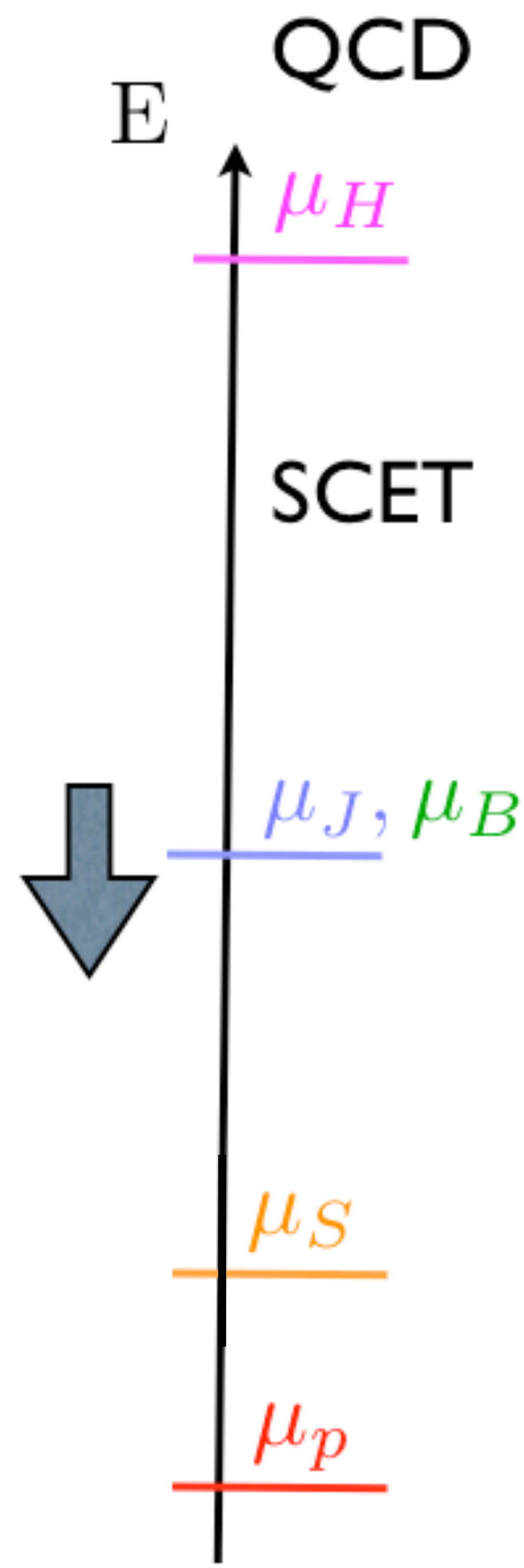
Quarks and Gluons
Form Jets

Key Simplifying Principle is to Exploit the Hierarchy of Scales



Hard-collinear factorization





Operators are built of building block fields:

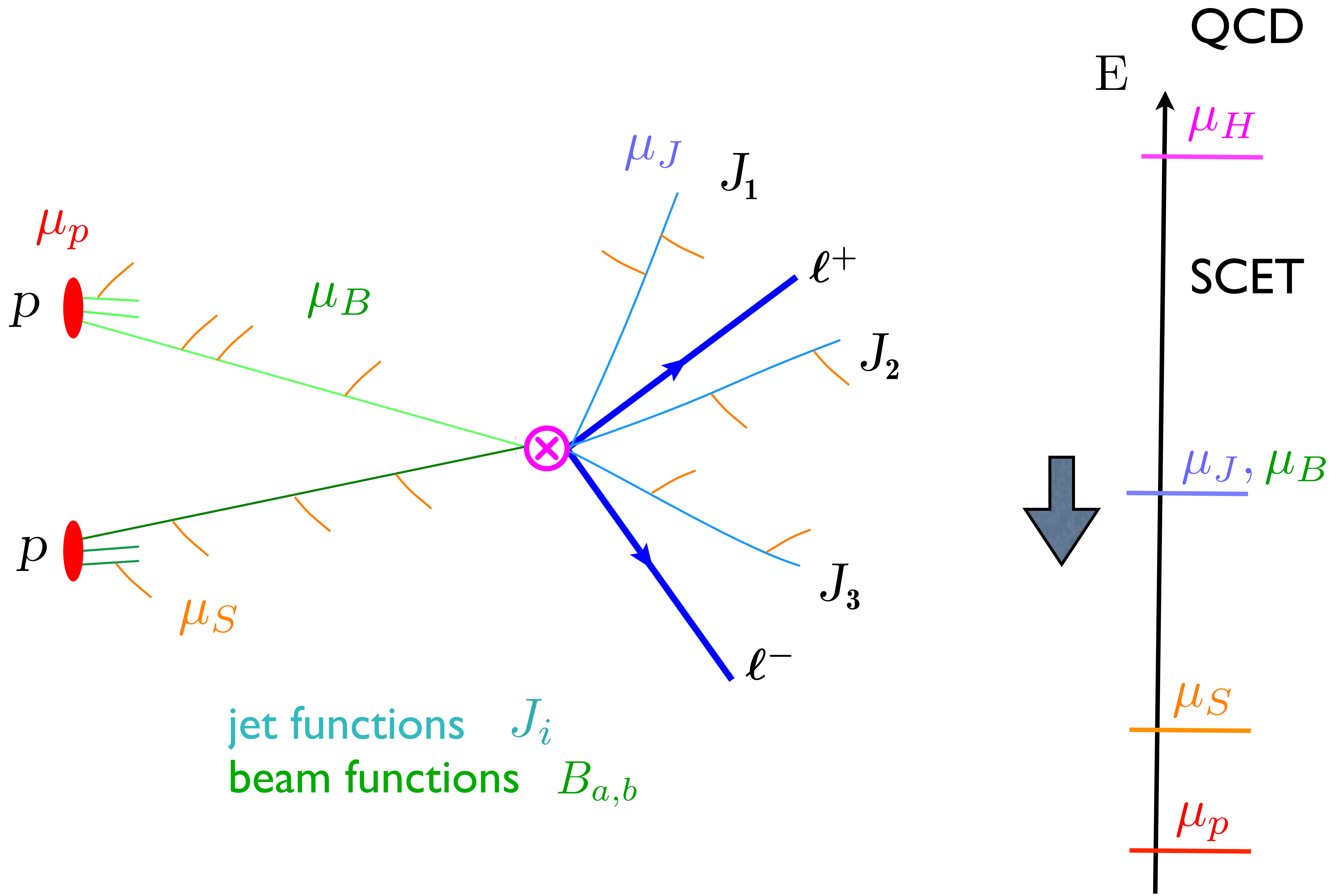
$$\mathcal{O} = (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})$$

$\chi_n = (W_n^\dagger \xi_n)$ "quark jet"

$\mathcal{B}_{n\perp}^\mu = [W_n^\dagger i D_\perp^\mu W_n]$ "gluon jet"

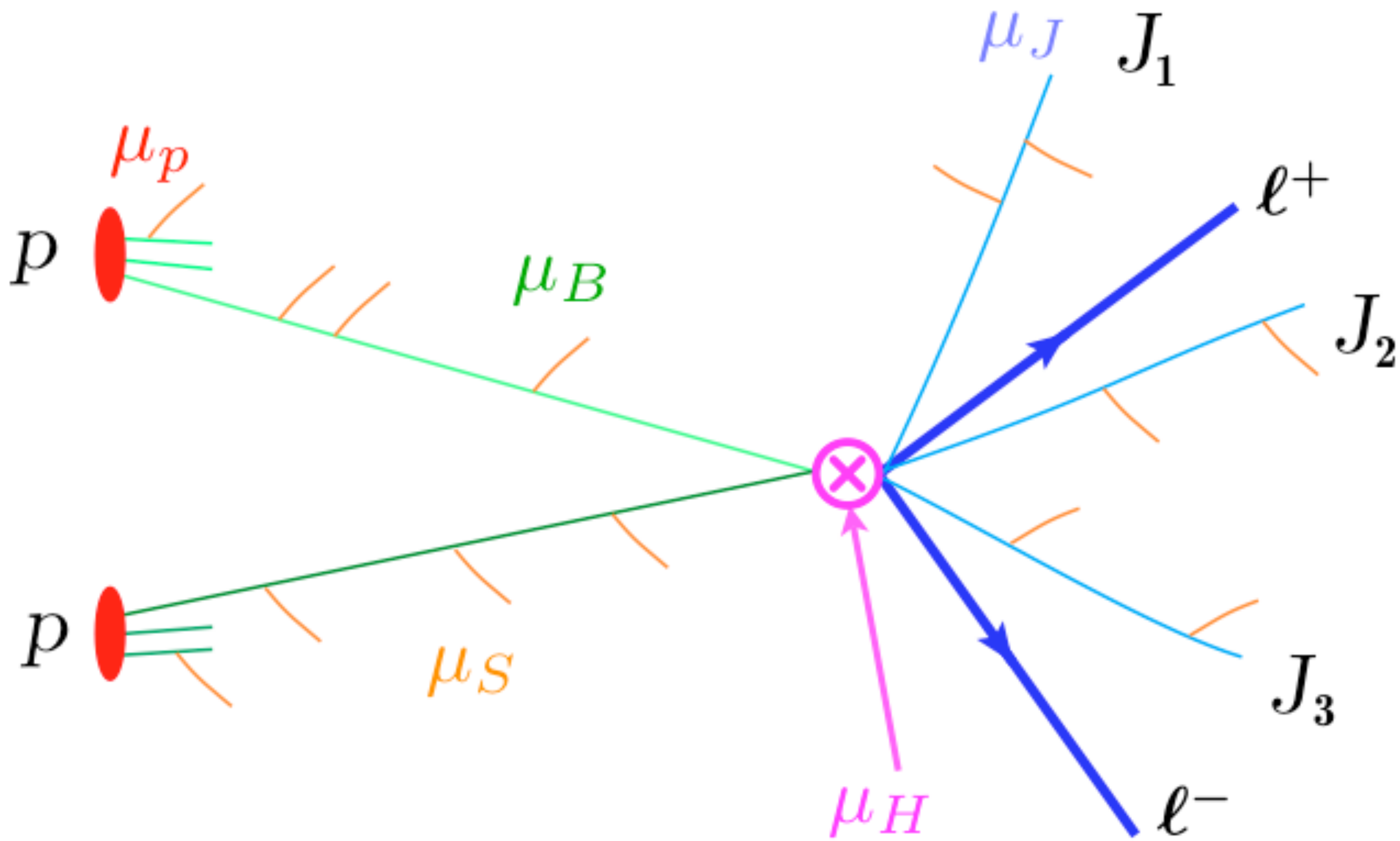
Wilson lines

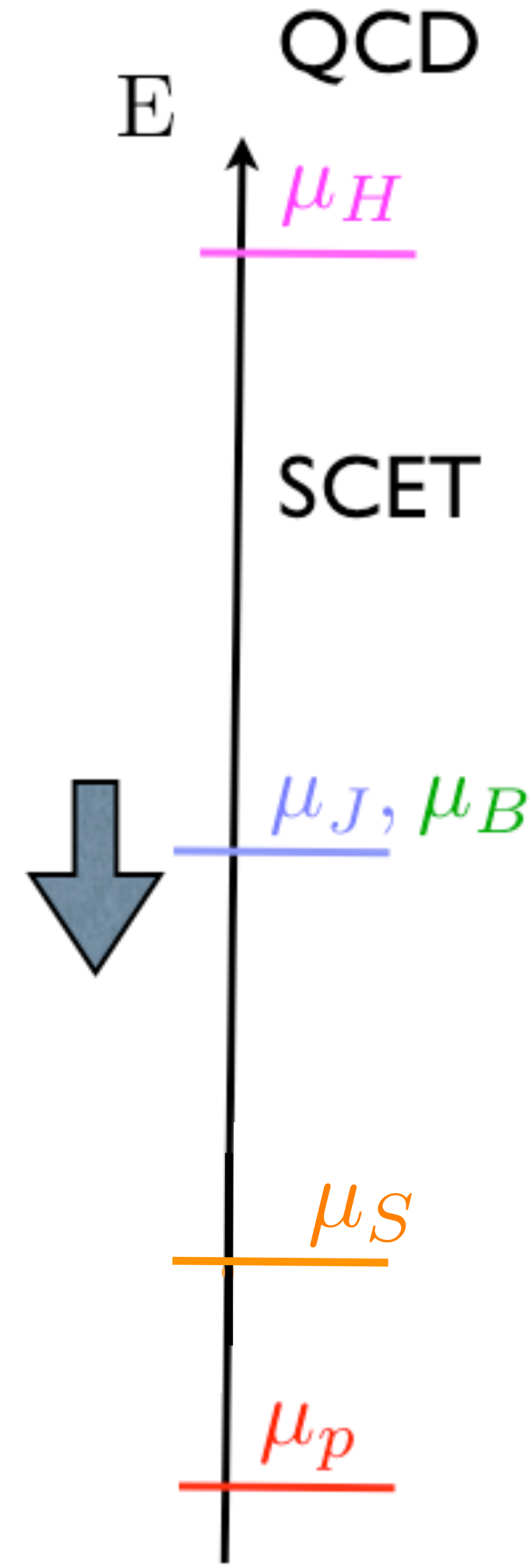
$$W_n = P \exp\left(ig \int_{-\infty}^{0} ds\, \bar{n} \cdot A_n(x + \bar{n}s)\right)$$



$$d\sigma = B_{a,b} \otimes H_j \otimes \prod_i J_i \otimes (\text{longer distance dynamics})$$

Soft-collinear factorization

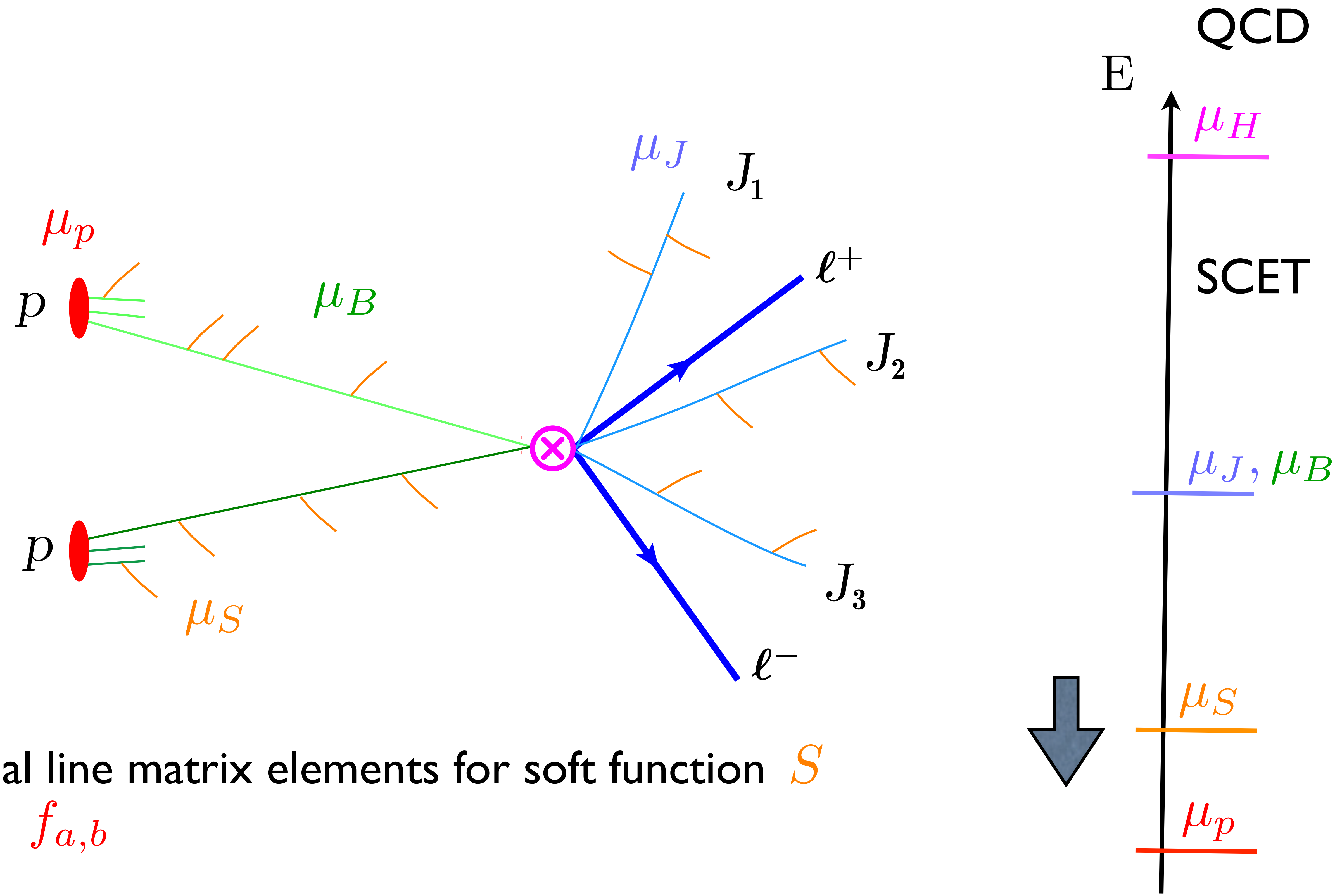




Soft radiation knows only about bulk properties of radiation in the jets

$$(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3})$$

Soft Wilson Lines

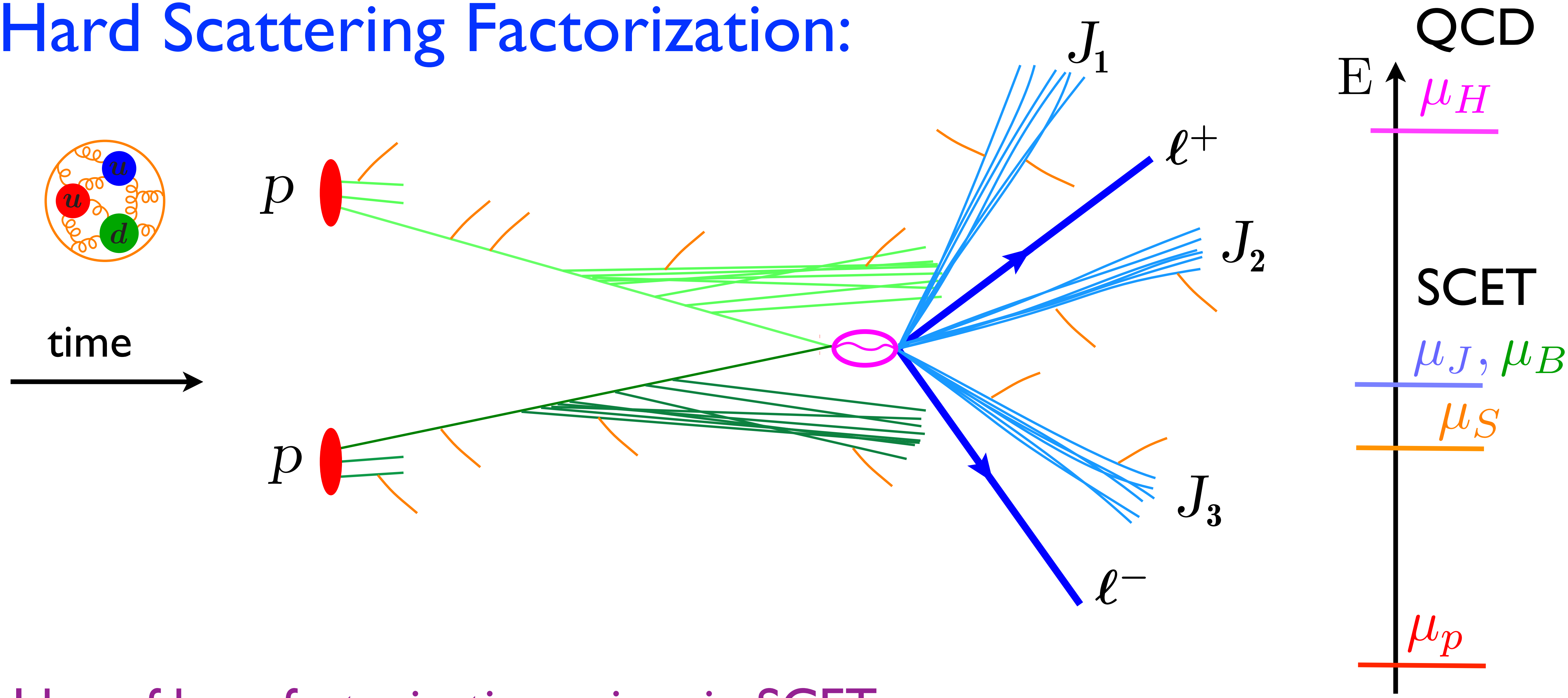


eikonal line matrix elements for soft function S

PDFs $f_{a,b}$

Factorization: $d\sigma = f_{a,b} \otimes \mathcal{I}_{a,b} \otimes H_j \otimes \prod_i J_i \otimes S$

Hard Scattering Factorization:



Idea of how factorization arises in SCET:

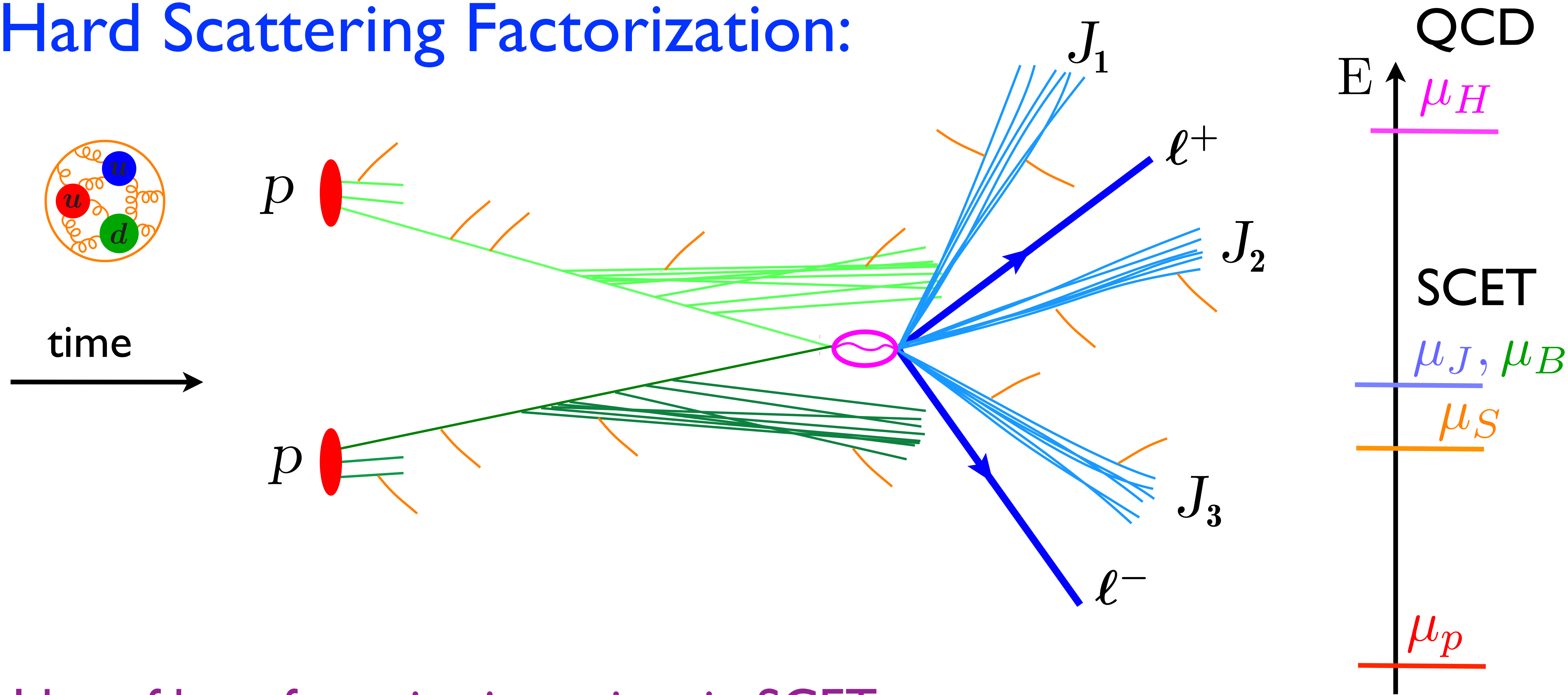
factorized Lagrangian: $\mathcal{L}^{(0)}_{\mathrm{SCET_{II}},\mathrm{S},\{\mathrm{n_i}\}} = \mathcal{L}^{(0)}_S(\psi_S, A_S) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i})$ ~~$+\mathcal{L}^{(0)}_G$~~

Glauber Lagrangian:

$$\mathcal{L}^{(0)}_G = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}^{iB}_n \frac{1}{\mathcal{P}^2_\perp} \mathcal{O}^{BC}_s \frac{1}{\mathcal{P}^2_\perp} \mathcal{O}^{jC}_{\bar{n}} + \sum_n \sum_{i,j=q,g} \mathcal{O}^{iB}_n \frac{1}{\mathcal{P}^2_\perp} \mathcal{O}^{j_n B}_s$$

Rothstein, IS

Hard Scattering Factorization:



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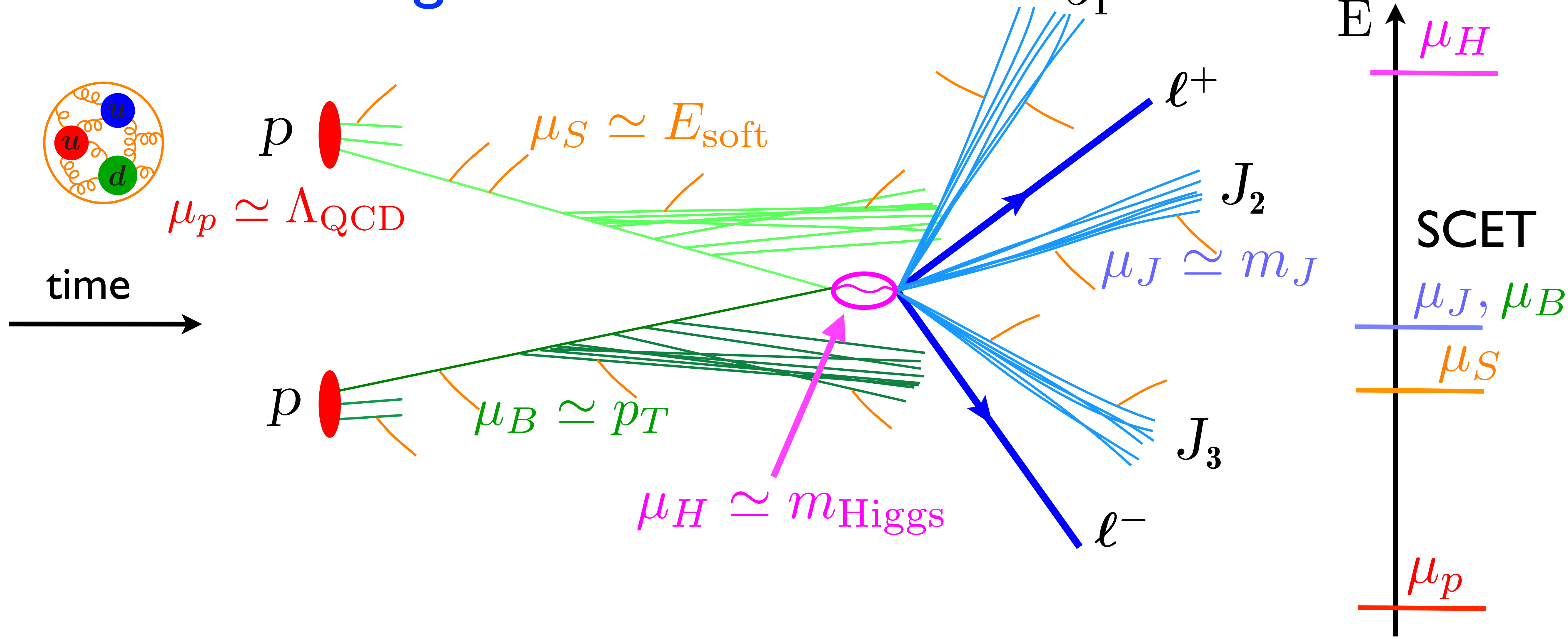
factorized Hard Ops: $C \otimes (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})(\mathcal{Y}_{n_a}\mathcal{Y}_{n_b}\mathcal{Y}_{n_1}Y_{n_2}Y_{n_3})$

factorized Measurement $\delta(\tau - \tau_{n_a} - \tau_{n_b} - \tau_{n_1} - \tau_{n_2} - \tau_{n_3} - \tau_s)$

→ factorized squared matrix elements defining jet, soft, … functions

Hard Scattering Factorization:



Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$ ← hadronization

$\mu_p \simeq \Lambda_{\rm QCD}$

(In some cases by Operators, or is power suppressed)

Perturbative: $\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$

Used to Sum Logs

Universal Functions:	μ_B beam	μ_H hard	μ_J jet	μ_S pert. soft

Examples of Factorization:

- Inclusive Higgs production $\quad pp \to \text{Higgs} + \text{anything}$

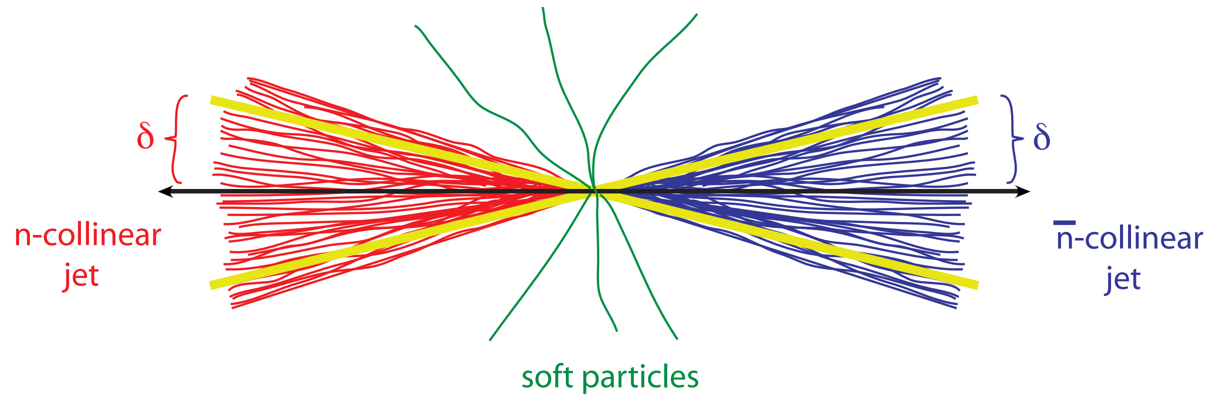
$$d\sigma = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu)\, H_{ij}^{\text{incl}}\Big(\frac{m_H e^{Y}}{E_{\text{cm}}\xi_a}, \frac{m_H e^{-Y}}{E_{\text{cm}}\xi_b}, m_H, \mu\Big)$$

(Collins, Soper, Sterman)

(PDFs contribute, No Glaubers, No Softs)

- Dijet production $\quad e^+e^- \to 2 \text{ jets}$

thrust $\quad \tau \ll 1$



$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, d\ell'\, J_T(Q^2\tau - Q\ell, \mu)\, S_T(\ell - \ell', \mu)\, F(\ell')$$

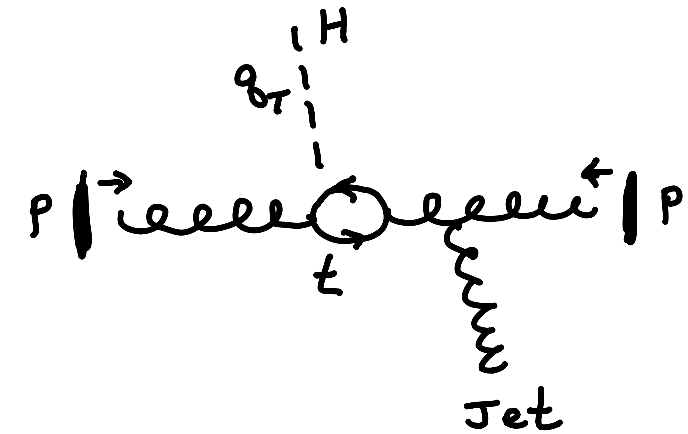
hard function — jet functions — perturbative soft function — non-perturbative soft function

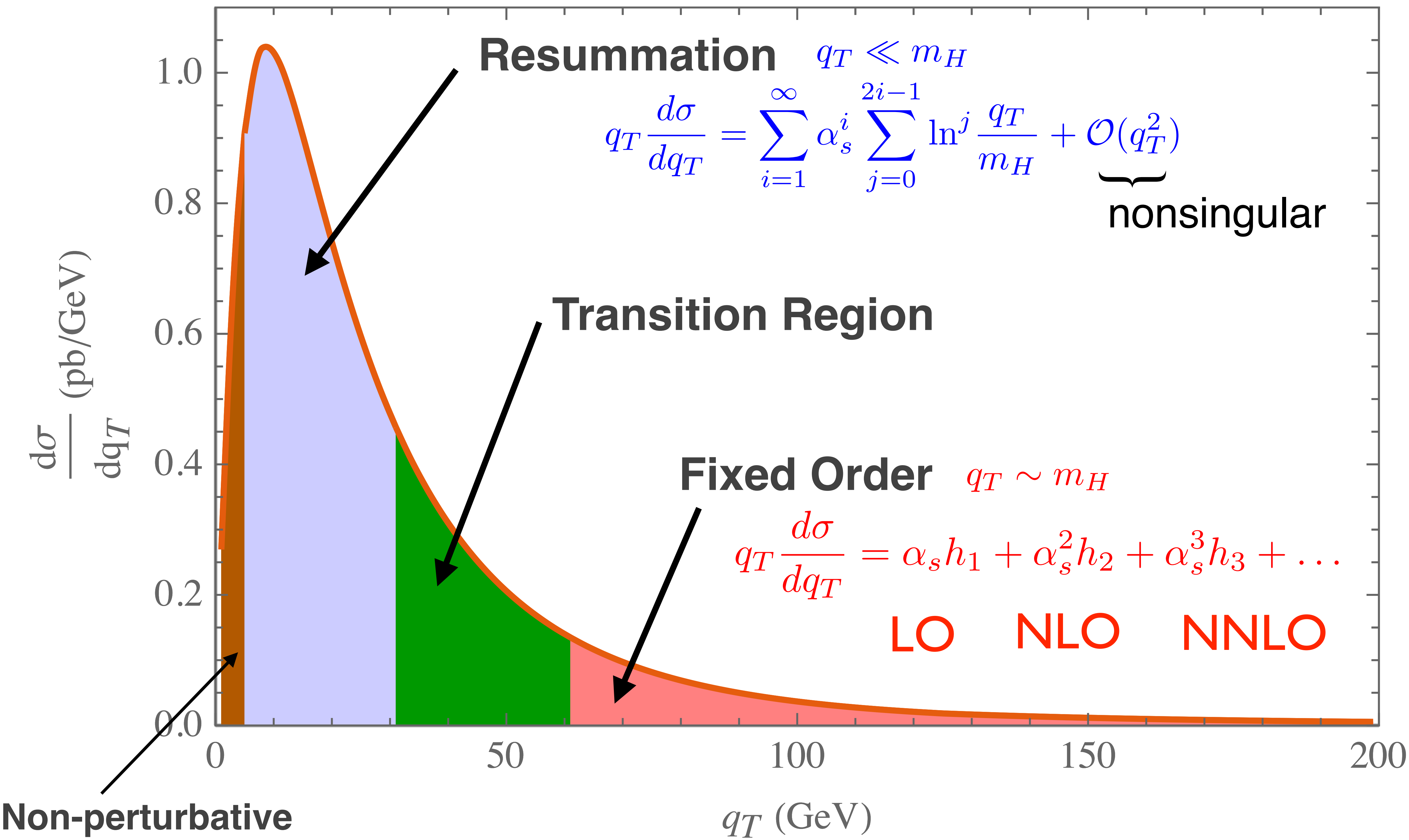
(No PDFs, No Glaubers, Softs contribute)

Higgs q_T spectrum

gluon fusion

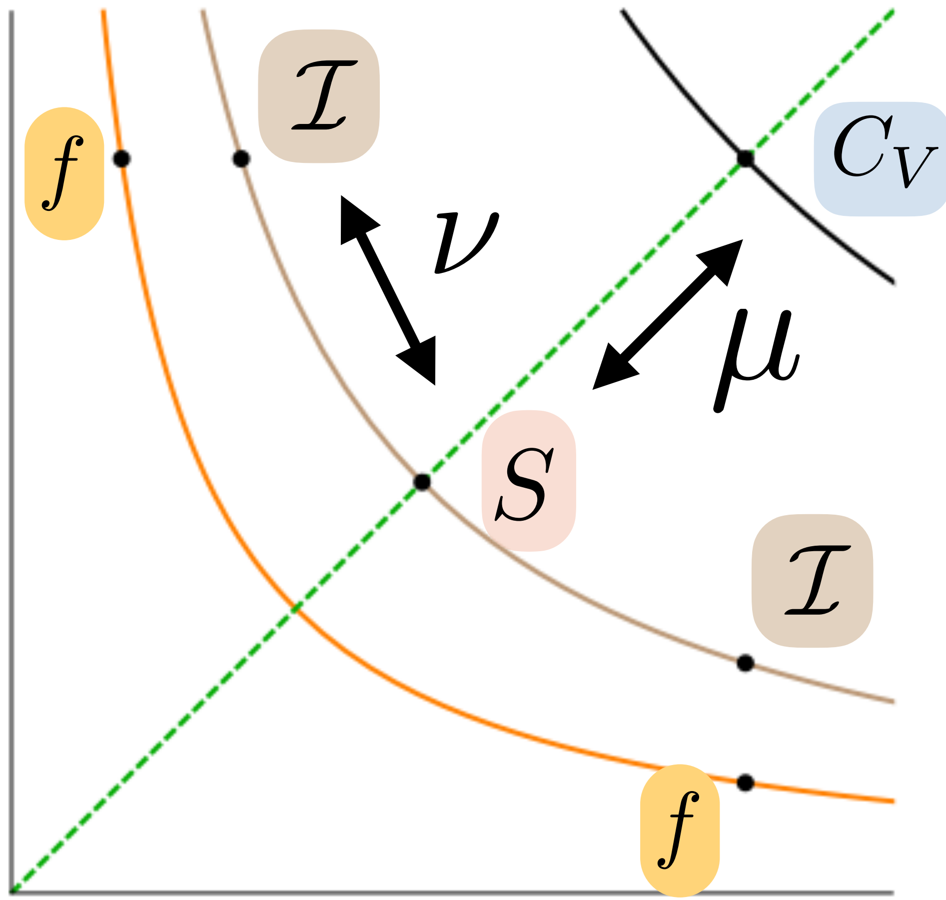
Higgs recoils against Jets





Small q_T factorization in SCET

$$\frac{d^2\sigma}{d^2\vec{q}_T} = \int dx_a dx_b\, \sigma_0 \delta\Big(x_a x_b - \frac{m_H^2}{s}\Big) \int \frac{d^2\vec{b}}{(2\pi)^2}\, e^{i\vec{b}\cdot\vec{q}_T}\, W(x_a, x_b, m_H, \vec{b}) + \frac{d^2\sigma}{d^2\vec{q}_T}\Big|_{\text{non-sing.}}$$



μ = invariant mass scale

ν = "rapidity" RGE scale

(dimension-1)

Chiu, Jain, Neill, Rothstein (1202.0814)

Perturbative Ingredients from the literature:

3 loop anomalous dimensions

3 loop hard and soft functions, 2 loop beam fn + 3 loop logs

In particular 3-loop rapidity anom.dim calculation:

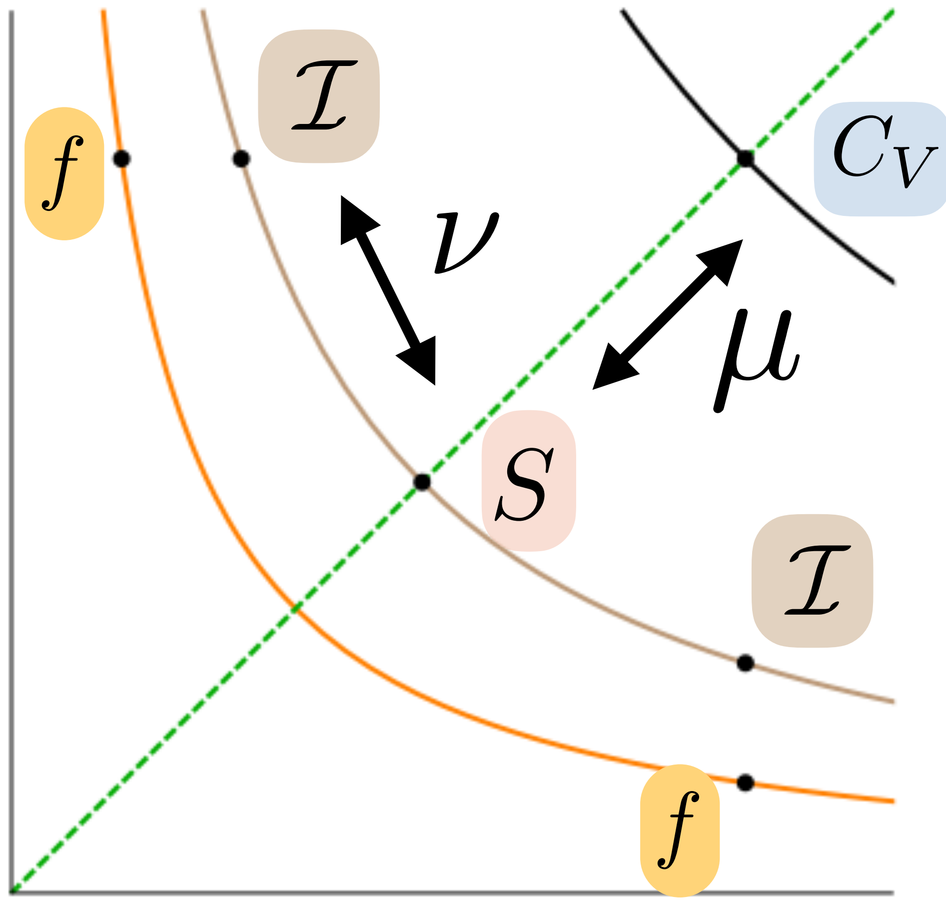
Li, Neill, Zhu (1604.00392, 1604.01404)

$$W(x_a, x_b, m_H, \vec{b}) = \Big|C_V\Big(m_t, m_H, \mu\Big)\Big|^2 S(\vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_1}(x_a, Q, \vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_2}(x_b, Q, \vec{b}, \mu, \nu)$$

$$B^{\alpha\beta}_{g/N}(x, Q, \vec{b}, \mu, \nu) = \sum_k \int \frac{d\xi}{\xi}\, \mathcal{I}^{\alpha\beta}_{gk}\Big(\frac{x}{\xi}, \vec{b}, \mu, \nu\Big) f_{k/N}(\xi, \mu) + \mathcal{O}\big(\Lambda^2_{\rm QCD}\vec{b}^{\,2}\big)$$

Small q_T factorization in SCET

$$\frac{d^2\sigma}{d^2\vec{q}_T} = \int dx_a dx_b\, \sigma_0 \delta\left(x_a x_b - \frac{m_H^2}{s}\right) \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{q}_T} W(x_a, x_b, m_H, \vec{b}) + \frac{d^2\sigma}{d^2\vec{q}_T}\bigg|_{\text{non-sing.}}$$



Single Scale Functions:

hard function	$\ln \frac{Q^2}{\mu^2}$	
beam function	$\ln(b^2\mu^2)$	$\ln \frac{Q^2}{\nu^2}$
soft function	$\ln(b^2\mu^2)$	$\ln(b^2\nu^2)$

$$W(x_a, x_b, m_H, \vec{b}) = \left|C_V\left(m_t, m_H, \mu\right)\right|^2 S(\vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_1}(x_a, Q, \vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_2}(x_b, Q, \vec{b}, \mu, \nu)$$

Resummation:

$$\ln W = L \sum_k (\alpha_s L)^k + \sum_k (\alpha_s L)^k + \alpha_s \sum_k (\alpha_s L)^k + \alpha_s^2 \sum_k (\alpha_s L)^k$$

LL NLL NNLL N3LL

$$L = \ln(m_H b)$$

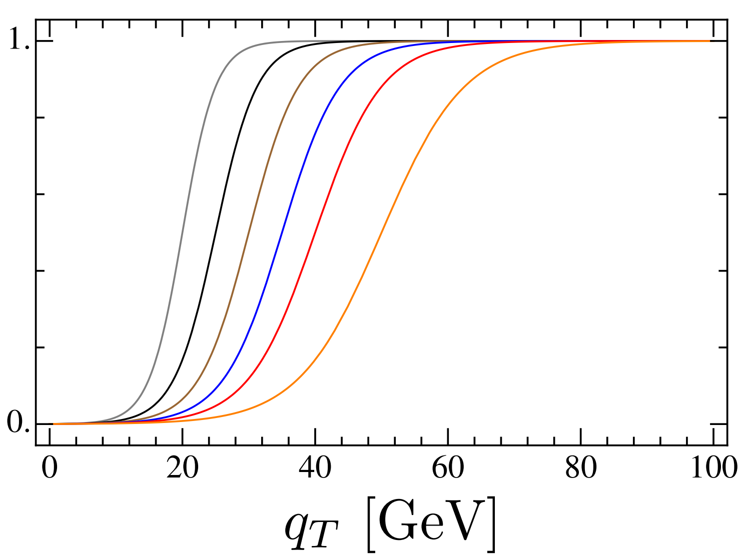
$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{\text{resum}}}{dq_T} + \frac{d\sigma^{\text{FO}}}{dq_T} - \frac{d\sigma^{\text{sing}}}{dq_T}$$

resummation | fixed order | singular (overlap)

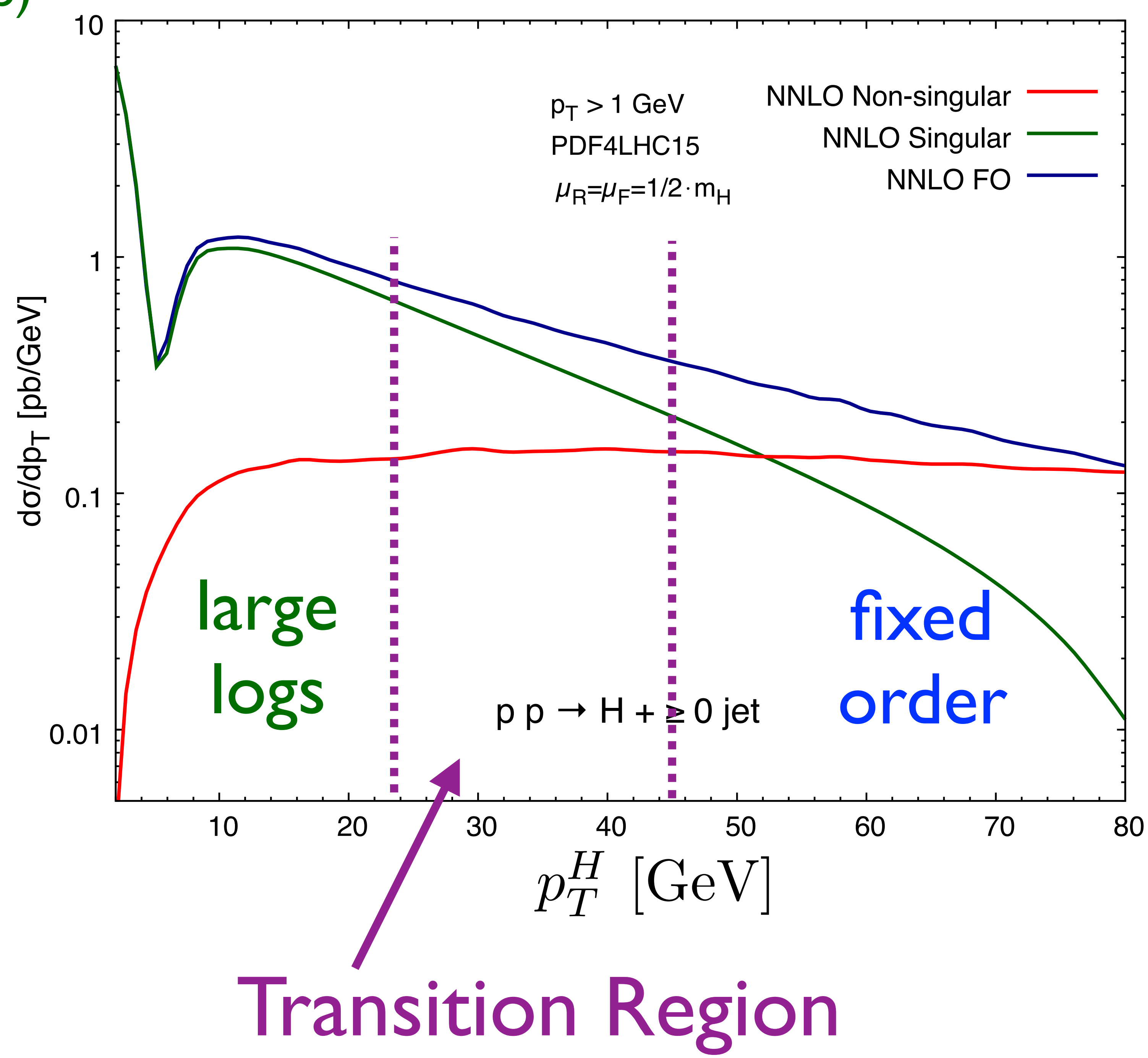
(fixed order − singular) = nonsingular

use profiles to smoothly turn off resummation

$$\mu_i(b, q_T),\ \ \nu_i(b, q_T)$$

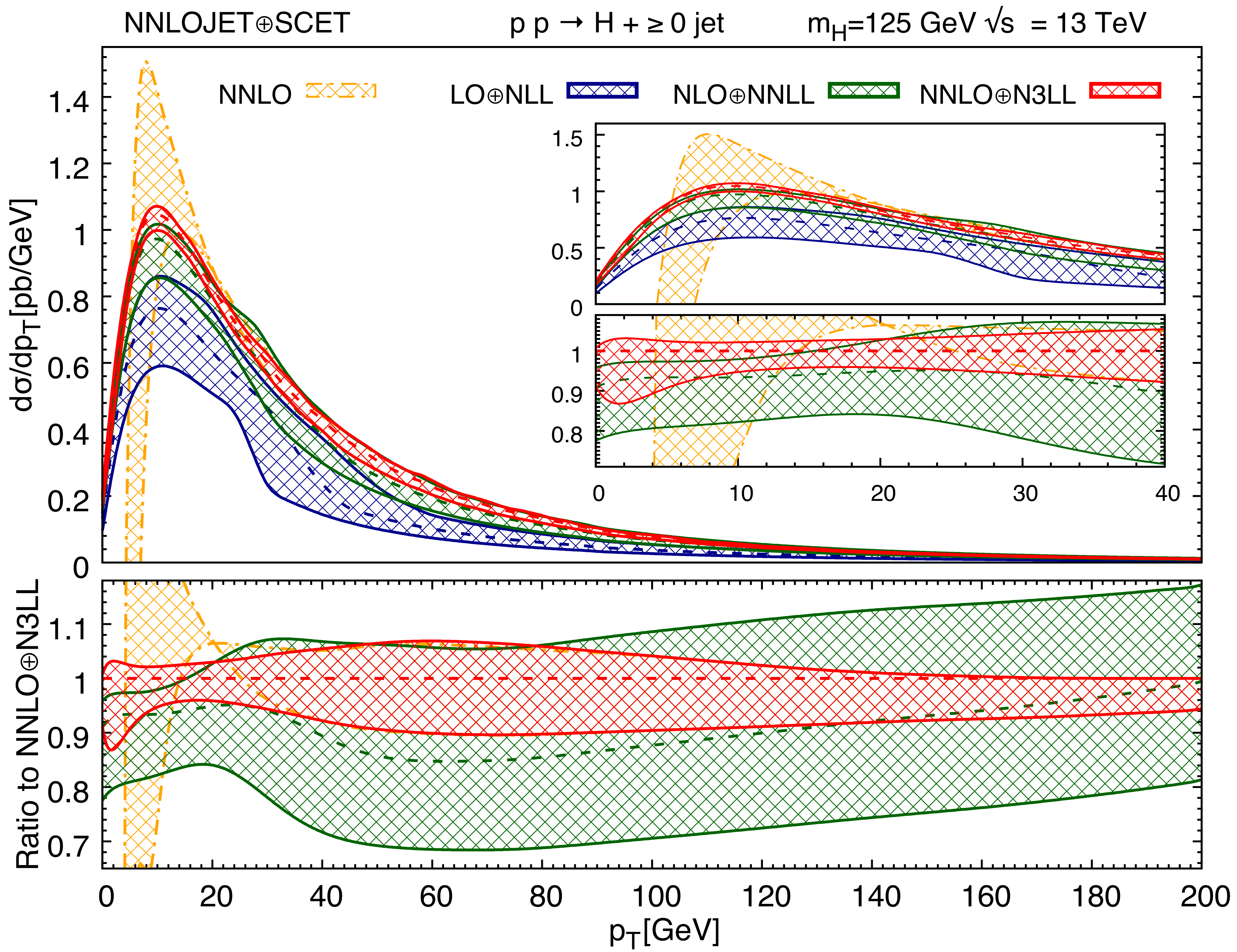


guaranteed that shape dependence only effects terms beyond N3LL+NNLO



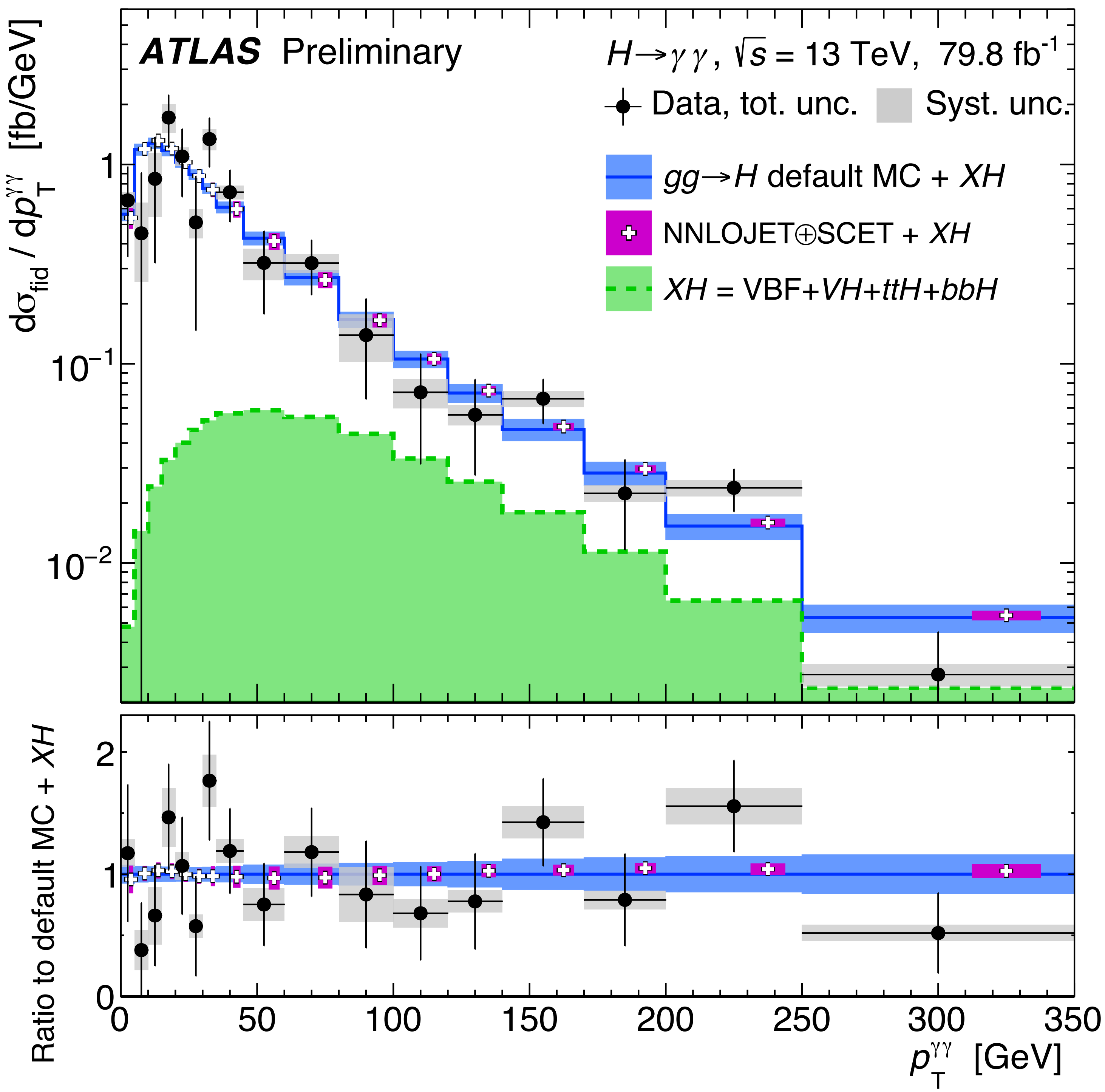
New N3LL+NNLO result

Chen, Gehrmann, Glover, Huss,
Li, Neill, Schulze, IS, Zhu (1805.00736)



Compared to qT spectrum from ATLAS with $H \to \gamma\gamma$

ATLAS-CONF-2018-028 July 2018



Jet Cross Sections and Distributions

Inclusive Jet Cross Sections

$$pp \longrightarrow \text{jet} + \text{anything}$$

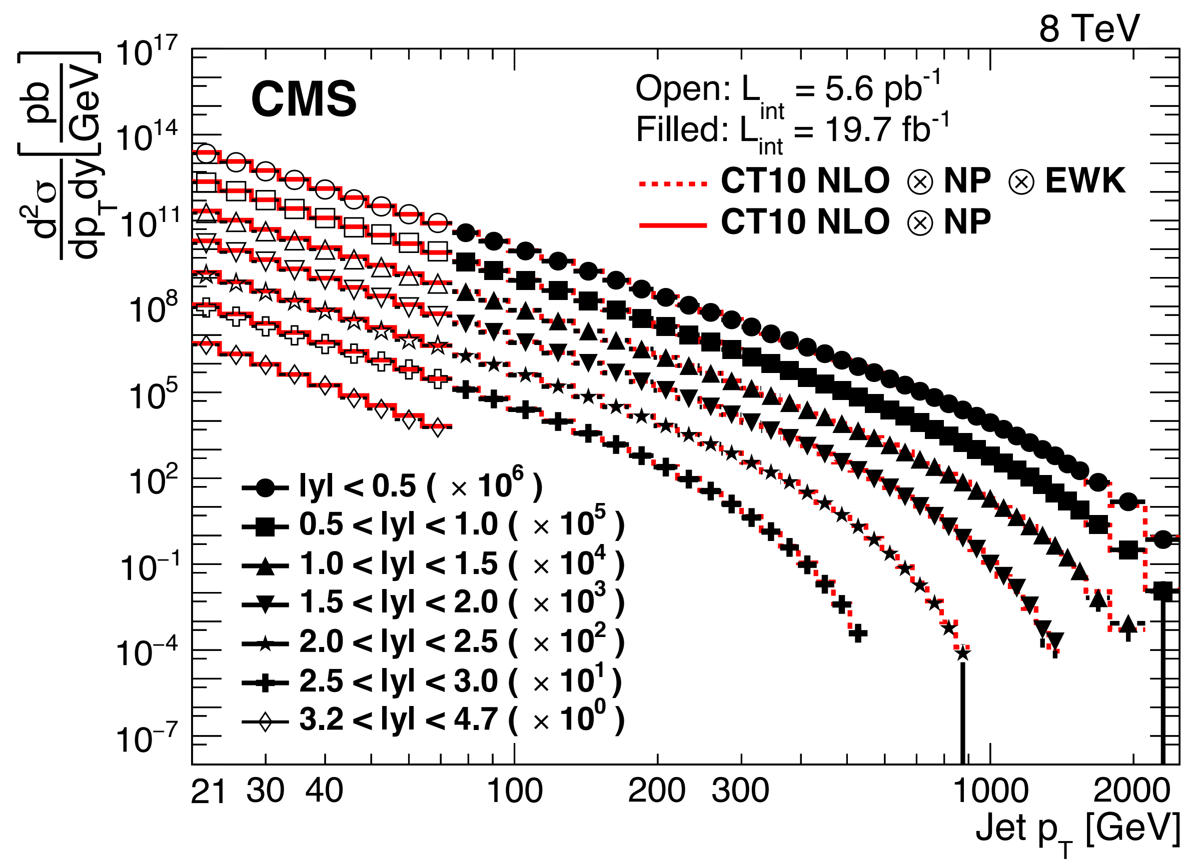
one or more jets

measure jet kinematics

(pT, rapidity)

$$\frac{d\sigma}{dp_T dy} = \sum_{ij} \frac{d\hat{\sigma}_{ij}}{dp_T dy} \otimes f_i \otimes f_j$$

constrain PDF, measure α_s



Exclusive Jet Cross Sections

$$pp \longrightarrow \text{N-jets}$$

Theoretically have different factorization formula

Jet Bins:

$H \longrightarrow WW$
- 0–jets
- 1–jet
- 2–jets

$H \longrightarrow \tau\tau$
- 0–jets
- 1–jet
- 2–jets

$H \longrightarrow \gamma\gamma$

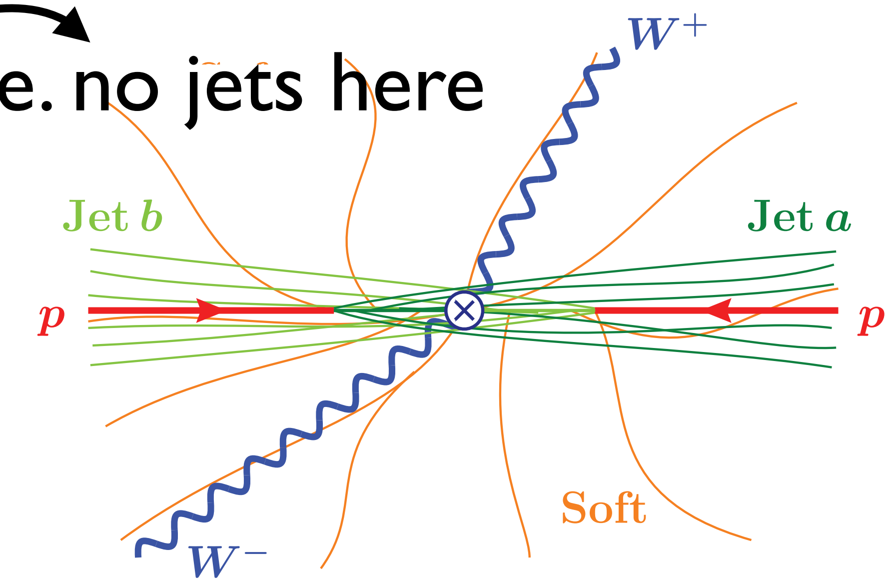
$H \longrightarrow ZZ$
- inclusive
- 2–jets

Control backgrounds and enhance sensitivity

Jet Veto

eg. $pp \to H \to WW + 0\text{-jets}$

restrict jets $p_T^{\rm jet} \le p_T^{\rm cut}$

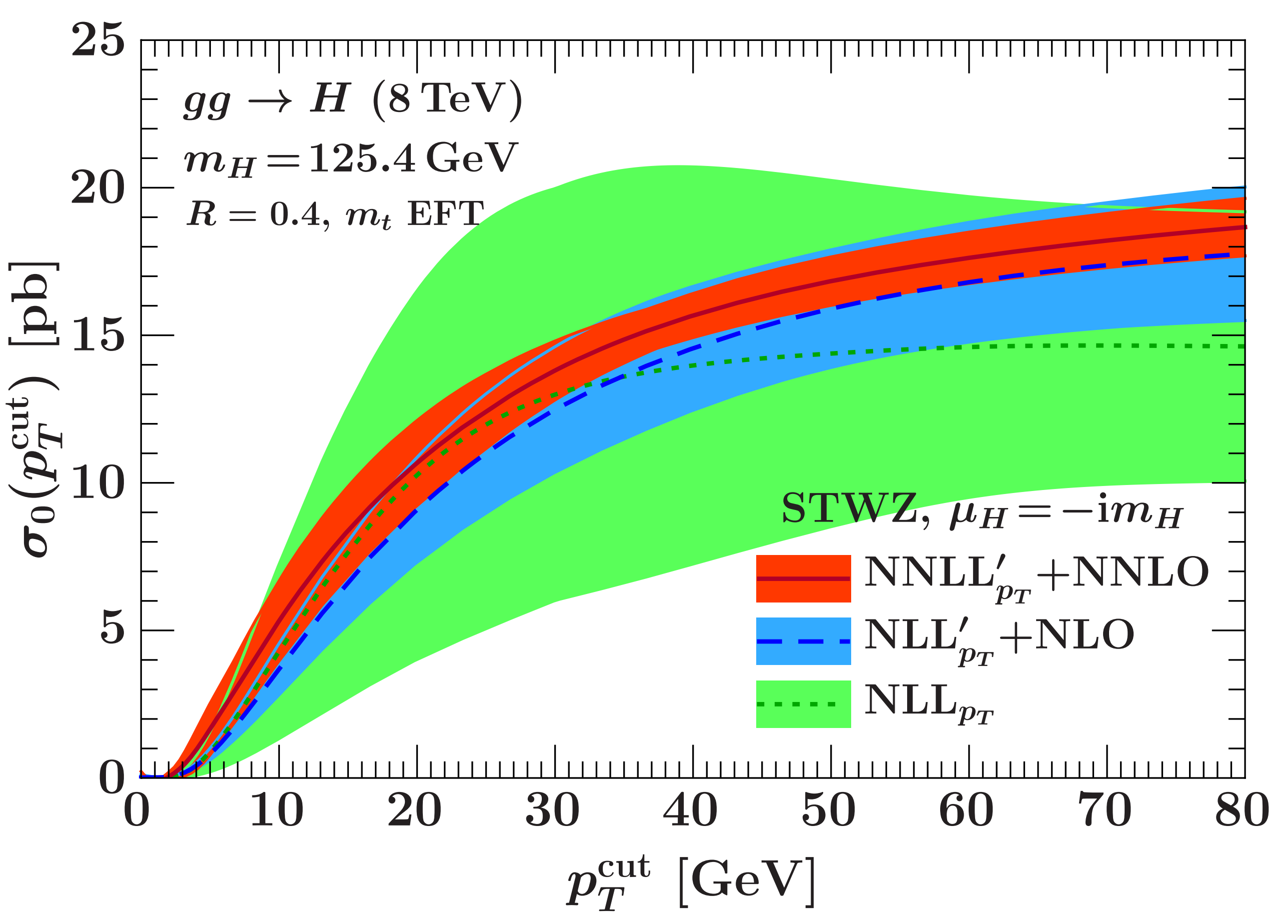


Resummation of jet-veto logs

Factorization: $p_T^{\rm cut} \ll m_H$ $\quad \sigma_0(p_T^{\rm cut}) = H_{gg}(m_H) \times [B_g(m_H, p_T^{\rm cut}, R)]^2 \times S_{gg}(p_T^{\rm cut}, R)$

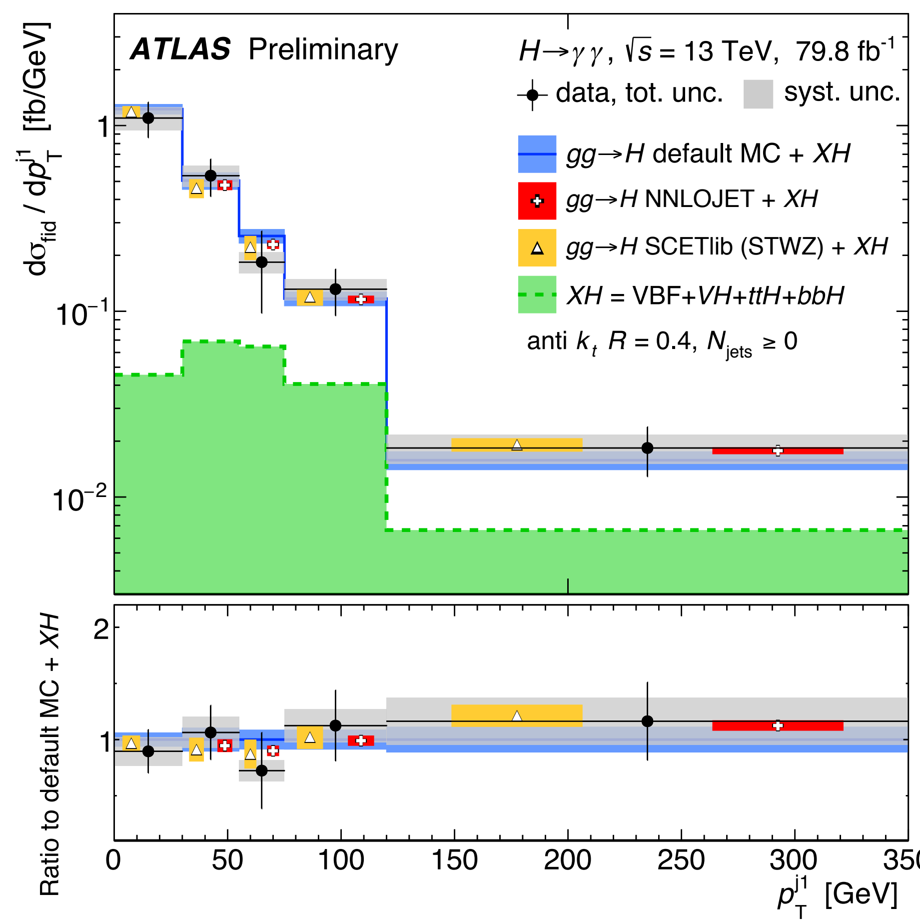
$B_g = \mathcal{I}_{gj}(m_H, p_T^{\rm cut}, R) \otimes f_j$

Banfi, Monni, Salam, Zanderighi (1206.4998, 1203.5773)
Becher, Neubert (1205.3806, 1307.0025)
Stewart, Tackmann, Walsh, Zuberi (STWZ) (1307.1808)



$$\frac{d\sigma_{\ge 1}}{dp_T^{j1}} = -\frac{d\sigma_0(p_T^{j1})}{dp_T^{j1}}$$

ATLAS-CONF-2018-028



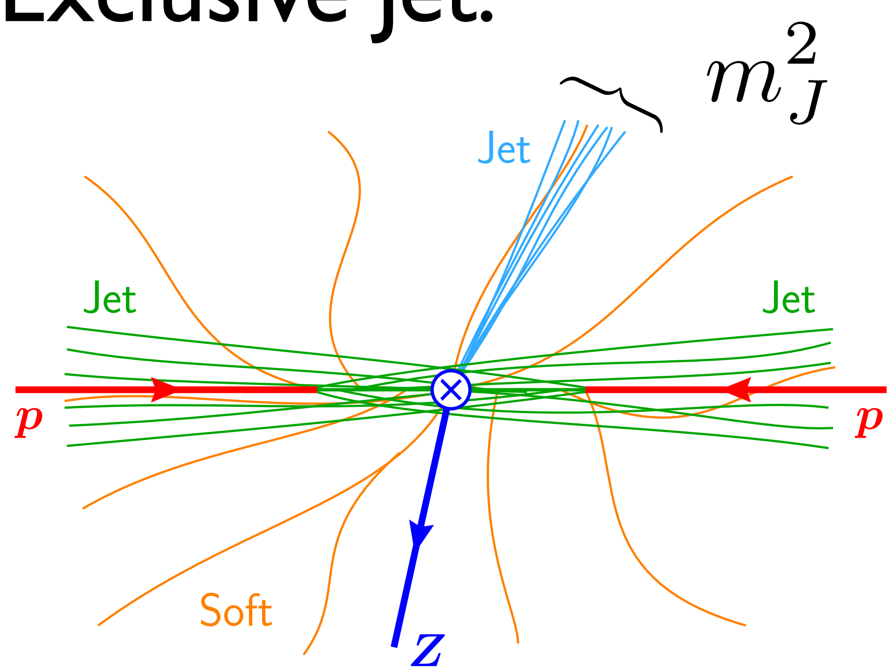
Chen, Cruz-Martinez Gehrmann, Glover, Jaquier (NNLOJET) (1408.5325,1607.08817)

Jet Mass

$$m_J^2 = \Big(\sum_{i\in J} p_i^\mu\Big)^2$$

depends on: soft radiation, jet radius R, jet algorithm, hard process (q vs. g)

Exclusive Jet:



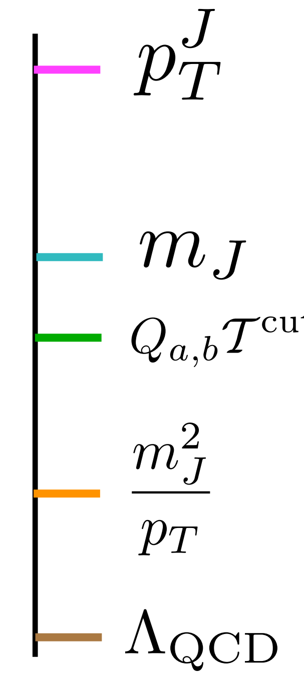
eg. $R \sim 1$

$$\frac{d\sigma(\mathcal{T}^{\rm cut})}{dm_J d\Phi_J} = \sum_\kappa H_{\kappa_H}^{\rm 1jet\text{-}Z}(\Phi_J) \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b)$$

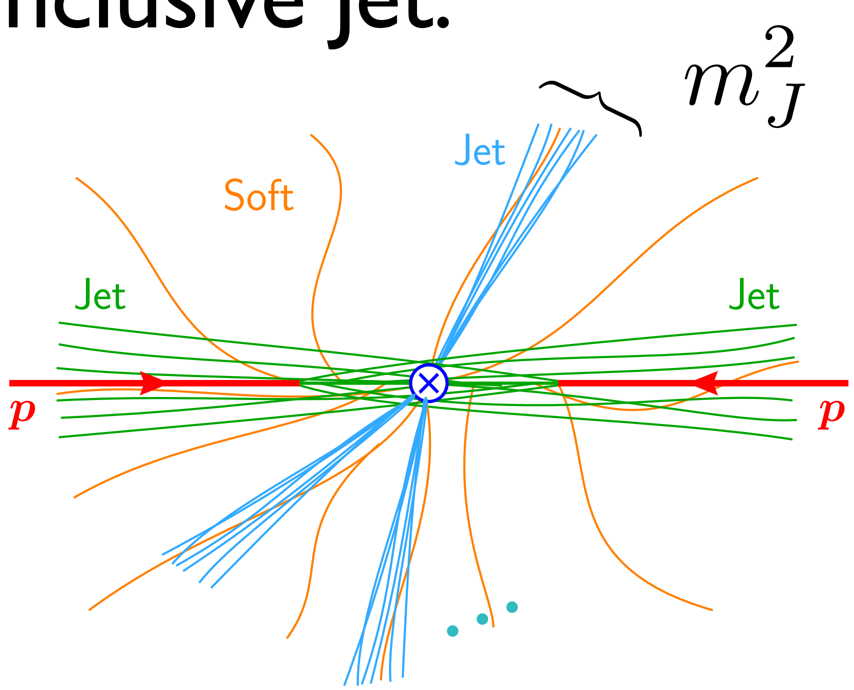
$$\times \int ds_J\, J_{\kappa_J}(s_J) S_1^\kappa\Big(\mathcal{T}^{\rm cut} - \frac{t_a}{Q_a} - \frac{t_b}{Q_b}, \frac{m_J^2 - s_J}{Q_1}, \{\hat{q}_i \cdot \hat{q}_j\}, R\Big)$$

$$\otimes F_{\kappa_\Lambda}$$

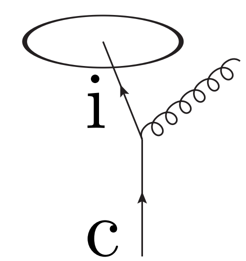
IS, Tackmann, Waalewijn (1302.0846)



Inclusive Jet:



eg. $\frac{R}{2} \ll 1$

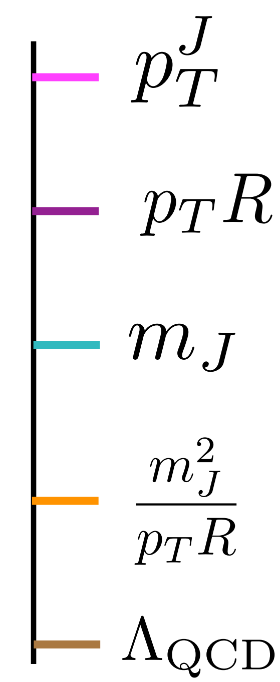


$$\frac{d\sigma}{dm_J d\Phi_J} = \sum_{a,b,c} f_a f_b \otimes H_{ab}^c(x_{a,b}, \eta, p_T/z)$$

$$\otimes \sum_i \mathcal{H}_{c\to i}(z, p_T R) J_i(m_J) \otimes S_i(m_J^2/p_T, R)$$

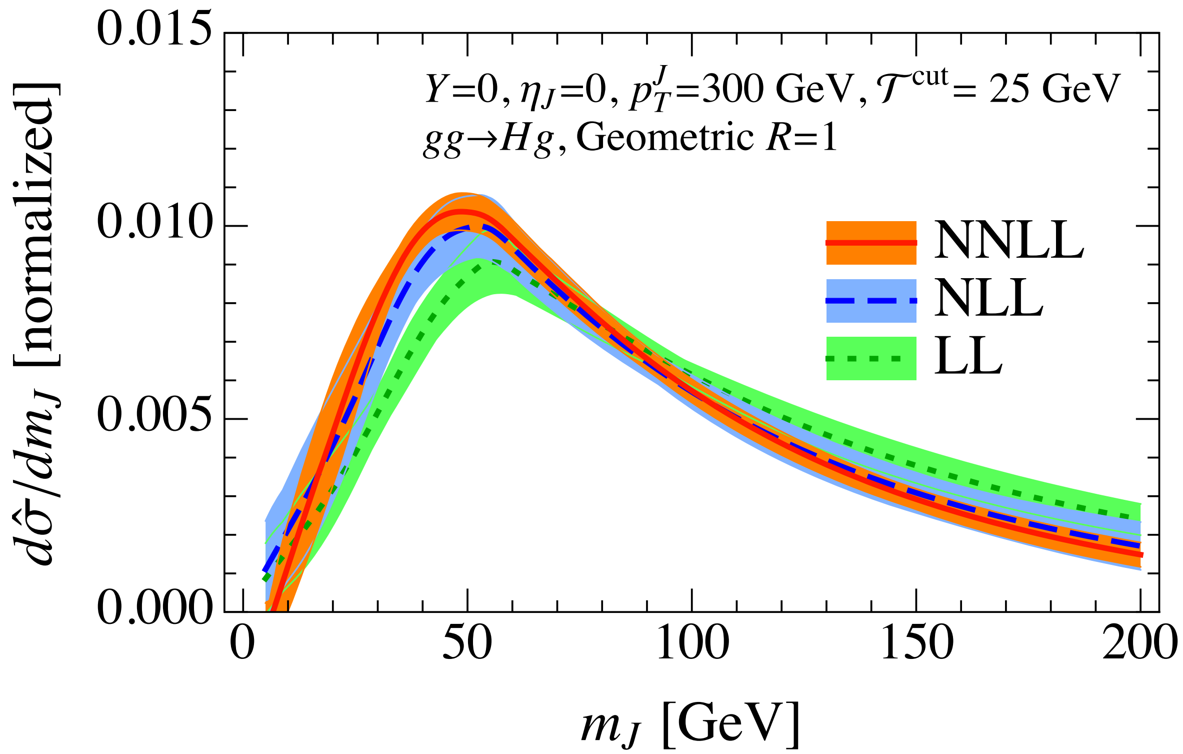
$$\otimes F_i$$

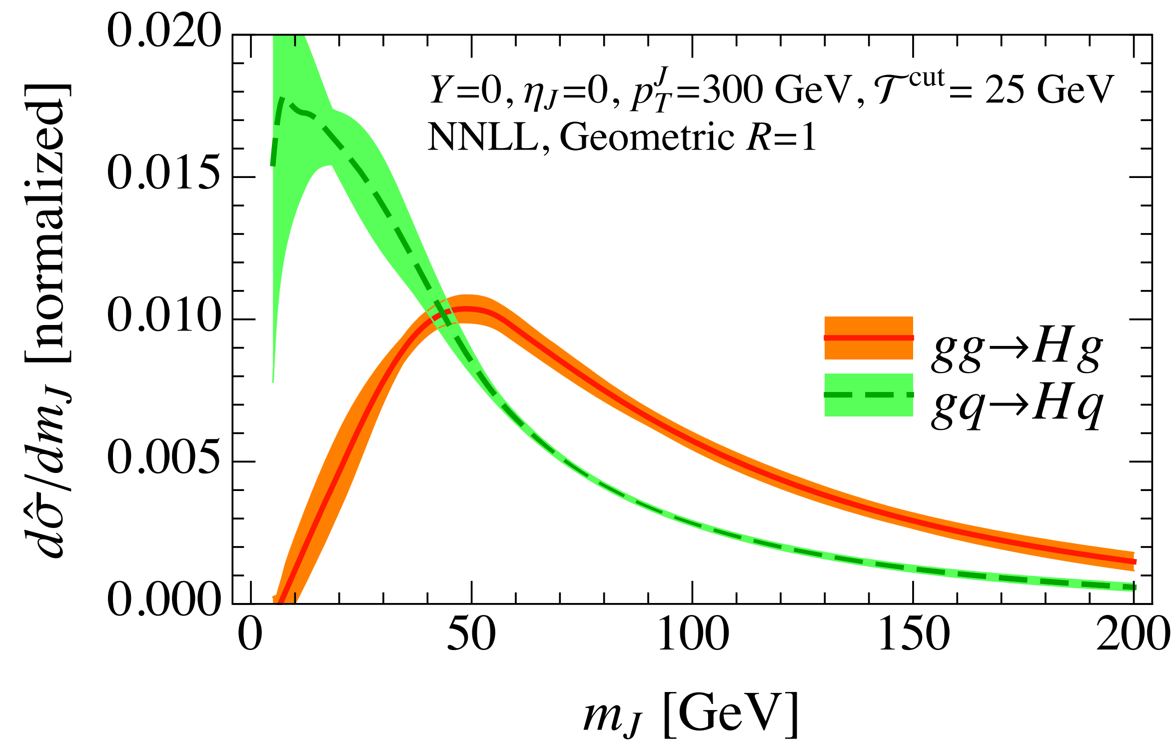
Kang, Ringer, Vitev (1701.05839)
Kang, Lee, Liu, Ringer (1803.03645)



NNLL results are available for exclusive jets

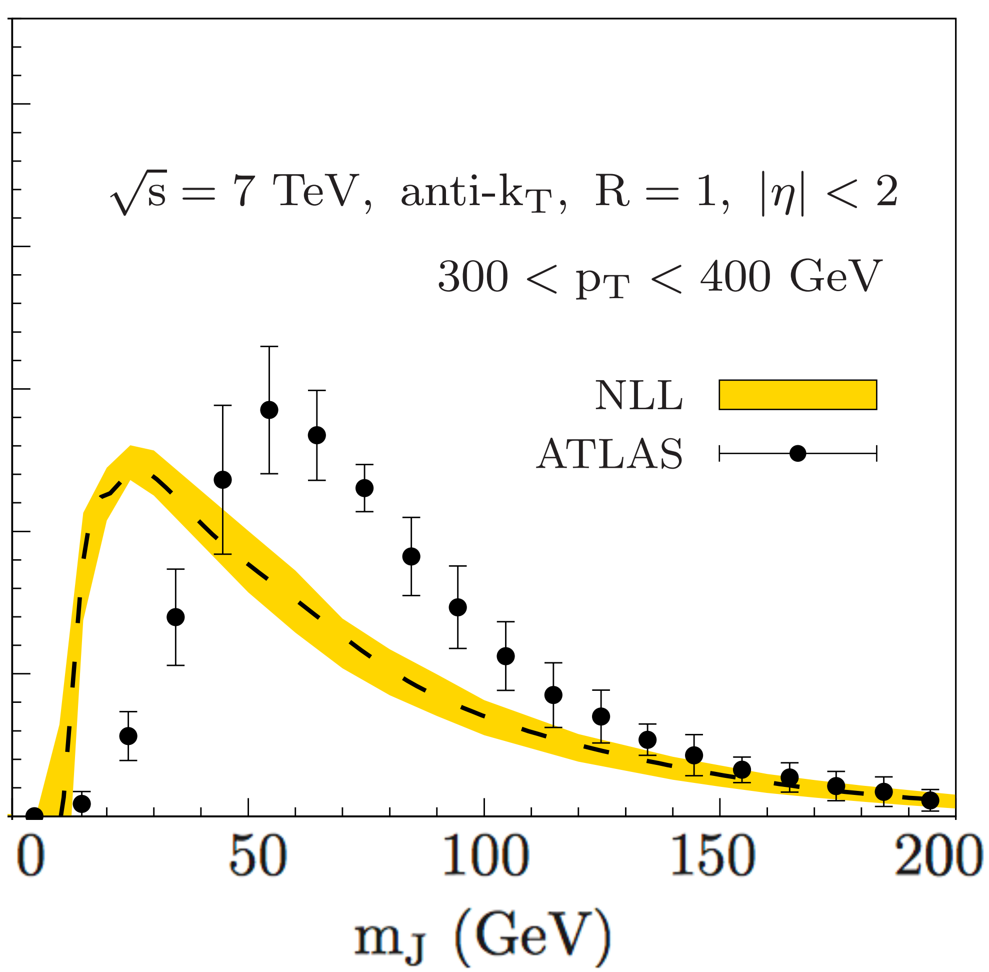
Jouttenus, IS, Tackmann, Waalewijn (1302.0846)

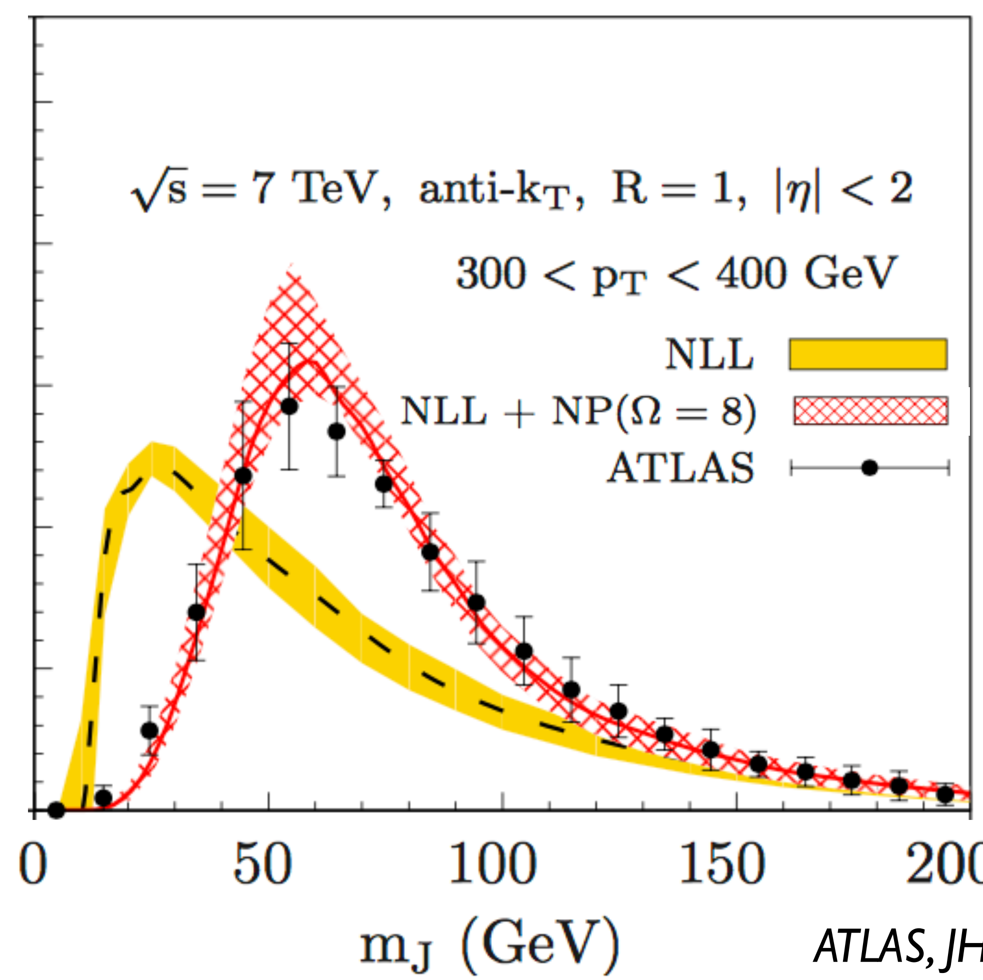




NLL results and Data are available for inclusive jets

Kang, Lee, Liu, Ringer (1803.03645)





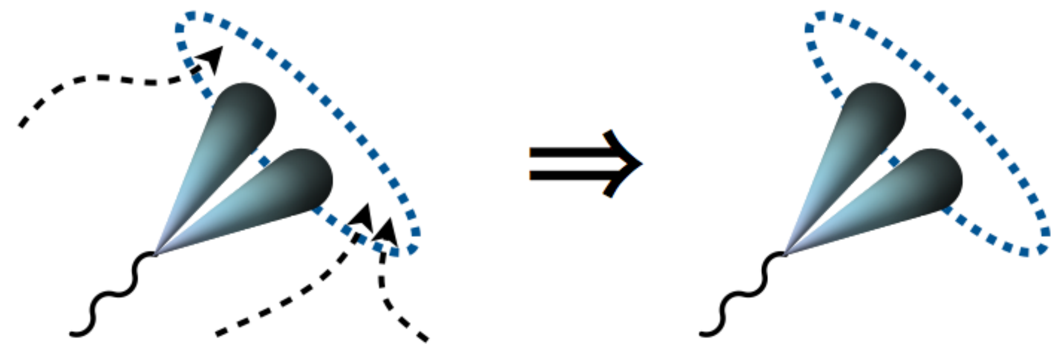
large
nonpert.
shift

ATLAS, JHEP 05 (2012) 128

Jet Substructure

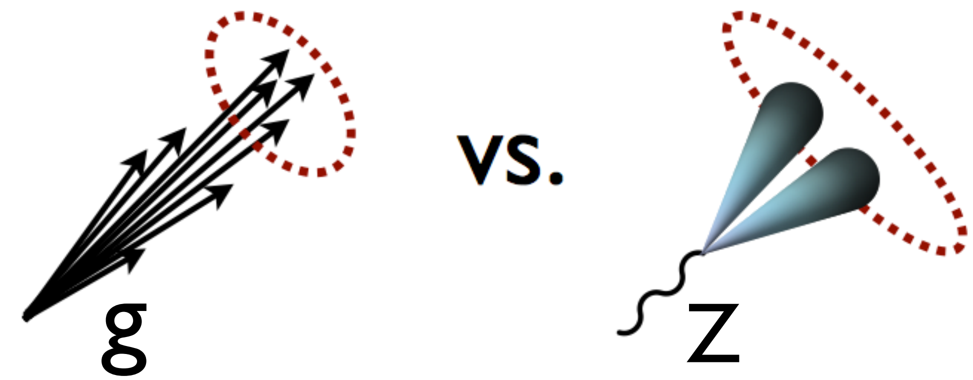
- grooming jets

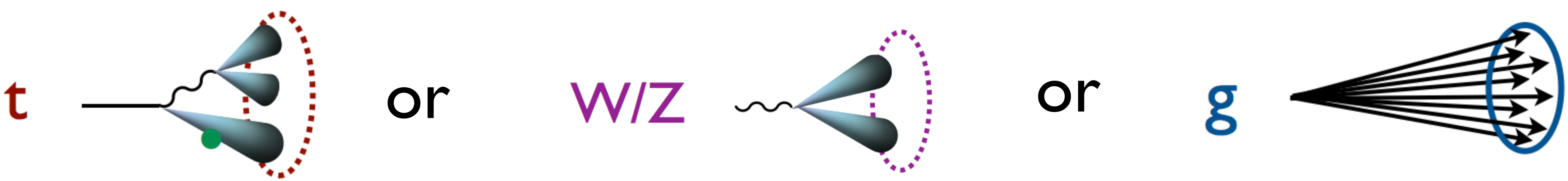
  remove soft contamination from jets



- tagging subjets

  boosted particles have collimated decay products





Soft Drop

Larkoski, Marzani, Soyez, Thaler (1402.2657)

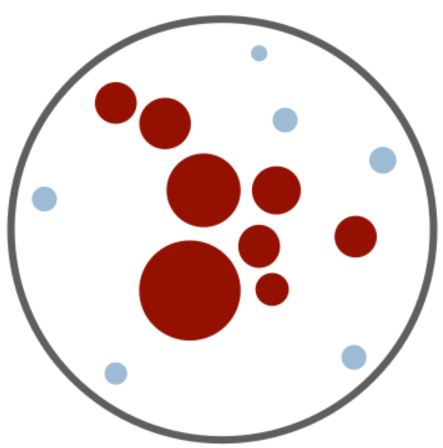
Grooms soft radiation from the jet

$$\frac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\rm cut} \left(\frac{\Delta R_{ij}}{R_0}\right)^{\beta}$$

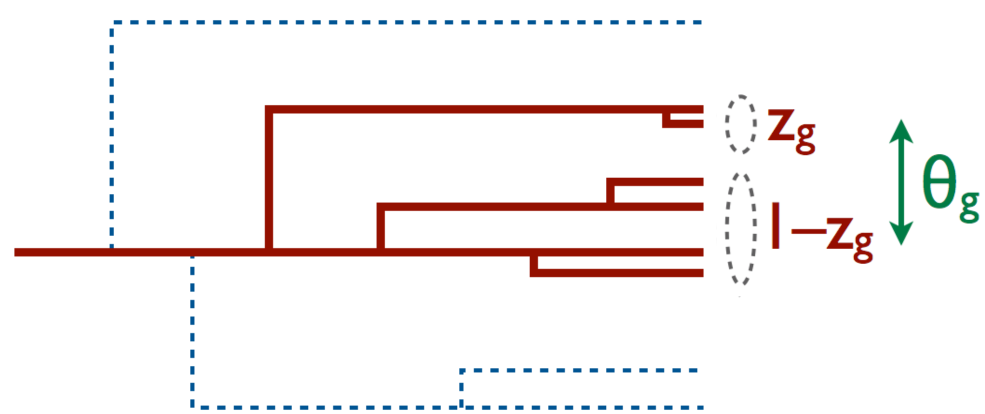
$$z > z_{\rm cut}\, \theta^{\beta}$$

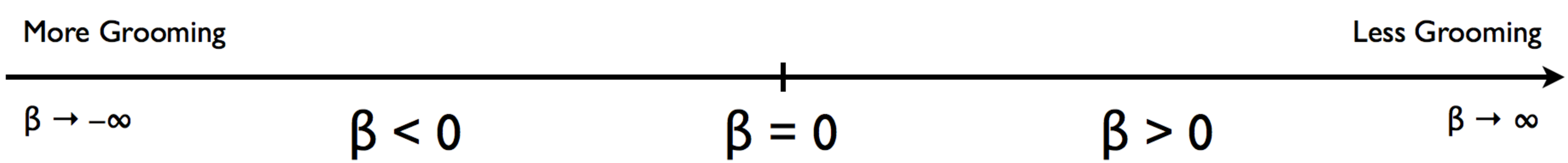
two grooming parameters

Groomed Jet



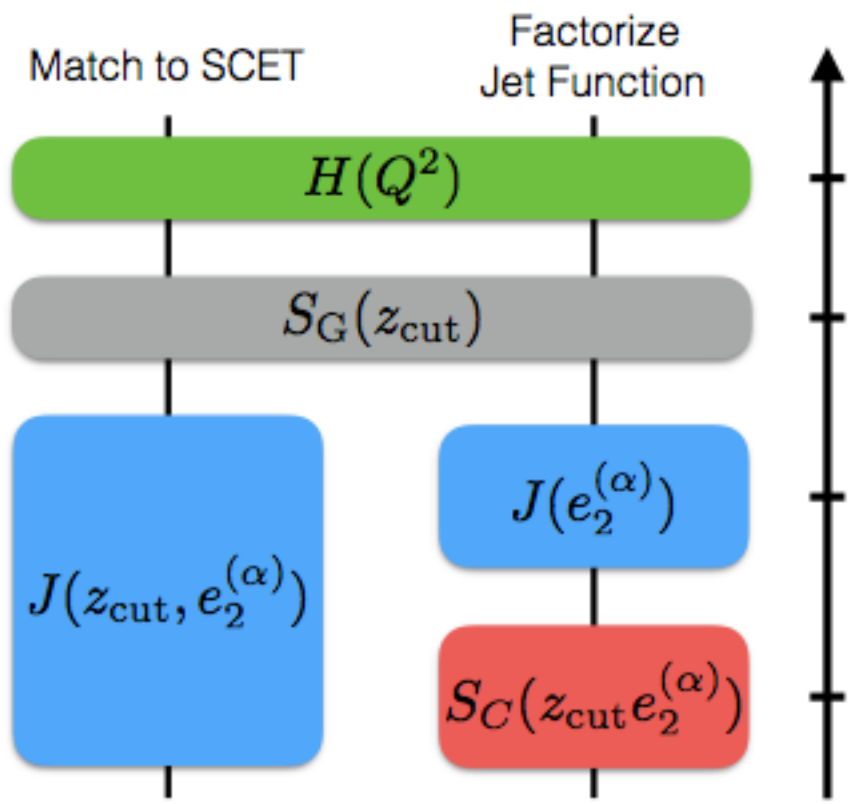
Groomed Clustering Tree

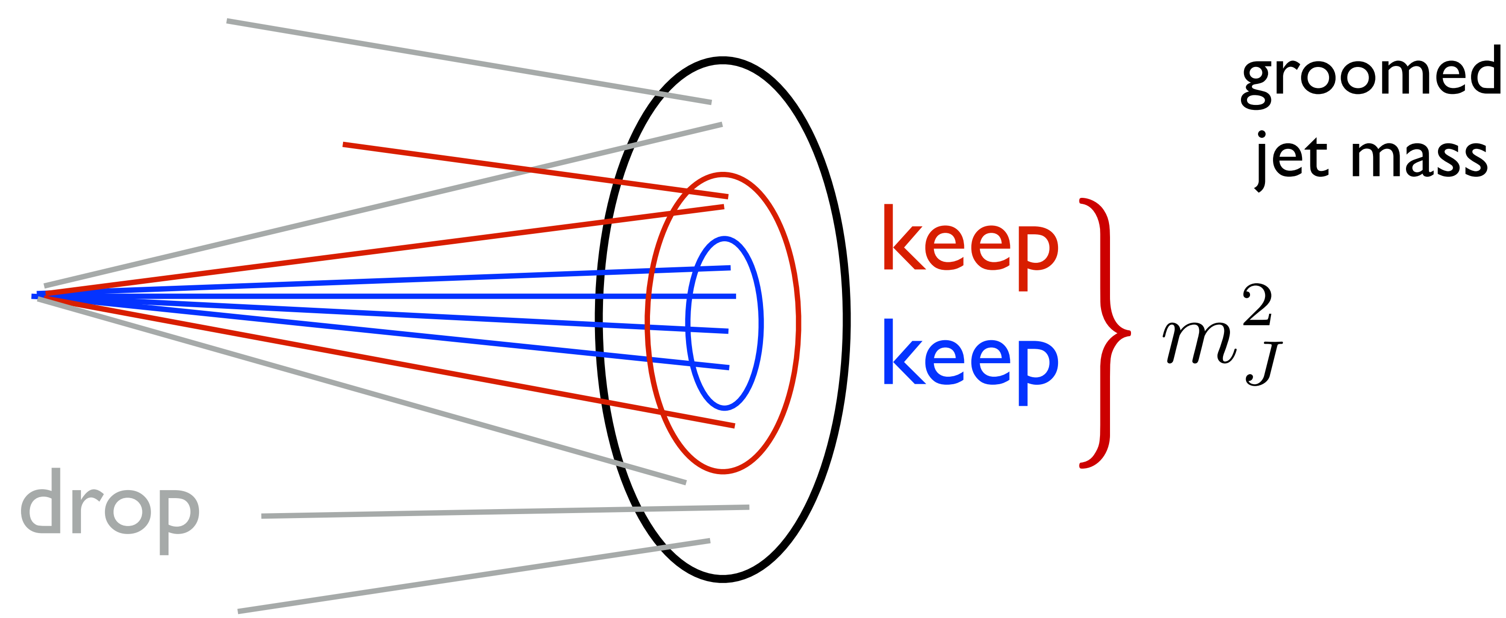




Soft Drop Factorization

Frye, Larkoski, Schwartz, Yan (1603.09338)

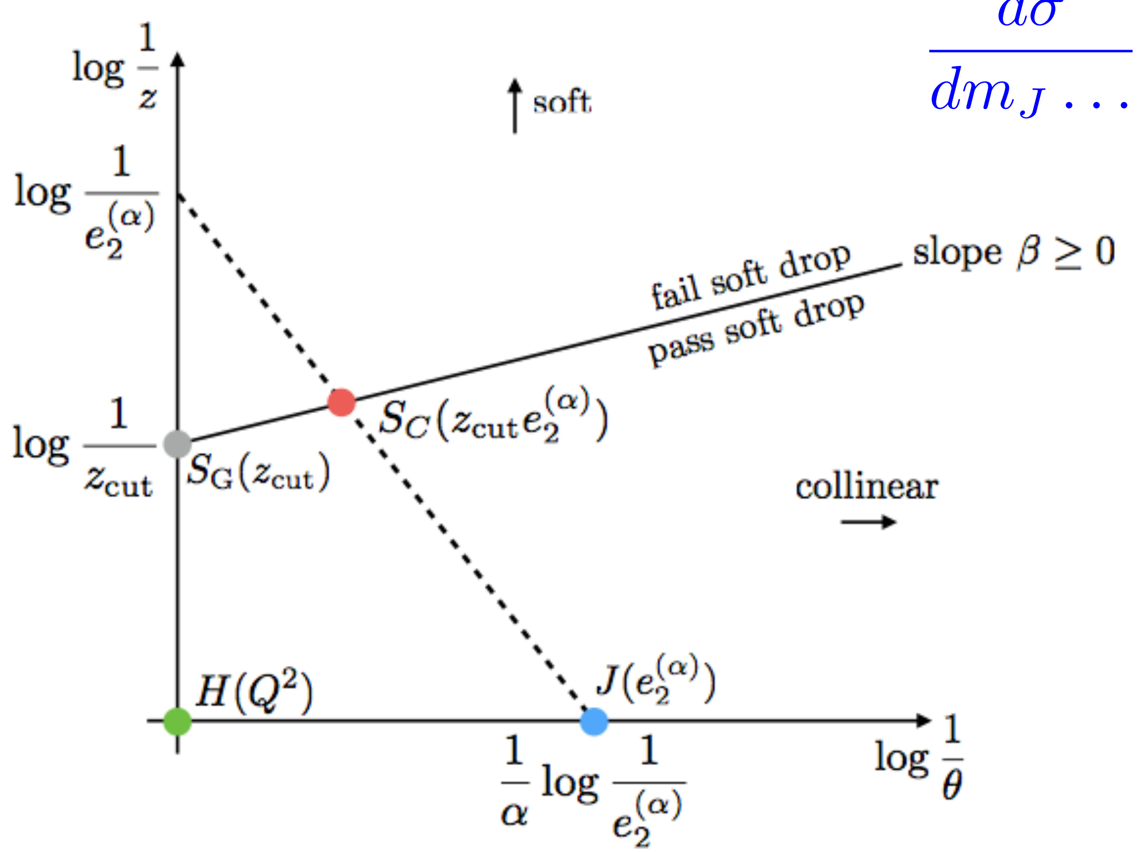




$$\frac{m_J^2}{Q^2} \ll z_{\rm cut} \ll 1$$

$$\frac{d\sigma}{dm_J \ldots} = \sum_\kappa \underbrace{D_\kappa(Q^2, z_{\rm cut}, \beta, \ldots)}_{H \times S_G} \left[S_C(m_J^2/Q, Q z_{\rm cut}, \beta) \otimes J(m_J) \right]$$

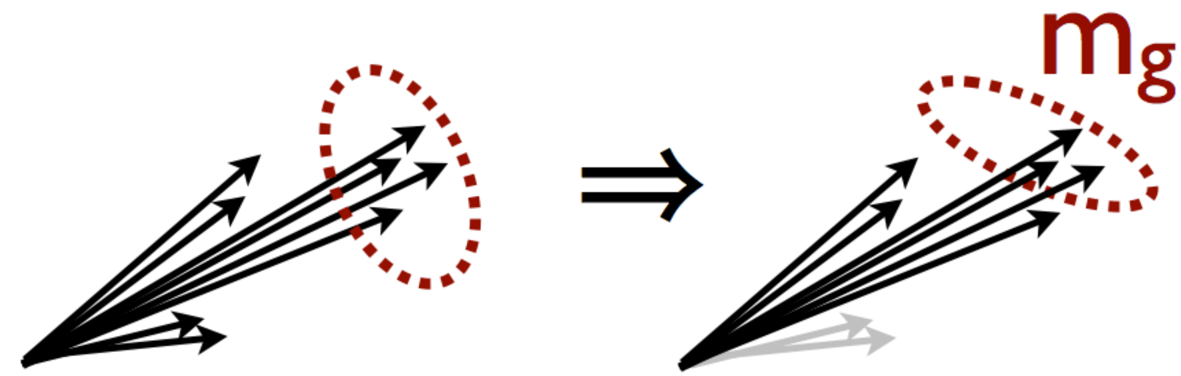
Soft-Collinear function

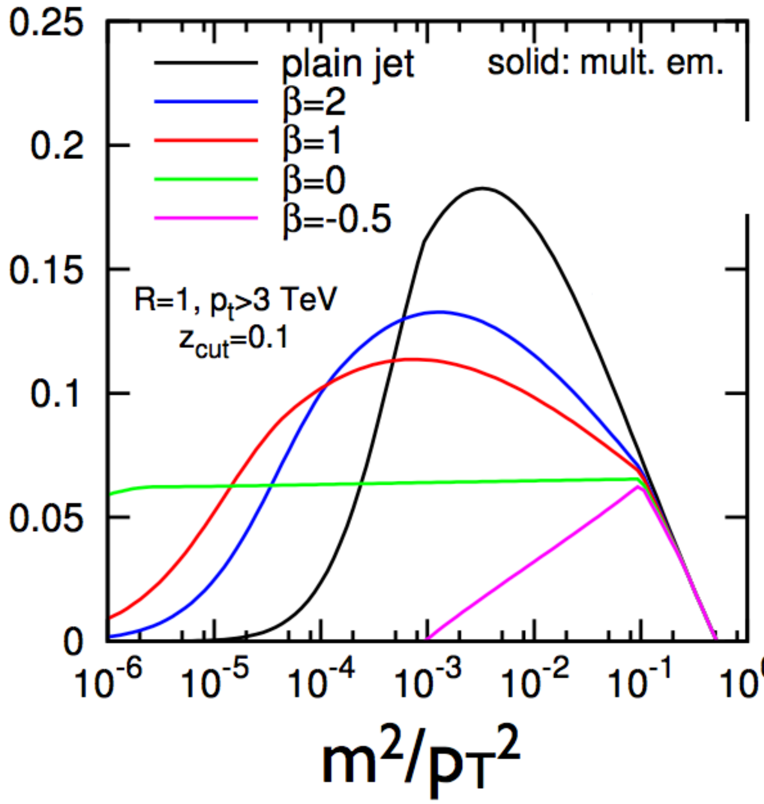


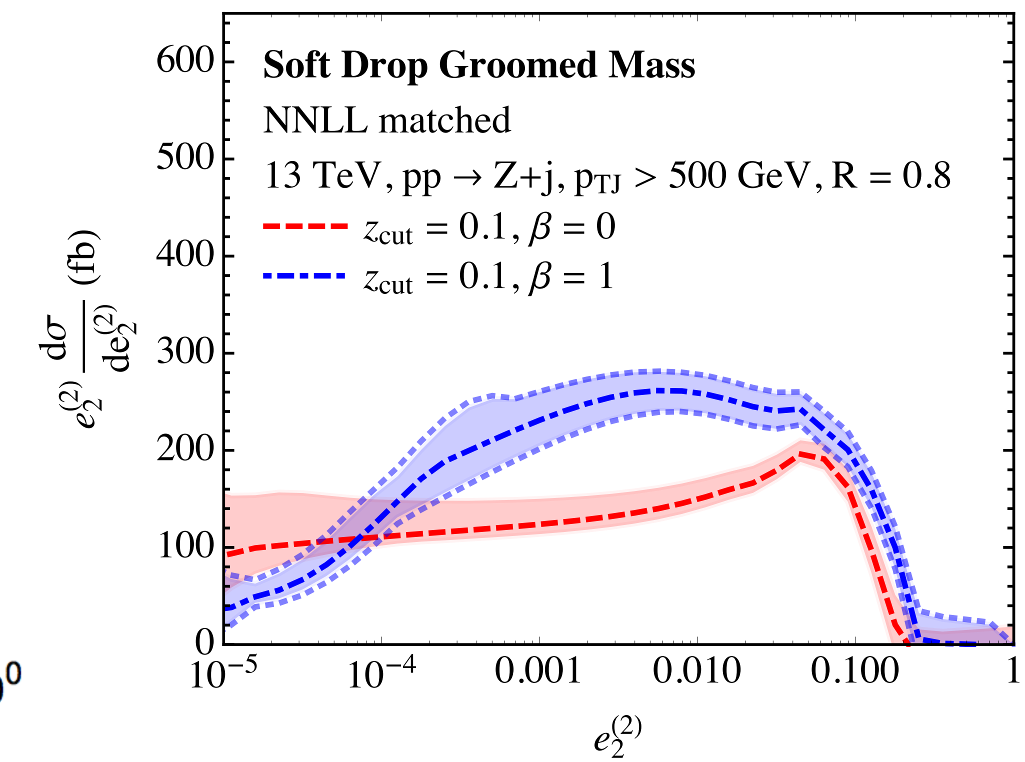
isolates measurement

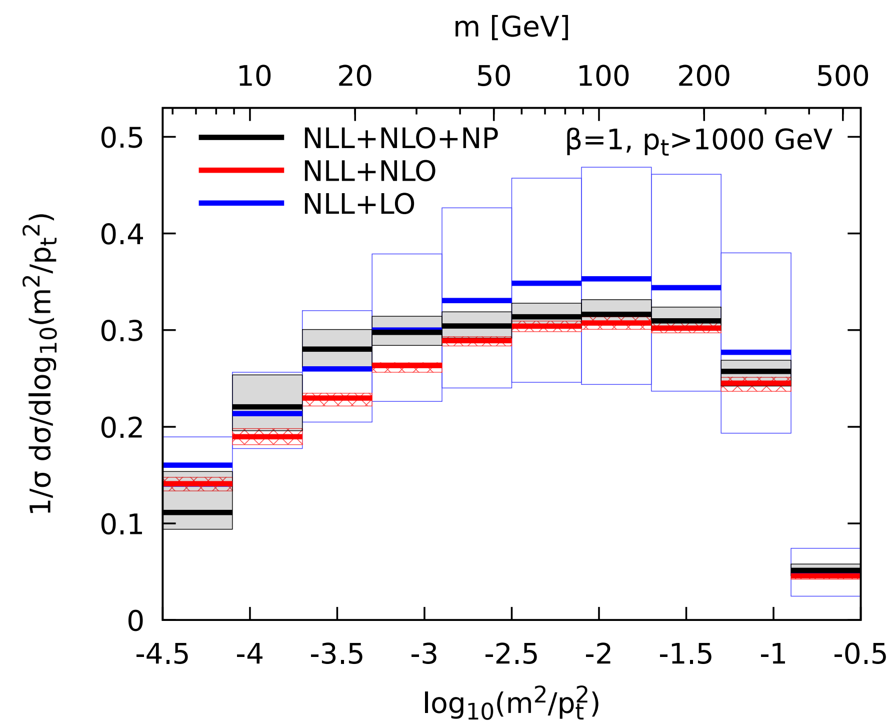
enables NNLL precision

Groomed Jet Mass (Soft Drop)









Pert. QCD at $\simeq$ NLL

NNLL+LO

NLL+NLO

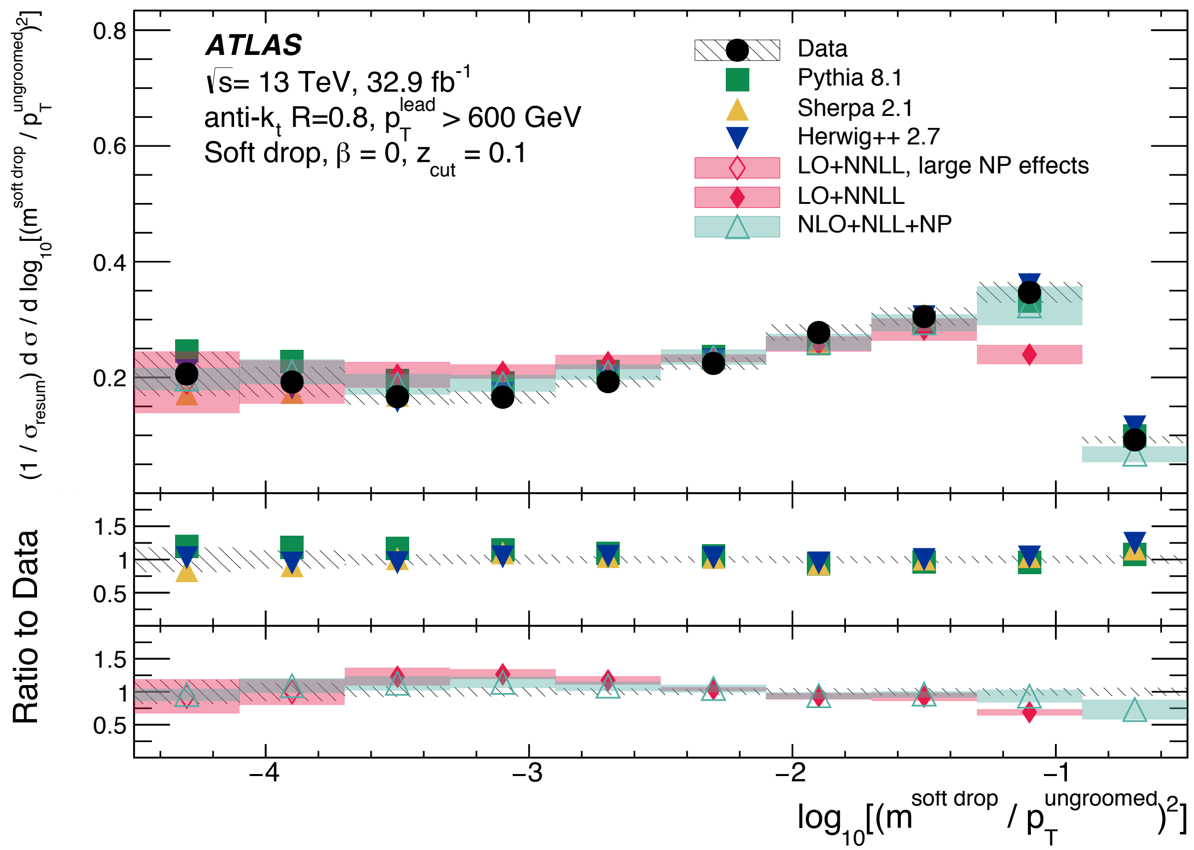
Larkoski, Marzani, Soyez, Thaler 2014

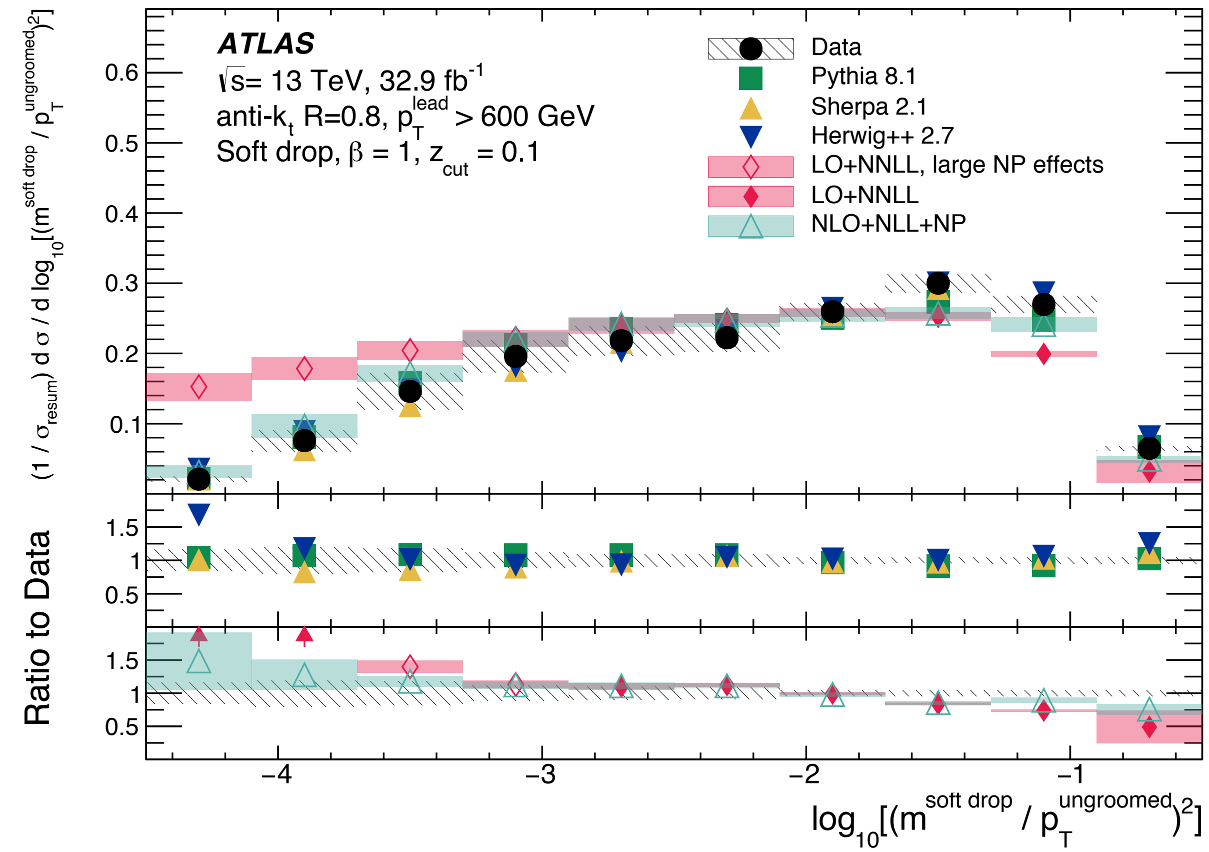
Frye, Larkoski, Schwartz, Yan 2016

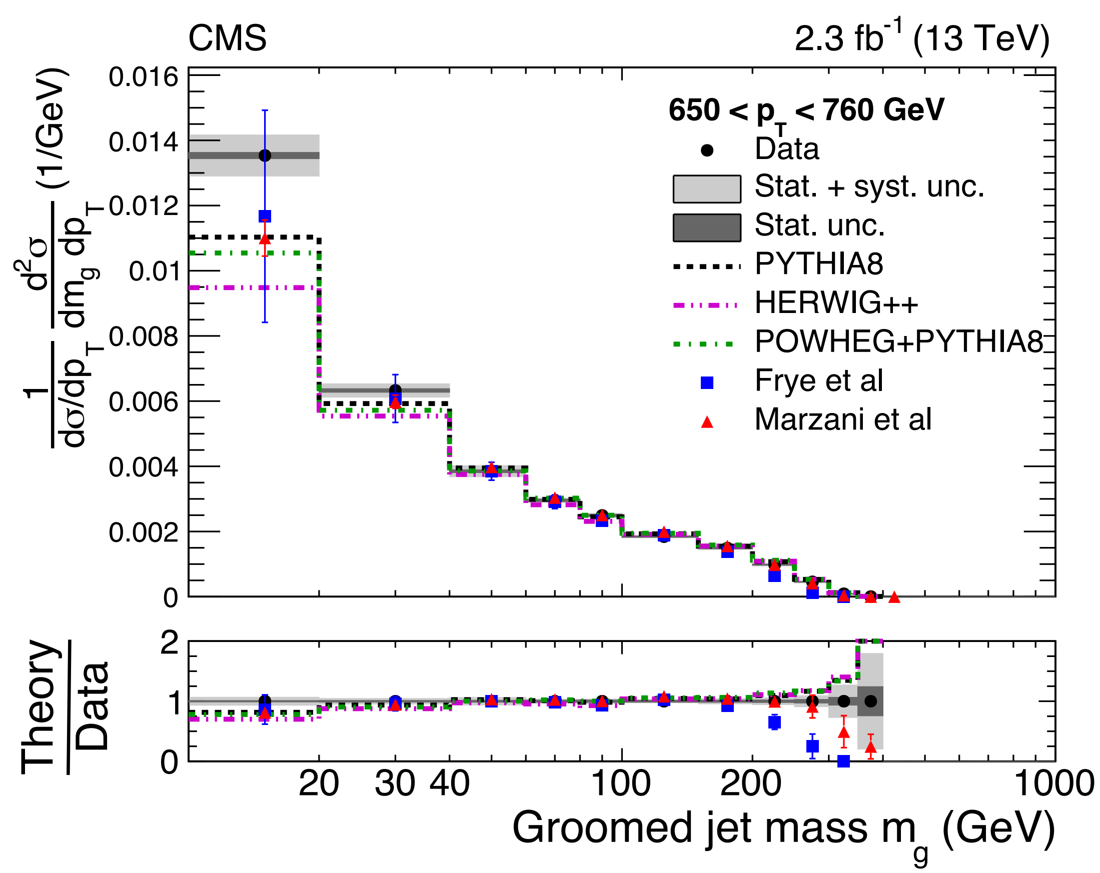
Marzani, Schunk, Soyez 2017

Comparison with Measurements

ATLAS 1711.08341







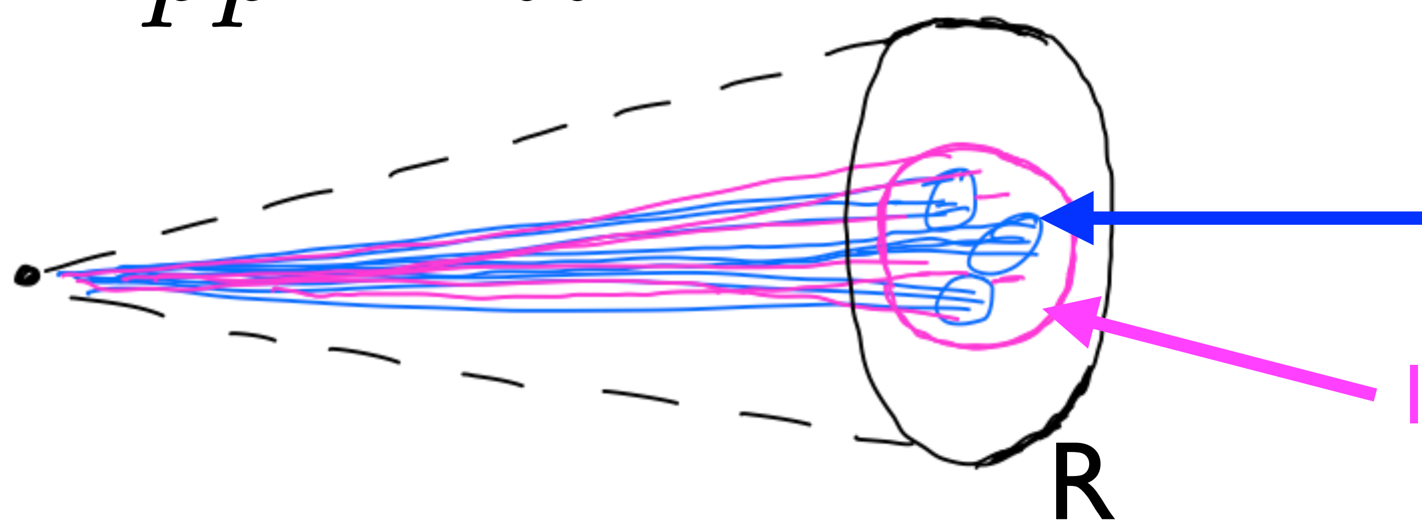
CMS 1807.05974

Boosted Top Jets

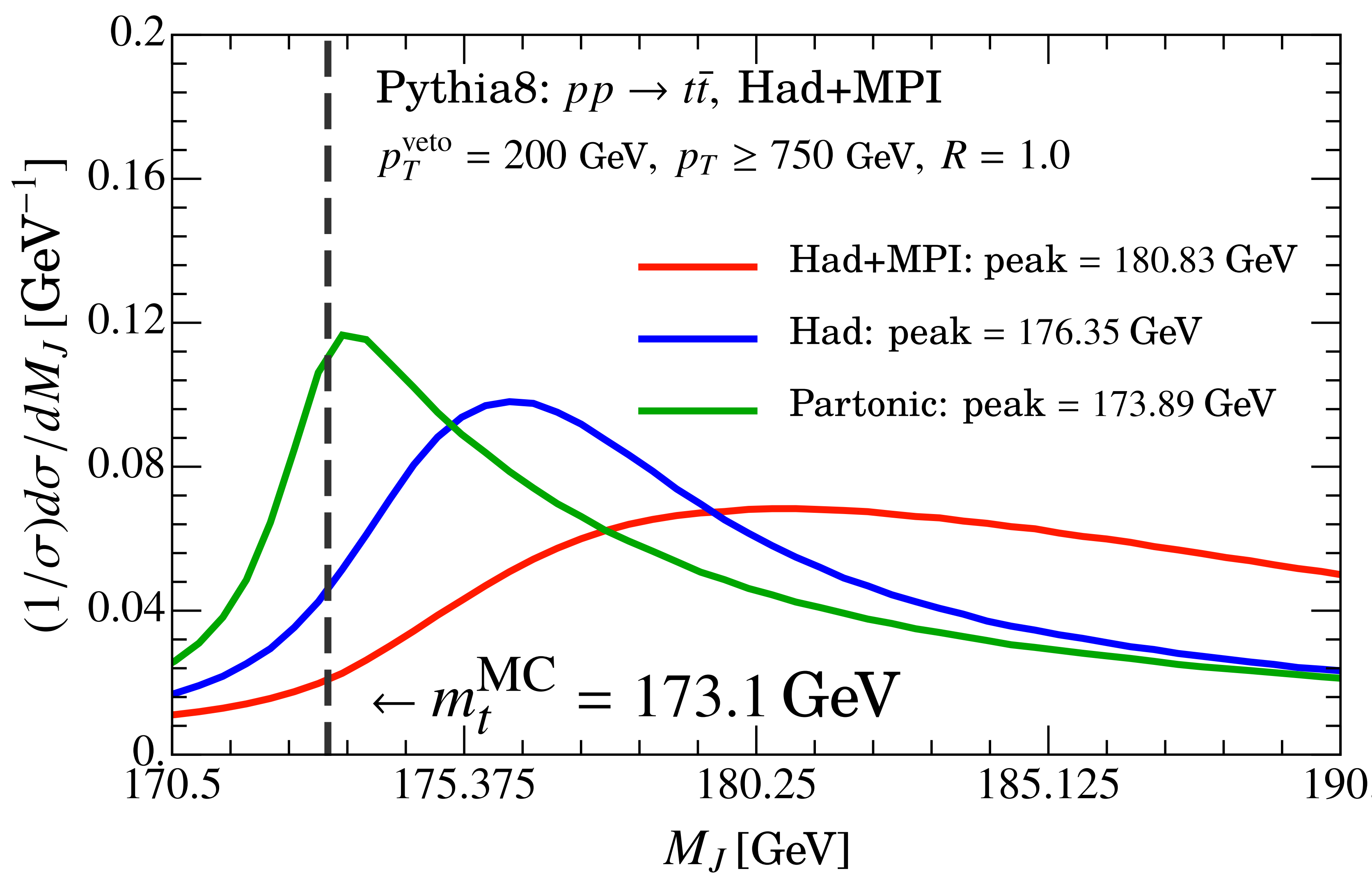
Hoang, Mantry, Pathak, Stewart (1708.02586)

$pp \to t\bar{t}$

Soft drop also removes contamination for boosted top jets



Ungroomed:



Boosted Top Jets

Hoang, Mantry, Pathak, Stewart (1708.02586)

$$pp \to t\bar{t}$$

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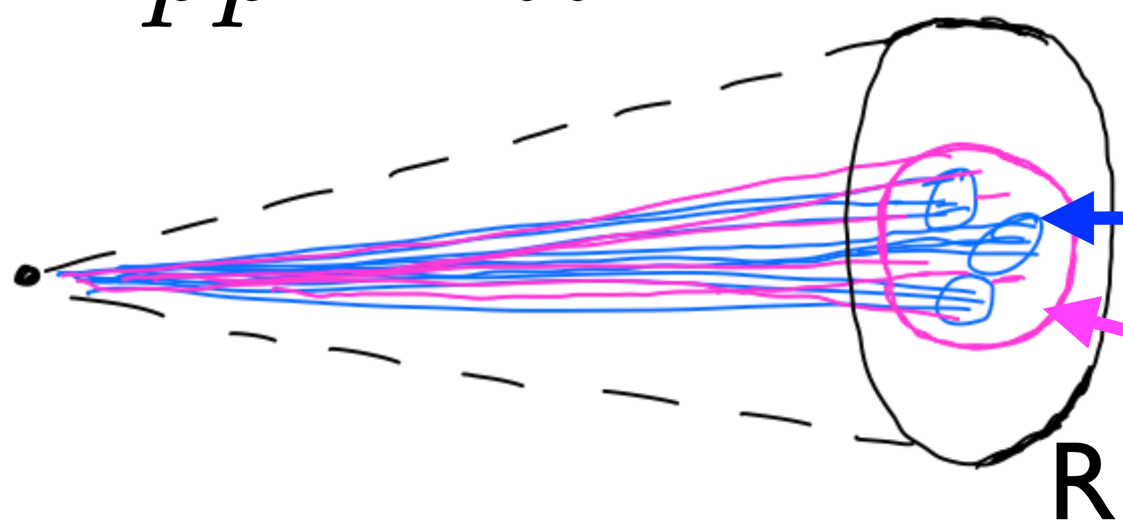
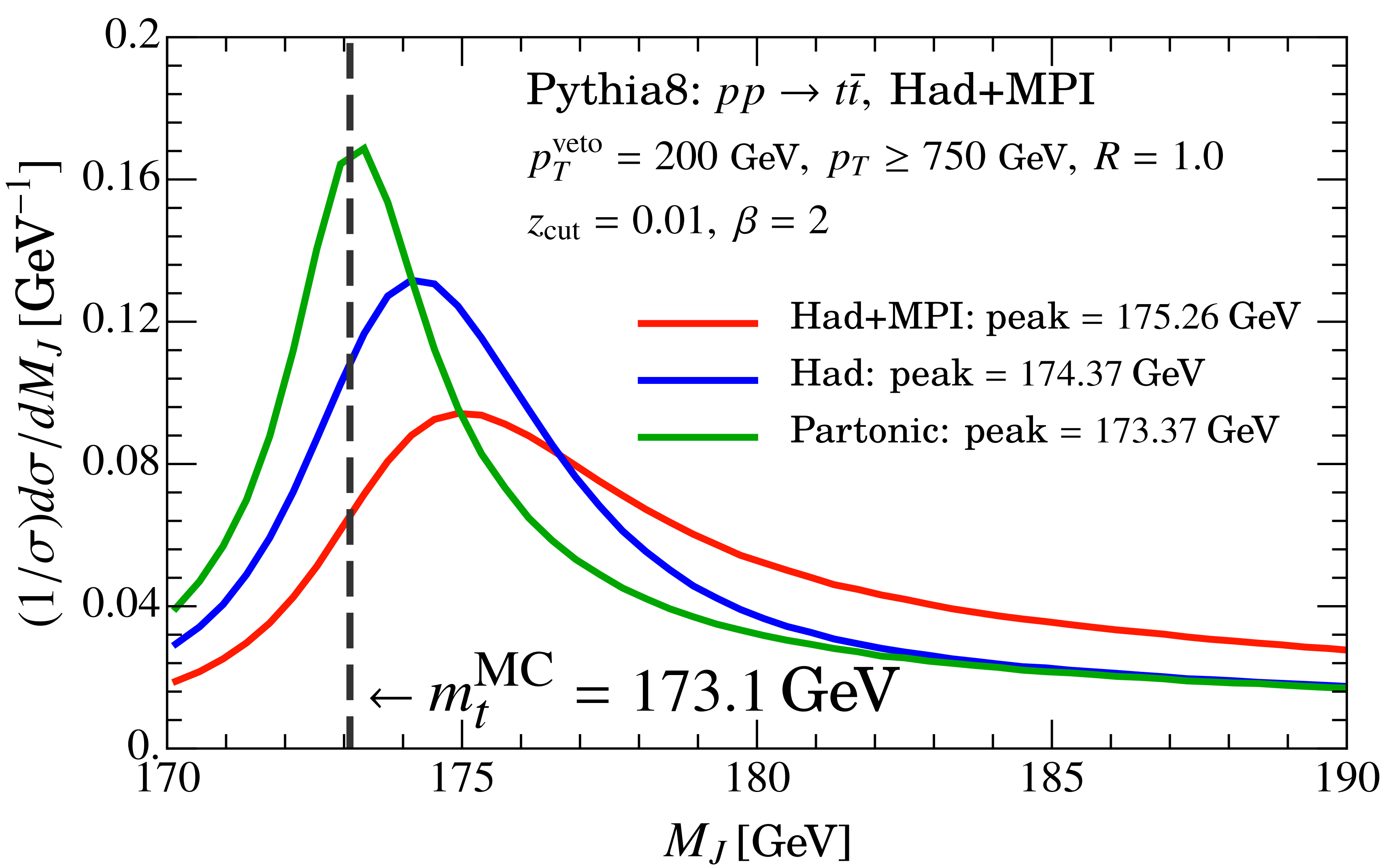


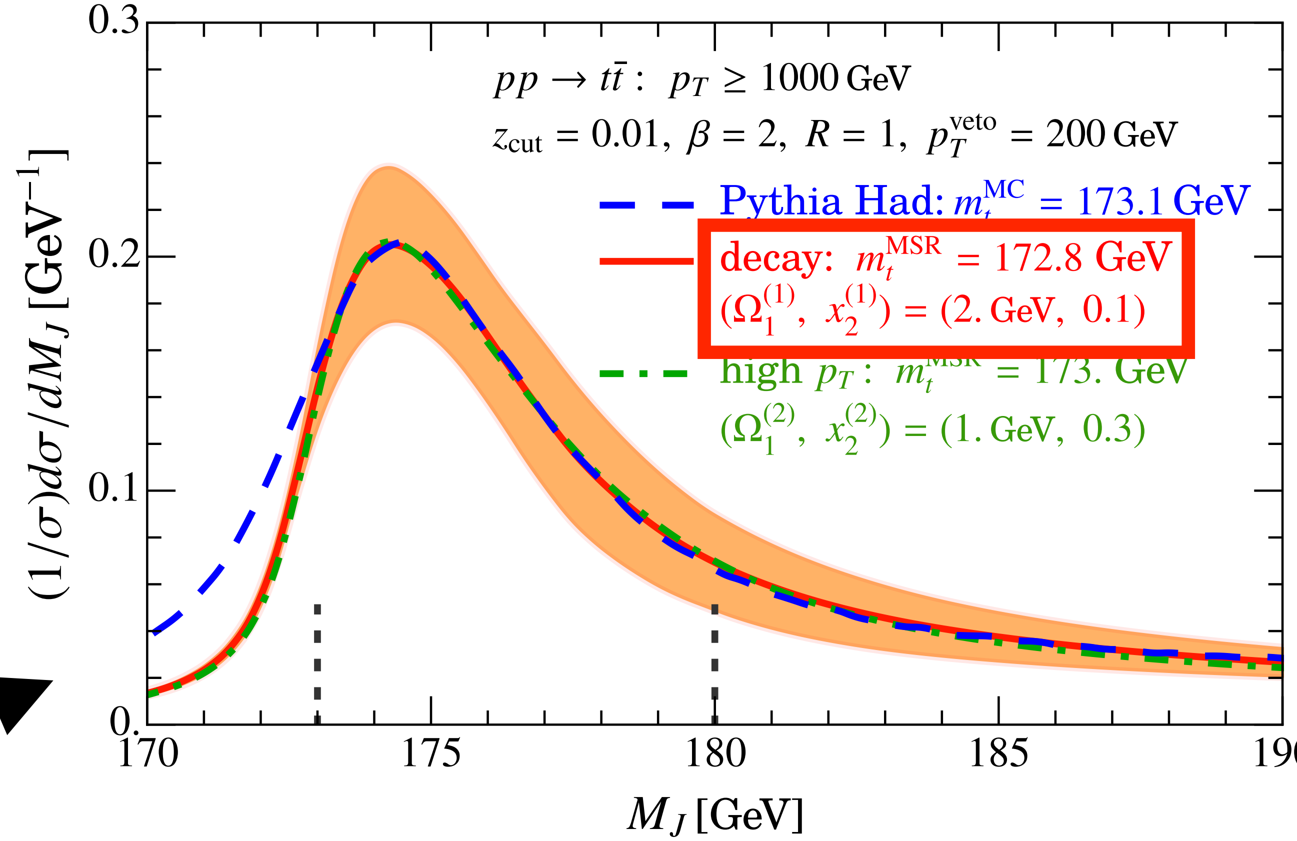
Lightly Groomed:

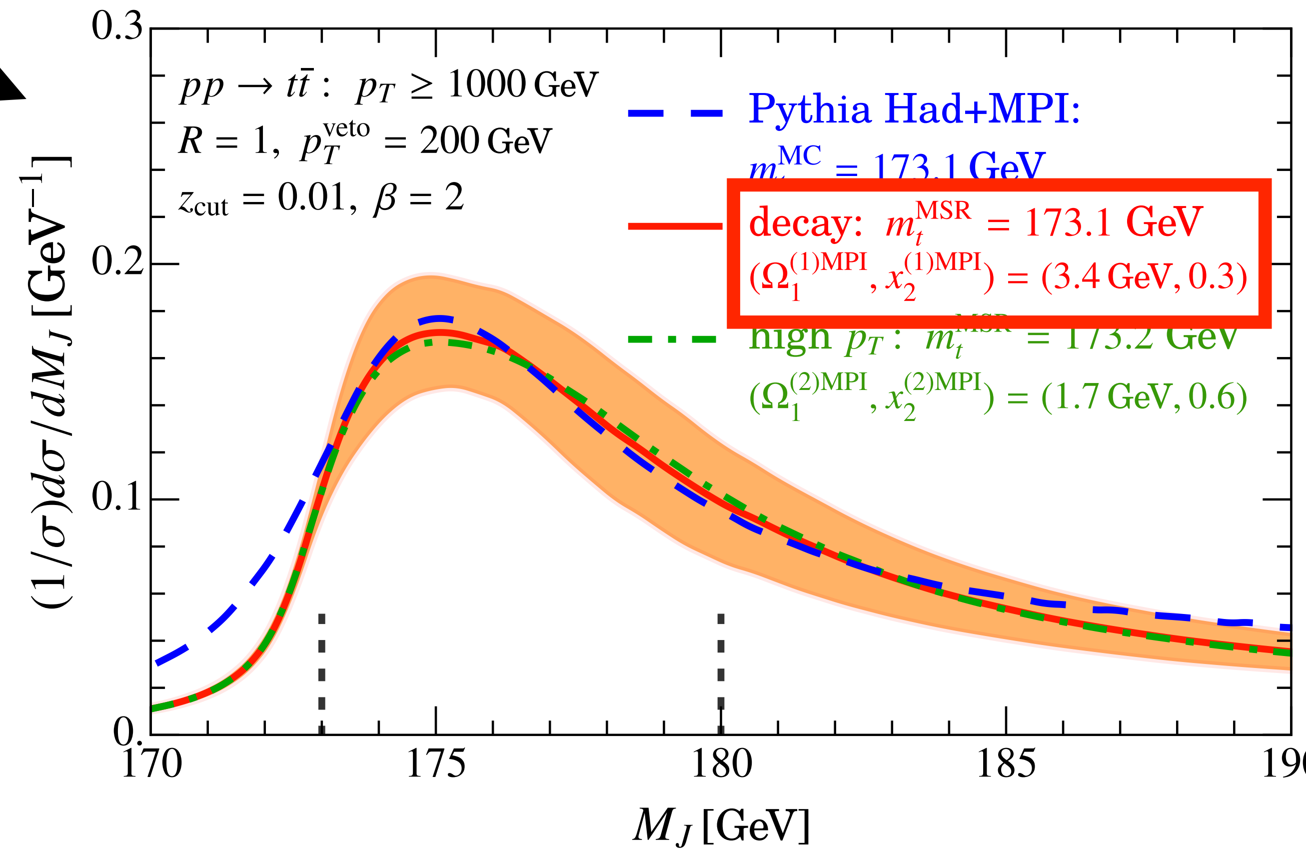


Factorization formula can also be derived for groomed jet mass for this case

Enables top mass measurement in a short distance scheme with control over soft effects

Tested by NLL comparison with Pythia





Power Corrections

$\tau \ll 1$

$$\tau \frac{d\sigma}{d\tau} = \sum_{i,j} c_{i,j}^{(0)}\, \alpha_s^i \, \ln^j \tau + \sum_{i,j} c_{i,j}^{(1)}\, \alpha_s^i \, \tau \ln^j \tau + \ldots$$

Leading Power — Next to Leading Power

logs generated by power corrections to soft and collinear limits

Interesting:

- Formal questions: Factorization? Universality of functions? Universality of anomalous dimensions?
- Sudakov suppression at subleading power?
- Improve Fixed Order Calculations (subtractions)
- Examples where subleading power is needed (high precision, B's)

EFT framework (SCET) is ideal for studying power corrections

[Stewart, Bauer, Pirjol] [Beneke, Feldman, . . .]
[Neubert, Becher, Paz, Hill]

Subleading Power in SCET

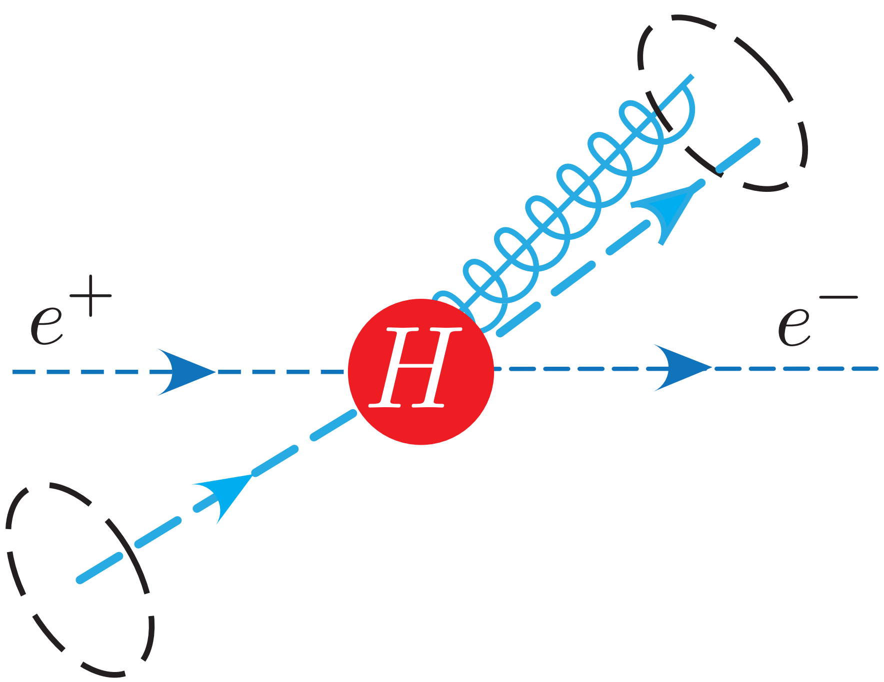
systematic power expansion
about soft & collinear limits

$\lambda \ll 1$

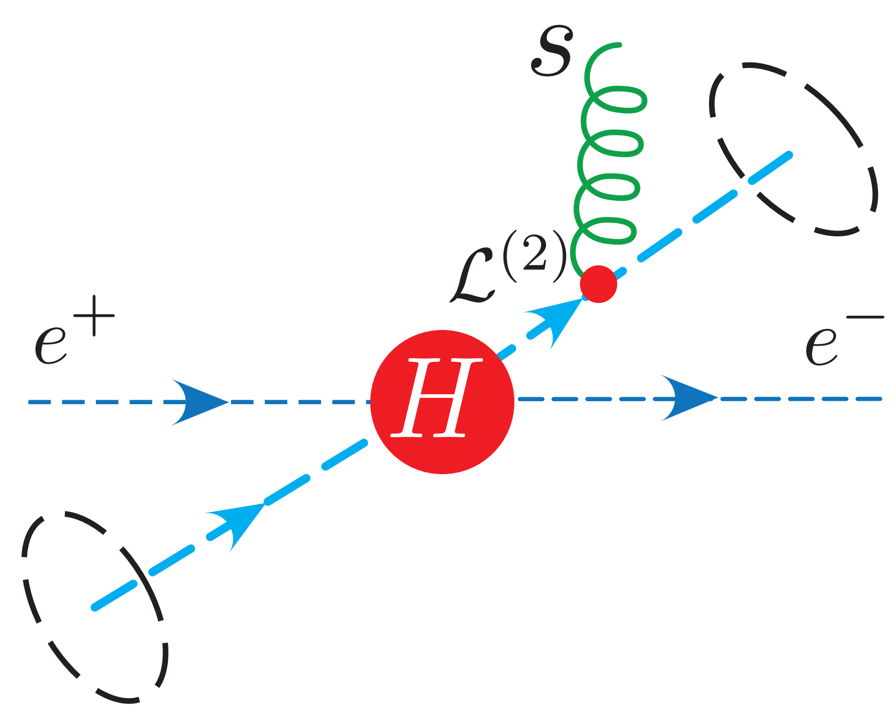
$\mathcal{O}(\lambda^i)$

$$\mathcal{L}_{\mathsf{SCET}} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm dyn} = \sum_{i\geq 0} \mathcal{L}^{(i)}_{\rm hard} + \sum_{i \geq 0} \mathcal{L}^{(i)}$$

Subleading Hard Scattering Operators



Subleading Lagrangians



N-Jettiness Subtraction Method for NNLO

(for N jets)

- IR divergences in fixed order calculations can be regulated using event shape observables. [Boughezal, Focke, Petriello, Liu], [Gaunt, Stahlhofen, Tackmann, Walsh]

$$\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

want (N)NLO

predict with factorization

resolved, only need extra emission at (N)LO

error goes like:

$$\Delta\sigma^{\rm NLO}(\tau_{\rm cut}) \sim \alpha_s \, \tau_{\rm cut} \ln \tau_{\rm cut}$$

$$\Delta\sigma^{\rm NNLO}(\tau_{\rm cut}) \sim \alpha_s^2 \, \tau_{\rm cut} \ln^3 \tau_{\rm cut}$$

can improve factorization result by computing these terms

rule of thumb: each log computed gains an order of magnitude in precision (or computing time) [MCFM @ NNLO]

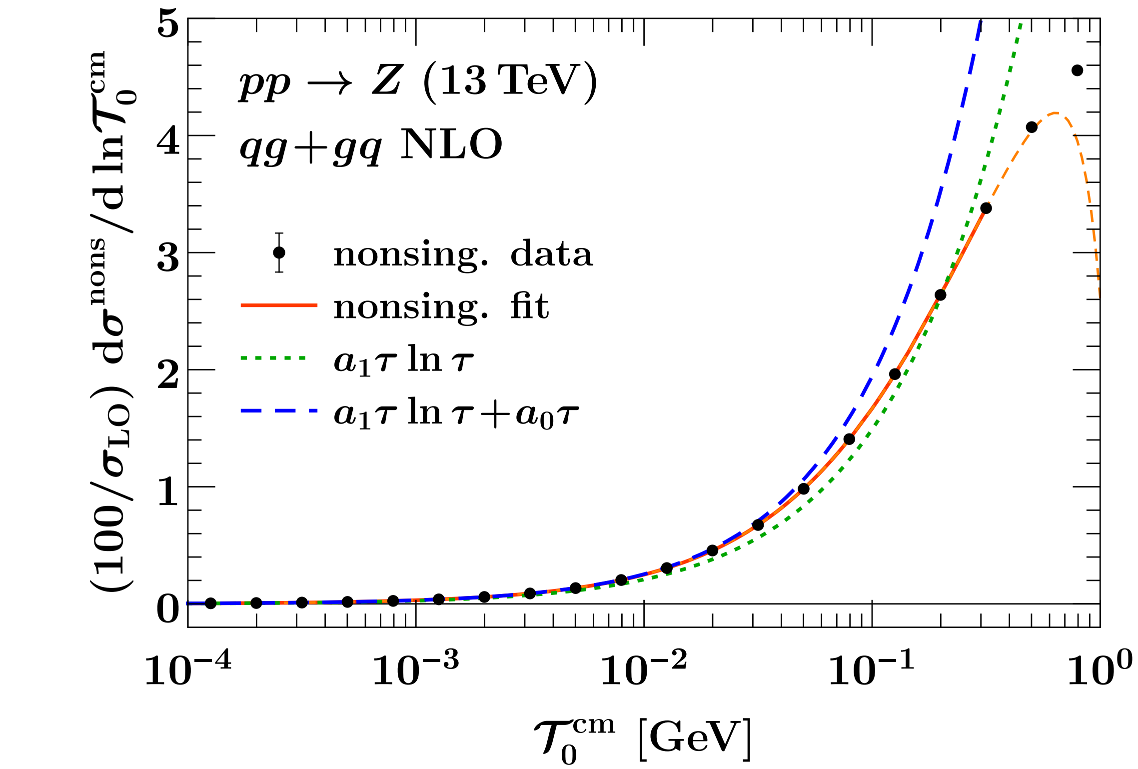
Power suppressed calculations for Z+0-Jets

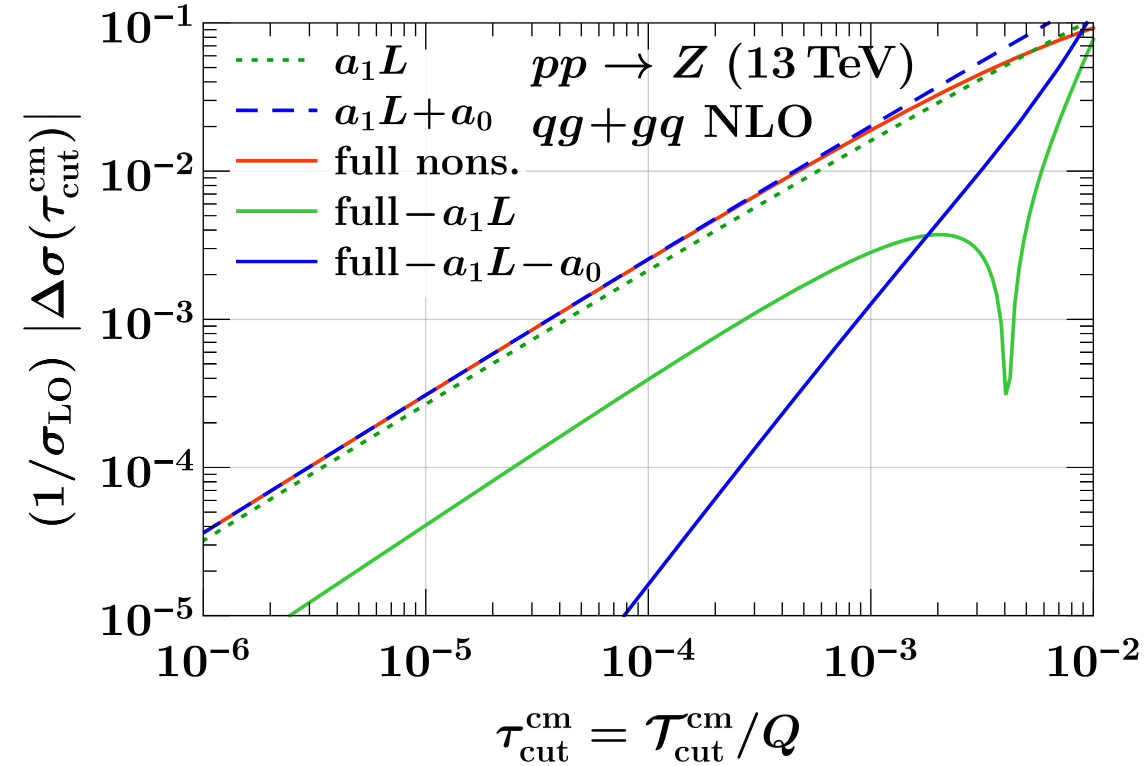
Moult, Rothen, IS, Tackmann, Zhu (1612. 00450)

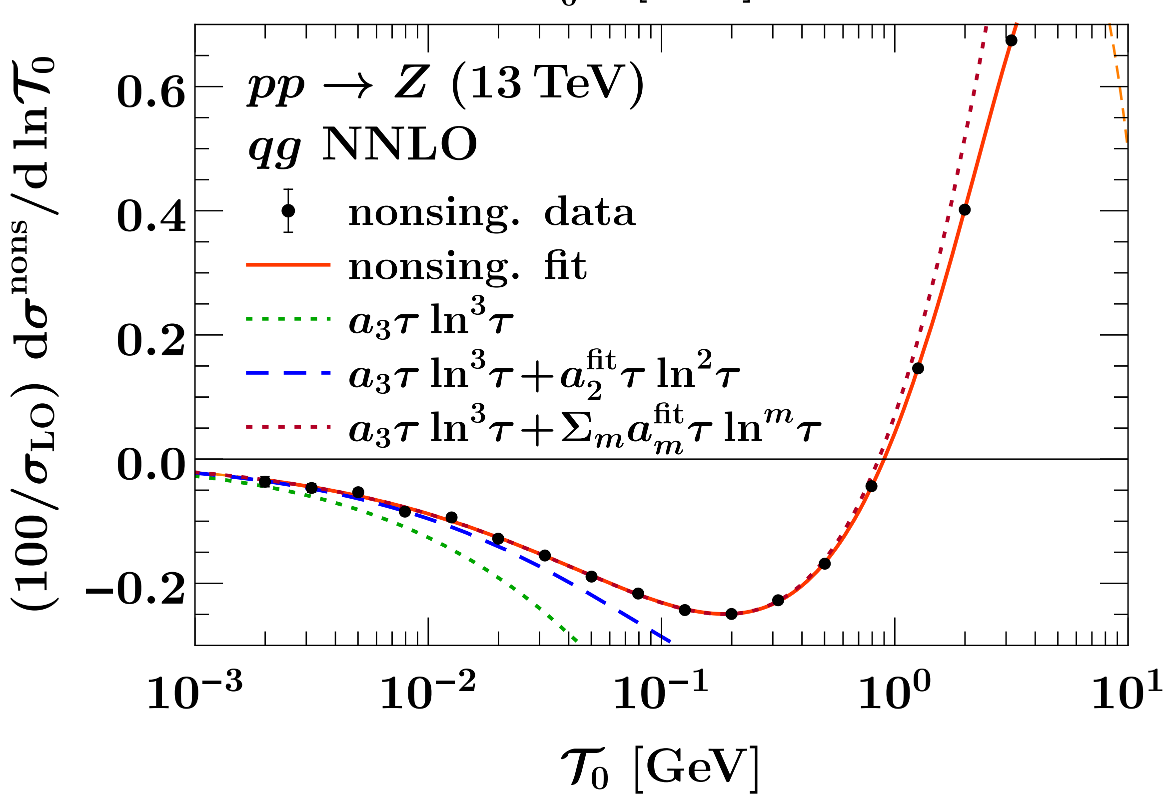
Ebert, Moult, IS, Tackmann, Vita, Zhu (1807.10764)

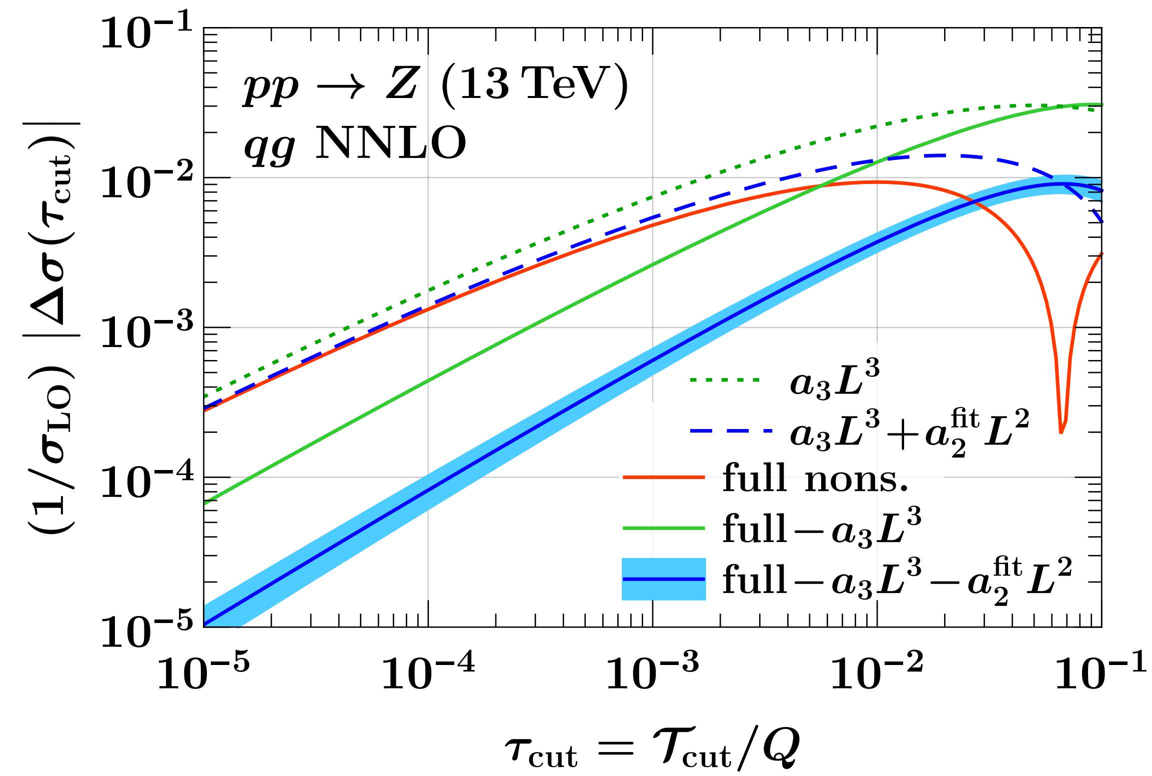
calculation of: $\alpha_s\,\tau\ln\tau$ and $\alpha_s\,\tau$, $\alpha_s^2\,\tau\ln^3\tau$

(validated with MCFM)





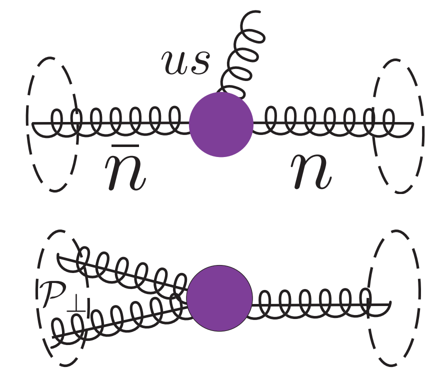




First Subleading Power Resummation for an Event Shape

LL resummed result for pure glue $H \to gg$ thrust

Moult, IS, Vita, Zhu (1804.04665)



Sources:

- Power corrections to scattering amplitudes.
- Power corrections to kinematics.

- The subleading jet and soft functions satisfy a 2×2 mixing RG

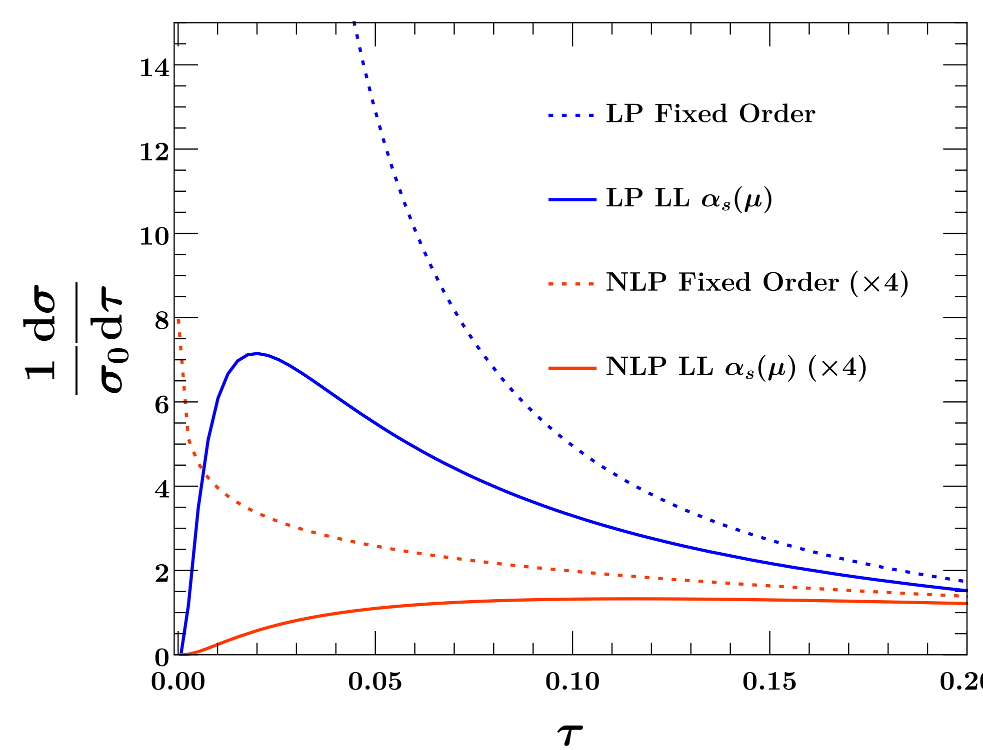
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{11}(y,\mu) & \gamma_{12} \\ 0 & \gamma_{22}(y,\mu) \end{pmatrix} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix}$$

Result:

$$\frac{\tau}{\sigma_0} \frac{d\sigma^{\rm NLP}_{\rm LL}}{d\tau} = \frac{\alpha_s}{4\pi} 8 C_A \, \underbrace{\tau \ln \tau}_{} \, e^{\overbrace{-\frac{\alpha_s}{4\pi}(4C_A)\ln^2\tau}} \tau$$

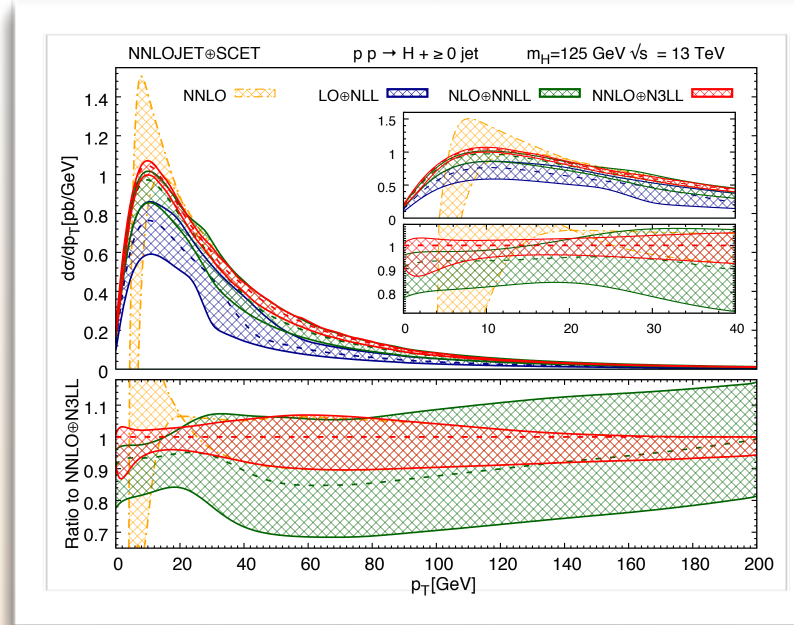
single log from mixing

Sudakov like leading power

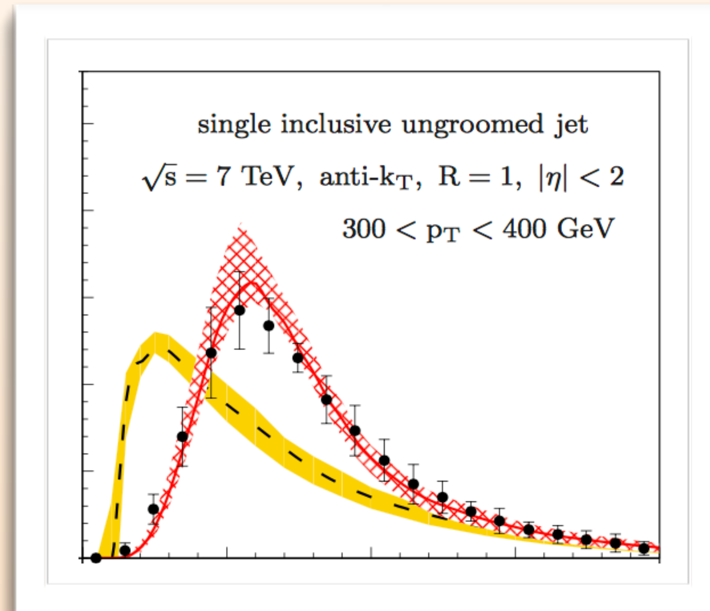


Summary:

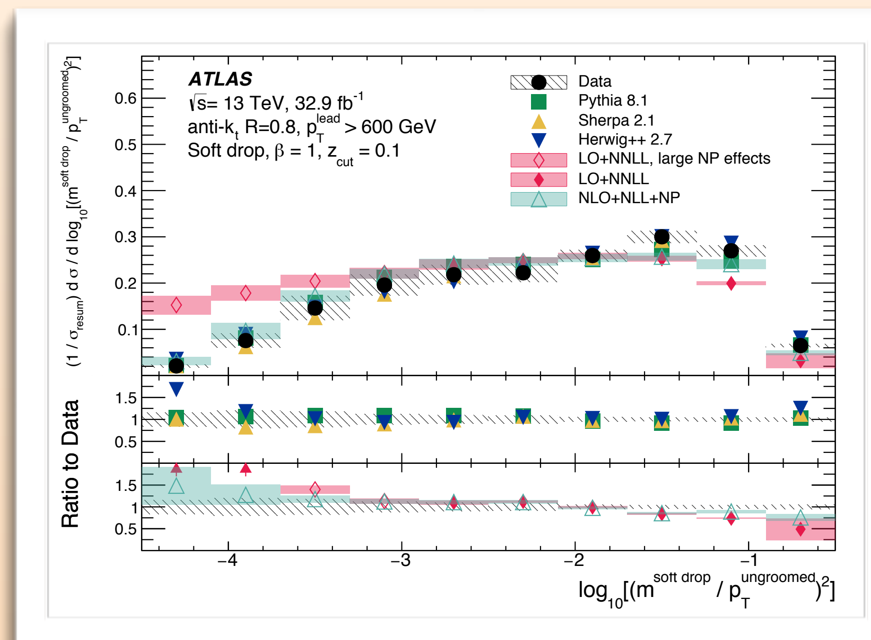
- Precision QCD for LHC (Higgs qT)



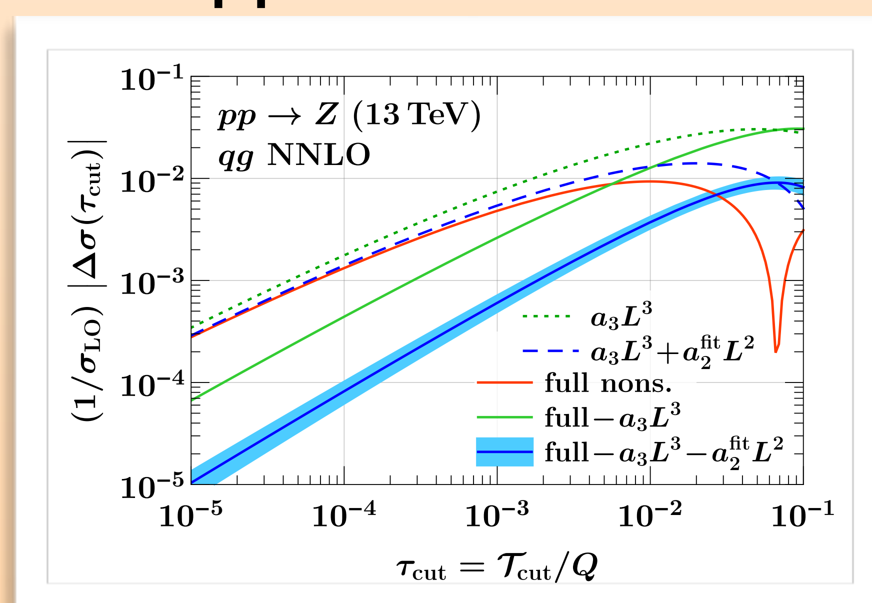
- Jet Mass Distribution



- Soft Drop Groomed Jets



- Power Suppressed Corrections



- Effective Field Theory is a powerful tool for Jets