# Loop functions in thermal QCD

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# Bibliography: lattice


(1) TUMQCD coll.
*Color screening in 2+1 flavor QCD*
arXiv:1804.10600

(2) TUMQCD coll.
*Polyakov loop in 2+1 flavor QCD from low to high temperatures*
Phys. Rev. D93 (2016) 114502 arXiv:1603.06637

# Bibliography: perturbation theory


(1) M. Berwein, N. Brambilla, P. Petreczky and A. Vairo
*Polyakov loop correlator in perturbation theory*
Phys. Rev. D96 (2017) 014025 `arXiv:1704.07266`

(2) M. Berwein, N. Brambilla, P. Petreczky and A. Vairo
*Polyakov loop at next-to-next-to-leading order*
Phys. Rev. D93 (2016) 034010 `arXiv:1512.08443`

(3) M. Berwein, N. Brambilla and A. Vairo
*Renormalization of Loop Functions in QCD*
Phys. Part. Nucl. 45 (2014) 656 `arXiv:1312.6651`

(4) M. Berwein, N. Brambilla, J. Ghiglieri and A. Vairo
*Renormalization of the cyclic Wilson loop*
JHEP 1303 (2013) 069 `arXiv:1212.4413`

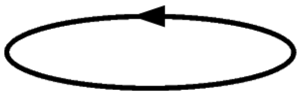
(5) N. Brambilla, J. Ghiglieri, P. Petreczky and A. Vairo
*The Polyakov loop and correlator of Polyakov loops at next-to-next-to-leading order*
Phys. Rev. D82 (2010) 074019 `arXiv:1007.5172`

# Loop functions

# Loop functions: Polyakov loop

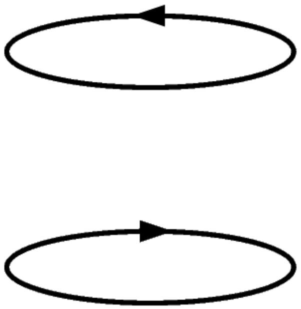
- Polyakov loop average in a thermal ensemble at a temperature $T$

  $P(T)|_R \equiv \frac{1}{d_R}\langle \mathrm{Tr}\ L_R \rangle = e^{-F_Q/T}$ (R $\equiv$ color representation)

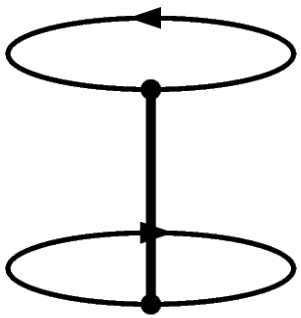
  $d_A = N^2 - 1$, $d_F = N$ and $L_R(\mathbf{x}) = \mathcal{P} \exp \left( ig \int_0^{1/T} d\tau A^0(\mathbf{x}, \tau) \right)$

# Loop functions: correlators



- Polyakov loop correlator

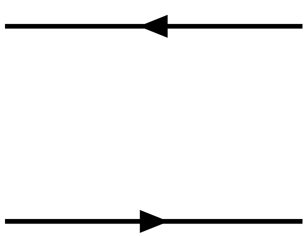
$$P_c(r,T) \equiv \frac{1}{N^2}\langle \mathrm{Tr} L_F^\dagger(\mathbf{0}) \mathrm{Tr} L_F(\mathbf{r})\rangle = e^{-F_{Q\bar{Q}}/T}$$



- Cyclic Wilson loop

$$W_c(r,T) \equiv \frac{1}{N}\langle \mathrm{Tr} L_F^\dagger(\mathbf{0}) U^\dagger(1/T) L_F(\mathbf{r}) U(0)\rangle = e^{-F_{W_c}/T}$$

where $U(1/T) = \mathcal{P} \exp\left(ig \int_0^1 ds\, \mathbf{r}\cdot \mathbf{A}(s\mathbf{r}, 1/T)\right) = U(0)$.



- Singlet correlator in Coulomb gauge

$$W_s(r,T) \equiv \frac{1}{N}\langle \mathrm{Tr} L_F^\dagger(\mathbf{0}) L_F(\mathbf{r})\rangle = e^{-F_S/T}$$

# Polyakov loop

# Polyakov loop at $\mathcal{O}(g^3)$

$$P = 1 + \frac{C_R \alpha_s m_D}{2T}, \qquad F_Q = -\frac{C_R \alpha_s m_D}{2}$$

where $C_R$ is the quadratic Casimir of the representation $R$.
The Debye mass $m_D$ is given by

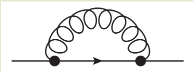
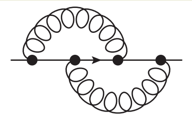
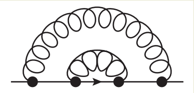
$$\Pi_{00}(|\mathbf{k}| \ll T) \approx m_D^2 = \frac{C_A + T_F n_f}{3} g^2 T^2$$

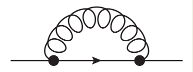
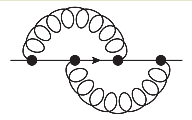
In the weak coupling one assumes the hierarchy of scales

$$T \gg m_D \sim gT \gg m_M \sim g^2 T$$

where $m_M$ is the magnetic mass.

# Exponentiation

$P = 1 + C_R$  $+ C_R^2$  $+ C_R\left(C_R - \frac{C_A}{2}\right)$  $+ C_R^2$  $+ \ldots$

$= \exp\Big[C_R$  $- \frac{1}{2} C_R C_A$  $+ \ldots\Big] = \exp\left(D_1 + D_2 + \ldots\right)$

- Dots stand for order $g^6$ contributions.
- The free energy is proportional to $C_R$: Casimir scaling.

$$D_1 = -\frac{C_R g^2}{2T} \int_k D_{00}(0, \boldsymbol{k})$$

$$D_2 = -\frac{C_R C_A g^4}{4T} \int_{k,\, q} \Bigg[\frac{1}{12T} D_{00}(0, \boldsymbol{k}) D_{00}(0, \boldsymbol{q}) - \sum_{k_0}{}' \frac{1}{k_0^2} D_{00}(k_0, \boldsymbol{k}) \big(2 D_{00}(0, \boldsymbol{q}) - D_{00}(k_0, \boldsymbol{q})\big)\Bigg]$$

# Polyakov loop at $\mathcal{O}(g^4)$

$$P = 1 + \frac{C_R\alpha_{\rm s}}{2}\frac{m_D}{T} + \frac{C_R\alpha_{\rm s}^2}{2}\left[C_A\left(\ln\frac{m_D^2}{T^2} + \frac{1}{2}\right) - n_f \ln 2\right]$$

- The logarithm, $\ln m_D^2/T^2$, signals that an infrared divergence at the scale $T$ has canceled against an ultraviolet divergence at the scale $m_D$.


○ Burnier Laine Vepsäläinen JHEP 1001 (2010) 054

Brambilla Ghiglieri Petreczky Vairo PR D82 (2010) 074019

## Polyakov loop at $\mathcal{O}(g^5)$

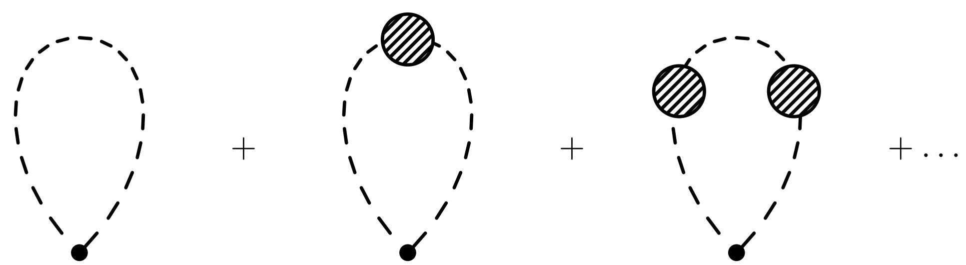
$$
\begin{aligned}
-\frac{F_Q}{T} = \ln P = & \frac{C_R \alpha_s(\mu)\, m_D}{2T} + \frac{C_R \alpha_s^2}{2}\left[C_A\left(\frac{1}{2} + \ln\frac{m_D^2}{T^2}\right) - 2T_F n_f \ln 2\right] \\
& + \frac{3 C_R \alpha_s^2 m_D}{16\pi T}\left[3C_A + \frac{4}{3}T_F n_f\,(1 - 4\ln 2) + 2\beta_0\left(\gamma_E + \ln\frac{\mu}{4\pi T}\right)\right] \\
& - \frac{C_R C_F T_F n_f \alpha_s^3 T}{2 m_D} \\
& - \frac{C_R C_A^2 \alpha_s^3 T}{m_D}\left[\frac{89}{48} + \frac{\pi^2}{12} - \frac{11}{12}\ln 2\right]
\end{aligned}
$$

- Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

# Checks

- Feynman gauge and Coulomb gauge.
- Static gauge: $\partial_0 A_0 = 0$:

$$P = \frac{1}{d_R}\mathrm{Tr}\langle \exp\left(\frac{igA^0(\mathbf{x})}{T}\right)\rangle =$$



- Phase-space Coulomb gauge:

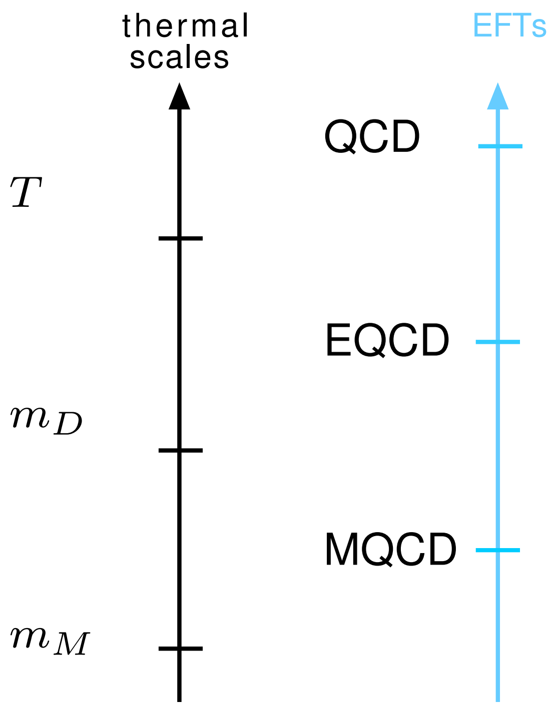
$$e^{-S} = \exp\left[-\int_0^{1/T} d\tau \int d^3x \left(\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{2}F^a_{0i}F^a_{0i}\right)\right]$$

$$= \mathcal{N}^{-1}\int \mathcal{D}E_i \exp\left[-\int_0^{1/T} d\tau \int d^3x \left(\frac{1}{4}F^a_{ij}F^a_{ij} + iE^a_i F^a_{0i} + \frac{1}{2}E^a_i E^a_i\right)\right]$$

  - Andrasi EPJ C37 (2004) 307

- Dimensionally reduced effective field theories.

# Dimensionally reduced EFTs




Polyakov loop

$$P_{\rm EQCD} = Z_0^E - Z_2^E \frac{g^2}{2 d_R T} {\rm Tr}\,\langle \tilde{\rm A}_0^2 \rangle + \dots$$

$$P_{\rm MQCD} = Z_0^M + \frac{Z_1^M}{2 m_D^3} \langle \tilde{F}_{ij}^a \tilde{F}_{ij}^a \rangle + \dots$$

- The Polyakov loop may be calculated relying mostly on known results.
  - Braaten Nieto PR D53 (1996) 3421
    Kajantie Laine Rummukainen Shaposhnikov NP B503 (1997) 357, ...
- Non-perturbative contributions carried by $m_M$ are of order $g^7$ ($Z_1^M \sim \alpha_{\rm s}^2$).

# Magnetic mass contributions

MQCD shows that magnetic mass contributions appear at $\mathcal{O}(g^7)$.

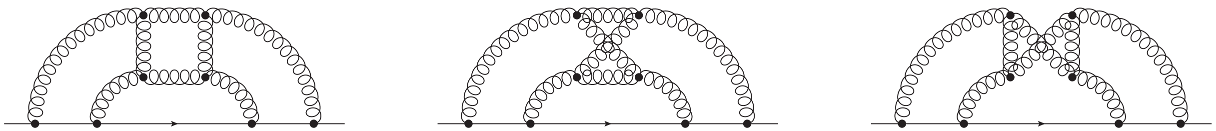
- At order $g^5$ the following two diagrams cancel when the spatial gluon carries a momentum of order $m_M$:

- The explicit cancellation of the magnetic mass contributions at order $g^6$ has also been checked.

# Casimir scaling

Casimir scaling holds up to $\mathcal{O}(g^7)$ (including $m_M$ contributions).

Possible Casimir scaling violations may happen at $\mathcal{O}(g^8)$, through diagrams like



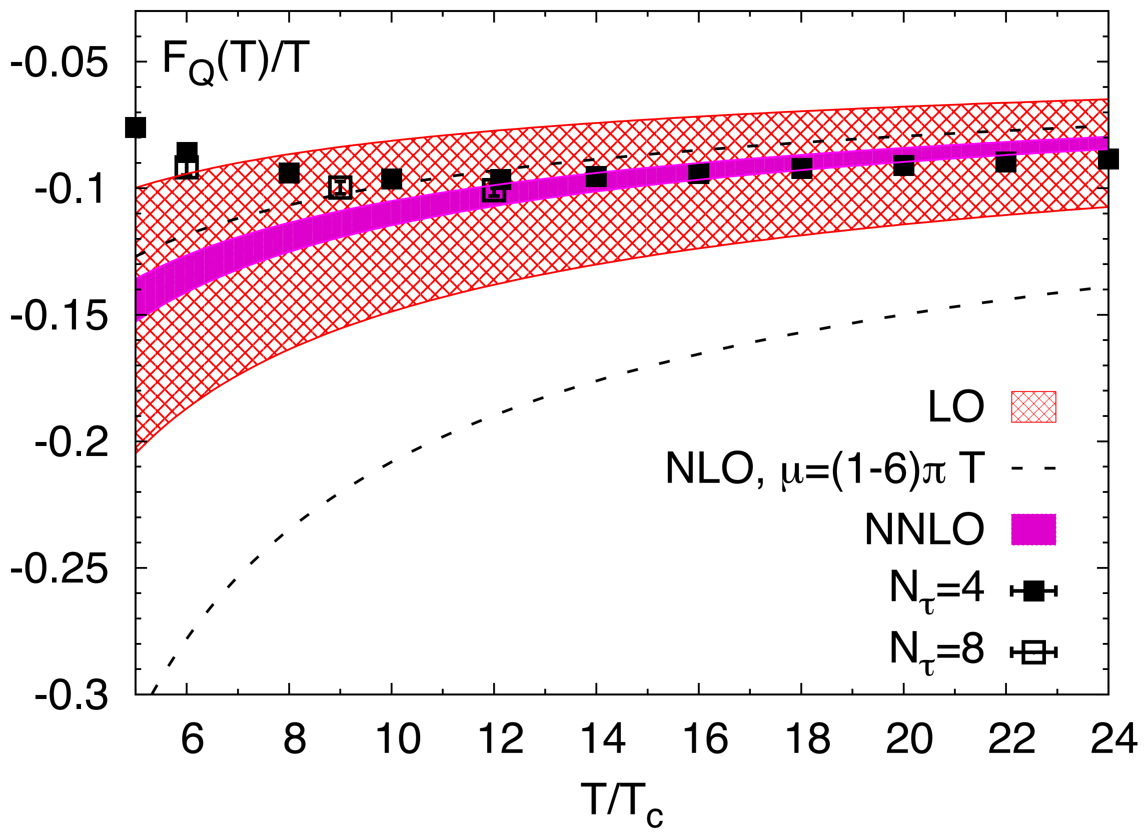
+ 4 gluon vertex diagrams + light quark loop diagrams.

These are proportional to

$$C_R^{(4)} = f^{i_1 a_1 i_2} \cdots f^{i_4 a_4 i_1} \frac{1}{d_R} \mathrm{Tr}\left[T_R^{a_1} \cdots T_R^{a_4}\right], \qquad \text{with} \quad \frac{C_F^{(4)}}{C_A^{(4)}} = \frac{C_F}{C_A} \frac{N^2+2}{N^2+12}$$

○ Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

# Free energy vs quenched lattice data

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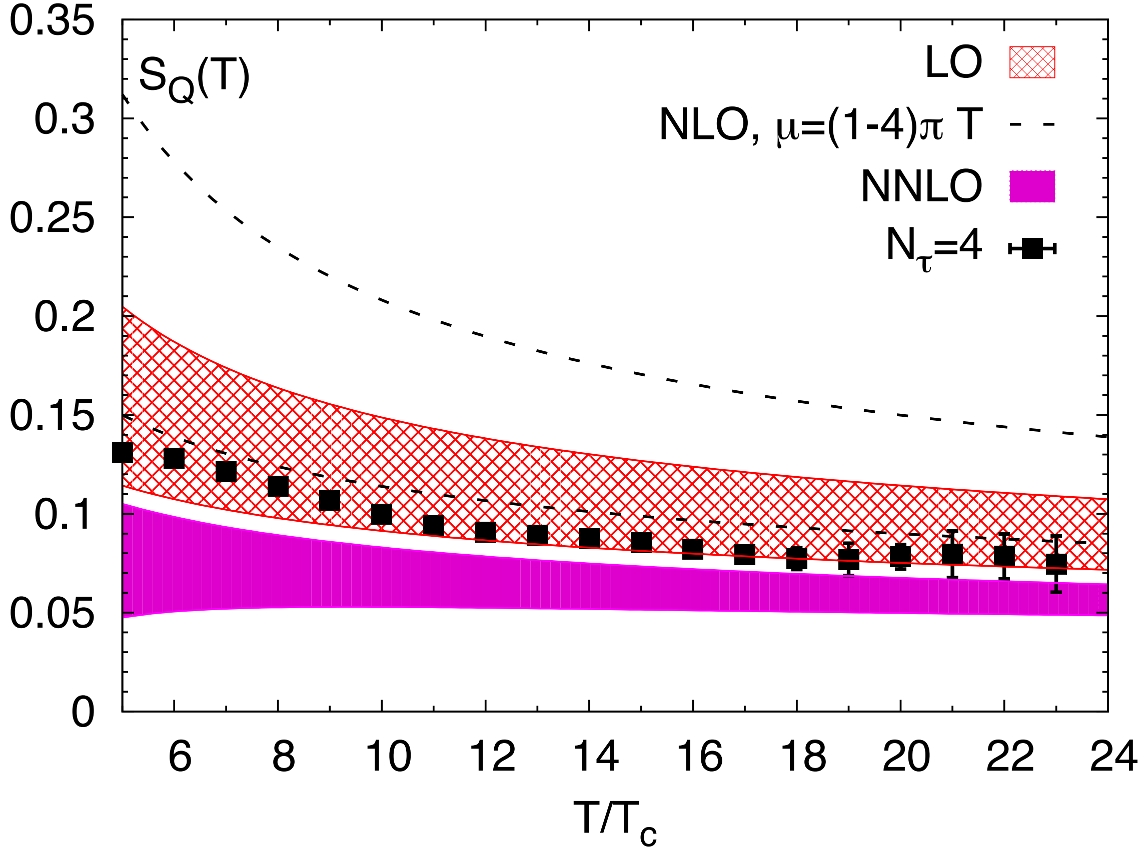
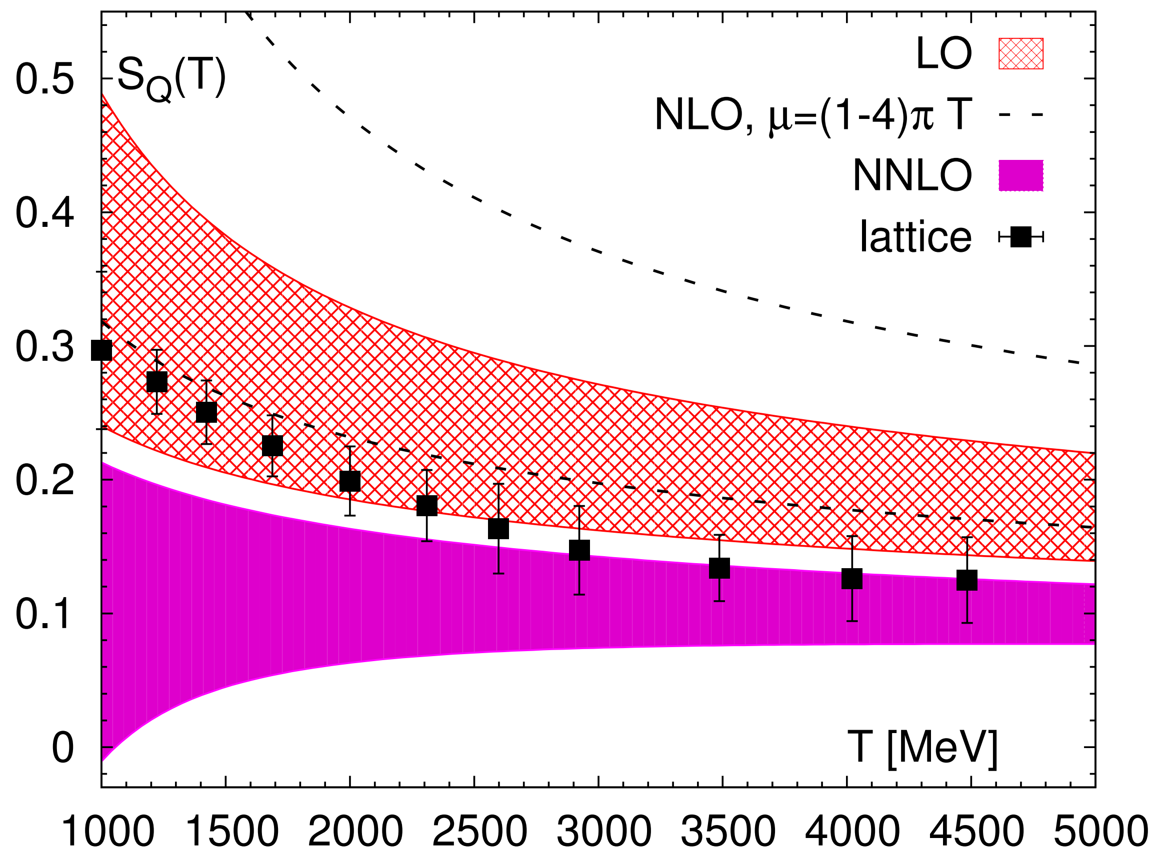
# Entropy vs quenched lattice data

The entropy does not depend on the normalization shift: $S_Q = -\frac{\partial F_Q(T)}{\partial T}$.




○ Berwein Brambilla Petreczky Vairo PR D93 (2016) 034010

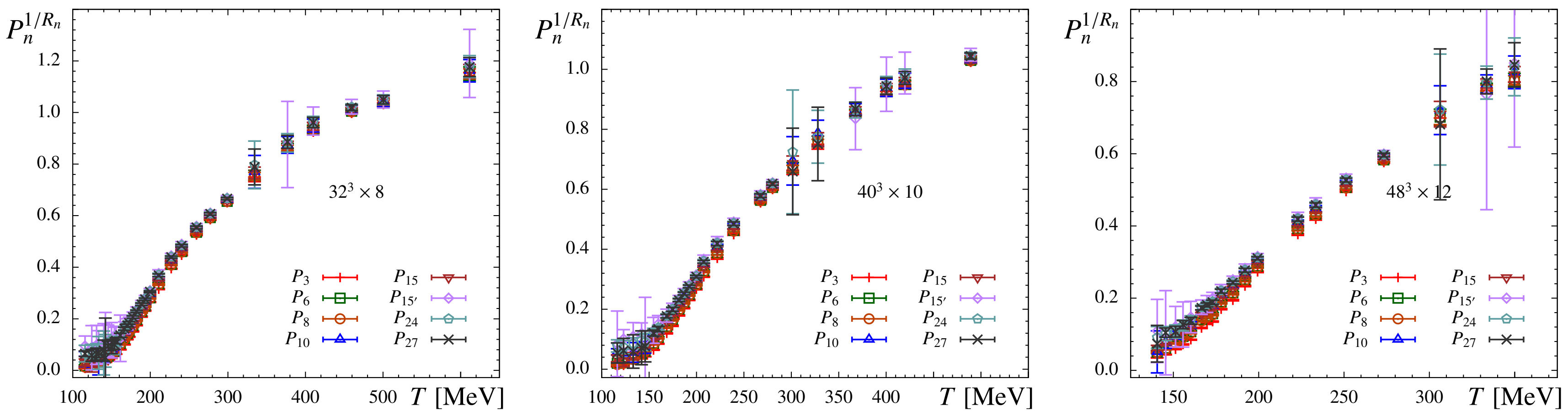
# Entropy vs 2+1 flavor lattice data




Position of the entropy peak: $T_S = 153^{6.5}_{-5}$ MeV.

- Bazavov Brambilla Ding Petreczky Schadler Vairo Weber
  PR D93 (2016) 114502

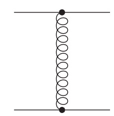
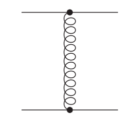
# Casimir scaling

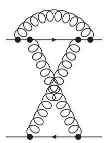
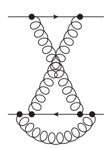
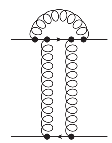
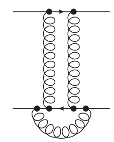
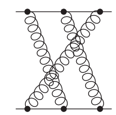
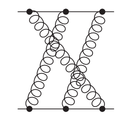
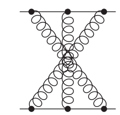
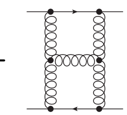
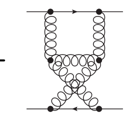


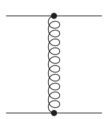
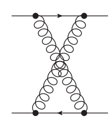
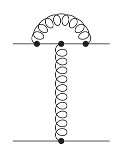
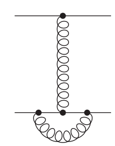

- Petreczky Schadler PR D92 (2015) 094517

# Polyakov loop correlator

# Polyakov loop correlator in perturbation theory

$\exp\left[\frac{2F_Q - F_{Q\bar{Q}}}{T}\right] = 1 + \frac{N^2-1}{8N^2}\mathcal{K}^2 \Bigg($  $\Bigg)$ $+ \frac{(N^2-1)(N^2-2)}{48N^3}\mathcal{K}^3 \Bigg($  $\Bigg)$

$+ \frac{N^2-1}{4N}\mathcal{K} \Bigg($  $+$  $+$  $+$  $+$  $+$  $+$  $-$  $-$  $\Bigg)$

$- \frac{N^2-1}{8N}\mathcal{K} \Bigg($  $\Bigg) \mathcal{K} \Bigg($  $+$  $+$  $\Bigg) + \mathcal{O}\left(\alpha_s^4\right)$

## Polyakov loop correlator up to $g^6$ at short distances, $m_D \gg \alpha_s/r$

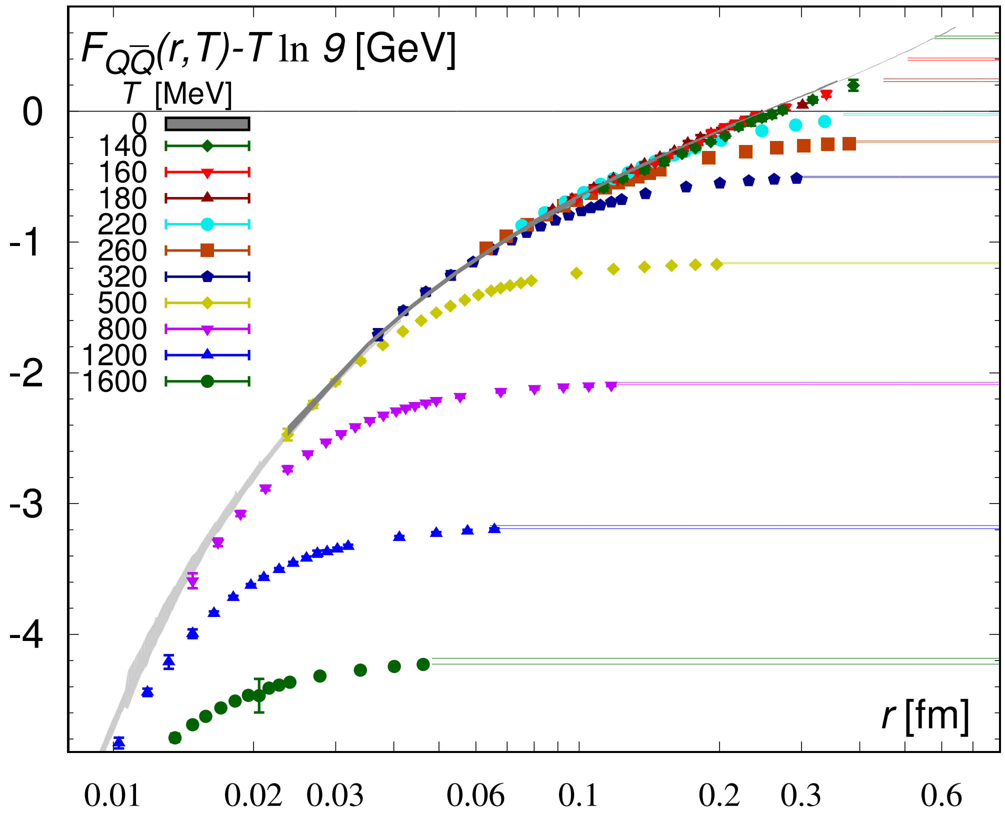
$$
\begin{aligned}
\exp\left[\frac{2F_Q - F_{Q\bar{Q}}}{T}\right]_{\text{up to } g^6} = 1 + \frac{N^2-1}{8N^2}\Bigg\{ & \frac{\alpha_s^2(1/r)}{r^2T^2} - \frac{2\alpha_s(1/r)\alpha_s(4\pi T)m_D(4\pi T)}{rT^2} \\
& + \frac{N^2-2}{6N}\frac{\alpha_s^3(1/r)}{r^3T^3} + \frac{\alpha_s(1/r)\alpha_s^2}{2\pi r^2T^2}\left(\frac{31}{9}N - \frac{10}{9}n_f + 2\beta_0\gamma_E\right) \\
& + \frac{2\alpha_s(1/r)\alpha_s^2}{rT}\left[N\left(1 - \frac{\pi^2}{8} + \ln\frac{T^2}{m_D^2}\right) + n_f\ln 2\right] \\
& - \frac{2\pi N\alpha_s(1/r)\alpha_s^2}{9} + \frac{\alpha_s^2(4\pi T)m_D^2(4\pi T)}{T^2} \\
& + 2\alpha_s(1/r)\alpha_s^2\left(\frac{4}{3}N + n_f\right)\zeta(3)rT \\
& - 2\pi\alpha_s(1/r)\alpha_s^2\left(\frac{22}{675}N + \frac{7}{270}n_f\right)(r\pi T)^2\Bigg\} + \mathcal{O}\left(g^6(r\pi T)^4\right)
\end{aligned}
$$

- Berwein Brambilla Petreczky Vairo PR D96 (2017) 014025

## Polyakov loop correlator at $g^7$ at short distances, $m_D \gg \alpha_s/r$

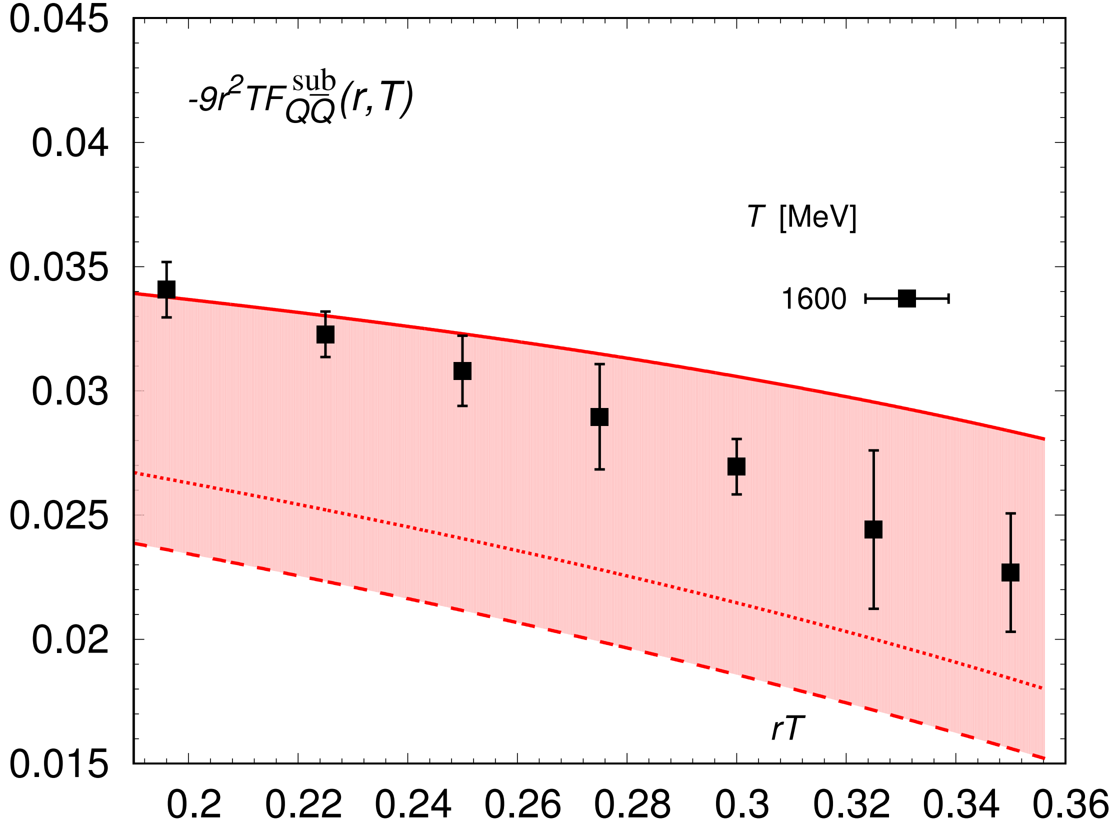
$$
\begin{aligned}
\exp\left[\frac{2F_Q - F_{Q\bar{Q}}}{T}\right]_{g^7} = \frac{N^2-1}{8N^2}\Bigg\{&-\frac{N^2-2}{2N}\,\frac{\alpha_s^2(1/r)\alpha_s(4\pi T)m_D(4\pi T)}{r^2T^3} \\
&-\frac{2\alpha_s^2\alpha_s(4\pi T)m_D(4\pi T)}{4\pi r T^2}\left(\frac{31}{9}N - \frac{10}{9}n_f + 2\beta_0\gamma_E\right) \\
&-\frac{3\alpha_s(1/r)\alpha_s^2 m_D}{4\pi r T^2}\left[3N + \frac{2}{3}n_f(1-4\ln 2) + 2\beta_0\gamma_E\right] \\
&+\frac{(N^2-1)n_f}{2N}\,\frac{\alpha_s(1/r)\alpha_s^3}{r m_D} + \frac{2N^2\alpha_s(1/r)\alpha_s^3}{r m_D}\left[\frac{89}{24} + \frac{\pi^2}{6} - \frac{11}{6}\ln 2\right] \\
&-\frac{2\alpha_s^2\alpha_s(4\pi T)m_D(4\pi T)}{T}\left[N\left(-\frac{1}{2} + \ln\frac{T^2}{m_D^2}\right) + n_f\ln 2\right] \\
&-\frac{\alpha_s(1/r)\alpha_s m_D^3}{3T^3}rT + \frac{2\pi N\alpha_s^2\alpha_s(4\pi T)m_D(4\pi T)}{9T}rT \\
&-\frac{2\alpha_s^2\alpha_s(4\pi T)m_D(4\pi T)}{T}\left(\frac{4}{3}N + n_f\right)\zeta(3)(rT)^2 \\
&+\frac{2\alpha_s^2\alpha_s(4\pi T)m_D(4\pi T)}{T}\left(\frac{22}{675}N + \frac{7}{270}n_f\right)(r\pi T)^3\Bigg\} + \mathcal{O}\left(g^7(r\pi T)^4\right)
\end{aligned}
$$

## $F_{Q\bar{Q}}$ on the lattice



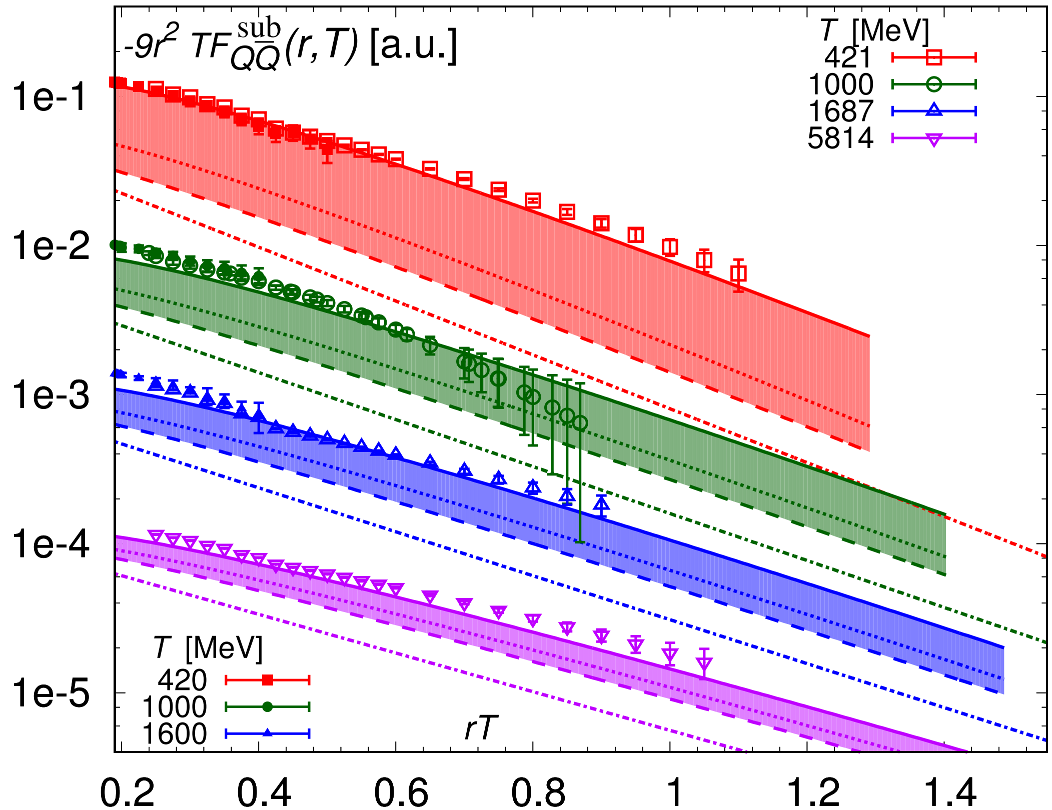

- TUMQCD coll. arXiv:1804.10600

# $F_{Q\bar{Q}}$ at short distances




- TUMQCD coll. arXiv:1804.10600

# $F_{Q\bar{Q}}$ at screening distances




- TUMQCD coll. arXiv:1804.10600

# Polyakov loop correlator in pNRQCD

In an EFT/pNRQCD framework $P_c(r,T)$ can be put in the form

$$P_c(r,T) = \frac{1}{N^2}\left[e^{-f_s(r,T,m_D)/T} + (N^2-1)e^{-f_o(r,T,m_D)/T} + \mathcal{O}\left(\alpha_s^3 (rT)^4\right)\right]$$

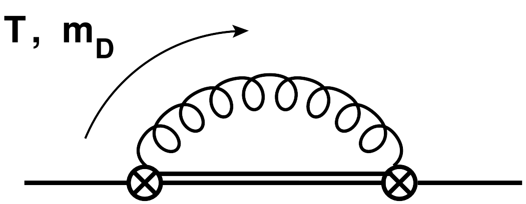
$f_s = Q\bar{Q}$-color singlet free energy, $f_o = Q\bar{Q}$-color octet free energy
to be matched from the singlet and octet pNRQCD propagators

$$\frac{\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle}{\mathcal{N}} = e^{-V_s(r)/T}(1+\delta_s) \equiv e^{-f_s(r,T,m_D)/T}$$

$$\frac{\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle}{\mathcal{N}} = e^{-V_o(r)/T}\left[(N^2-1)\langle P_A\rangle + \delta_o\right] \equiv (N^2-1)e^{-f_o(r,T,m_D)/T}$$

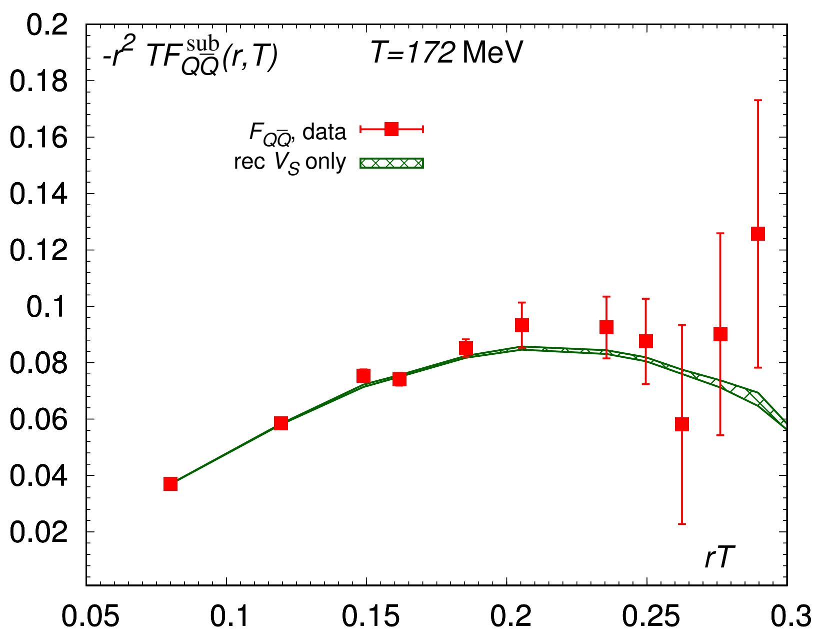
where $\delta_s$ and $\delta_o$ stand for thermal loop corrections to the singlet/octet propagators:

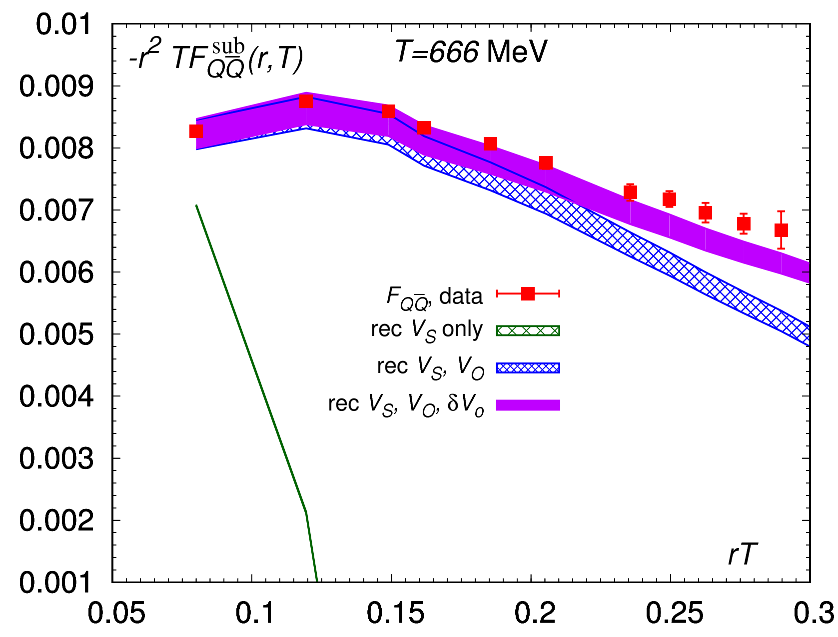
$\delta_s =$




○ Brambilla Ghiglieri Petreczky Vairo PR D78 (2008) 014017

# $F_{Q\bar{Q}}$ on the lattice and pNRQCD



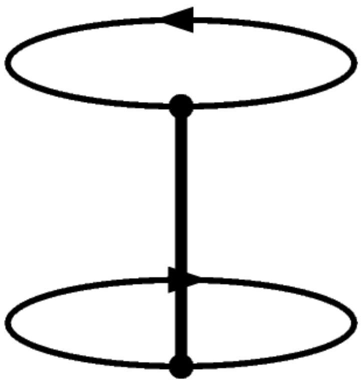




- TUMQCD coll. arXiv:1804.10600

# Cyclic Wilson loop

## Divergences of the cyclic Wilson loop

Differently from $P(T)$ and $P_c(r,T)$, $W_c(r,T)$ is divergent after charge and field renormalization. This divergence is due to intersection points.



Although it may seem that the cyclic Wilson loop has a continuously infinite number of intersection points, one needs to care only about the two endpoints, for the Wilson loop contour does not lead to divergences in the other ones.

# How to renormalize intersection divergences

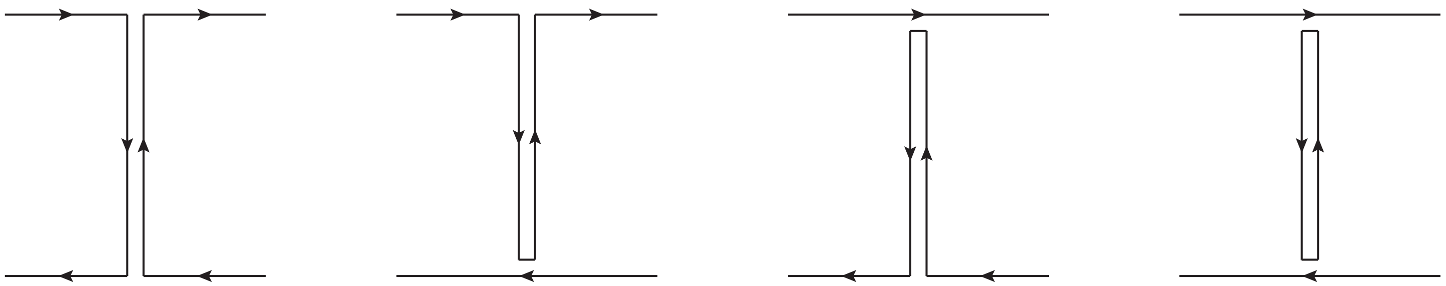
For intersection points connected by 2 Wilson lines (angles $\theta_k$) and cusps (angles $\varphi_l$):

$$W^{(R)}_{i_1 i_2 \dots i_r} = Z_{i_1 j_1}(\theta_1) Z_{i_2 j_2}(\theta_2) \cdots Z_{i_r j_r}(\theta_r) Z(\varphi_1) Z(\varphi_2) \cdots Z(\varphi_s) W_{j_1 j_2 \dots j_r}$$

- The indices $i_k$ and $j_k$ label the different possible path-ordering prescriptions.
- The loop functions are color-traced and normalized by the number of colours.
- This ensures that all loop functions are gauge invariant.
- The coupling in $W^{(R)}_{i_1 i_2 \dots i_r}$ is the renormalized coupling.
- The matrices $Z$ are the renormalization matrices.

○ Brandt Neri Sato PR D24 (1981) 879

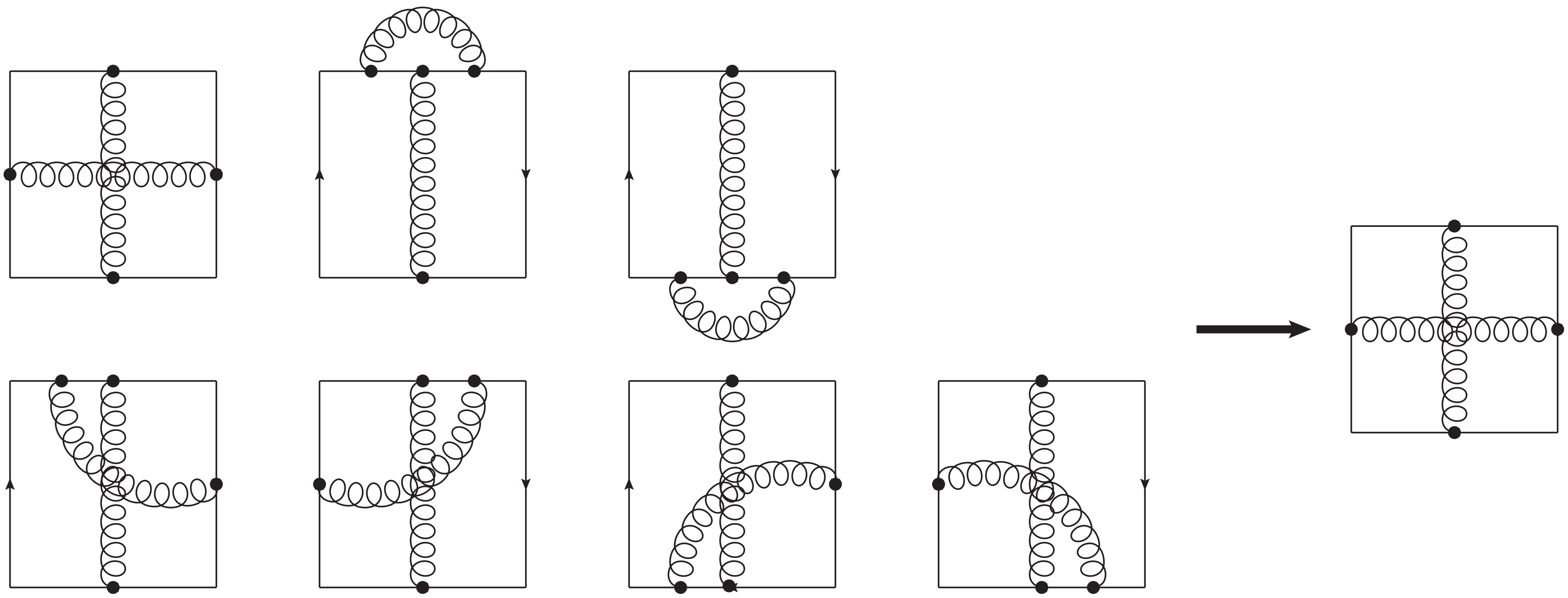
# How to renormalize the cyclic Wilson loop



$$\begin{pmatrix} W_c^{(R)} \\ P_c \end{pmatrix} = \begin{pmatrix} Z & 1-Z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} W_c \\ P_c \end{pmatrix}$$

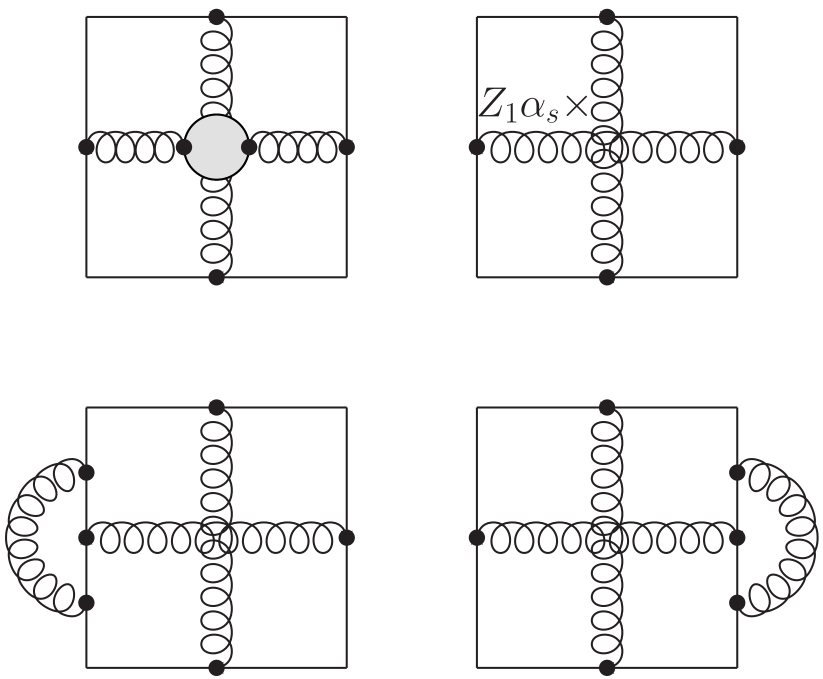
$$Z = 1 + Z_1 \alpha_s \mu^{-2\varepsilon} + Z_2 \left(\alpha_s \mu^{-2\varepsilon}\right)^2 + \mathcal{O}(\alpha_s^3)$$

## $Z_1$



$$Z_1 = -\frac{C_A}{\pi}\frac{1}{\bar{\varepsilon}}$$

## $Z_2$




$Z_2$ reabsorbs all divergences of the type $\alpha_{\rm s}^3/(rT)$.

All other divergences at $\mathcal{O}(\alpha_{\rm s}^3)$ are reabsorbed by $Z_1$ (combined with $P_c(r,T)$ at $\mathcal{O}(\alpha_{\rm s}^2)$)!

# Renormalization group equation at one loop

$$\begin{cases} \mu \dfrac{\mathrm{d}}{\mathrm{d}\mu} \left( W_c^{(R)} - P_c \right) = \gamma \left( W_c^{(R)} - P_c \right) \\ \mu \dfrac{\mathrm{d}}{\mathrm{d}\mu} \alpha_{\mathrm{s}} = -\dfrac{\alpha_{\mathrm{s}}^2}{2\pi} \beta_0 \end{cases}$$

$\gamma$ is the anomalous dimension of $W_c^{(R)} - P_c$:

$$\gamma \equiv \frac{1}{Z} \mu \frac{\mathrm{d}}{\mathrm{d}\mu} Z = 2C_A \frac{\alpha_{\mathrm{s}}}{\pi} + \mathcal{O}(\alpha_{\mathrm{s}}^2)$$

$$\left( W_c^{(R)} - P_c \right)(\mu) = \left( W_c^{(R)} - P_c \right)(1/r) \left( \frac{\alpha_{\mathrm{s}}(\mu)}{\alpha_{\mathrm{s}}(1/r)} \right)^{-4C_A/\beta_0}$$

## $W_c$ for $1/r \gg T \gg m_D \gg g^2/r$

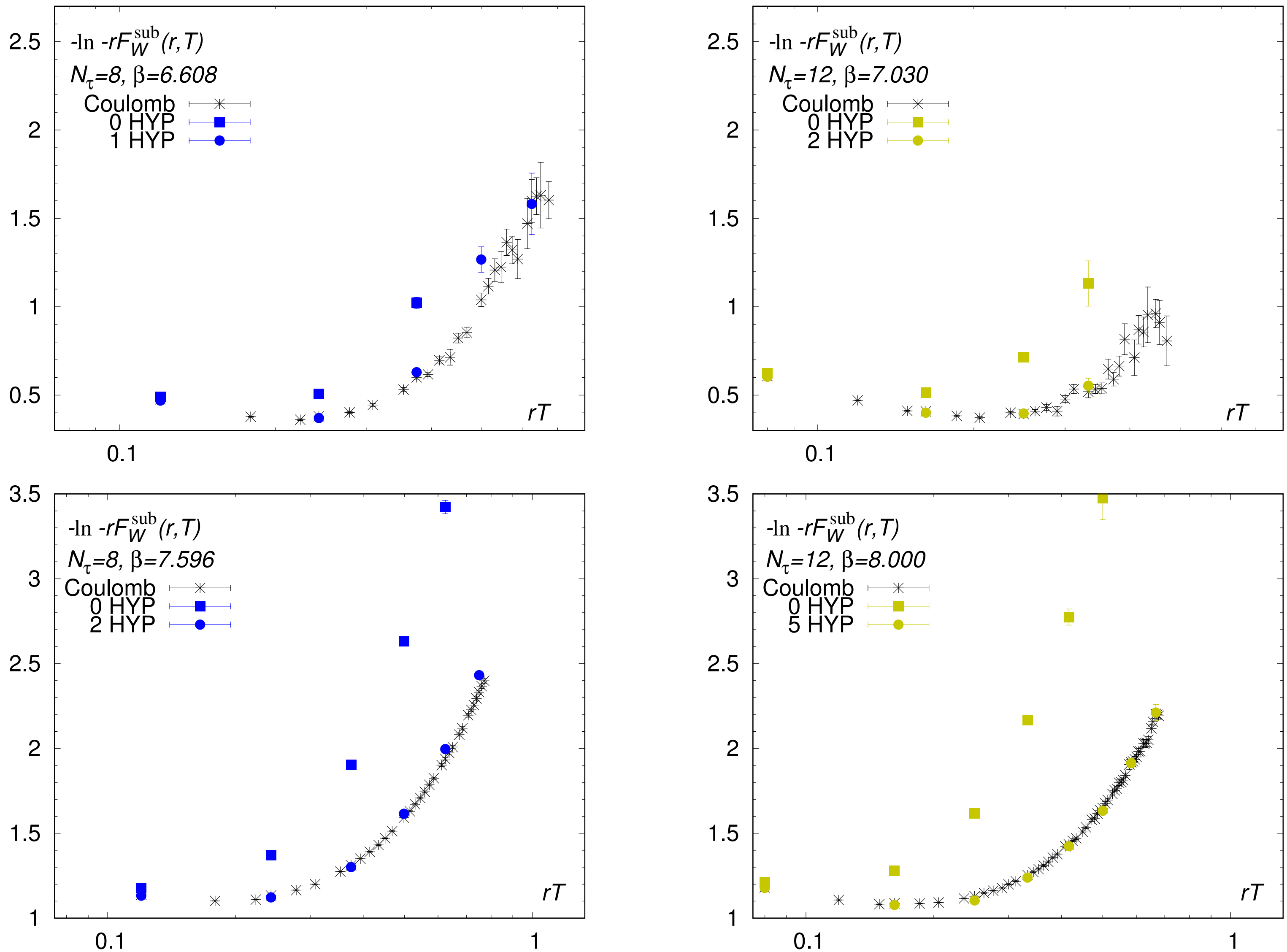
In $\overline{\rm MS}$ at NLO and LL accuracy (i.e. including all terms $\alpha_s/(rT) \times (\alpha_s \ln \mu r)^n$), we have

$$
\begin{aligned}
\ln W_c^{(R)} \quad = \quad & \frac{C_F \alpha_s(1/r)}{rT} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[ \left( \frac{31}{9} C_A - \frac{10}{9} n_f \right) + 2\,\beta_0 \gamma_E \right] \right. \\
& \left. + \frac{\alpha_s C_A}{\pi} \left[ 1 + 2\gamma_E - 2\ln 2 + \sum_{n=1}^{\infty} \frac{2(-1)^n \zeta(2n)}{n(4n^2-1)} (rT)^{2n} \right] \right\} \\
& + \frac{4\pi \alpha_s C_F}{T} \int \frac{d^3k}{(2\pi)^3} \left( e^{i\mathbf{r}\cdot\mathbf{k}} - 1 \right) \left[ \frac{1}{\mathbf{k}^2 + \Pi_{00}^{(T)}(0,\mathbf{k})} - \frac{1}{\mathbf{k}^2} \right] + C_F C_A \alpha_s^2 \\
& + \frac{C_F \alpha_s}{rT} \left[ \left( \frac{\alpha_s(\mu)}{\alpha_s(1/r)} \right)^{-4C_A/\beta_0} - 1 \right] + \mathcal{O}\left(g^5\right)
\end{aligned}
$$

$\Pi_{00}^{(T)}(0,\mathbf{k})$ = (known) thermal part of the gluon self-energy in Coulomb gauge.

∘ Berwein Brambilla Ghiglieri Vairo JHEP 1303 (2013) 069

# $W_c$ from smeared lattices vs singlet correlator




- TUMQCD coll. arXiv:1804.10600

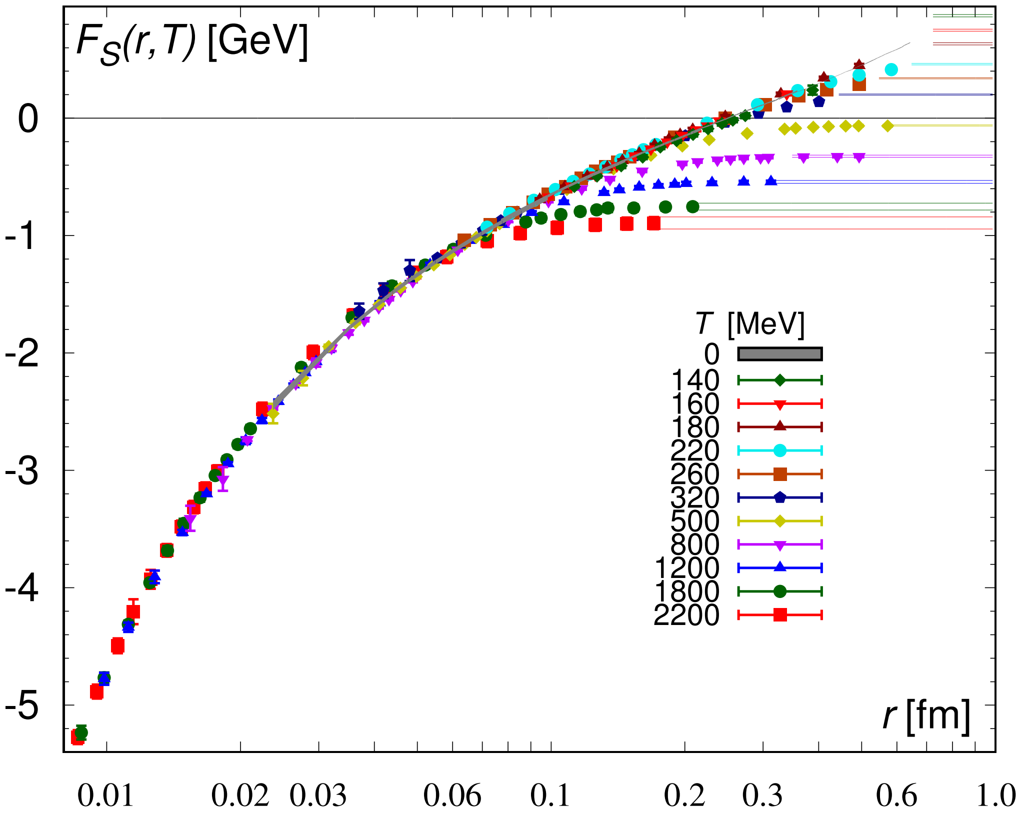
# Singlet correlator in Coulomb gauge

# The singlet free energy up to $g^4$ at short distances

$$
\begin{aligned}
\frac{F_S}{T} = & -\frac{N^2-1}{2N}\frac{\alpha_s(1/r)}{rT}\left[1+\frac{\alpha_s}{4\pi}\left(\frac{31}{9}N-\frac{10}{9}n_f+2\beta_0\gamma_E\right)\right]+\frac{1}{18}\left(N^2-1\right)\alpha_s^2 r\pi T \\
& -\frac{N^2-1}{2N}\left(\frac{4}{3}N+n_f\right)\zeta(3)\alpha_s^2 r^2 T^2+\frac{N^2-1}{12N}\frac{\alpha_s m_D^3}{T^3}r^2T^2 \\
& +\frac{N^2-1}{2N}\left(\frac{22}{675}N+\frac{7}{270}n_f\right)\alpha_s^2(r\pi T)^3+\mathcal{O}\left(\alpha_s^2(r\pi T)^5,\alpha_s^3\right)
\end{aligned}
$$

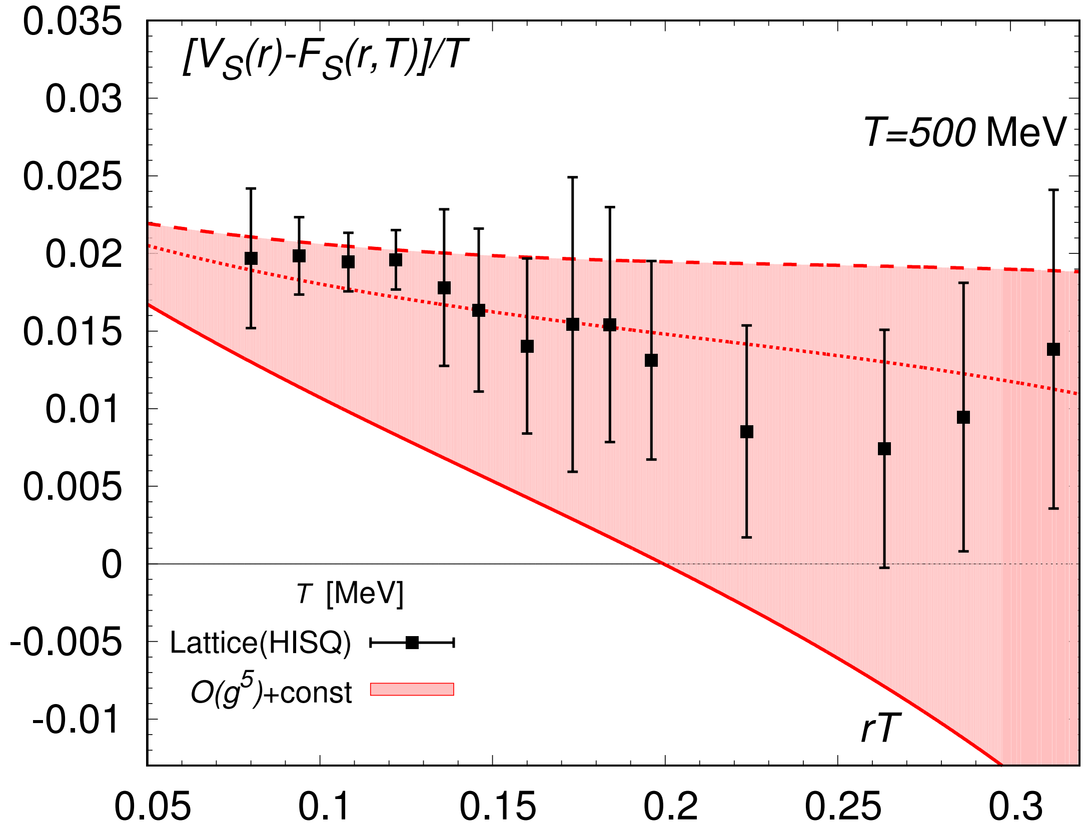
- Burnier Laine Vepsäläinen JHEP 1001 (2010) 054
  Berwein Brambilla Petreczky Vairo PR D96 (2017) 014025

# $F_S$ on the lattice



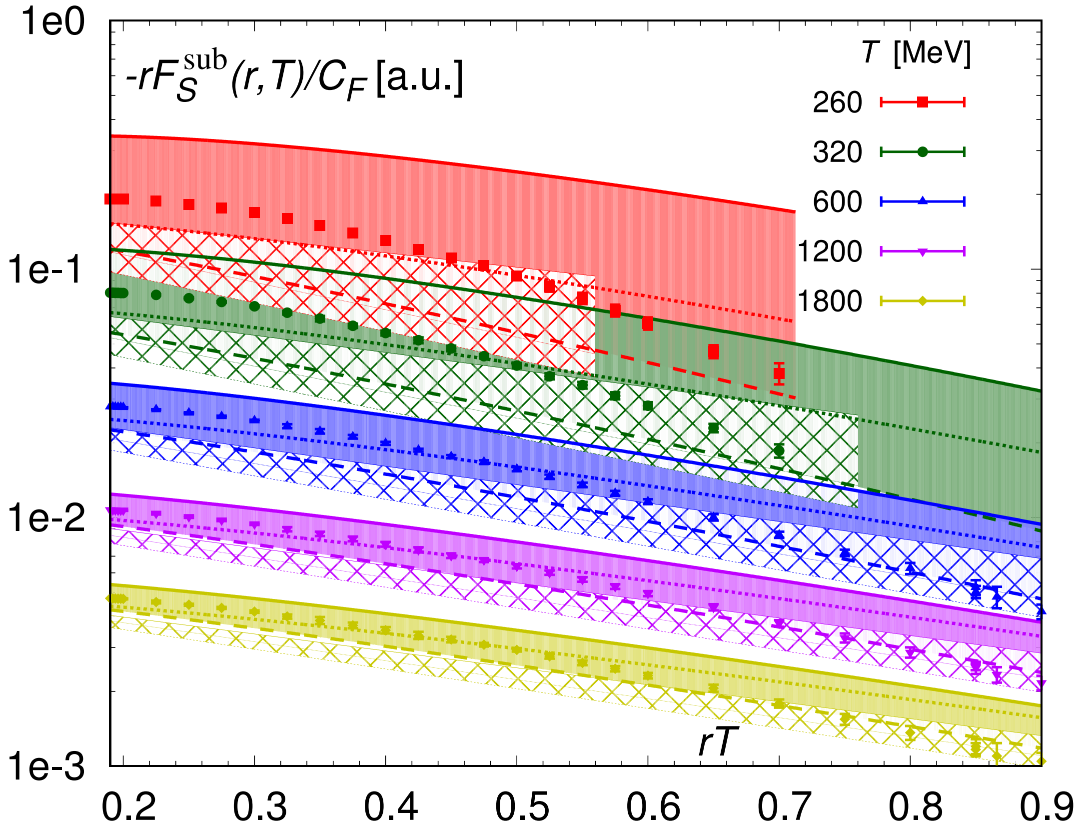

○ TUMQCD coll. arXiv:1804.10600

## $F_S$ at short distances




○ TUMQCD coll. arXiv:1804.10600

## $F_S$ at screening distances




- TUMQCD coll. arXiv:1804.10600

# Conclusions

- The Polyakov loop has been computed up to order $g^5$.
- The (subtracted) $Q\bar{Q}$ free energy has been computed at short distances up to corrections of order $g^7(rT)^4$, $g^8$.
- The (subtracted) $Q\bar{Q}$ free energy has been computed at screening distances up to corrections of order $g^8$.
- The cyclic Wilson loop free energy has been computed at short distances up to corrections of order $g^5$ + LL resummation.
- The singlet free energy has been computed at short distances up to corrections of order $g^4(rT)^5$, $g^6$.
- The singlet free energy has been computed at screening distances up to corrections of order $g^5$.

- Lattice calculations are consistent with weak-coupling expectations.