Transport in Dense Nuclear Matter

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Ermal Rrapaj (University of Guelph), Sanjay Reddy (INT Seattle)

[S. Stetina, in preparation]
Outer layer of neutron star cores

- homogeneous plasma of electrons, muons, protons, and neutrons
- stable homogeneous nuclear matter
  - degenerate QED plasma ($e^-, \mu^-, p^+$)
  - $p, n$ form strongly interacting Fermi liquid
- $\beta$ equilibrium and charge neutrality
  \[ \mu_n - \mu_p = \mu_e = \mu_{\mu}, \quad n_e + n_{\mu} = n_p \]
- critical densities
  - stability of hom. phase (spinodal point)
  \[ n_c \sim 0.6 \, n_0 \]
  - onset of muons ($\mu_e = m_\mu$)
  \[ n_\mu \sim 0.75 \, n_0 - 0.8 \, n_0 \]
- electrons under NS conditions are always relativistic, degenerate, weakly interacting
- important contribution to transport
Neutron star phenomenology

Transport phenomena in the outer core of neutron stars

transport is determined by

- electromagnetic response:
  - screening & damping
  - collective modes
- correlations of strong & EM int.
- Scattering rates of fermions

transport is relevant for

- neutron stars \((T \ll \mu)\)
- damping of hydro/R modes
- spin evolution
- thermal relaxation

supernovae
Energy loss of $e^-$ and $\mu^-$ in high density matter

Separation of scales in degenerate plasma:

- hard region (free space limit): $p = (p_0, \mathbf{p}) \sim k_f$
- soft region (medium effects): $p \sim e k_f$

transport in *cold and dense* matter (e.g., at $n = n_0$: $\mu_e \sim 120$ MeV, $T < 1$ MeV)

= scattering close to the Fermi surface

$\rightarrow$ *fermions (and holes)* $p \sim k_f$ are always *on-shell*, there is no damping at order $\alpha_f$

$\rightarrow$ *photon* is either hard (large angle) or soft (small) angle

**collective modes** in the soft region: Photon (transverse), Plasmon (longitudinal)

Relevant contribution to the fermion self energy: bare fermions + resummed photon

with $\imath$ = $\imath_\uparrow + \imath_\downarrow$
RPA photon propagator

Relativistic one-loop resummation (“Random Phase Approximation”, RPA)

- Dressed photon propagator (Coulomb Gauge):

\[
\tilde{D}^{\mu \nu}(q_0, q) = \frac{q^2}{q^2} (q^2 - \Pi_L)^{-1} P_L^{\mu \nu} + (q^2 - \Pi_\perp)^{-1} P_\perp^{\mu \nu}, \quad \Pi^{\mu \nu} \text{ photon pol. tensor}
\]

→ **Weak screening** approximation of longitudinal/transverse propagators:

\[
D_L \propto \frac{1}{q^2 - m_D^2}, \quad D_\perp \propto \frac{1}{q^2 - i \frac{q_0}{|q|} q_\perp^2}, \quad m_D^2 = \frac{4\alpha_f}{\pi} \mu k_f, \quad q_\perp^2 = \alpha_f k_f^2
\]


→ **Hard dense loop** (HDL) approximation \( q \ll k_f \) (leading order contribution in soft region)

Damping rate of degenerate fermions

\[
-2\text{ Im}
\begin{bmatrix}
\begin{array}{c}
\cdot \\
\cdot
\end{array}
\end{bmatrix}
= \left|
\begin{array}{c}
\begin{array}{c}
\cdot \\
\cdot
\end{array}
\end{array}
\right|^2
\]

optical theorem: int. rate. \( \Gamma \propto \text{Im} \Sigma \)

\[
\Gamma_+ = \frac{1}{2} \text{Tr} [\Lambda_+ \gamma_0 \text{Im} \Sigma_R] = -\frac{1}{2p_0} \text{Tr}[(\gamma \cdot p + m) \text{Im} \Sigma_R (p_0, p)], \quad p_0 = \epsilon_p
\]

photon spectrum \( \rho^{\mu\nu} = \rho_L P_L^{\mu\nu} + \rho_\perp P_\perp^{\mu\nu} \)

→ week screening & close to Fermi surface \( \epsilon_p - \mu \ll m_D \), \( u = q_0 / |q| \)

\[
\Gamma_L \approx \frac{e^2}{4\pi} \frac{m_D^2}{v_f^2} \int_0^\infty du \ u \int_0^\infty d|q| \frac{1}{(m_D^2 + q^2)^2} = \frac{e^2}{32} \frac{1}{m_D v_f^2} (\epsilon_p - \mu)^2
\]

\[
\Gamma_\perp \approx \frac{e^2}{4\pi} \frac{m_D^2}{v_f^2} \int_0^\infty du \ u \int_0^\infty d|q| |q| \frac{4q^2}{16q^6 + u^2 \pi^2 m_D^2 v_f^2} = \frac{e^2}{12\pi} v_f |\epsilon_p - \mu|
\]

Damping: weak screening vs. HDL vs. full RPA

→ **nonrelativistic**: electric interactions dominate, magnetic interactions are down by \( \left( \frac{v}{c} \right)^2 \)

→ **relativistic**: damping due to the exchange of plasmons and photons is equally important


electrons at \( n = n_0 \)

![Graphs showing damping vs. momentum for electrons at \( n = n_0 \).]

- solid: full one-loop
- dashed: HDL
- dot-dashed: weak screening

→ HDL approximations work much better in the longitudinal channel!
Longitudinal and transverse damping (II)

→ **nonrelativistic:** electric interactions dominate, magnetic interactions are down by $\left(\frac{v}{c}\right)^2$

→ **relativistic:** damping due to the exchange of plasmons and photons is equally important


**muons at n = n₀**

→ $|q| \ll k_f$ hard to fulfill, HDL don’t work really well in either channel

→ $\Gamma_L$ overtakes $\Gamma_\perp$
RPA photon propagator: multi species

Photon propagator in the presence of several fermion species

\[ \tilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{q^2} (q^2 - Tr [\Pi_L])^{-1} P_L^{\mu\nu} + (q^2 - Tr [\Pi_\perp])^{-1} P_\perp^{\mu\nu}, \quad \Pi \rightarrow \text{diag}(\Pi_e, \Pi_\mu, \Pi_p) \]

\[ \sum_{i = e, \mu, p} \Pi_i \]

RPA resummation in multi-component plasma is well established:


Protons are quasiparticles (strongly interacting Fermi liquid)

→ Proton fraction \( n_p \), effective masses \( m_p^* \), residual interactions \( V_{pp}, V_{pn} \)

extracted from Landau energy functional based on Skyrme type interactions

→ Here: NRAPR, SKRA, SQMC700, LNS, KDE0v1

QED + strong interactions

What's the role of the neutrons within RPA?

- Coupling strength to photon tiny in free space

- BUT: Interactions induced by the polarizability of protons

- use resummed RPA polarization tensor for protons

\[ \Pi_p = \frac{\Pi_p (1 - V_{nn} \Pi_n)}{1 - V_{nn} \Pi_n - V_{pp} \Pi_p + (V_{nn} V_{pp} - V_{np}^2) \Pi_n \Pi_p} \]

\[ \overline{D}^{\mu\nu}(q_0, q) = \frac{q^2}{q^2} (q^2 - \Pi_{e,L} - \Pi_{\mu,L} - \overline{\Pi}_{p,L})^{-1} P_L^{\mu\nu} + (q^2 - \Pi_{e,\perp} - \Pi_{\mu,\perp} - \overline{\Pi}_{p,\perp})^{-1} P_{\perp}^{\mu\nu} \]

effective interaction

\[ L_{\gamma-n} = e^2 V_{np} (\overline{\gamma} \gamma \mu \ n) A_{\nu} (\Pi_{L,p} P_L^{\mu\nu} + \Pi_{\perp,p} P_{\perp}^{\mu\nu}) \]

\[ \rightarrow \text{changes to transverse spectrum are negligible since for protons } \Pi_{\perp} \ll \Pi_L \].
“Induced” (strong) screening

TD definition: \[ \tilde{\Pi}_{L,p}(q_0 = 0) = \left[ \frac{\partial \mu_p(\mu_n)}{\partial n_p} \right]^{-1} = \frac{m_p^2 (1 + V_{nn} m_n^2)}{1 + V_{nn} m_n^2 + V_{pp} m_p^2 + (V_{nn} V_{pp} - V_{np}^2) m_n^2 m_p^2} \]

Impact of induced interactions most pronounced at densities close to the crust-core boundary

\[ \rightarrow \text{Impact of induced interactions most pronounced at densities close to the crust-core boundary} \]

homogeneous nuclear matter unstable

pure QED (NRAPR)

\[ n = 0.65 \, n_0 \]
Damping rate of electrons: multiple species

Energy loss of electrons due to collisions with other electrons, muons, and protons

\[ -2 \text{Im} \begin{matrix} e, \mu, p \end{matrix} e \begin{cases} e \end{cases} \begin{cases} e \end{cases} + \begin{cases} e \end{cases} \begin{cases} \mu \end{cases} + \begin{cases} e \end{cases} \begin{cases} p \end{cases} \end{matrix} \begin{cases} e \end{cases} \begin{cases} \mu \end{cases} \begin{cases} p \end{cases} \begin{cases} 2 \end{cases} \]

→ Total screening mass:

\[ M_D^2 = \sum_a m_{D,a}^2 \]

\[ \rho_L = -\frac{1}{\pi} \frac{\text{Tr}[\text{Im} \Pi_L]}{(\text{Tr}[\text{Re} \Pi_L] - q^2)^2 + (\text{Tr}[\text{Im} \Pi_L])^2} \]

- Total scattering rate of electrons close to the crust-core boundary
- Easy to implement in transport calculations (fit as function of \( \epsilon_p - \mu \))
Impact of induced interactions

- $\Gamma_L$ strongly modified near crust-core boundary
- follows the evolution of the induced screening

**Electrons**

- For $n = 0.61 \, n_0$
- For $n = n_0$
- For $n = 2 \, n_0$

**Muons**

- For $n = n_0$
- For $n = 2 \, n_0$
Outlook

Where to go from here:

• Improve on implementation of nuclear interactions (dynamical screening)

• Account for proton superconductivity \( \rightarrow \) Meissner effect

\[ D_\perp = \frac{1}{q^2 - \Pi_{\perp, e} - \Pi_{\perp, \mu} - \Pi_{\perp, p}} \]

\( \rightarrow \) induced \( e^- \rightarrow n \) scattering dominates


• include magnetic fields

[S. Stetina, E. Rrapaj, S. Reddy, work in progress]
go raibh maith agat!
(Thank you!)