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Transport in Dense Nuclear Matter

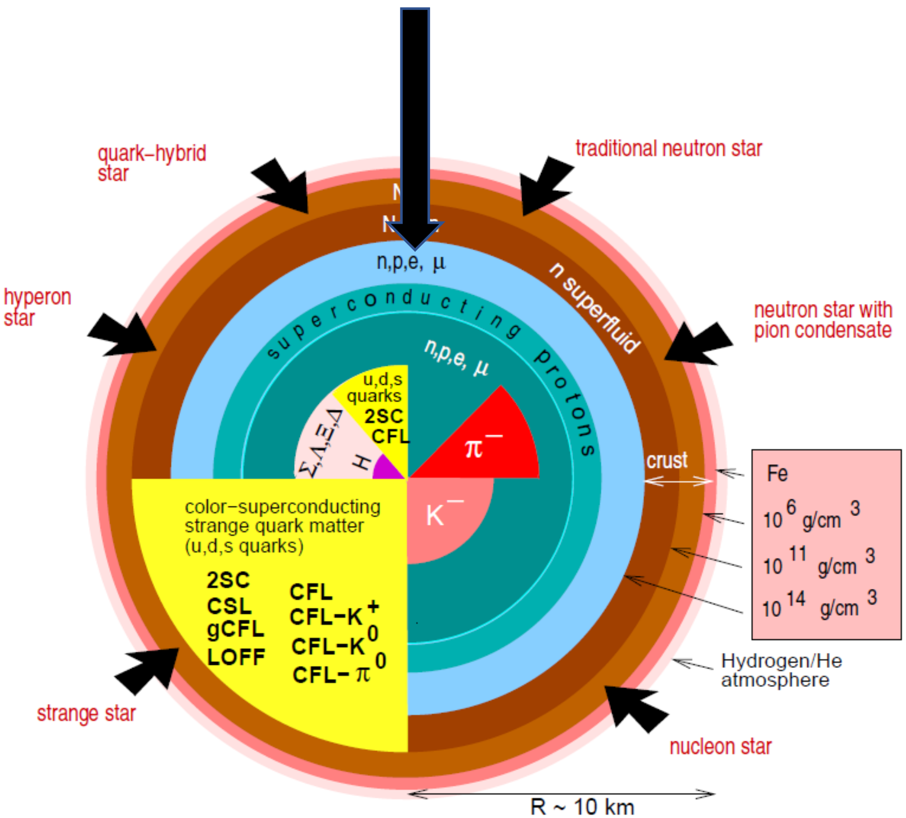
Quark Confinement and the Hadron Spectrum XIII, 2018, Maynooth University

Ermal Rrapaj (University of Guelph), Sanjay Reddy (INT Seattle)

[S. Stetina, E. Rrapaj , S. Reddy, Phys.Rev. C97 (2018) no.4, 045801]
[S. Stetina, in preparation]

Outer layer of of neutron star cores

homogeneous plasma of electrons, muons, protons, and neutrons



[Weber, J. Phys. G27, 465 (2001)]

stable homogeneous nuclear matter

- *degenerate QED plasma* $(e^-,\ \mu^-,\ p^+)$
- p , n form *strongly interacting Fermi liquid*

→ *ß equilibrium* and *charge neutrality*

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ , \qquad n_e + n_\mu = n_p$$

critical densities

→ stability of hom. phase (spinodal point)

$$n_c \sim 0.6\, n_0$$

→ onset of muons $(\mu_e = m_\mu)$

$$n_\mu \sim 0.75\, n_0 - 0.8\, n_0$$

→ **electrons under NS conditions are always relativistic, degenerate, weakly interacting**

→ **important contribution to transport**

Neutron star phenomenology

Transport phenomena in the outer core of neutron stars

transport is relevant for

neutron stars ($T \ll \mu$)

- damping of hydro/R modes
- spin evolution
- thermal relaxation

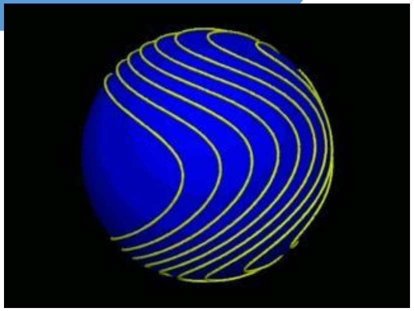
transport is determined by

electromagnetic response:

- screening & damping
- collective modes

correlations of strong & EM int.

Scattering rates of fermions



supernovae

Energy loss of e^- and μ^- in high density matter

Separation of scales in degenerate plasma:

- hard region (free space limit): $\mathrm{p} = (p_0,\ \boldsymbol{p}) \sim\ k_f$
- soft region (medium effects): $\mathrm{p}\ \sim e\, k_f$

transport in *cold and dense* matter (e.g., at $n = n_0$: $\mu_e \sim 120$ MeV, $T < 1$ MeV)

=

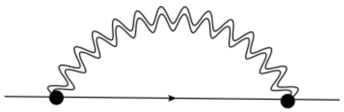
scattering close to the Fermi surface

→ **fermions (and holes)** $p \sim k_f$ are always *on-shell*, there is no damping at order α_f

→ **photon** is either hard (large angle) or soft (small) angle

collective modes in the soft region: Photon (transverse), Plasmon (longitudinal)

Relevant contribution to the fermion self energy: bare fermions + resummed photon



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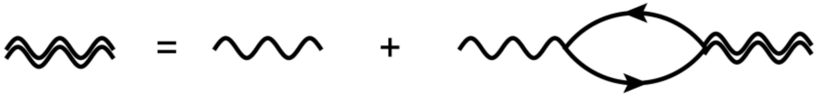


RPA photon propagator

Relativistic one-loop resummation ("Random Phase Approximation" , RPA)

- Dressed photon propagator (*Coulomb Gauge*):

$$\widetilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{\boldsymbol{q}^2}(\boldsymbol{q}^2 - \Pi_L)^{-1} P_L^{\mu\nu} + (q^2 - \Pi_\perp)^{-1} P_\perp^{\mu\nu} ,$$

$\Pi^{\mu\nu}$ photon pol. tensor

→ **Weak screening** approximation of longitudinal/transverse propagators:

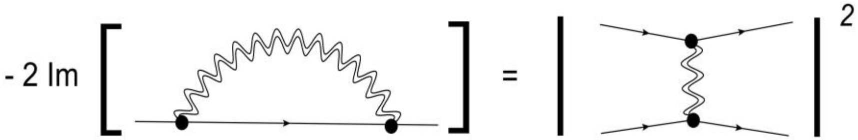
$$D_L \propto \frac{1}{q^2 - m_D^2} , \qquad D_\perp \propto \frac{1}{q^2 - i\left(\frac{q_0}{|\boldsymbol{q}|}\right) q_{\mathrm{f}}^2} , \qquad m_D^2 = \frac{4\alpha_f}{\pi}\mu k_f , \qquad q_{\mathrm{f}}^2 = \alpha_f k_f^2$$

[E. Flowers and N. Itoh, Astrophys. J.206, 218 (1976)] Transport in dense matter
[P.S. Shternin, D.G. Yakovlev Phys.Rev.D78 (2008), 063006] Shear viscosity in NS cores
[P.S. Shternin, D.G. Yakovlev Phys.Rev.D75 (2007), 103004] Electron-muon heat conduction in NS cores

→ **Hard dense loop** (HDL) approximation $\boldsymbol{q} \ll \boldsymbol{k_f}$ (leading order contribution in soft region)

[P.S. Shternin, D.G. Yakovlev, Phys.Rev.D74 (2006), 043004] Transport in degenerate electron plasma
[H. Heiselberg and C. J. Pethick, Phys.Rev.D48 (1993)] Transport in QCD plasma
[A. Harutyunyan, A. Sedrakian, Phys. Rev. C 94, 025805 (2016)] Transport in NS crust

Damping rate of degenerate fermions



optical theorem: int. rate. $\Gamma \propto \boldsymbol{Im\,\Sigma}$

$$\Gamma_+ = \frac{1}{2}\, Tr\,[\Lambda_+\, \gamma_0\, \mathrm{Im}\,\Sigma_\mathrm{R}] = -\frac{1}{2p_0}\, Tr[(\gamma\cdot p + m)\, \mathrm{Im}\,\Sigma_R\,(p_0, \boldsymbol{p}\,)]\,, \qquad p_0 = \epsilon_{\boldsymbol{p}}$$

photon spectrum $\rho^{\mu\nu} = \rho_L\, P_\mathrm{L}^{\mu\nu} + \rho_\perp\, P_\perp^{\mu\nu}$

→ *week screening & close to Fermi surface* $\epsilon_{\boldsymbol{p}} - \mu \ll m_D\,,$ $\qquad u = q_0 \,/\, |\boldsymbol{q}|$

$$\Gamma_L \simeq \frac{e^2}{4\pi}\frac{m_D^2}{v_f^2}\int_0^{|\epsilon_{\boldsymbol{p}}-\mu|} du\; u \int_0^\infty d|\boldsymbol{q}|\, \frac{1}{(m_D^2 + \boldsymbol{q}^2)^2} = \frac{e^2}{32}\frac{1}{m_D\, v_f^2}\,(\epsilon_{\boldsymbol{p}} - \mu)^2$$

$$\Gamma_\perp \simeq \frac{e^2}{4\pi} m_D^2\, v_f^2 \int_0^{|\epsilon_{\boldsymbol{p}}-\mu|} du\; u \int_0^\infty d|\boldsymbol{q}|\; |\boldsymbol{q}|\, \frac{4\,\boldsymbol{q}^2}{16\,\boldsymbol{q}^6 + u^2\,\pi^2 m_D^2\, v_f^2} = \frac{e^2}{12\pi}\, v_f |\epsilon_{\boldsymbol{p}} - \mu|$$

compare to: [C. Manuel, Phys.Rev. D62 (2000) 076009]

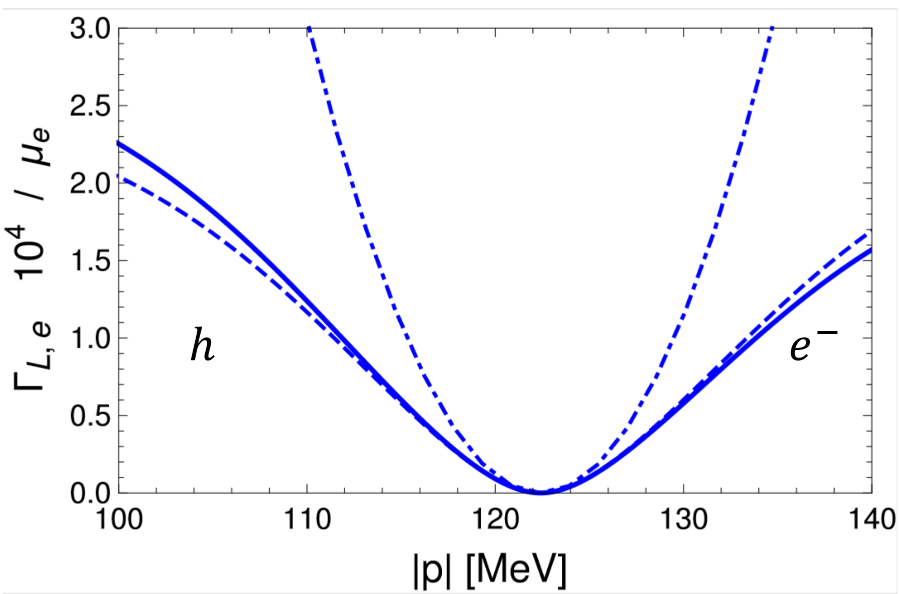
Damping: weak screening vs. HDL vs. full RPA

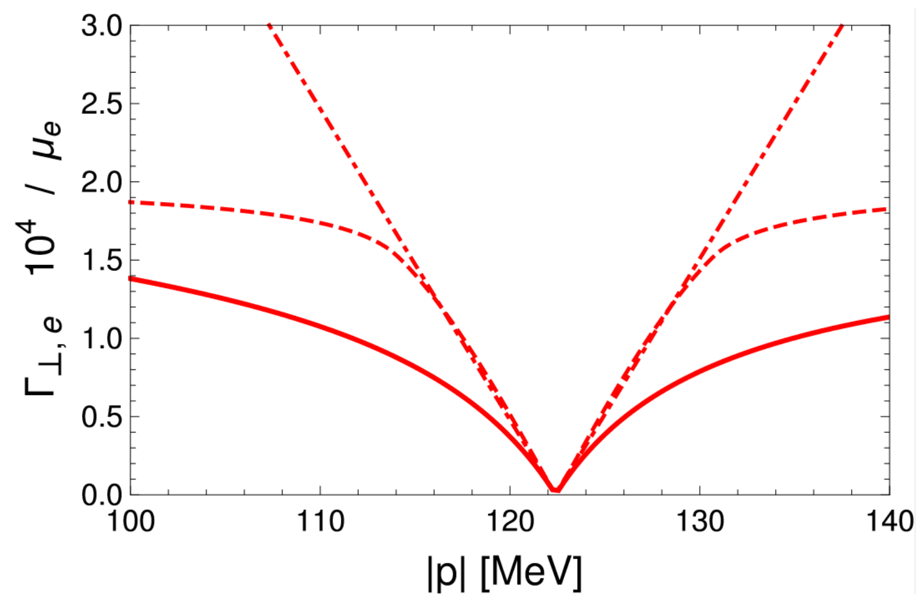
→ **nonrelativistic**: electric interactions dominate, magnetic interactions are down by $\left(\frac{v}{c}\right)^2$

→ **relativistic:** damping due to the exchange of plasmons and photons is equally important

[H. Heiselberg, G. Baym, C. J. Pethick, J. Popp, Nuc. Phys. A 544 (1992)]

electrons at n = n0





solid: full one-loop dashed: HDL dot-dashed: weak screening

→ **HDL approximations work much better in the longitudinal channel!**

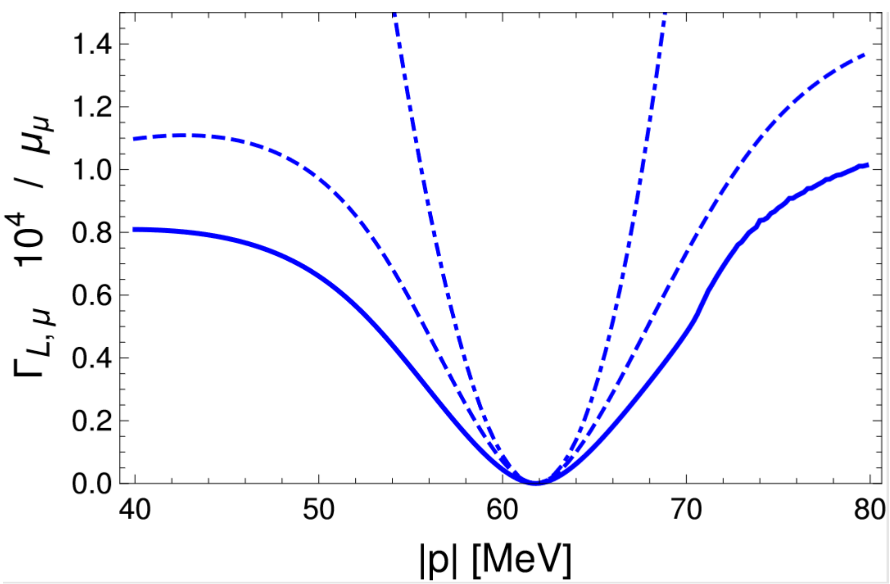
Longitudinal and transverse damping (II)

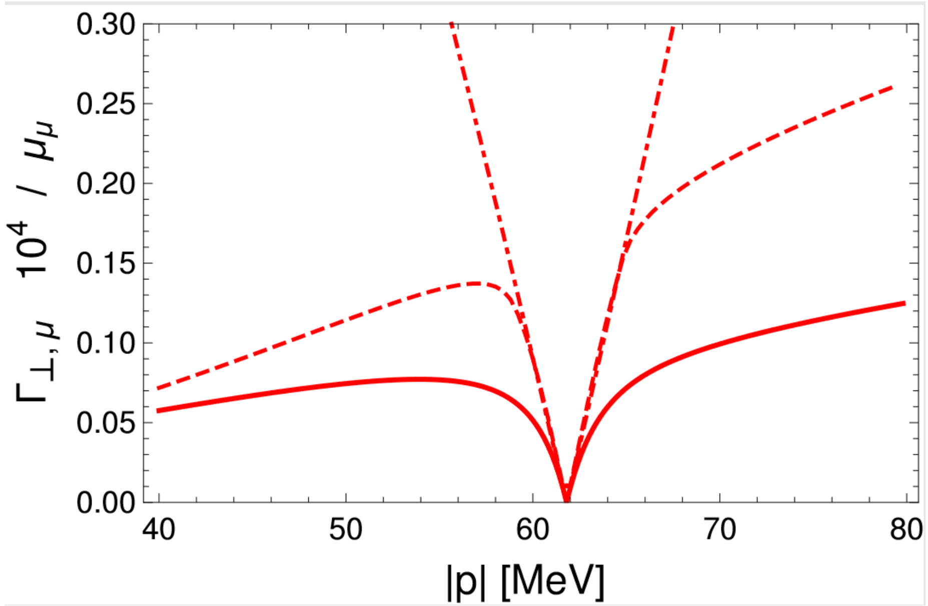
→ **nonrelativistic**: electric interactions dominate, magnetic interactions are down by $\left(\frac{v}{c}\right)^2$

→ **relativistic:** damping due to the exchange of plasmons and photons is equally important

[H. Heiselberg, G. Baym, C. J. Pethick, J. Popp, Nuc. Phys. A 544 (1992)]

muons at n = n0





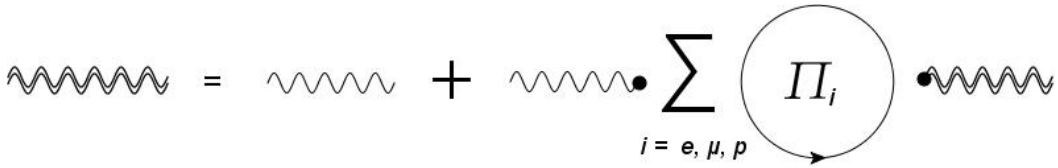
→ $|q| \ll k_f$ **hard to fulfill, HDL don't work really well in either channel**

→ Γ_L **overtakes** $\Gamma_\perp$

RPA photon propagator: multi species

Photon propagator in the presence of several fermion species

$$\widetilde{D}^{\mu\nu}(q_0,q) = \frac{q^2}{\boldsymbol{q}^2}(\boldsymbol{q}^2 - Tr\,[\Pi_L])^{-1}\, P_L^{\mu\nu} + (q^2 - Tr[\,\Pi_\perp])^{-1}\, P_\perp^{\mu\nu}, \qquad \Pi \rightarrow \mathrm{diag}\big(\Pi_e, \Pi_\mu, \Pi_p\big)$$



RPA resummation in multi-component plasma is well established:

[C. Horowitz, K. Wehrberger, Nucl. Phys. A 531, 665 (1991)]

[S. Reddy, M. Prakash, J.M. Lattimer, J.A. Pons, PRC 59, 2888 (1999)]

Protons are quasiparticles (strongly interacting Fermi liquid)

→ Proton fraction n_p, effective masses m_p^* , residual interactions V_{pp} , V_{pn} extracted from **Landau energy functional** based on **Skyrme type interactions**

→ Here: **NRAPR, SKRA, SQMC700, LNS, KDE0v1**

[M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, P. D. Stevenson , PRC 85 ,035201]

QED + strong interactions

What's the role of the neutrons within RPA?

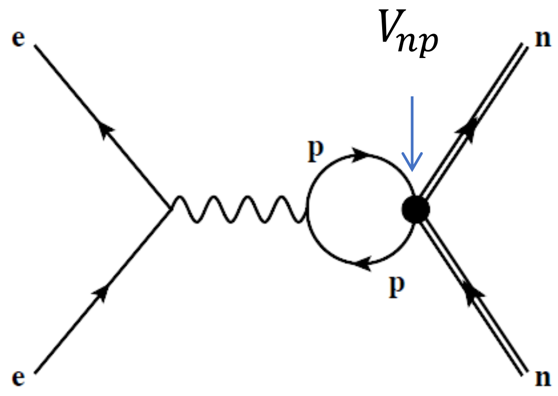
- Coupling strength to photon tiny in free space

- BUT: Interactions induced by the polarizability of protons

[B. Bertoni, S. Reddy, E. Rrapaj, Phys. Rev. C 91, 025806 (2015)]

- use *resummed* RPA polarization tensor for protons

[S. Reddy, M. Prakash, J.M. Lattimer, J.A. Pons, PRC 59, 2888 (1999)]



$$\widetilde{\Pi}_p = \frac{\Pi_p(1 - V_{nn}\Pi_n)}{1 - V_{nn}\Pi_n - V_{pp}\Pi_p + \left(V_{nn}V_{pp} - V_{np}^2\right)\Pi_n\Pi_p}$$

$$\widetilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{\mathbf{q}^2}\left(q^2 - \Pi_{e,L} - \Pi_{\mu,L} - \widetilde{\Pi}_{p,L}\right)^{-1} P_L^{\mu\nu} + \left(q^2 - \Pi_{e,\perp} - \Pi_{\mu,\perp} - \widetilde{\Pi}_{p,\perp}\right)^{-1} P_\perp^{\mu\nu}$$

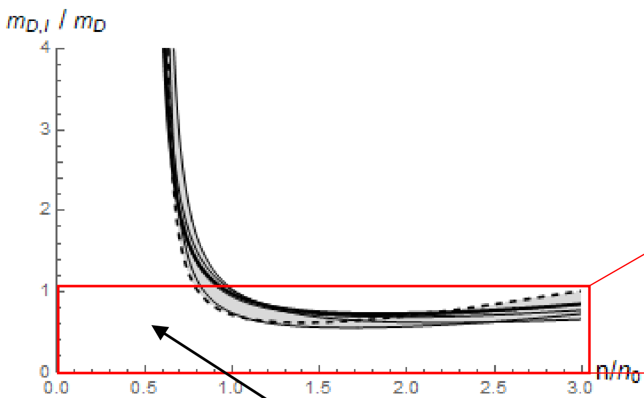
effective interaction

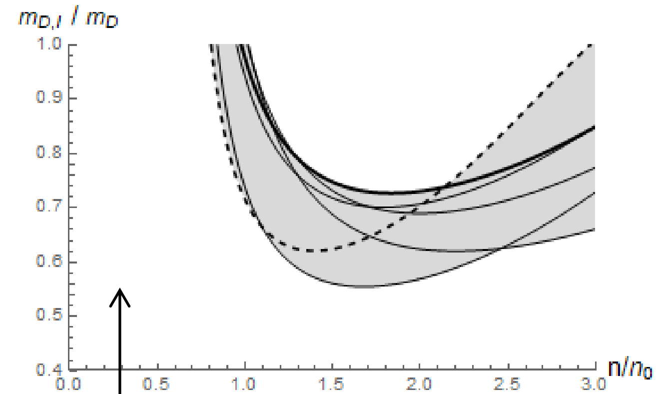
$$L_{\gamma-n} = e^2\, V_{np}\left(\bar{n}\,\gamma_\mu\, n\right)\; A_\nu\,(\Pi_{L,p}\, P_L^{\mu\nu} + \Pi_{\perp,p}\, P_\perp^{\mu\nu})$$

→ changes to transverse spectrum are negligible since for protons $\Pi_\perp \ll \Pi_L$.

“Induced” (strong) screening

TD definition: $\widetilde{\Pi}_{L,\,p}(q_0 = 0\,) = \left[\frac{\partial \mu_p(\mu_n)}{\partial n_p}\right]^{-1} = \frac{m_p^2\,(1+V_{nn}\,m_n^2\,)}{1+V_{nn}\,m_n^2+V_{pp}\,m_p^2+\left(V_{nn}V_{pp}-V_{np}^2\right)m_n^2\,m_p^2}$

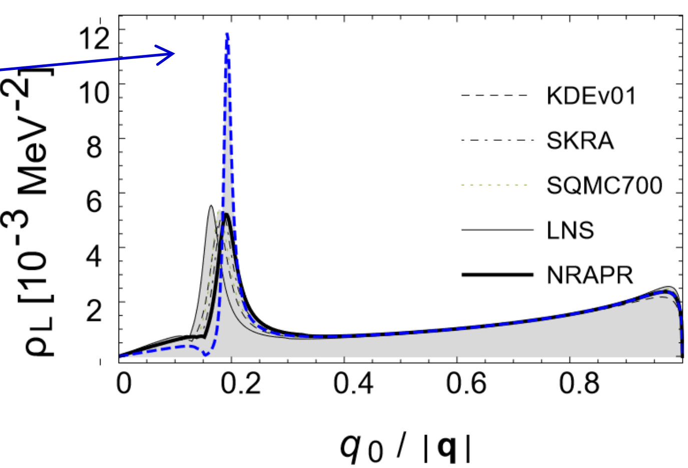




homogeneous nuclear matter unstable

n = 0.65 n_0

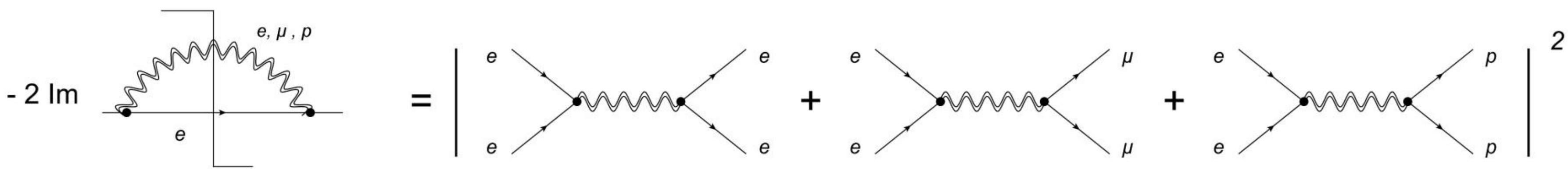
pure QED (NRAPR)



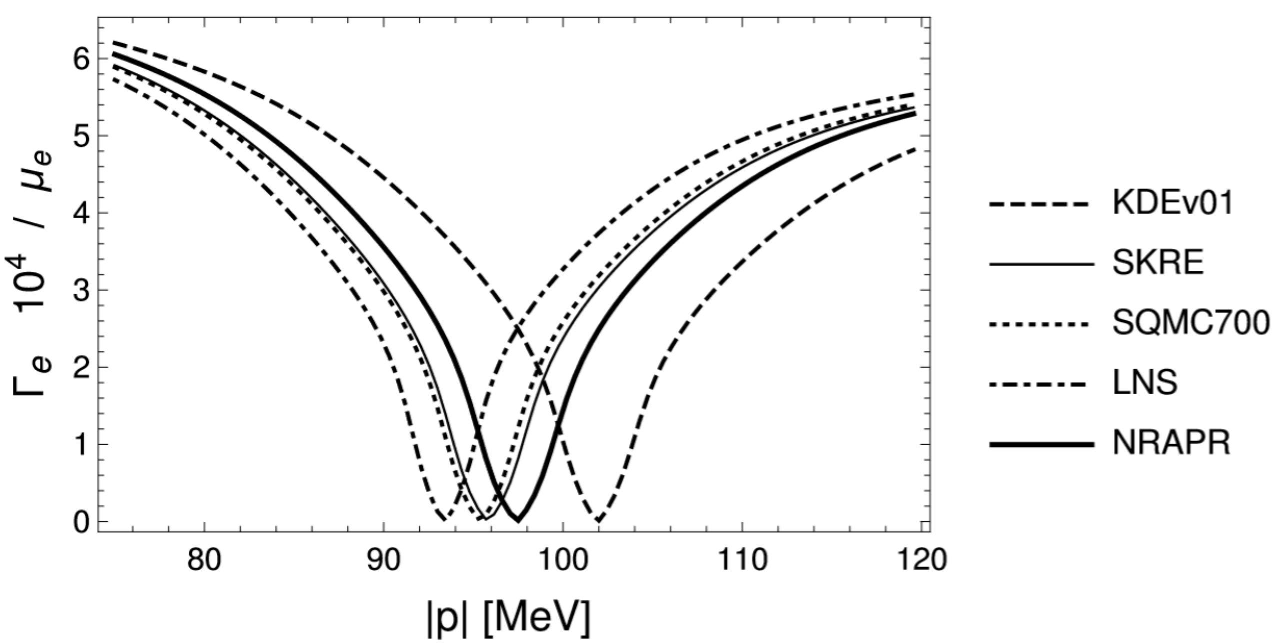
→ **Impact of induced interactions most pronounced at densities close to the crust-core boundary**

Damping rate of electrons: multiple species

energy loss of electrons due to collisions with other electrons, muons, and protons



→ total screening mass: $M_D^2 = \sum_a m_{D,a}^2$ $\rho_L = -\frac{1}{\pi}\frac{Tr[Im\,\Pi_L]}{(Tr[Re\Pi_L] - \boldsymbol{q}^2)^2 + (Tr[Im\,\Pi_L])^2}$



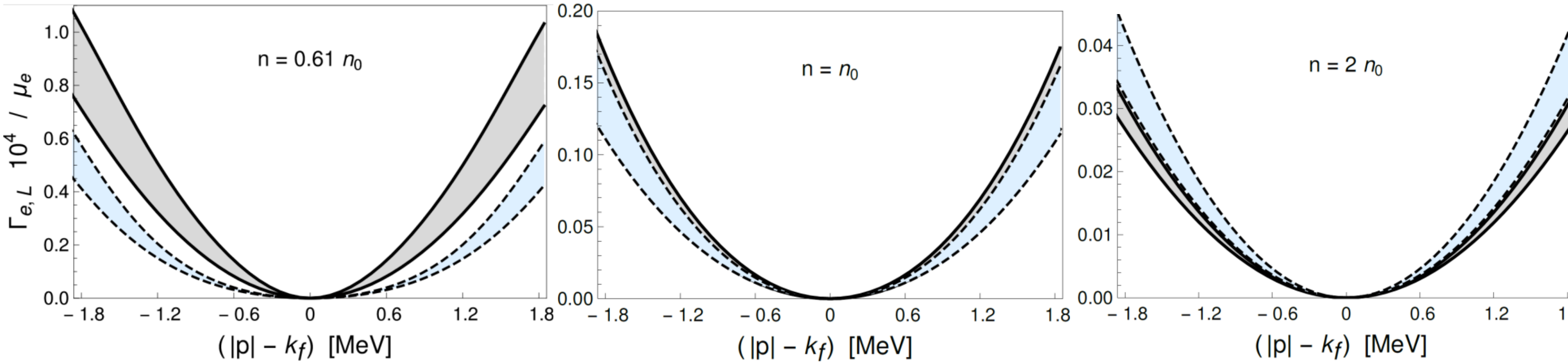
- **total scattering rate** of electrons close to the crust-core boundary
- easy to implement in transport calculations (fit as function of $\epsilon_{\boldsymbol{p}} - \mu$)

Impact of induced interactions

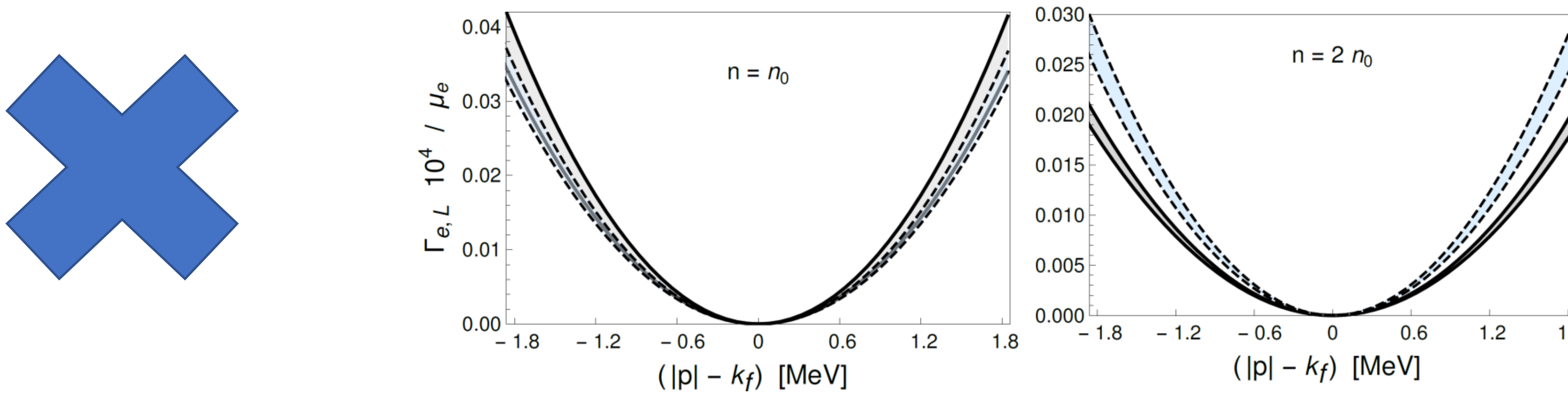
→ Γ_L **strongly modified** near **crust-core boundary**

→ follows the evolution of the induced screening

electrons



muons



Outlook

- Existing calculations of transport effects in dense nuclear matter can be refined by taking into account dynamical screening effects and induced interactions

[S. Stetina, E. Rrapaj, S. Reddy, work in progress]

Where to go from here:

- **Improve on implementation of nuclear interactions (dynamical screening)**
- **Account for proton superconductivity → Meissner effect**

$$D_{\perp} = \frac{1}{q^2 - \Pi_{\perp,e} - \Pi_{\perp,\mu} - \Pi_{\perp,\mathrm{p}}}$$

→ **induced $e^- - n$ scattering dominates**

[B. Bertoni, S. Reddy, E. Rrapaj, Phys. Rev. C 91, 025806 (2015)]

- **include magnetic fields**

go raibh maith agat !

(Thank you !)