

Determining the Strong Coupling Status & Challenges

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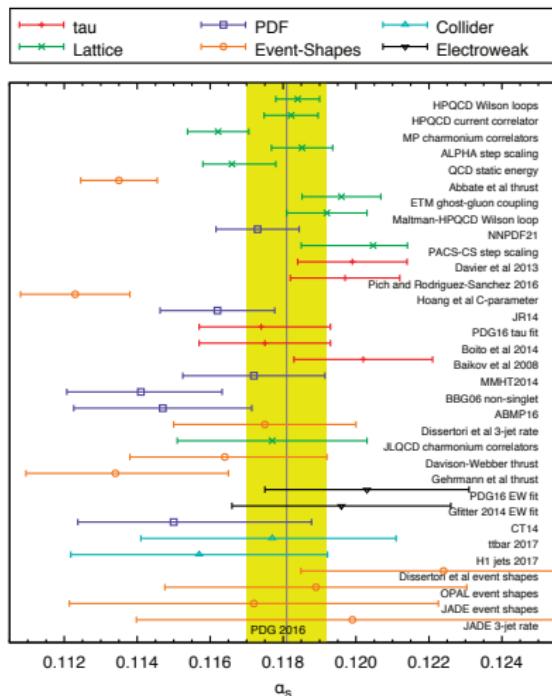
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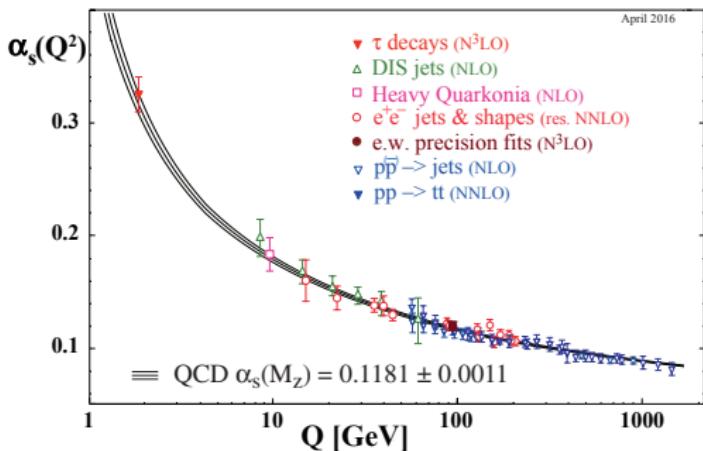
XIIIth Quark Confinement and the Hadron Spectrum
Maynooth, Ireland, 1–6 August 2018

Most Recent Compilations

G. Salam, 2017

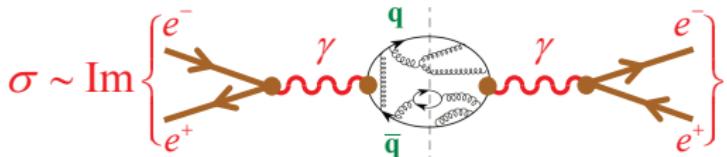


S. Bethke, G. Dissertori, G. Salam, PDG2016



See also: S. Alekhin et al., arXiv:1512.05194 [hep-ph]

Inclusive Observables: $R_{ee}(s)$



$$R_{ee}(s) = 12\pi \operatorname{Im} \Pi_{\text{em}}(s)$$

$$\Pi_{\text{em}}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T [J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0)] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{\text{em}}(q^2)$$

$$R_{ee} \equiv \frac{\Gamma(e^+e^- \rightarrow \text{hadrons})}{\Gamma(e^+e^- \rightarrow e^+e^-)} = \sum_q Q_q^2 N_C \left\{ 1 + \sum_{n \geq 1} F_n \left[\frac{\alpha_s(M_Z^2)}{\pi} \right]^n \right\} + \mathcal{O}\left(\frac{m_q^2}{s}, \frac{\Lambda^4}{s^2}\right)$$

Perturbative series known to $\mathcal{O}(\alpha_s^4)$: ($n_F = 5$)

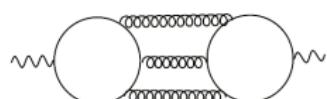
Baikov-Chetyrkin-Kühn-Rittinger

$$F_1 = 1 , \quad F_2 = 1.9857 - 0.1153 n_F = 1.4092$$

$$F_3 = -6.63694 - 1.20013 n_F - 0.00518 n_F^2 - 1.2395 \eta = -12.805$$

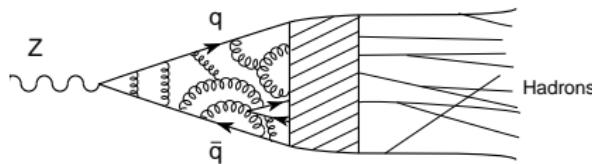
$$F_4^{\text{NS}} = -156.608 + 18.7748 n_F - 0.7974 n_F^2 + 0.0215 n_F^3 - 14.952 \eta = -80.434$$

$$\eta = \left(\sum_q Q_q \right)^2 / \left(N_C \sum_q Q_q^2 \right)$$



Singlet contributions

Z Hadronic Width



Vector + Axial

$$R_Z \equiv \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow e^+ e^-)} = R_Z^{\text{EW}} N_C \left\{ 1 + \sum_{n \geq 1} \tilde{F}_n \left[\frac{\alpha_s(M_Z^2)}{\pi} \right]^n \right\} + \mathcal{O}\left(\frac{m_q^2}{M_Z^2}, \frac{\Lambda^4}{M_Z^4}\right)$$

Perturbative series known to $\mathcal{O}(\alpha_s^4)$

Baikov-Chetyrkin-Kühn-Rittinger

$$\left[+ \mathcal{O}\left(\alpha_s^2 \frac{m_b^4}{M_Z^4}, \alpha_s^2 \frac{m_b^2}{m_t^2}, \alpha_s^3 \frac{M_Z^2}{m_t^2}\right) \right]$$

Z-pole data (EW fit)

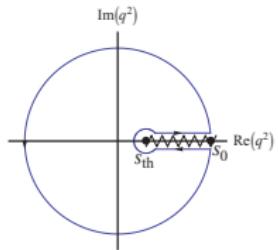
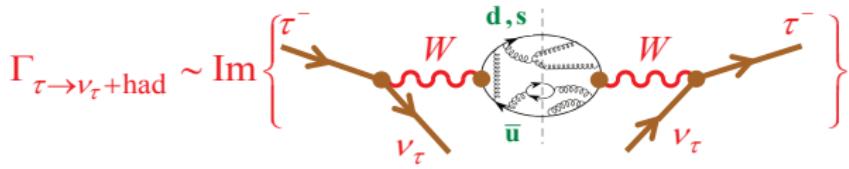


$$\alpha_s(M_Z^2) = 0.1196 \pm 0.0030$$

Gfitter 2014

Assumes validity of the EW Standard Model

τ Hadronic Width: R_τ



$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right) + |V_{us}|^2 \Pi_{us,V+A}^{(J)}(s)$$

Braaten-Narison-Pich '92

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau + \text{hadrons}]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e]} = N_C S_{EW} (1 + \delta_P + \delta_{NP})$$

$$= 6\pi i \oint_{|x|=1} dx (1-x)^2 \left[(1+2x) \Pi^{(0+1)}(m_\tau^2 x) - 2x \Pi^{(0)}(m_\tau^2 x) \right]$$

$$\delta_P = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\%$$

Baikov-Chetyrkin-Kühn '08

$$a_\tau \equiv \alpha_s(m_\tau^2)/\pi , \quad S_{EW} = 1.0201(3)$$

Marciano-Sirlin, Braaten-Li, Erler

$$\delta_{NP} = -0.0064 \pm 0.0013 \quad (\text{fitted from data})$$

Davier et al '14

Comprehensive analysis of ALEPH data

Rodríguez-Sánchez, Pich, arXiv:1605.06830

Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments ¹	$0.339^{+0.019}_{-0.017}$	$0.319^{+0.017}_{-0.015}$	$0.329^{+0.020}_{-0.018}$
Mod. ALEPH moments ²	$0.338^{+0.014}_{-0.012}$	$0.319^{+0.013}_{-0.010}$	$0.329^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments ³	$0.336^{+0.018}_{-0.016}$	$0.317^{+0.015}_{-0.013}$	$0.326^{+0.018}_{-0.016}$
s_0 dependence ⁴	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform ⁵	$0.328^{+0.014}_{-0.013}$	$0.318^{+0.015}_{-0.012}$	$0.323^{+0.015}_{-0.013}$
Combined value	0.335 ± 0.013	0.320 ± 0.012	0.328 ± 0.013



$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$1) \quad \omega_{kl}(x) = (1+2x)(1-x)^{2+k}x^l \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

$$2) \quad \tilde{\omega}_{kl}(x) = (1-x)^{2+k}x^l \quad (k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

$$3) \quad \omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^m (k+1)x^k = 1 - (m+2)x^{m+1} + (m+1)x^{m+2}, \quad 1 \leq m \leq 5$$

$$4) \quad \omega^{(2,m)}(x) \quad 0 \leq m \leq 2, \quad 1 \text{ single moment in each fit}$$

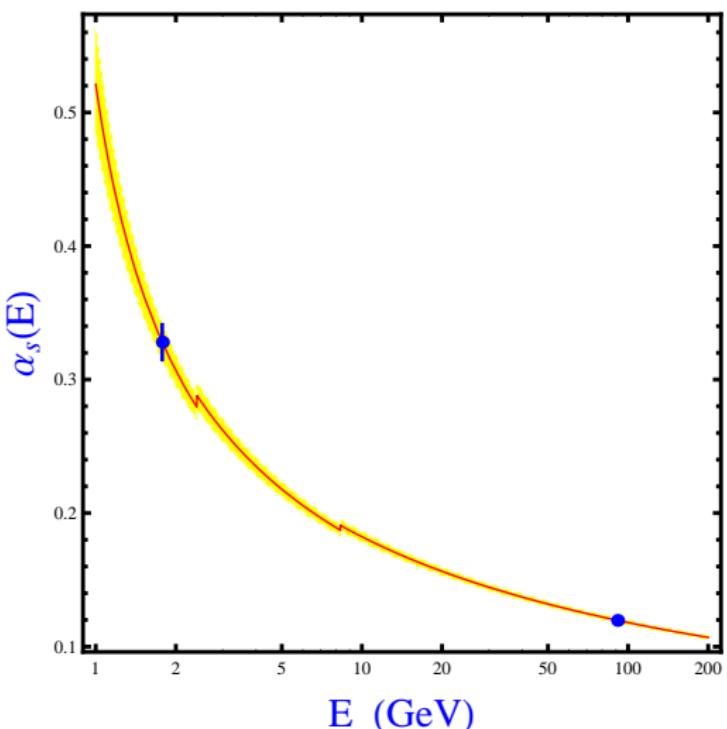
$$5) \quad \omega_a^{(1,m)}(x) = (1-x^{m+1})e^{-ax} \quad 0 \leq m \leq 6$$

Inclusive Status

$\mathcal{O}(\alpha_s^4)$ accuracy (N³LO)

- R_{ee} : Too large experimental uncertainties
- R_Z : Small theoretical errors & high experimental precision
 - Possible improvements with more precise data
 - Sensitive to new-physics contributions
- R_τ : High sensitivity to α_s (low scale m_τ)
 - Improved control of δ_{NP} with more precise data
 - Uncertainty dominated by perturbative errors

α_s at N³LO from τ and Z



$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$

$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{Z \text{ width}} = 0.1196 \pm 0.0030$$

Very precise test of Asymptotic Freedom

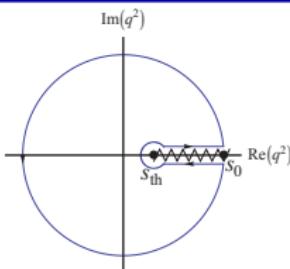
$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0001 \pm 0.0015_\tau \pm 0.0030_Z$$

Backup



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Non-Perturbative Contribution



$$A_J^\omega(s_0) \equiv \int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_J^{(J)}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_J^{(J)}(s)$$

$$\Pi_J^{(J)}(s) \approx \Pi_J^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,J}^{(J)}}{(-s)^{D/2}}$$

$$A_J^{\omega,\text{NP}}(s_0) = \pi \sum_D a_{-1,D} \frac{\mathcal{O}_{D,J}^{(J)}}{s_0^{D/2}} \quad , \quad \omega(-s_0 x) = \sum_n a_{n,D} x^{n+D/2}$$

- Strong power suppression at $s_0 = m_\tau^2$: $\sim (\Lambda_{\text{QCD}}/m_\tau)^D$, $D \geq 4$

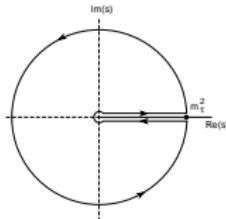
$$\mathcal{O}_{4,V/A} \approx 4\pi^2 \left\{ \frac{1}{12\pi} \langle \alpha_s GG \rangle + (m_u + m_d) \langle \bar{q}q \rangle \right\} \approx [(6.7 \pm 3.2) - 0.6] \cdot 10^{-3} \times m_\tau^4$$

- R_τ : $\omega(x) = 1 - 3x^2 + 2x^3 \rightarrow \delta_{\text{NP}} = -3 \frac{\mathcal{O}_{6,V+A}}{m_\tau^6} - 2 \frac{\mathcal{O}_{8,V+A}}{m_\tau^8}$

Additional chiral suppression in $\mathcal{O}_{6,V+A}$

- Sensitivity to \mathcal{O}_D with different $\omega(x)$ \rightarrow Measure δ_{NP}

R_τ suitable for a precise α_s determination



$$R_\tau = 6\pi i \oint_{|x|=1} (1-x)^2 \left[(1+2x) \Pi^{(0+1)}(m_\tau^2 x) - 2x \Pi^{(0)}(m_\tau^2 x) \right]$$

$$\Pi_{\mathcal{J}}^{(J)}(s) \approx \Pi_{\mathcal{J}}^{(J)}(s)^{\text{OPE}} = \sum_D \frac{\mathcal{O}_{D,\mathcal{J}}^{(J)}}{(-s)^{D/2}}$$

- Known to $\mathcal{O}(\alpha_s^4)$. Sizeable $\delta_P \sim 20\%$. Strong sensitivity to α_s
- m_τ large enough to safely use the OPE. Flat V + A distribution
- OPE only valid away from real axis: $(1-x)^2$ pinched at $s = m_\tau^2$
- $m_{u,d} = 0 \rightarrow s \Pi^{(0)} = 0 \rightarrow R_{\tau,V+A} = 6\pi i \oint_{|x|=1} (1-3x^2+2x^3) \Pi_{ud,V+A}^{(0+1)}(m_\tau^2 x)$
 $\rightarrow \delta_{NP} \sim 1/m_\tau^6$ Strong suppression of non-perturbative effects
- $D=6$ OPE contributions have opposite sign for V & A. Cancellation
- δ_{NP} can be determined from data: $\delta_{NP} = -0.0064 \pm 0.013$

Davier et al

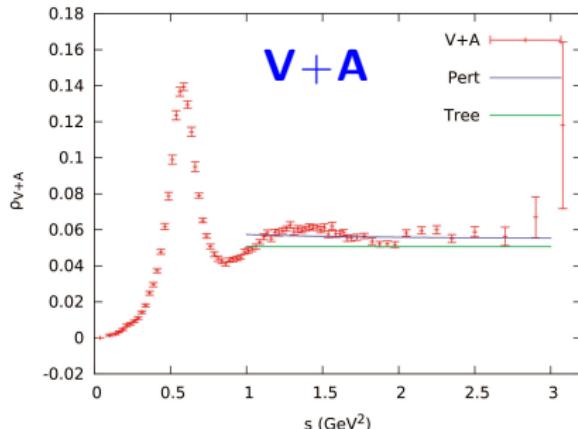
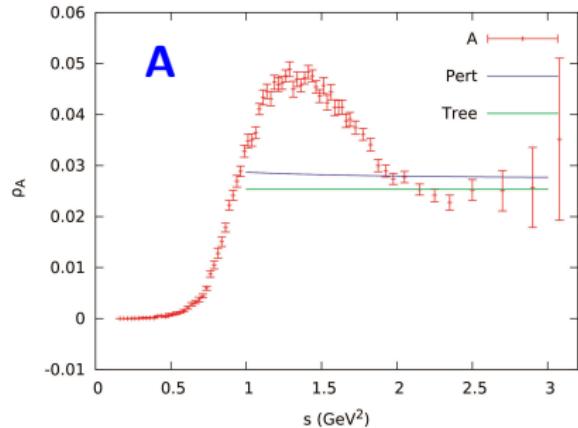
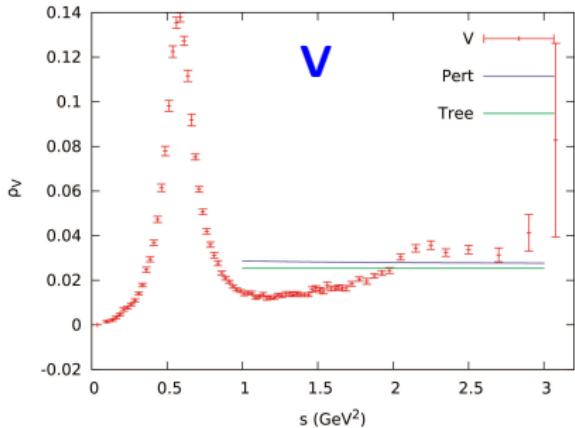
R_τ

$$\alpha_s(m_\tau^2) = 0.331 \pm 0.013$$

Pich 2014

ALEPH Spectral Functions

Davier et al. 2014



$$\textcolor{blue}{\alpha_s(m_\tau^2) = 0.329}$$

— Parton Model