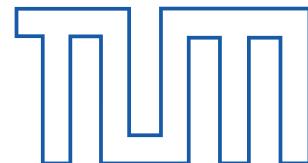


# $\alpha_s$ from static energy and force @ Confinement XIII

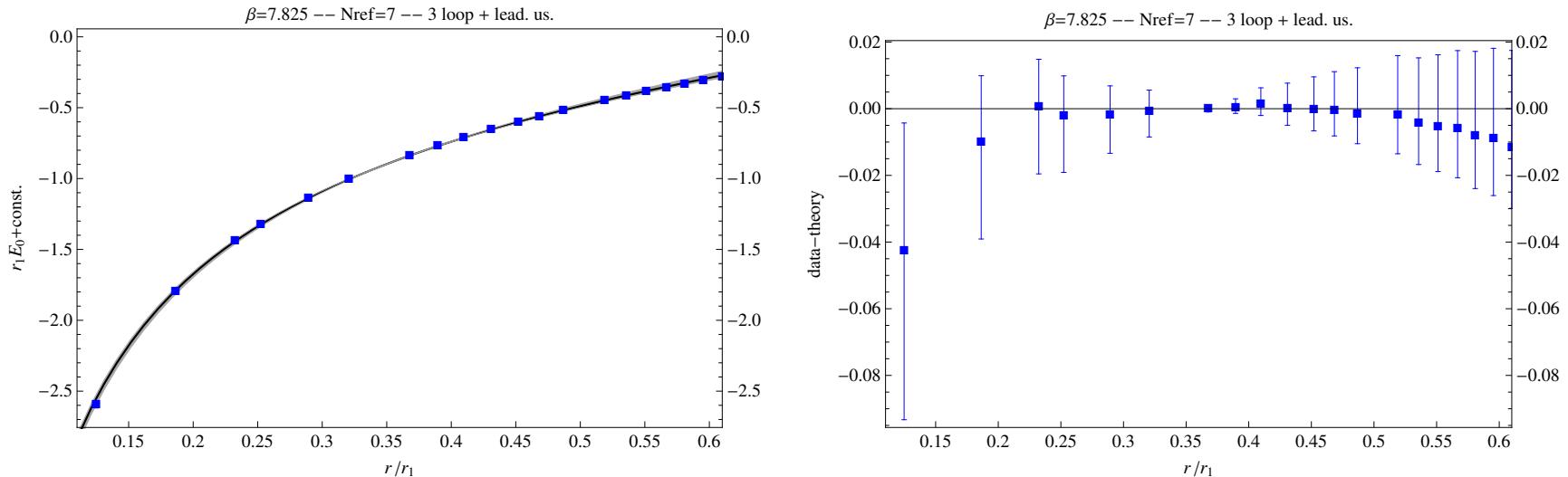
Antonio Vairo

Technische Universität München



# Status

## Static energy vs lattice data



- Perturbation theory ( $N^3\text{LO} + \text{RG}$ ) agrees with lattice data from 0.05 fm up to 0.14 fm.
- Using data for  $\beta = 7.373, 7.596, 7.825$  one gets

$$\Lambda_{\overline{\text{MS}}} = 315^{+18}_{-12} \text{ MeV} \quad \text{which converts to} \quad \alpha_s(M_Z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$

## Energy, potential, force

The **static energy**,  $E_0$ , is related to the **potential**,  $V_s$ , through:  $E_0 = V_s + \Lambda_s + u_s$

The perturbative expansion of  $V_s$  is affected by a renormalon ambiguity of order  $\Lambda_{\text{QCD}}$ . This ambiguity does not affect the slope of the potential (and the extraction of  $\alpha_s$ ).

It may be eliminated from the perturbative series

- either by subtracting a (constant) series in  $\alpha_s$  to  $V_s$  and reabsorb it in a redefinition of the residual mass  $\Lambda_s$ ,
- or by considering the **force**:

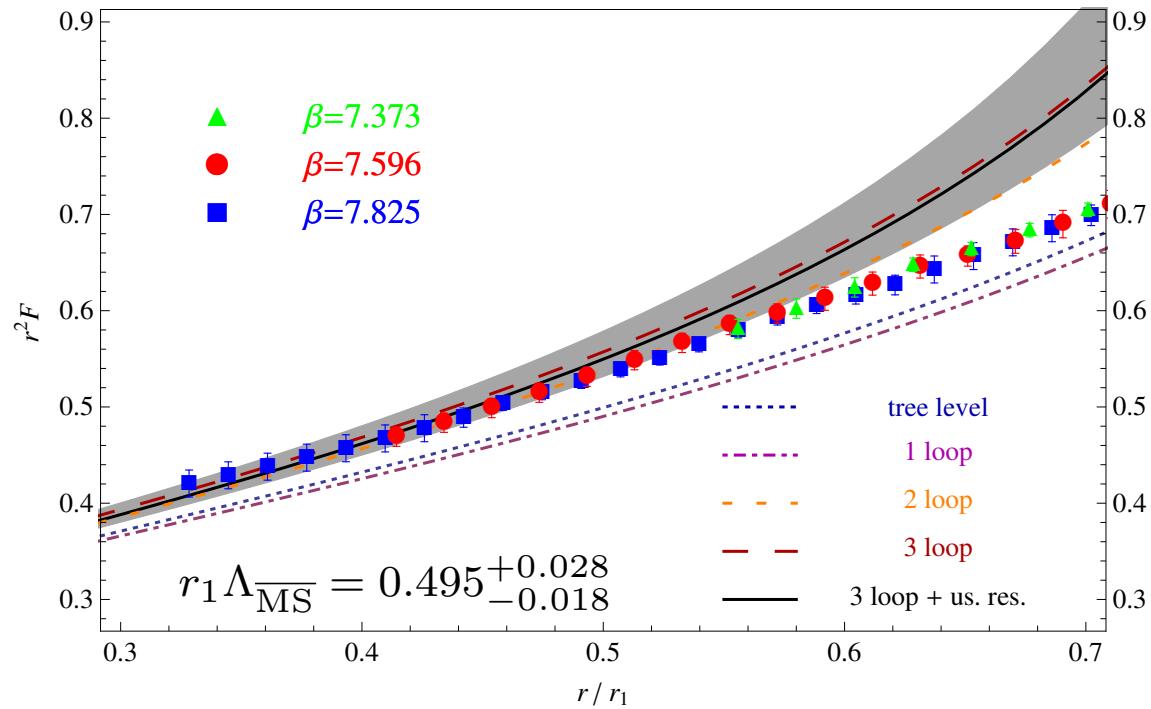
$$F(r, \alpha_s(\nu)) = \frac{d}{dr} E_0(r, \alpha_s(\nu))$$

- The force  $F(r, \alpha_s(1/r))$  could be directly compared with lattice,
- or integrated and compared with the static energy

$$E_0(r) = \int_{r_*}^r dr' F(r', \alpha_s(1/r'))$$

up to an irrelevant constant fixed by the overall normalization of the lattice data.

## Force vs lattice data

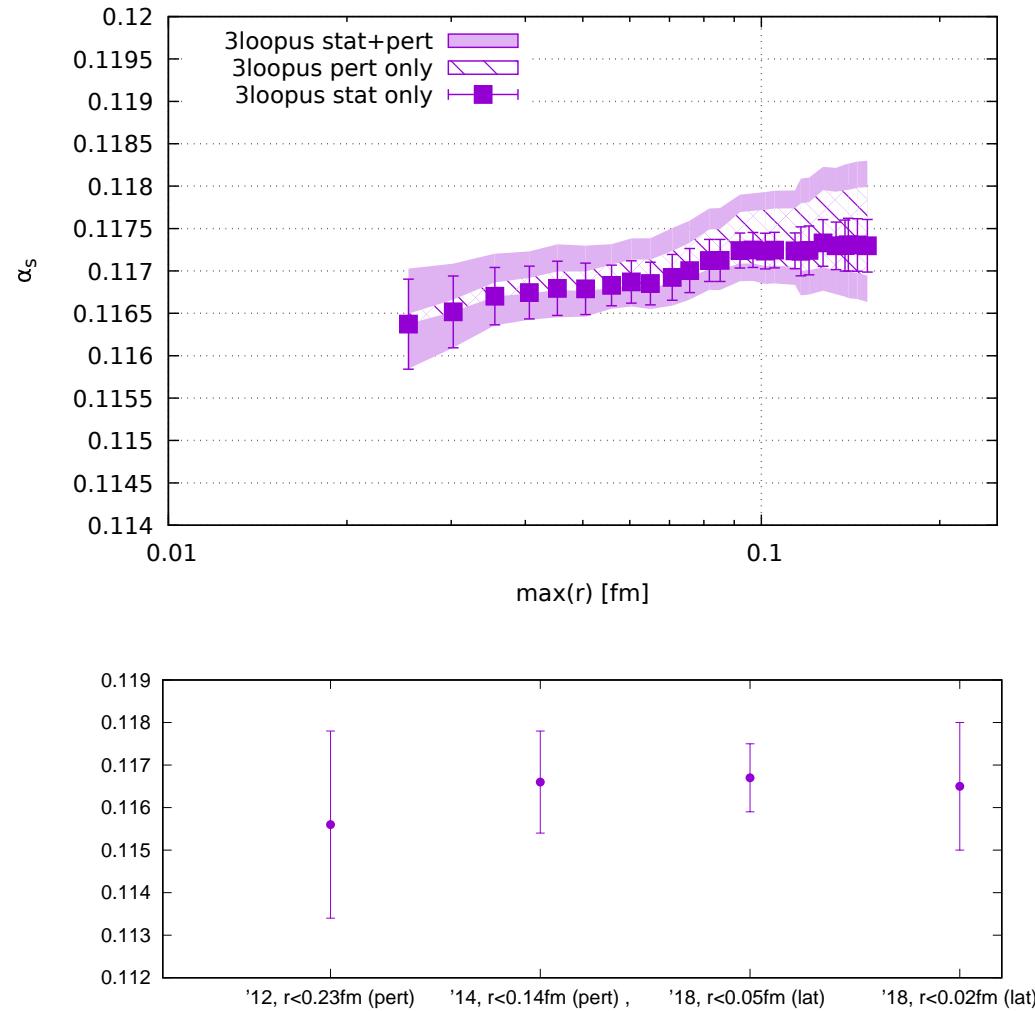


○ Bazavov Brambilla Garcia Petreczky Soto Vairo PRD 90 (2014) 074038

# Perspectives

- Not all of the presently available perturbative information has been used:  
 $N^3LL$ , finite mass effects, ...  
because the data are not sensitive to it.
- Also the data do not seem sensitive to short-range nonperturbative effects  
(e.g., condensates  $\sim r^3 \langle E(0)^2 \rangle$ , or correlators  $\sim r^2 \int dt \langle E(t)E(0) \rangle$ ).
- Can data at shorter distances, in 2+1+1 lattices, from different observables complement or extend our present knowledge of  $\alpha_s$  from the QCD potential?

## $\alpha_s$ from the static energy at very short distances



## Charm mass effects

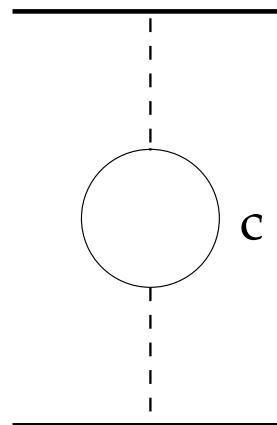
- $m_c \gg \Lambda_{\text{QCD}}$   
Charm-mass effects happen at a perturbative scale.
- $m_c \approx 1.5 \text{ GeV} \approx 0.13 \text{ fm}$

The charm mass affects

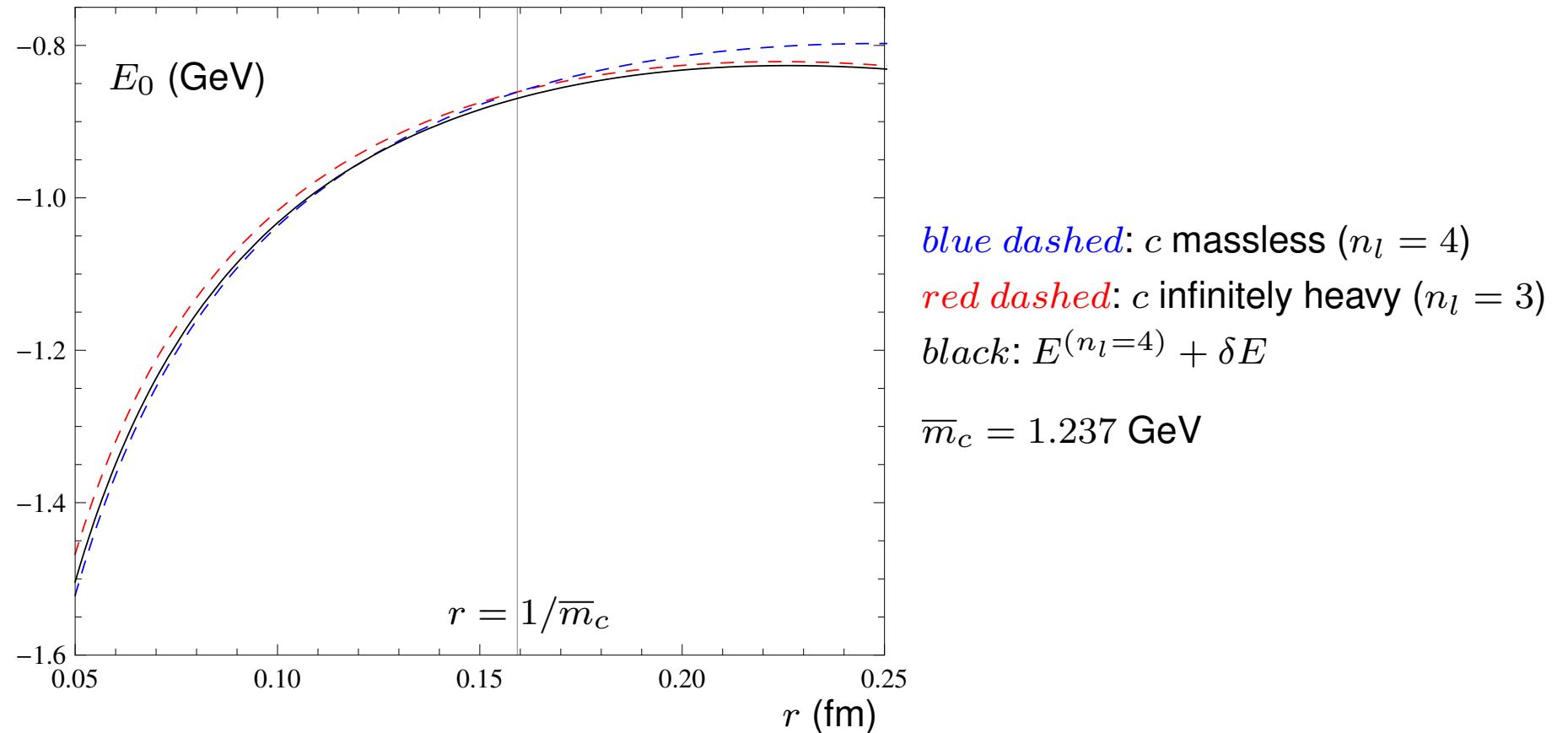
\* bottomonium physics:  $m_b v_b \sim m_c$

\* 2+1+1 lattice computations of the static energy at short distances.

- Charm mass effects in the static energy are well known:



# Charm mass effects in the static energy



## $\alpha_s$ from the force

It is possible to compute the force directly from the lattice:

$$F(r) = - \lim_{T \rightarrow \infty} \frac{\left\langle \text{Tr P } \hat{\mathbf{r}} \cdot g \mathbf{E}(t, \mathbf{r}) \exp \left\{ ig \oint_{r \times T} dz^\mu A_\mu \right\} \right\rangle}{\left\langle \text{Tr P} \exp \left\{ ig \oint_{r \times T} dz^\mu A_\mu \right\} \right\rangle}$$

- Vairo MPLA 31 (2016) 1630039