QCD coupling from the lattice

Rainer Sommer

John von Neumann Institute for Computing, DESY
&
Humboldt University, Berlin

Confinement Conf., Maynooth, August 2, 2018

Lattice vs. Phenomenology

Lattice	Phenomenology
▸ Euclidean short distance observables - no hadronization	▸ Minkowski scattering / decays - hadronization corrections - duality violations
▸ Input from low energy - tiny exp. uncertainties	▸ Input from high energy - often limits precision
▸ Often difficult PT	▸ Often high order PT
▸ Limiting: QED effects (heavy quarks)	

FLAG review

Eur. Phys. J. C (2017) 77:112
DOI 10.1140/epjc/s10052-016-4509-7

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Review

Review of lattice results concerning low-energy particle physics

Flavour Lattice Averaging Group (FLAG)

S. Aoki[1], Y. Aoki[2,3,17], D. Bečirević[4], C. Bernard[5], T. Blum[3,6], G. Colangelo[7], M. Della Morte[8,9], P. Dimopoulos[10,11], S. Dürr[12,13], H. Fukaya[14], M. Golterman[15], Steven Gottlieb[16], S. Hashimoto[17,18], U. M. Heller[19], R. Horsley[20], A. Jüttner[21,a], T. Kaneko[17,18], L. Lellouch[22], H. Leutwyler[7], C.-J. D. Lin[22,23], V. Lubicz[24,25], E. Lunghi[16], R. Mawhinney[26], T. Onogi[14], C. Pena[27], C. T. Sachrajda[21], S. R. Sharpe[28], S. Simula[25], R. Sommer[29], A. Vladikas[30], U. Wenger[7], H. Wittig[31]

FLAG review

- Review of lattice results
- Averages / Ranges for interesting quantities

Eur. Phys. J. C (2017) 77:112
DOI 10.1140/epjc/s10052-016-4509-7

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Review

Review of lattice results concerning low-energy particle physics

Flavour Lattice Averaging Group (FLAG)

S. Aoki[1], Y. Aoki[2,3,17], D. Bečirević[4], C. Bernard[5], T. Blum[3,6], G. Colangelo[7], M. Della Morte[8,9], P. Dimopoulos[10,11], S. Dürr[12,13], H. Fukaya[14], M. Golterman[15], Steven Gottlieb[16], S. Hashimoto[17,18], U. M. Heller[19], R. Horsley[20], A. Jüttner[21,a], T. Kaneko[17,18], L. Lellouch[22], H. Leutwyler[7], C.-J. D. Lin[22,23], V. Lubicz[24,25], E. Lunghi[16], R. Mawhinney[26], T. Onogi[14], C. Pena[27], C. T. Sachrajda[21], S. R. Sharpe[28], S. Simula[25], R. Sommer[29], A. Vladikas[30], U. Wenger[7], H. Wittig[31]

FLAG review

- Review of lattice results
- Averages / Ranges for interesting quantities
- α_s since FLAG 2013

  working group

  Tetsuya Onogi
  Roger Horsley
  RS

Eur. Phys. J. C (2017) 77:112
DOI 10.1140/epjc/s10052-016-4509-7

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Review

Review of lattice results concerning low-energy particle physics

Flavour Lattice Averaging Group (FLAG)

S. Aoki[1], Y. Aoki[2,3,17], D. Bečirević[4], C. Bernard[5], T. Blum[3,6], G. Colangelo[7], M. Della Morte[8,9], P. Dimopoulos[10,11], S. Dürr[12,13], H. Fukaya[14], M. Golterman[15], Steven Gottlieb[16], S. Hashimoto[17,18], U. M. Heller[19], R. Horsley[20], A. Jüttner[21,a], T. Kaneko[17,18], L. Lellouch[22], H. Leutwyler[7], C.-J. D. Lin[22,23], V. Lubicz[24,25], E. Lunghi[16], R. Mawhinney[26], T. Onogi[14], C. Pena[27], C. T. Sachrajda[21], S. R. Sharpe[28], S. Simula[25], R. Sommer[29], A. Vladikas[30], U. Wenger[7], H. Wittig[31]

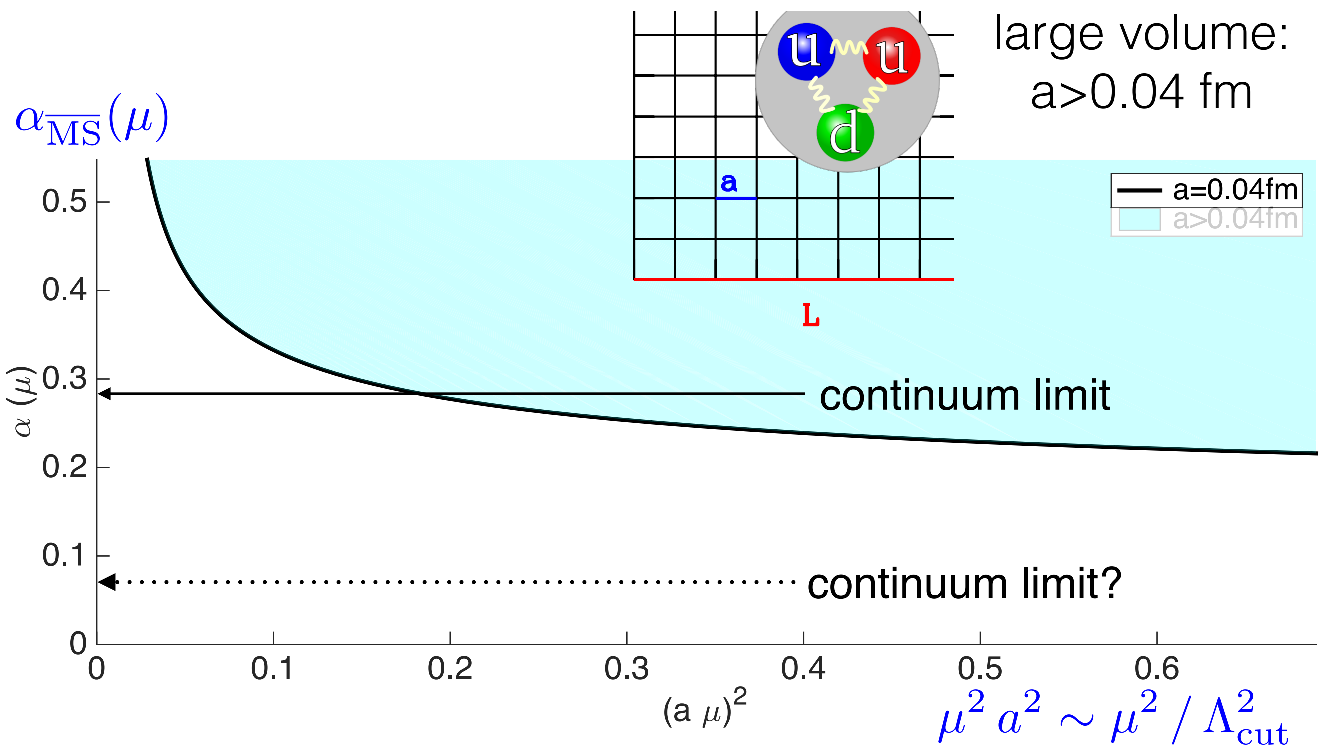
FLAG review

- Review of lattice results
- Averages / Ranges for interesting quantities
- α_s since FLAG 2013
  working group
  Tetsuya Onogi
  Roger Horsley
  RS
- Quality criteria

- Renormalization scale $r = 1/\mu$ small, α_s small
  - ★ all points relevant in the analysis have $\alpha_{\rm eff} < 0.2$
  - ○ all points have $\alpha_{\rm eff} < 0.4$ and at least one $\alpha_{\rm eff} \leq 0.25$
  - ■ otherwise
- Perturbative behaviour $n_l =$ loop order: high
  - ★ verified over a range of a factor 4 change in $\alpha_{\rm eff}^{n_l}$ without power corrections or alternatively $\alpha_{\rm eff}^{n_l} = 0.01$ is reached
  - ○ agreement with perturbation theory over a range of a factor 2.25 in $\alpha_{\rm eff}^{n_l}$ possibly fitting with power corrections or alternatively $\alpha_{\rm eff}^{n_l} = 0.02$ is reached
  - ■ otherwise
- Continuum extrapolation lattice spacing small: $a \ll r\,,\quad a\mu \ll 1$
  At a reference point of $\alpha_{\rm eff} = 0.3$ (or less) we require
  - ★ three lattice spacings with $\mu a < 1/2$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \leq 1/4$ and 2-loop $\mathcal{O}(a)$ improvement, or $\mu a \leq 1/8$ and 1-loop $\mathcal{O}(a)$ improvement
  - ○ three lattice spacings with $\mu a < 1.5$ reaching down to $\mu a = 1$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \leq 1/4$ and one-loop $\mathcal{O}(a)$ improvement
  - ■ otherwise

FLAG review + my illustration

Challenge



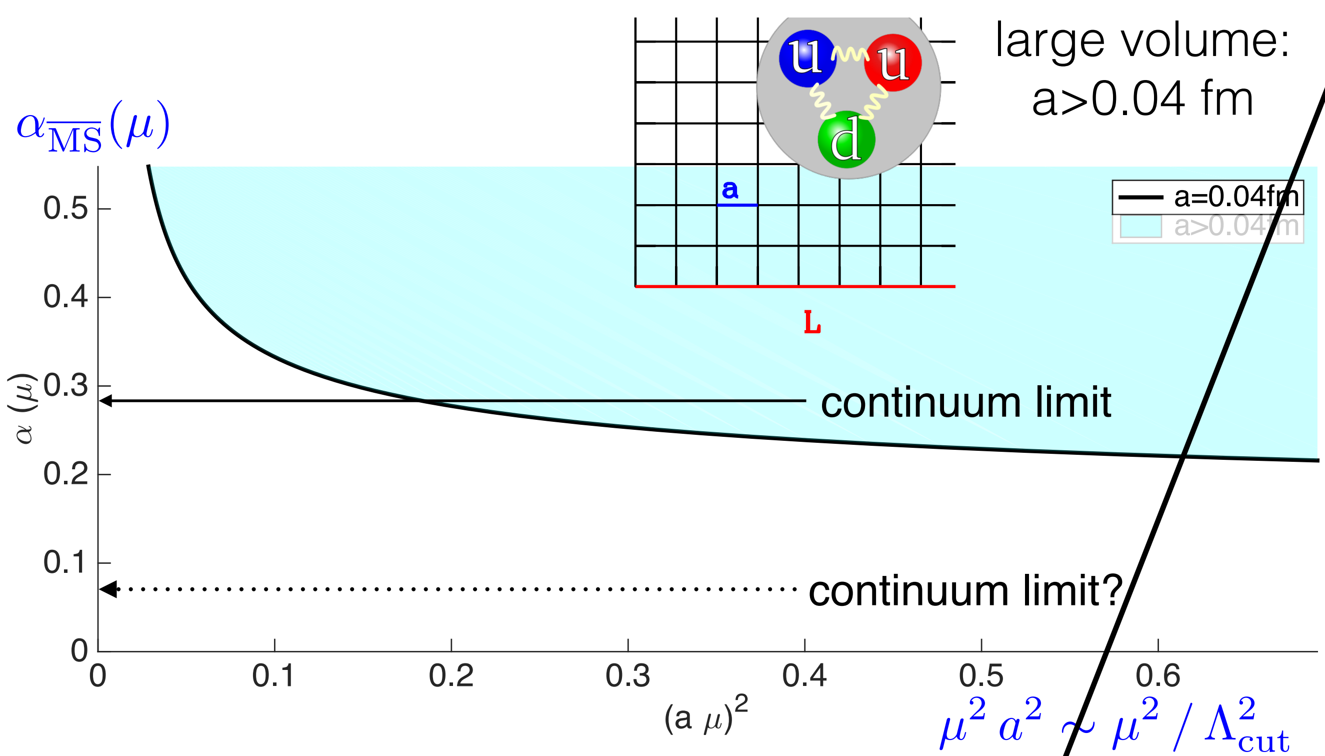
- Renormalization scale $r = 1/\mu$ small, α_s small
  - ★ all points relevant in the analysis have $\alpha_{\rm eff} < 0.2$
  - ○ all points have $\alpha_{\rm eff} < 0.4$ and at least one $\alpha_{\rm eff} \leq 0.25$
  - ■ otherwise
- Perturbative behaviour $n_l =$ loop order: high
  - ★ verified over a range of a factor 4 change in $\alpha_{\rm eff}^{n_l}$ without power corrections or alternatively $\alpha_{\rm eff}^{n_l} = 0.01$ is reached
  - ○ agreement with perturbation theory over a range of a factor 2.25 in $\alpha_{\rm eff}^{n_l}$ possibly fitting with power corrections or alternatively $\alpha_{\rm eff}^{n_l} = 0.02$ is reached
  - ■ otherwise

lattice spacing small: $a \ll r\,,\quad a\mu \ll 1$

- Continuum extrapolation
  At a reference point of $\alpha_{\rm eff} = 0.3$ (or less) we require
  - ★ three lattice spacings with $\mu a < 1/2$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \leq 1/4$ and 2-loop $\mathcal{O}(a)$ improvement, or $\mu a \leq 1/8$ and 1-loop $\mathcal{O}(a)$ improvement
  - ○ three lattice spacings with $\mu a < 1.5$ reaching down to $\mu a = 1$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \leq 1/4$ and one-loop $\mathcal{O}(a)$ improvement
  - ■ otherwise

FLAG review + my illustration

Challenge



- Renormalization scale $r = 1/\mu$ small, α_s small

  ★ all points relevant in the analysis have $\alpha_{\rm eff} < 0.2$

  ○ all points have $\alpha_{\rm eff} < 0.4$ and at least one $\alpha_{\rm eff} \leq 0.25$

  ■ otherwise

- Perturbative behaviour $n_l =$ loop order: high

  ★ verified over a range of a factor 4 change in $\alpha_{\rm eff}^{n_1}$ without power corrections or alternatively $\alpha_{\rm eff}^{n_1} = 0.01$ is reached

  ○ agreement with perturbation theory over a range of a factor 2.25 in $\alpha_{\rm eff}^{n_1}$ possibly fitting with power corrections or alternatively $\alpha_{\rm eff}^{n_1} = 0.02$ is reached

  ■ otherwise

  lattice spacing small:

- Continuum extrapolation $a \ll r,\ a\mu \ll 1$

  At a reference point of $\alpha_{\rm eff} = 0.3$ (or less) we require

  ★ three lattice spacings with $\mu a < 1/2$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \leq 1/4$ and 2-loop $\mathcal{O}(a)$ improvement, or $\mu a \leq 1/8$ and 1-loop $\mathcal{O}(a)$ improvement

  ○ three lattice spacings with $\mu a < 1.5$ reaching down to $\mu a = 1$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \leq 1/4$ and one-loop $\mathcal{O}(a)$ improvement

  ■ otherwise

Compromise between good

physical scale = renormalization s.

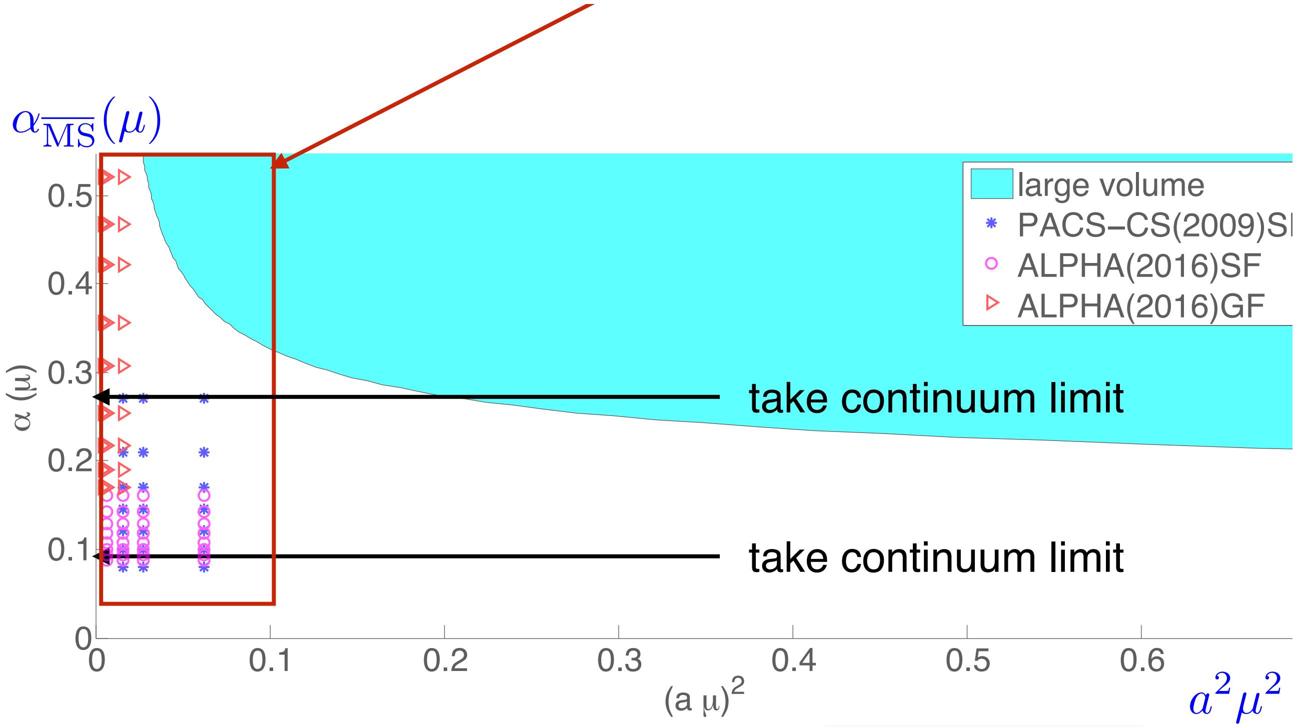
and

lattice spacing

FLAG review + my illustration

Finite volume couplings (see A.R.)

no compromise is necessary



but continuum extrapolations remain the biggest headaches

- Renormalization scale $r = 1/\mu$ small, α_s small
  - ★ all points relevant in the analysis have $\alpha_{\rm eff} < 0.2$
  - ○ all points have $\alpha_{\rm eff} < 0.4$ and at least one $\alpha_{\rm eff} \le 0.25$
  - ■ otherwise
- Perturbative behaviour $n_l =$ loop order: high
  - ★ verified over a range of a factor 4 change in $\alpha_{\rm eff}^{n_l}$ without power corrections or alternatively $\alpha_{\rm eff}^{n_l} = 0.01$ is reached
  - ○ agreement with perturbation theory over a range of a factor 2.25 in $\alpha_{\rm eff}^{n_l}$ possibly fitting with power corrections or alternatively $\alpha_{\rm eff}^{n_l} = 0.02$ is reached
  - ■ otherwise

lattice spacing small: $a \ll r\,,\quad a\mu \ll 1$

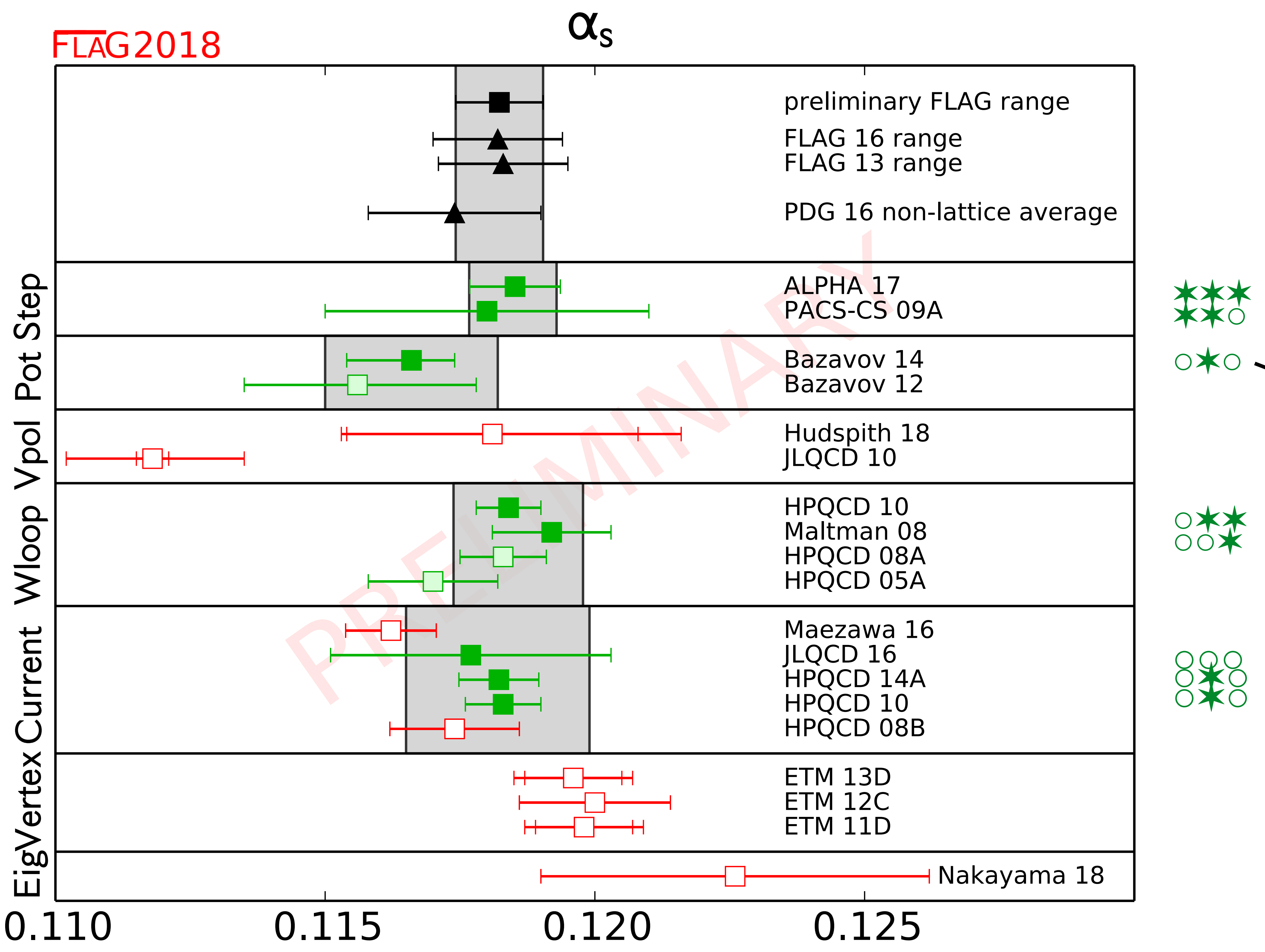
- Continuum extrapolation
  At a reference point of $\alpha_{\rm eff} = 0.3$ (or less) we require
  - ★ three lattice spacings with $\mu a < 1/2$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \le 1/4$ and 2-loop $\mathcal{O}(a)$ improvement, or $\mu a \le 1/8$ and 1-loop $\mathcal{O}(a)$ improvement
  - ○ three lattice spacings with $\mu a < 1.5$ reaching down to $\mu a = 1$ and full $\mathcal{O}(a)$ improvement, or three lattice spacings with $\mu a \le 1/4$ and one-loop $\mathcal{O}(a)$ improvement
  - ■ otherwise

Different categories (methods)

Reviewed in detail in the FLAG reports

- Step scaling: finite volume couplings, PT at O(100 GeV) see A. Ramos — $\mu = 1/L$
- Potential: from QQ-potential, see P. Petretzky, A. Vairo — $\mu = 1/r$
- Vacuum polarization: light quark <J J> correlator, large Q^2 — $\mu = 1/Q$
- Wilson loops: RxT Wilson loops, (R,T)=(a,a), (a,2a) small. — $\mu = 1/a = 1/\Lambda_{cut}$
- Current 2-pt functions: moments of heavy quark <J J> correlators — $\mu = 1/M$
- Vertex: ghost-gluon vertex etc., fixed gauge — $\mu = 1/Q$
- Eigenvalue distribution: eigenvalue density of Dirac operator for large eigenvalues — $\mu = 1/\lambda$

Towards FLAG 2019



limited by

statistics, continuum extrapolation

compromise $a\mu \ll 1$

PT, lattice effects *

compromise $a\mu \ll 1$

* Small Wloops are special perturbation theory at $a\mu$=1 (non-universal)

preliminary FLAG2018 $\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.11823(81)$ is quite precise

Where does PT apply? Test where Λ is constant

$$\Lambda_{\text{eff}}(\alpha) \;=\; \Lambda \;\times\; (1 + \mathrm{O}(\alpha^{n}))$$

$1 + n$ - loop β-fct

Where does PT apply? Test where Λ is constant

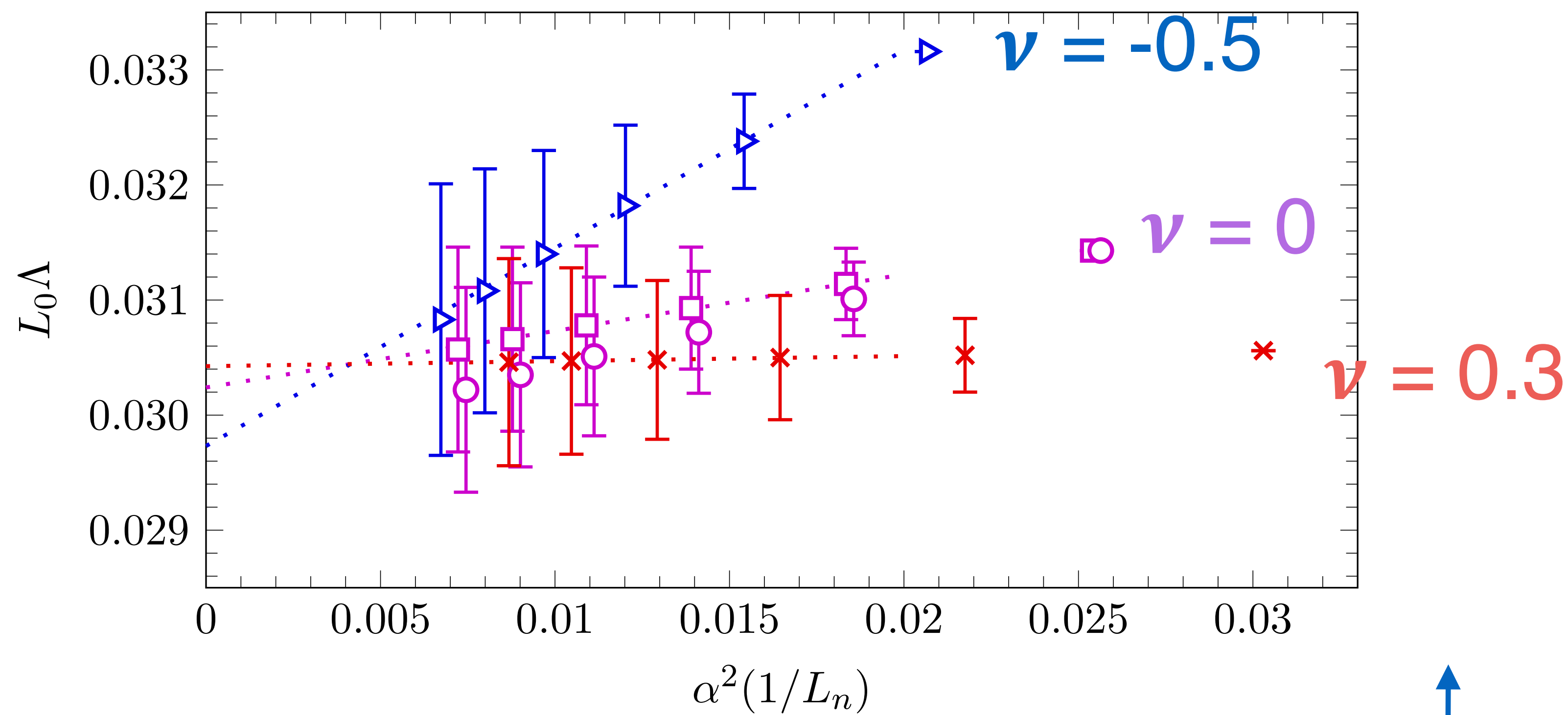
$$\Lambda_{\text{eff}}(\alpha) \;=\; \Lambda \;\times\; (1 + \mathrm{O}(\alpha^n))$$

$1 + n$ - loop β-fct

- SF coupling

  (ν-dependent schemes)

  [M. Dalla Brida et al. 2017]

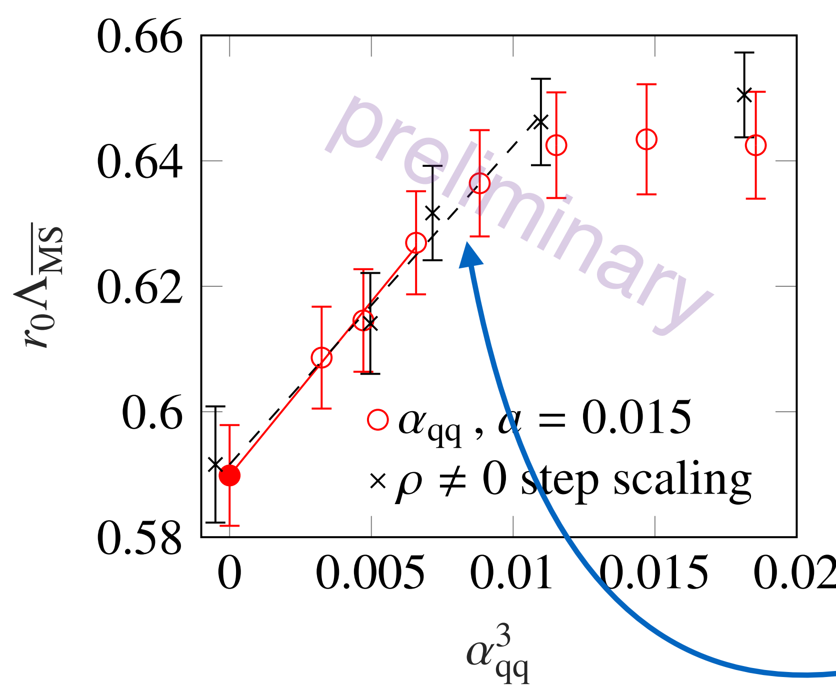


- qq-coupling

  pure gauge

  large vol. small a (0.02fm)

  [Husung, Koren, Krah, S. 2017]



$\alpha = 0.2$

***α* needs to be small!**

Personal opinion where we should go

- A good strategy is: divide quantities into
  - simple, perturbative, precise, large momentum scale
    —> determine α
  - more difficult or less precise
    —> test perturbation theory + resummations + …
- Mainly a sociological problem to put this into practice
- Some Quality criteria also for continuum phenomenology would be a step forward
  - small α
  - simple theory
- Lattice (FLAG) quality criteria should become more stringent