On the order of the thermal transition in QCD as function of the number of quark flavours and their masses

Francesca Cuteri, Owe Philipsen and Alessandro Sciarra

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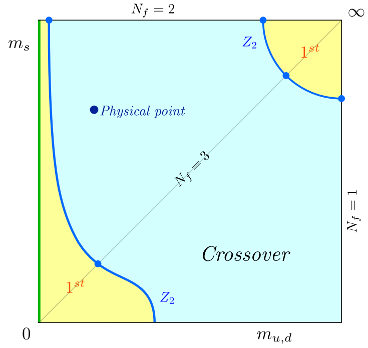
Standard $(m_s, m_{u,d})$ Columbia plot

Dependence of the order of the QCD thermal phase transition on the quark masses



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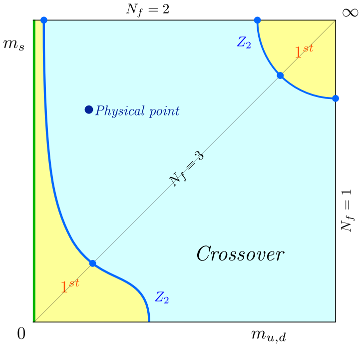
- Restored global $SU_L(N_f) \times SU_R(N_f)$ for $m_{u,d}, m_s \to 0$ Pisarski, Wilczek (1984)



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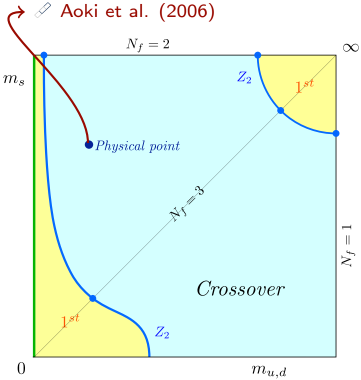

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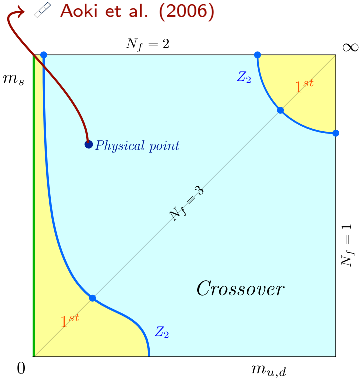

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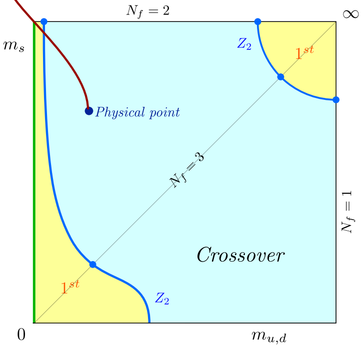

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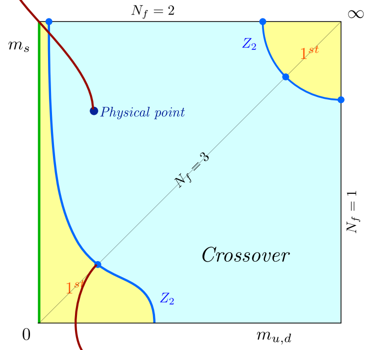

de Forcrand, Philipsen (2007)
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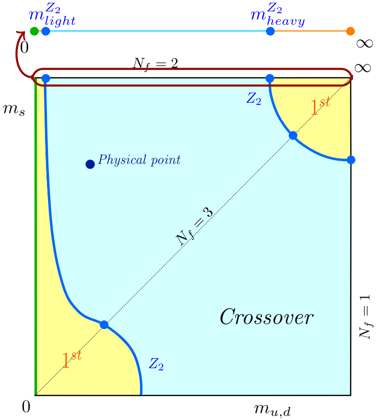
- $m_{u,d}$ very small. Transition affected by remnants of the chiral universality class?
- Relevance of the strength of the $U(1)_A$ anomaly at T_c Pisarski, Wilczek (1984)
- Strong cut-off and discretization dependence of chiral Z_2 boundary
  - Critical quark masses unreachably small for highly improved actions

$(m_s, m_{u,d})$ Columbia plot in the continuum

- Columbia plot from the "unimproved viewpoint". Focus on $N_f = 2$.
  - Chiral 1^{st} order region shrinks towards the continuum limit



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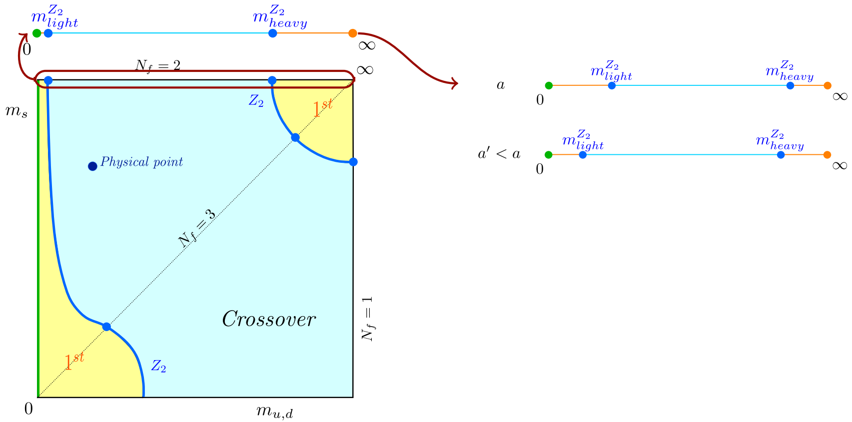

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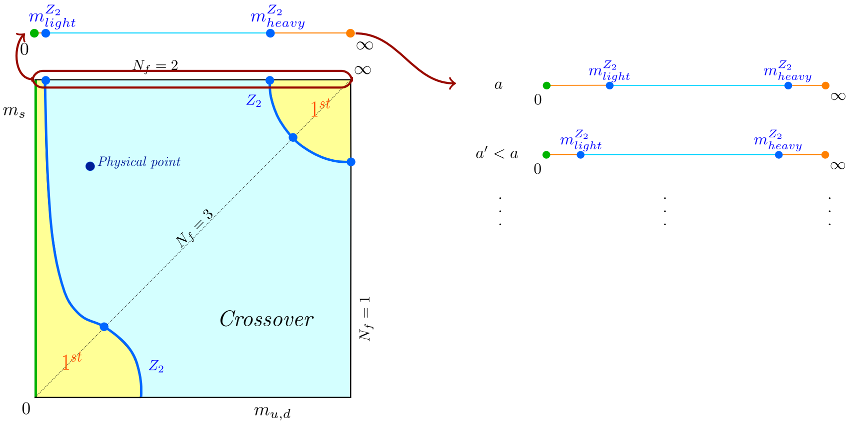

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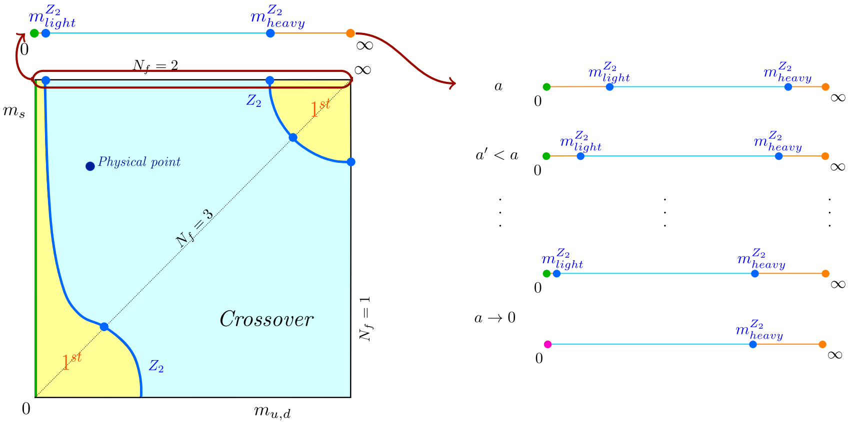
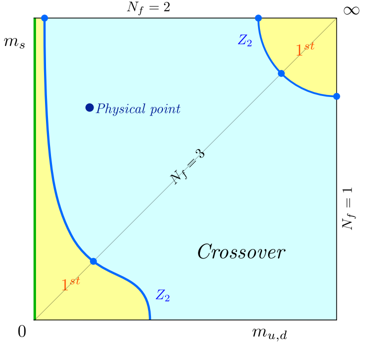
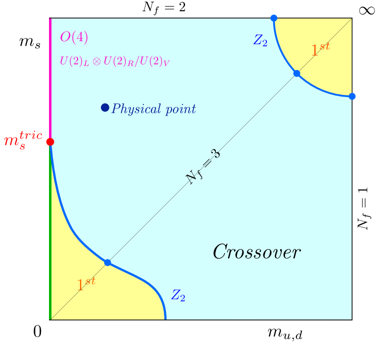


$(m_s, m_{u,d})$ Columbia plot in the continuum

At least two possible scenarios for the nature of $N_f = 2$ chiral transition





- “Direct approach" $\mu = 0$, $N_f = 2$ and $m_{u,d} \to 0$ proved to be too expensive

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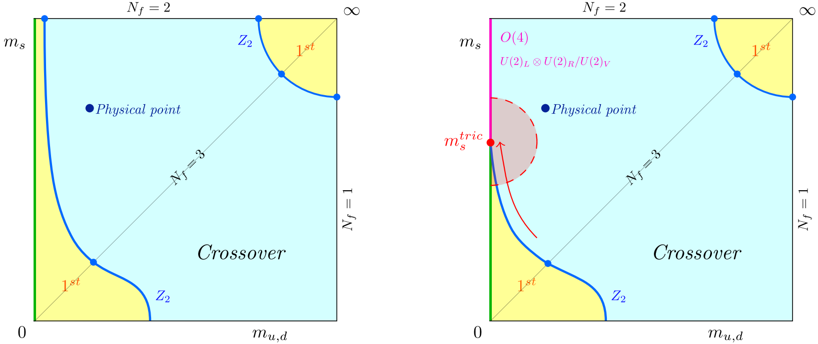
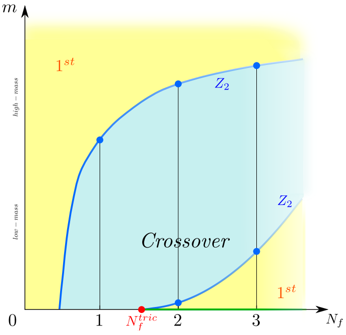


- "Direct approach" $\mu = 0$, $N_f = 2$ and $m_{u,d} \to 0$ proved to be too expensive
- "Indirect approaches" exploit tricritical scaling laws for controlled extrapolations to the chiral limit
  - From imaginary chemical potential $\mu = i\mu_i$
  - At $\mu = 0$ and $(2 + 1)N_f$

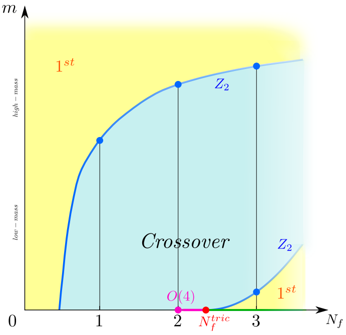
Alternative (m, N_{F}) Columbia plot

Yet another “indirect approach", promoting N_{f} to continuous real parameter

$$Z_{N_{\mathrm{f}}=2.\#} = \int \mathcal{D}U \left[\det M(U,m)\right]^{2.\#} e^{-S_G}$$



$N_{\mathrm{f}}^{tric} < 2$



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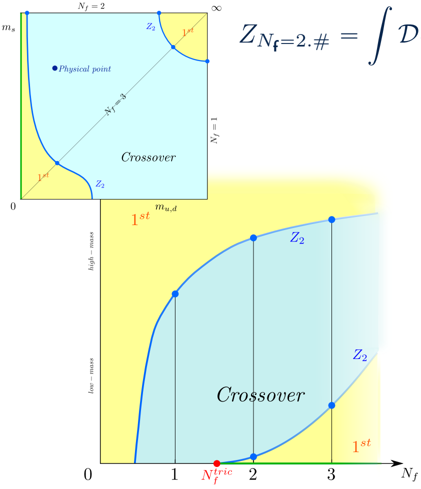

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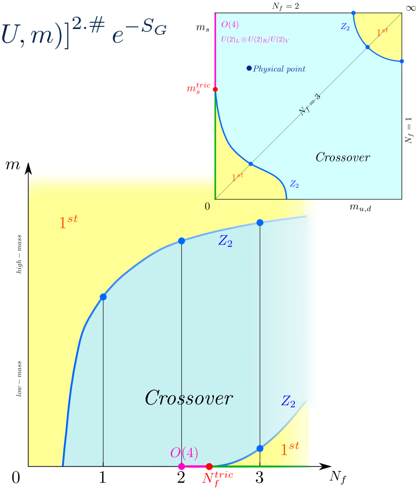
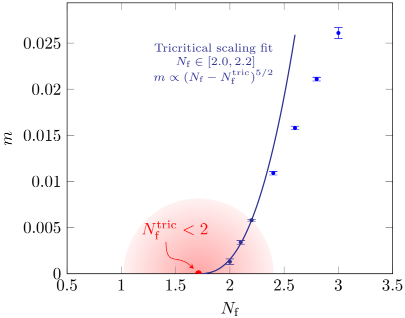
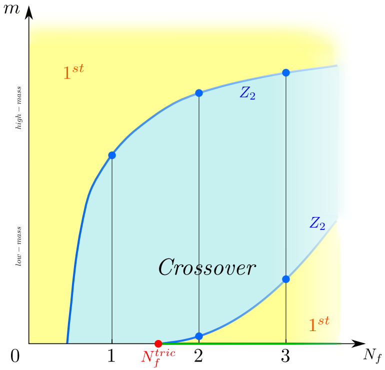


$N_f^{tric} > 2$

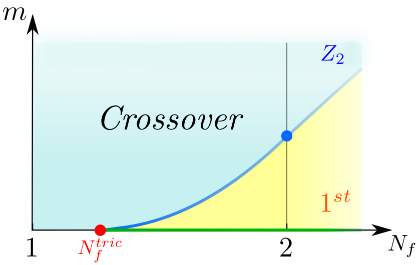
Conclusion from the tricritical scaling





Z_2 boundary linear in some N_F range?

- If it is reasonable to expect both linearity within some range in N_f and tricritical scaling closer to the chiral limit
  - make use of a linear extrapolation to $m = 0$ to get an upper bound N_f^{lin} for N_f^{tricr}, without the need to enter the tricritical scaling region
    - lower cost
    - no need for simulations at non integer number of flavors?!



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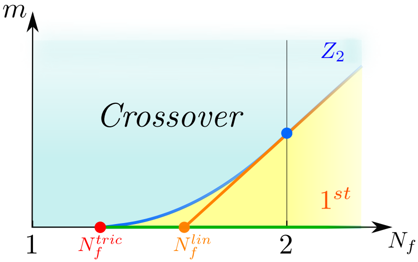
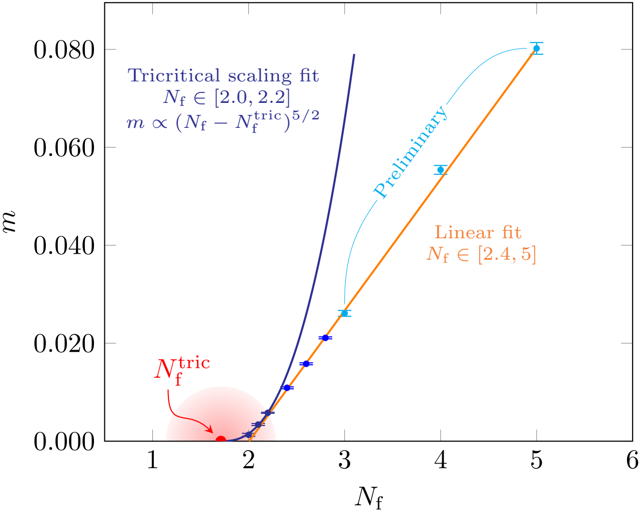


- If $N_f^{lin} < 2$, while one simulates at larger and larger N_τ towards the continuum limit
  - Transition in the $N_f = 2$ chiral limit keeps being a first order one
- As soon as $N_f^{lin} \gtrsim 2$
  - No conclusion can be drawn

Results, so far...



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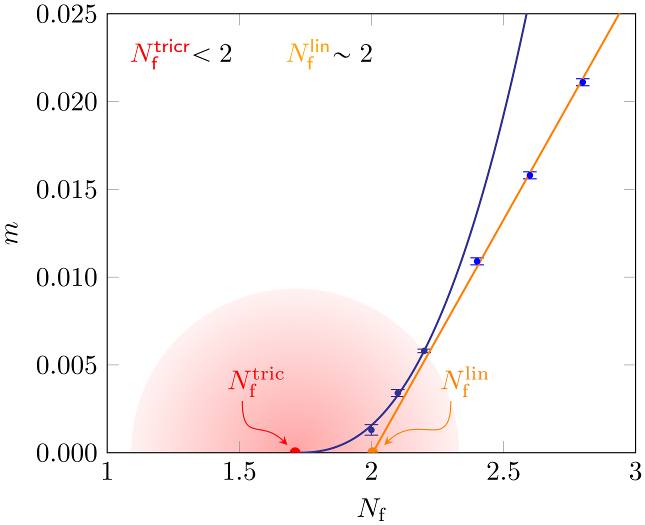

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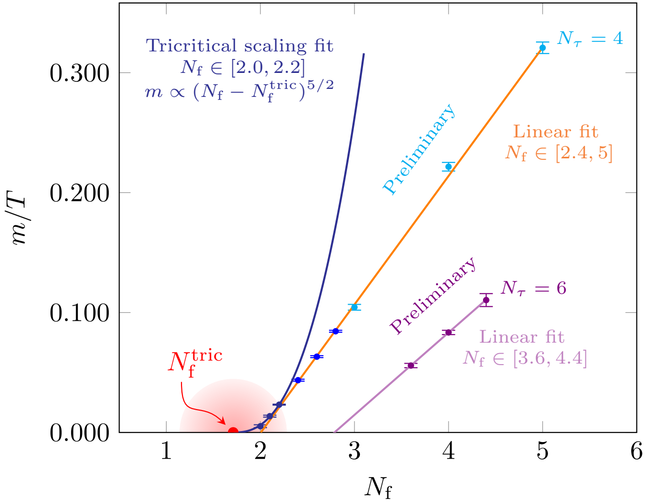

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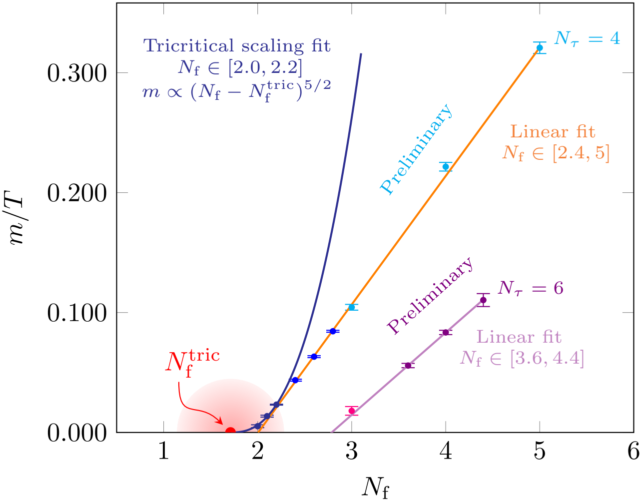

$N_f = 3$, $N_\tau = 6$ from de Forcrand, Kim, Philipsen (2007)

Conclusions, so far...

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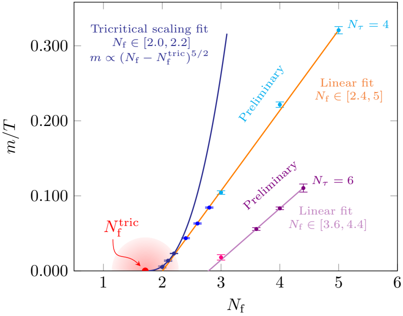

- **On the one hand**
  - Nature of the chiral phase transition at $N_f = 2$ remains elusive to our extrapolation already at $N_\tau = 6$.
  - While being interesting on its own, linearity does not seem to help in resolving the $N_f = 2$ puzzle.
- **On the other hand**
  - If we think of the size of the shift in the boundary, the first order scenario would look more and more contrived with larger and larger N_τ values.
  - It would be interesting to obtain similar results for other fermion discretizations based on this method.

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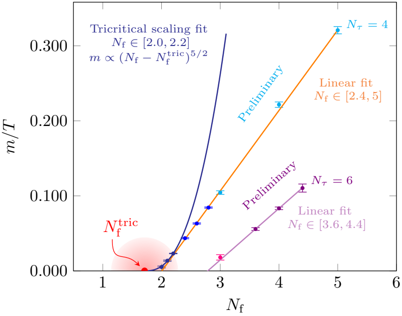
$N_\mathsf{f}^\mathsf{lin} \lesssim 3$ @ $N_\tau = 6$



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Backup