Spatial distribution of colour fields in the $SU(3)$ flux tube

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Introduction

Theoretical arguments

- Quantum ChromoDynamics (QCD) expected to accommodate all strong interaction phenomena, yet, colour confinement is escaping a theoretical explanation.
- Not yet clear which feature of the QCD vacuum, hence which of the field configurations contributing to the QCD vacuum partition function, are responsible for the linear confining potential between a static quark and antiquark at large distances.

Numerical facts

- Area-law scaling of large Wilson loops and linear potential between a static $q\bar{q}$ pair at distances above ~ 0.5 fm, up to distances of ~ 1.4 fm with dynamical quarks, where string breaking should take place.
- Dominant component of the colour fields created by a static $q\bar{q}$ pair is the chromoelectric one distributed in "flux tube" structures.

Plan on the basis of numerical findings

Shed light via a scan of *all* components of the colour field over *all* possible two-dimensional planes transverse to the axis joining the sources.

- Confirmed suppression of the chromomagnetic field components as well as of the transverse components of the chromoelectric field, as observed already in the earlier SU(2) studies.
  - Large-distance, nonperturbative and confining dynamics encoded in the chromoelectric field longitudinal to the line connecting the static sources
  - Transverse chromoelectric fields account only for the short-distance, perturbative and Coulomb-like part of the colour field.

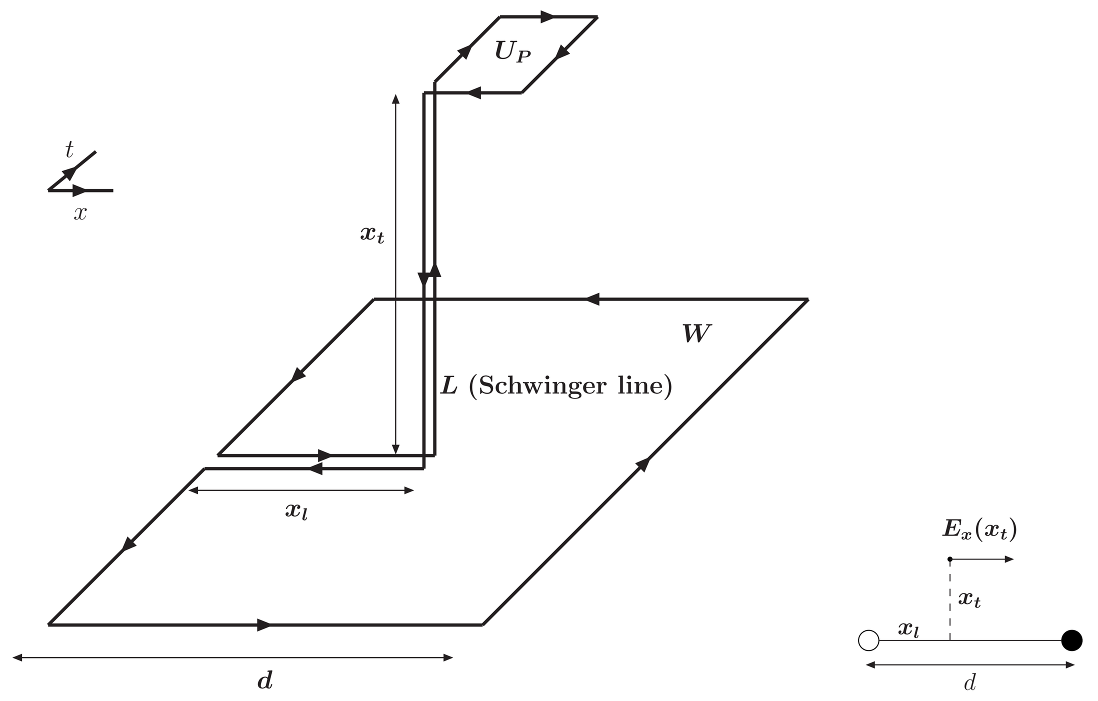
Based on this observation a simple procedure can be devised for the extraction of the non perturbative field:

- Use the transverse components to profile the Coulomb contribution,
- Subtract the Coulomb part from the longitudinal chromoelectric field, thus isolating its (dominating) nonperturbative portion.

Strategy to extract colour fields

- The field configurations generated by a static quark-antiquark pair can be probed by calculating on the lattice the vacuum expectation value of the connected correlator

$$\rho^{\text{conn}}_{W,\,\mu\nu} = \frac{\langle \mathrm{tr}\left(W L U_P L^\dagger\right)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(U_P)\mathrm{tr}(W)\rangle}{\langle \mathrm{tr}(W)\rangle} .$$



- The Wilson loop connected to the plaquette is the source of a colour field which points onto an unknown direction n^a in colour space.
- We measure the average projection of the colour field onto that direction. Schwinger lines realize the colour parallel transport between the source loop and the "probe" plaquette.
- In the naive continuum limit [Di Giacomo, Maggiore, Olejnik (1990)]

$$\rho^{\text{conn}}_{W,\,\mu\nu} \xrightarrow{a\to 0} a^2 g\left[\left\langle n^a F^a_{\mu\nu}\right\rangle_{q\bar{q}} - \left\langle n^a F^a_{\mu\nu}\right\rangle_0\right] .$$

$$n^a F^a_{\mu\nu}(x) = \frac{1}{a^2 g}\,\rho^{\text{conn}}_{W,\,\mu\nu}(x) ,$$

as a necessary consequence of the gauge-invariance of $\rho^{\text{conn}}_{W,\,\mu\nu}$ and of its linear dependence on the colour field in the continuum limit.

Simulation algorithm & lattice setup

- Simulations are performed in pure gauge $SU(3)$, making use of the publicly available MILC code, modified to introduce the relevant observables.

β	lattice	distance d	statistics	smearing steps
6.370	48^4	0.95 fm	1000	80
6.240	48^4	1.14 fm	4000	100
6.136	48^4	1.33 fm	16000	120

- The lattice discretization for the pure gauge $SU(3)$ is the standard Wilson action.
- Configurations are evolved through heatbath and overrelaxation sweeps.
- Physical scale is set assuming for the string tension the standard value of $\sqrt{\sigma} = 420$ MeV and using the parameterization [Edwards, Heller, Klassen (1998)]

$$\begin{aligned}(a\sqrt{\sigma})(g) &= f_{\mathrm{SU}(3)}(g^2)\,\{1 + 0.2731\,\hat{a}^2(g)\\ &- 0.01545\,\hat{a}^4(g) + 0.01975\,\hat{a}^6(g)\,\}/0.01364 ,\end{aligned}$$

$$\hat{a}(g) = \frac{f_{\mathrm{SU}(3)}(g^2)}{f_{\mathrm{SU}(3)}(g^2(\beta=6))} ,\ \beta = \frac{6}{g^2} ,\quad 5.6 \le \beta \le 6.5 ,$$

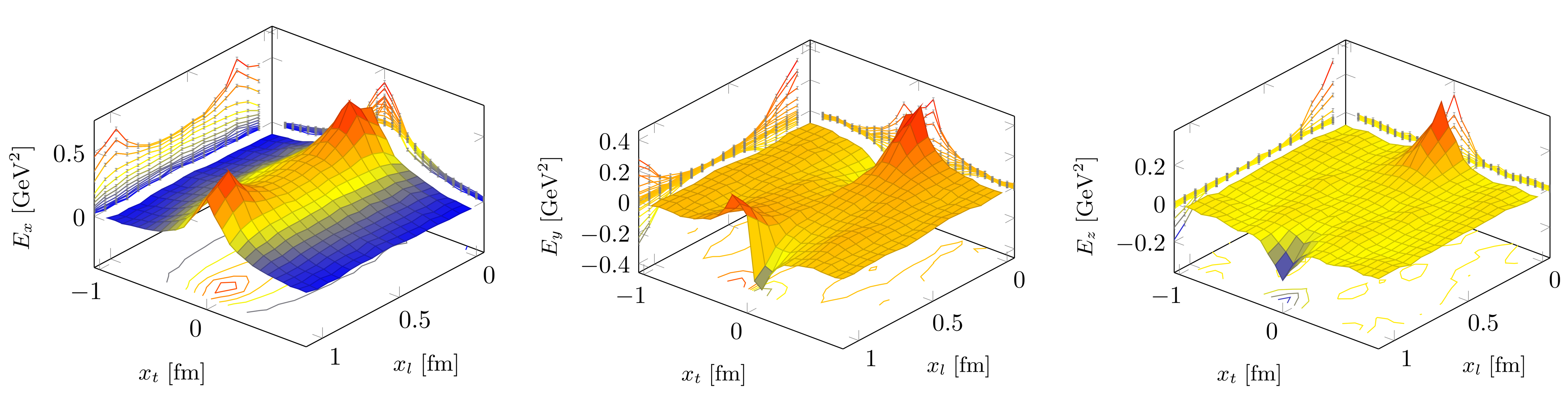
with

$$f_{\mathrm{SU}(3)}(g^2) = \left(b_0 g^2\right)^{-b_1/2b_0^2} e^{-\frac{1}{2b_0 g^2}} ,\ b_0 = \frac{11}{(4\pi)^2} ,\ b_1 = \frac{102}{(4\pi)^4} .$$

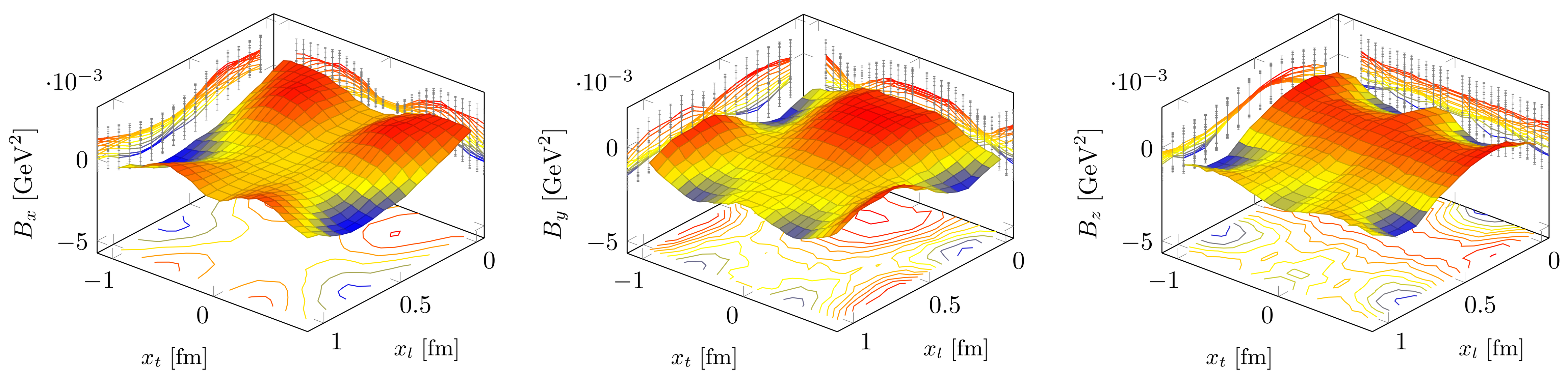
- To improve the signal-to-noise ratio and extract the physical information carried by fluctuations at the physical scale configurations are smoothed out by the *smearing* procedure
  - One step of HYP smearing on the temporal links, with smearing parameters $(\alpha_1, \alpha_2, \alpha_3) = (1.0, 0.5, 0.5)$,
  - N_{APE} steps of APE smearing on the spatial links, with smearing parameter $\alpha_{\text{APE}} = 0.167$.

Results before subtraction

The three components of the chromoelectric field at $d = 1.14$ fm

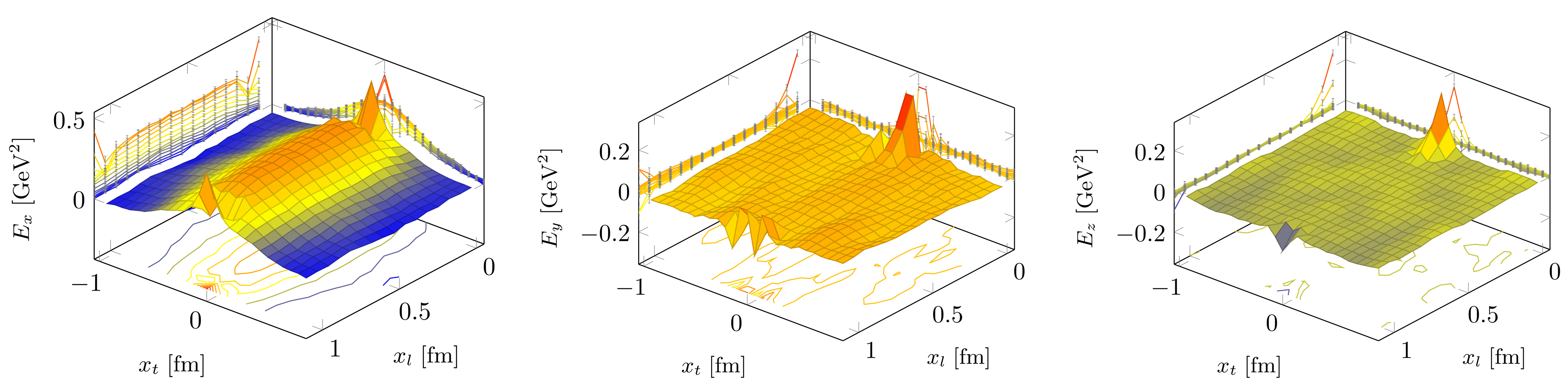


The three components of the chromomagnetic field at $d = 1.14$ fm



Results after subtraction

The three components of the subtracted, non-perturbative chromoelectric field at $d = 1.14$ fm



Extracting the non-perturbative field

- The expectation is that the measured chromoelectric field can be decomposed as

$$\vec{E}(\vec{r}) = \vec{E}_C(\vec{r}) + \vec{E}_{np}(\vec{r})$$

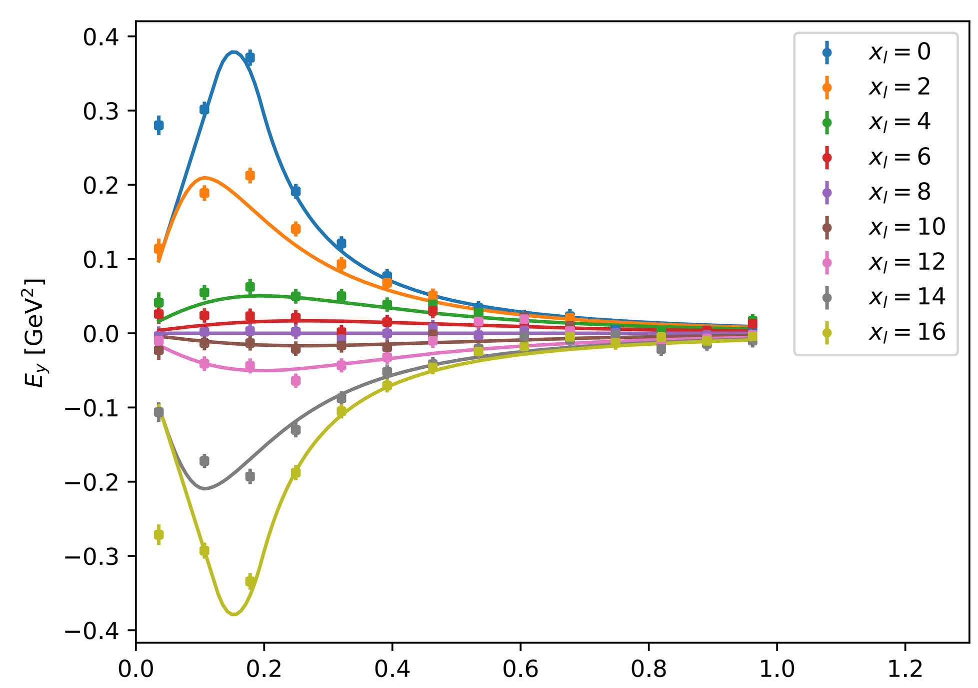
  - $\vec{E}_C(\vec{r})$ being the perturbative contribution behaving as a Coulomb electrostatic field;
  - $\vec{E}_{np}(\vec{r})$ being the non-perturbative confining contribution expected to be purely longitudinal far away from the sources.
- For $\vec{E}_C(\vec{r})$ one can use the ansatz

$$\vec{E}_C(\vec{r}) = \vec{E}_R(\vec{r}-\vec{r}_1, q) + \vec{E}_R(\vec{r}-\vec{r}_2, -q), \qquad \text{with}$$

$$\vec{E}_R(\vec{r}, q) = \frac{q\vec{r}}{\max(r^3, R^3)}$$

assuming the field to be produced by two uniformly charged spheres of radius R.

  - With this ansatz we try to account for the behavior of the field close to the sources.
- Non perturbative contribution extracted by a fit of E_y using the Coulomb ansatz



From the fit one can extract the two fit parameters.

- The electrostatic charge q;
- The radius R of the sources.

The fit procedure on preliminary data yields

β	q	R	χ^2_r
6.370	0.26(3)	0.13(3)	6.3
6.240	0.29(5)	0.16(2)	3.4
6.136	0.29(17)	0.21(13)	1.1

Conclusions

- While all components of the measured chromomagnetic field are compatible with zero everywhere, the chromoelectric field components could be extracted out of smeared gauge configurations and show some of the expected symmetries.
- The transverse components of the electromagnetic fields are found to be well described by a Coulomb ansatz at least not too close to the position of the two static sources.
- After the subtraction of the Coulomb component, the dominant longitudinal chromoelectric field is found to be constant in shape with respect to longitudinal displacements far enough from the two sources.